Pasinetti's hyper-integrated labour coefficients and the 'pure labour theory of value'

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Outline

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  - capital investment to meet growing demand
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- 'A complete generalization of the pure labour theory of value': $\mathbf{p}_i = \mathbf{v}_i^\star$ w

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- Is this right?

The special case: production-prices

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- Prices not completely reducible to labor costs

The distribution of real income

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- Workers spend what they earn

$$\mathbf{l}\mathbf{q}^{\mathsf{T}} w = \mathbf{p}\mathbf{w}^{\mathsf{T}}$$

- Capitalists spend what they earn

$$\mathbf{p}\mathbf{A}\mathbf{q}^{\mathsf{T}} r = \mathbf{p}\mathbf{c}^{\mathsf{T}}$$

An equivalent representation of production-prices

- Substitute $r = \frac{\mathbf{p}\mathbf{c}^{\mathsf{T}}}{\mathbf{p}\mathbf{A}\mathbf{q}^{\mathsf{T}}}$ into price equation $\mathbf{p} = \mathbf{p}\mathbf{A}(1+r) + \mathbf{l}w$

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where matrix $\mathbf{C} = [c_{i,j}]$, such that

$$c_{i,j} = \frac{\mathbf{p}\mathbf{A}^{(j)}}{\mathbf{p}\mathbf{A}\mathbf{q}^{\mathsf{T}}} c_i$$

The capitalist consumption matrix

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$$c_{i,j} = \mathbf{p}\mathbf{A}^{(j)} r \frac{c_i}{\mathbf{p}\mathbf{A}\mathbf{q}^{\mathsf{T}} r}$$

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- Hence $c_{i,j}$ is the quantity of commodity i distributed to capitalists per unit output of commodity j
- Matrix $\mathbf{C}$ is a 'capitalist consumption matrix'
- $\mathbf{C}$ has same units as $\mathbf{A}$, i.e. $\mathbf{C}$ is a 'physical' input-output matrix

Production-prices revisited

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Nonstandard labour-values

Definition

Nonstandard labour-values are $\tilde{\mathbf{v}} = \tilde{\mathbf{v}}\tilde{\mathbf{A}} + \mathbf{l}$, where $\tilde{\mathbf{A}} = \mathbf{A} + \mathbf{C}$ is the technique augmented by capitalist consumption.

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Theorem (Equivalence theorem)

Given an economy with production-prices, $\mathbf{p} = \mathbf{p}\mathbf{A}(1+r) + \mathbf{l}w$*, quantities,* $\mathbf{q} = \mathbf{q}\mathbf{A}^T + \mathbf{w} + \mathbf{c}$*, and zero saving (i.e. workers and capitalists spend what they earn) then*

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- $\mathbf{p}$ is wage bill of direct, indirect and super-indirect labour required to reproduce unit commodities.

Super-integrated labour coefficients

A family of equivalence theorems of increasing generality

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- Hyper-subsystems with 'natural' profit-rates: $\mathbf{p}_i = \mathbf{v}_i^{\star}w$, where $\mathbf{v}_i^{\star}$ are hyper-integrated labour coefficients (Pasinetti, 1988)

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- Hyper-subsystems with 'natural' profit-rates: $\mathbf{p}_i = \mathbf{v}_i^\star w$, where $\mathbf{v}_i^\star$ are hyper-integrated labour coefficients (Pasinetti, 1988)
- Pasinetti's growth model: $\mathbf{p}'' = \hat{\mathbf{v}}w$, where $\hat{\mathbf{v}}$ are super-integrated labour coefficients (see Theorem 1 in paper) (Wright, 2013)

General solution to Marx's transformation problem

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General solution to Marx's transformation problem

- Equivalence theorems have far-reaching consequences for theory of economic value
- Marx knew that production-prices do not represent Classical labour-values in a simple and direct manner
- But neither do they represent a re-weighting of Classical labour-values, as he sketched in Volume 3 of Capital
- In general, natural price structures represent *total labour costs*:

$$\mathbf{p} = \mathring{\mathbf{v}} w,$$

where $\mathring{\mathbf{v}}$ are 'total' in the sense they reduce *all* real costs to labour costs

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- Dual systems of price and labour-values employ different cost accounting conventions
- A 'category-mistake' (Ryle) to expect a partial measure of labor costs to be commensurate with a total measure of money costs
- Classical antinomies due to this category-mistake

Pasinetti's reproduction of the category-mistake

- Pasinetti's hyper-integrated labour coefficients are a partial measure of labor costs
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- Problem disappears with a total measure of labor costs, i.e. super-integrated labour coefficients
- Pasinetti's 'complete generalization' of Marx's TP reproduces, at a higher level of generality, the Classical category-mistake
- Conclusion: Pasinetti's restriction of the 'pure labour theory of value' to a 'logical frame of reference' is unwarranted

Questions?

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Related pre-thesis material:

- **Pasinetti's hyper-integrated labour coefficients and the 'pure labour theory of value'**. http://ssrn.com/abstract=2255732 (2013)
- **Sraffa's incomplete reductions to labour**. http://tinyurl.com/q8hkubg (2013)
- **A category-mistake in the Classical labour theory of value: identification and resolution**. http://ssrn.com/abstract=1963018 (2011)
- **Classical macrodynamics and the labour theory of value**. Open Discussion Papers in Economics, no. 76. Milton Keynes: The Open University. (2011)
- **Convergence to natural prices in simple production**. Open Discussion Papers in Economics, no. 75. Milton Keynes: The Open University (2011)
- **On nonstandard labour values, Marx's transformation problem and Ricardo's problem of an invariable measure of value**. *Boletim de Ciencias Economicas LII, Universidade de Coimbra* (2009)
- **The emergence of the law of value in a dynamic simple commodity economy**. Review of Political Economy, Vol. 20, No. 3, pages 367–391 (2008)

Dimensional analysis of capitalist consumption matrix

- We have $\mathbf{C} = [c_{i,j}]$, where

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- Interpretation: 'physical quantity of commodity i consumed per unit output of commodity j' (e.g., 'bushels of wheat per unit output of iron')
- Technique $\mathbf{A} = [a_{i,j}]$ has same units with interpretation 'physical quantity of commodity i used-up (as means-of-production) per unit output of commodity j'