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Marx's (not Ricardo's) 'transformation problem': a radical reconceptualization

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I. *Different Views of Economic Theories*

In recent years, two distinct bodies of literature have been concerned with the question of the relationship between Marxian and Ricardian economic theories. On the one hand, the Sraffa-based literature has shown a tendency to view economic theory as divisible into only two broad traditions or paradigms: the neoclassical marginalist method of analysis, and a generalized 'classical' framework, encompassing both Marxian and Ricardian variants.[1] A variety of recent writers, including such eminent Marxist scholars as Maurice Dobb and Ronald Meek, appear to have reached the conclusion that Sraffa-based analytics are fully congenial with Marxian concerns, and that the confrontation with neoclassical theory requires all critics to adopt a common 'classical' approach.[2] On the other hand, emerging from the French tradition in the history and philosophy of science,[3] Louis Althusser and others have produced a wide-ranging Marxist reevaluation of epistemological issues in theory, and of the related questions of theoretical methodology.[4] Althusser's work affirms the radical uniqueness of

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1. See, for example, John Eatwell, "The irrelevance of returns to scale in Sraffa's analysis," *Quarterly Journal of Economics* 15 (March 1977): 61–68; and Krishna Bharadwaj, "Maurice Dobb's critique of theories of value and distribution," *Cambridge Journal of Economics* 2 (June 1978).

2. See Dobb, "The Sraffa system and critique of the neo-classical theory of distribution," in *A critique of economic theory*, ed. E. K. Hunt and Jesse Schwartz (Harmondsworth, 1972), pp. 205–21; and Meek, "Introduction to the second edition," *Studies in the labor theory of value*, 2d ed. (New York, 1973), esp. p. xlii. The most polemical statement of this thesis is to be found in Ian Steedman, *Marx after Sraffa* (London, 1977), esp. chs. 1, 14.

3. The seminal works in this tradition are Gaston Bachelard, *La Formation de l'esprit scientifique* (Paris, 1938); idem, *Le Rationalisme appliqué* (Paris, 1949); idem, *Le Matérialisme rationnel* (Paris, 1953); and Georges Canguilhem, *La Connaissance de la vie* (Paris, 1965); idem, *Etudes d'histoire et de philosophie des sciences* (Paris, 1968).

4. Althusser's principal works are *For Marx*, trans. Ben Brewster (New York, 1970); *Reading Capital*, with Etienne Balibar, trans. Ben Brewster (London, 1970); and *Essays in self-criticism*, trans. Grahame Lock (London, 1976). Also related are Michael Foucault, *The archaeology of knowledge*, trans. A. M. Sheridan-Smith (New York, 1976), and Barry Hindess and Paul Q. Hirst, *Mode of production and social formation* (London, 1977). A useful attempt at a tentative critical overview of the project of this group is Dominique

the Marxian 'problematic' with respect to its predecessors and successors, including Ricardo, both in philosophy and in political economy.

Considering the sharp differences in the conclusions reached in these two bodies of literature, it is remarkable how little direct contact there has been between them. The Sraffa-based literature has examined the determinants of prices and profit rates, but has generalized its approach to include Marx without regard to the insights produced by the Althusserian tradition concerning the specificity of the Marxian method. But similarly, that Althusserian approach has thus far been applied almost exclusively to philosophical and historical issues within economics and should itself benefit from an explicit consideration of economic issues in Marxian theory, including the determination of prices and profit rates. In this study we begin what we expect to be an extended project, that of applying the perspectives and insights of the Althusserian tradition to the reinterpretation of Marx's theoretical and economic texts, here focusing explicitly on the 'transformation problem.'

Althusser's most important contributions have been presented in the form of an attack on reductionist or essentialist theoretical strategies: those which seek to comprehend the web of social causality by treating some aspects of society as phenomenal effects and other aspects as the essence(s) of those effects. In contrast, Althusser suggested 'overdetermination' as a term to designate the basic relational premise of Marx's approach.[5] This much-misunderstood concept implies, of course, mutual and reciprocal determination, but more is involved than simply the directions of causality. Overdetermination additionally implies relations of constitutivity, the power of each aspect of society not merely to affect other aspects, but also to effect them, constitute them, participate in determining the nature of, as well as the changes in, every other aspect. Any discourse premised upon overdetermination thus affirms a pointedly anti-essentialist strategy of theoretical argument.

This approach is important as the basis for an alternative view of Marx, one quite different from the continuist view which looks upon Marx as sharing and developing classical and specifically Ricardian concerns. In

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Lecourt, *Marxism and epistemology*, trans. Ben Brewster (London, 1975). Related, although in crucial ways divergent, arguments can be found in Thomas Kuhn, *The structure of scientific revolutions* (Chicago, 1962); Paul Feyerabend, *Against method* (London, 1975); and Imre Lakatos and Alan Musgrave, eds., *Criticism and the growth of knowledge* (Cambridge, 1970).

5. See especially the essays "Contradiction and overdetermination," and "On the materialist dialectic," in *For Marx*, pp. 87–128, 161–218. Althusser borrowed and developed this notion of overdetermination from Freud; see esp. Sigmund Freud, "The dream work," in *The basic writings of Sigmund Freud*, ed. and trans. A. A. Brill (New York, 1938), pp. 319–39. For a more extended discussion of the term as it is used here, see Stephen Resnick and Richard D. Wolff, "The theory of transitional conjunctures and the transition from feudalism to capitalism in western Europe," *Review of Radical Political Economy* 11 (Fall 1979): 3–22, 32–36.

much of the continuist literature, Marx and Ricardo are argued to have the same goal—the definition of a long-run distributional equilibrium. They are said to use the same mode of reasoning—from the existence 'in production' of a surplus to its distribution 'in commodity circulation' between wages and profits. Perhaps most importantly, they are treated as having fundamentally the same concept of value from which 'their' analysis proceeds. Mark Blaug, in 1978, summarized this common viewpoint: "The Marxian quest for the appropriate transformation of values into prices is nothing else than the Ricardian hunt for a perfect 'invariable standard of value.' The whole problem is derived from Ricardo."[6]

While, of course, no one can deny the existence of similarities between Ricardo and Marx, in our view their differences are much more significant. Indeed, we feel that this continuist view glosses over fundamental differences in the object, concepts, and theoretical strategies of Ricardian and Marxian approaches to political economy. Ricardo's effort was explicitly "to determine the laws which regulate . . . distribution" of the social product.[7] With product distribution as his 'entry point' into theory, Ricardo proceeded to explain those laws as the phenomenal effects produced by the interactions among (i) conditions of soil fertility, (ii) Malthusian population patterns, and (iii) capitalists' desires to accumulate. For Ricardo, these factors constitute the essence of the economic problem, and his theoretical strategy was to reduce other economic relations to expressions of this underlying essential core. Given that entry point and that strategy, Ricardo's concepts relating to classes functioned merely as *means* to the broader goal of specifying the essential determinants of social distribution.

For Marx, in contrast, the concept of 'class relations' was itself the point of entry, the object of his inquiry. The specification of capitalist distributional relations, important as that is for Marx, was not an end in itself, but rather a means to the fuller understanding of class relations.[8] This quite different object of discourse entails a correspondingly different strategy of argument, since class relations are in no sense the essence of the non-class aspects of society.[9] Rather, class relations must be understood as overdetermined or constituted in their particularity by all aspects of society. In other words, class relations have certain 'conditions of ex-

6. *Economic theory in retrospect* (Cambridge, 1978), pp. 244–45. A similar view, emphasizing Sraffa's standard commodity as the resolution of Marxian dilemmas inherited from Ricardo, can be found in Eatwell, "Controversies in the theory of surplus value: old and new," *Science and Society* 38 (Fall 1974): 281–303.

7. Ricardo, "Preface," *The Works and Correspondence of David Ricardo*, ed. Piero Sraffa, I (Cambridge, 1953), p. 5.

8. Cf. Althusser, *Reading Capital*, pp. 158–93.

9. For an elaboration of this point, see Resnick and Wolff, "Transitional conjunctures," pp. 3–11.

istence' provided by all the other aspects of society, which are in turn comparably overdetermined. Production and circulation are both overdetermined; neither is the essence of the other. The basic categories of Marx's theory of value are handled in a similarly non-reductionist fashion; the relation between value and value form (e.g., price of production) is complex, and in no sense can either be said to be the essential determinant.

A perspective such as this allows, we believe, a very different judgment of Marx's handling of value-price transformation, since that transformation itself cannot in this context be regarded as simply an extension of Ricardo's problems and concerns. Our approach to these issues represents a fundamental alternative or break from the customary (and, to us, Ricardian) view of commodity value as a pre-given and fully specified source for the deduction of competitive capitalist exchange ratios.

The traditional view of transformation sees the problem as one of providing a derivation of a particular sort: from one set of known quantities (values, surplus values, rate of surplus value) to one other set of initially unknown quantities (production prices, profits, rate of profit). It is now common to regard Marx's analysis as flawed, since he failed to reach the logically consistent answers derived first by Bortkiewicz, and more generally by Seton and Sraffa.[10] One of the more debated aspects of the problem concerns the fact that, when the necessary corrections are made in Marx's treatment, the mathematical necessity for a single normalization invalidates Marx's oft-stated claim of an aggregate equality both between surplus value and profit and between value and production price. Implicit in this interpretation is the position that the simultaneous satisfaction of the two aggregate equalities is a result of Marx's flawed analysis, a result which can be shown to be invalid by a correct application of Marx's own categories.

It seems to us that there is another quite different interpretation of the relevant passages. Rather than viewing the two aggregate equalities as a falsifiable result, we believe that they are components of the very *meanings* of the value categories Marx is developing in *Capital* III. For us, Marx's method of analysis necessarily involves different particular applications of general concepts like value, different concrete quantitative meanings, depending on the particular set of social processes which are assumed to constitute his theoretical object. Since *Capital* III represents a discretely different level of discourse from *Capital* I, involving new determinations (full inter-industry competition and capital mobility) and the integration of new concepts (the average rate of profit) corresponding to those determinations, we find it neither surprising nor somehow illegiti-

10. See Ladislaus von Bortkiewicz, "Value and price in the Marxian system," *International Economic Papers* 2 (1952): 5–60; and Francis Seton, "The transformation problem," *Review of Economic Studies* 29 (June 1957): 149–60.

mate that Marx should reformulate the quantitative application of the general definition of value in the light of the new and more complex object of his discourse. If the fundamental thrust of *Capital* I is to construct the basic class relations of capitalism and show on that basis how capitalist income derives from unpaid labor time, then the basic thrust of the transformation discussions in *Capital* III is to show that the particular form taken by that income (average profit) can be conceived as a specific relation between paid and unpaid labor time. Nowhere in his works did Marx carry this project, like some of his others, through to its completion, but we believe that his discussion contains all the elements needed to construct an internally consistent and distinctively Marxian theory of the average rate of profit, including its attendant transformation of value into price of production.

II. *Value and the Form of Value in Exchange*

Our contention is that at every level of his argument, Marx's concepts of value and the form of value in exchange are interdependent. Both depend on the specific set of social processes which Marx conceives as securing the conditions of existence for capitalist class relations. Each successive level of his argument allows the incorporation of a more complex conception of the economic processes which actualize the class relation. Within each level Marx proceeds by the concretization of the general concepts of value and form of value. To summarize his basic method: every set of economic processes which can be described as overdetermining capitalist class relations generates quantitative outcomes which can be conceived through the *joint* application of the categories of value and value form in some particular relationship. At each successive level of discourse the abstract definitions of these general concepts are concretized in a manner appropriate to the particular social processes incorporated at that level. Thus, each level involves, for Marx, the specification of both particular production processes and particular circulation processes, each dependent on (overdetermined by) the other. The quantitative investigation of the 'economy' so constituted at each level must proceed by constructing particular meanings for the quantitative categories specific to production (value) and circulation (form of value), each of which takes the other as a precondition.

The general definition of the value of a commodity, one which Marx consistently maintains across all levels of his argument in the three volumes of *Capital*, is the quantity of abstract labor time socially necessary for its reproduction. This general definition must be concretized at each level of Marxist discourse in terms of the concrete production and exchange processes specified and incorporated at that level. *Value*, then, is the quantity of social labor time 'attached to' the commodity in production, given the nature and functioning of the processes involved in commodity circulation. The *form of value* in exchange is, similarly, the quantity of social labor time 'attached to' the commodity in circulation, given the particular processes of production. The two general definitions are inherently interdependent; each takes the other as a precondition, which is just another way of saying that both, in their particular relationship, depend on the aggregate of social processes which are taken as the object of investigation.

The basic class relation of capitalism, the specifically capitalist mode of appropriating surplus labor, involves the circulation as well as the production of commodities. Therefore, any discursive specification of that class relation necessarily presupposes some explicit or implicit conception of commodity exchange. On the other hand, any discursive specification of capitalist exchange (or for that matter of capitalist production) requires for Marx the preliminary specification of the basic class relations. The particular and (we believe) different conceptions of the *magnitude* of commodity value in *Capital* I and III derive from this basic problem of discourse: exchange categories in *some* form are initially necessary to develop the capitalist relations of production, but *capitalist* exchange categories depend on the prior development of those class relations.

Now, for Marx, commodity circulation is always conceived as tending to establish the exchange of equivalents, the logic behind this being that any other result involves shortchanging and the subsequent alteration of the supplies and/or demands which established the market exchange ratios. The question at each level of discourse is, Equivalents in what sense? The answer defined in Part 1 of *Capital* I and maintained throughout volumes I and II is a carefully chosen device of discourse: Marx assumes that the particular circulation processes under consideration tend to establish exchange ratios defined by the equivalence of labor-time magnitudes determined *solely in production*. Thus, *Capital* I allows a progressively more developed notion of the processes of capitalist production, but deliberately maintains a simplistic, indeed non-capitalist, notion of the processes involved in the circulation sphere (no formation of an average rate of profit, etc.).

It is Marx's assumption of this particular sort of equivalent exchange which permits him to focus his attention where he wishes, on the sphere of production as the site of the performance and direct ('first') appropriation of surplus labor in capitalism.[11] Marx, in effect, develops a preliminary conception of the process of commodity circulation out of his assumption that each individual exchange of commodity for money is an exchange of equal *values*. The amount of social labor attached in produc-

11. See Marx, *Capital* I, trans. Ben Fowkes (New York, 1977), pp. 709–10.

tion to any commodity (its value) is realized in the money for which it is sold; any other commodity which can then be purchased with this money represents exactly that same value because it had attached to it in production exactly that same quantity of social labor time. The amount of labor time advanced in any purchase is thus in general the same as the amount of labor time received in the commodity so purchased; circulation functions here in quantitative terms as merely a neutral intermediation. Each exchange, and therefore the process of circulation encompassing all exchanges, is so specified as to effect no transformation of the quantity of value held by the commodity owner before and after the sale or purchase. The form of value in exchange is, by specific assumption, a pure reflection of the labor time expended in production. *Since this is so*, since exchange conditions are permitted no impact on any relevant magnitude, the only quantitative determinants involved in the specification of *either* value or value form are production conditions, the average technical conditions under which commodities are produced and reproduced: value and value form are of equal magnitude.

Similarly, because of the particular process of circulation which Marx assumes, the quantity of social labor time attached to commodities in production, the amount of socially necessary labor time involved, becomes identical to the quantity of abstract labor time physically/technically embodied in the commodity under average conditions of productivity. Marx's initial presupposition specifies a social situation in which, so far as quantities are concerned, anything that holds in production, holds in circulation; anything that holds for value, holds for value form; anything that holds for commodities, holds for the money for which they exchange, and therefore for capital as the advance of money. Without *some* such specific conception of exchange equivalence, it would be impossible to make any statements concerning the magnitude of capital advanced in relation to revenue realized. The particular device Marx adopts (value-equivalent exchange) is useful precisely because it allows him to get the circulation sphere 'out of the way,' to neutralize its effects, so that there can initially be an absolutely unambiguous focus on the practices and relationships within the sphere of production, where surplus labor is performed and directly appropriated by capital. Marx's method of class analysis requires him to proceed in this fashion, by levels of argument, through what he called the "intermediary stages."[12] For Marx, it was crucial to discuss the class relation and develop the notion of *what* it is that is distributed via circulation (commodities containing unpaid labor time) as the necessary prior step to the problem of *how* a capitalist distribution of those commod-

12. *Theories of surplus value* II (Moscow, 1968), p. 174. Marx often criticizes Ricardo for immediately assuming an average rate of profit, thereby presupposing its consistency with his concept of value rather than developing the relationship between value and an average rate of profit.

ities takes place. The *Capital* I assumptions were in turn necessary to accomplish this prior step.

Nevertheless, the value of a commodity (and therefore the value of commodity capital) in *Capital* I is a magnitude of directly and indirectly embodied labor time, not as an eternally fixed definition *à la* Ricardo, but because that is the only possible meaning of the concept of 'socially necessary labor time' under Marx's assumption concerning the equivalence of exchange at this initial level of his argument. The social necessity of labor in production is never *simply* identical to its technical necessity, just as the form of value in exchange is never *simply* identical to value itself; yet Marx assumes both initially, as a means to focus on class relations in production. He can then proceed in the rest of *Capital* I to examine the wage-labor relationship and discuss exploitation, knowing full well that the assumed process of circulation is inconsistent with a capitalist distribution of the commodities which emerge from the production process.

The level of argument which Marx reaches in *Capital* III can and does for the first time allow him to construct the interaction of capitalist class relations in the production process (volume I) with the commodity circulation process provisionally assumed in volume I and further developed in volume II. In Marx's own estimation: "The capitalist process of production as a whole represents a synthesis of the processes of production and circulation.. . . . The various forms of capital, as evolved in this book [vol. III], thus approach step by step the form which they assume on the surface of society."[13]

In Parts 1 and 2 of *Capital* III Marx focuses this interaction or 'synthesis' upon the 'conversion' of the rate of surplus value into an average rate of profit. This 'conversion' entails the corresponding 'transformation' of values into prices of production. In our interpretation the 'synthesis' and 'transformation' constituting this level of Marx's argument carry significant implications for his concepts of both value form and value itself.

Regarding the form of value, Marx makes two key points: first, the interaction of capitalist class relations in production with the process of commodity circulation generates a new form of value, 'price of production,' which differs from the form assumed in *Capital* I by its systematic deviation from equality with the magnitude of value. This first key point has been the focus of most of the controversy over the 'transformation problem.' But Marx made a second point which is central to our interpretation. He argued that the new form of value, as a constituent element of a capitalist economy and society, entered directly into the determination of commodity values, i.e., into the determination of the 'socially necessary' abstract labor time needed to reproduce such commodities. This second point about the form of value carries a significant implication about

13. *Capital* III (New York, 1967), p. 25.

Marx's concept of value itself. It shows—as detailed below—that 'socially necessary' is no longer simply equal to 'average physically embodied' abstract labor time as had been the provisional assumption of *Capital* I. The latter assumption of equality is thus shown to have been a special case of the general definition of value, one first chosen (volume I) and then superseded (volume III) according to the requirements of Marx's exposition of his theory.

Such an interpretation implies a formulation of the 'transformation problem' and a solution which differ sharply from all previous attempts in the continuing controversy on the subject.

III. *A New Interpretation of the 'Transformation Problem'*

Commodity circulation in *Capital* III is still conceived by Marx as tending toward an exchange of equivalents; the difference is in the nature of the equivalence. The equivalence of exchange in volume III must be constructed, no longer assumed as in volume I. More precisely, it must be constructed out of those processes specific to competitive capitalism which tend to establish a proportional distribution of unpaid labor time in the form of an average rate of profit on total capital.[14]

It is crucial to remember that in any exchange between money and commodities Marx always treats both the money and the commodities as representing magnitudes of social labor time. Thus, the magnitudes exchanged are always expressible in units of labor time. If the equivalence of exchange in *Capital* III is defined to mean that all exchanges occur in those ratios which result in the aggregate capital of each industry earning the same rate of profit, then in any equivalent exchange the magnitude of labor time which the money represents is the magnitude of labor time necessary to generate the average rate of profit for the industry producing the commodity in question.

This latter magnitude is defined as the price of production of the commodity, the specific form of value under the social conditons Marx as-

14. Marx conceives the determination of an average rate of profit—continually approximated but never actually realized—as the outcome of ceaseless interactions between production and circulation. The profit rate for any individual capital is tied to the market prices for its inputs and output. Market prices are determined by supply and demand, but supplies and demands vary continuously as new decisions are made on the basis of realized profit rates. Any overexpansion of supply relative to demand will lower the price of the commodity and the realized profit rates of its (differentially efficient) producers. Simultaneously, any sector which uses this good as an input will experience a lowered reproduction price for its capital and thus a higher average rate of profit. Such a situation would, in the context of *Capital* III, provoke changes in the allocation of capital perhaps sufficient to alter the technical basis of production, as less efficient capitals within low-profit industries shut down and capital flows to the more efficient techniques within high-profit sectors. In any case, there would certainly be changes in the scale of output in different industries, implying an altered pattern of reproduction demands, a new set of prices, different realized profit rates, etc.

sumes in *Capital* III. The point is crucial. The capitalist competitive criterion of equal profitability requires that the equivalence of exchange be defined in terms of value form; it is an equivalence between the prices of production of commodities with different values which nevertheless must exchange as equals.[15] Abstracting from the variations of supply and demand, the quantity of labor time each commodity represents in exchange—the amount of labor time *realized* by the producer—must be by definition an amount exactly sufficient to repurchase the means of production and labor power used up in its production, as well as to provide the average profit on the quantity of capital advanced. And of course, this magnitude of advanced capital is itself a price-of-production magnitude, the sum of the prices of production of the means of production and of subsistence required to produce this commodity.[16]

Thus, any commodity's price of production is a magnitude of labor time just exactly large enough (socially necessary) to reproduce the *capitalists* of its producing industry as capitalists on an equal footing with those in all other industries. As a magnitude, then, price of production—the form of value in exchange—will in general deviate from the value of the commodity (though they are dimensionally the same—both are magnitudes of social labor time), since value is here as always generally defined as the quantity of abstract labor time socially necessary to reproduce the *commodity*.

The particular meaning of this general definition must be constructed (concretized) with reference to the particular set of production and circulation processes under consideration in *Capital* III. This context requires, as Marx several times states, a consideration of the factors socially necessary to reproduce the commodity *as a product of capital*.[17] Thus, the specifically capitalist conditions of social circulation must have their own

15. It is worth noting that what Marx seeks is not an equilibrium solution, in the conventional sense of the term. It makes no sense to use such a term when there is no possibility of an equilibrium state, long- or short-run, being established by the process taken as Marx's theoretical object. But since there is assumed to be an equilibrating tendency in terms of distribution of profit between capitalist industries, one can always 'step into' the process at any moment and ask, given the conditions and scales of production and the historically determined real wage, what the general rate of profit would be if, hypothetically, demands were to mesh with supplies in just exactly the way necessary to generate such a proportional distribution.

This theoretical choice, to 'step into' an ongoing process of continuous change, generates results which have meaning only in a short-run sense—and a hypothetical short run at that. The factors taken as given are given only for the immediate period in question, since the responses made by capitalists to differential profit rates have the effect of changing the conditions from which the average rate of profit is derived. The average profit rate and its associated ratios are analogous to a target which moves each time the marksman takes aim at it; for Marx, equilibration and disequilibration are one and the same process.

16. See, for example, *Capital* III, pp. 161, 164–65.

17. See, for example, ibid., p. 175. Cf. *Capital* I, p. 969, where Marx argues: ". . . as the *result*, the *product of capital*, the commodity changes in form (and later on, in the price of production, it will be changed in *substance* too)."

impact on the social necessity of the labor time involved in production. To reproduce a commodity, it is necessary to combine means of production and living labor; therefore, to quantify the value of that commodity it is necessary to sum two magnitudes of labor time which express the size of these two constituent elements of the production process.

Whatever the wage, whatever the cost of the labor power involved, the contribution to commodity value made by living labor is the amount of that labor expressed as abstract labor time. This component of the value of output is solely determined in production and is utterly independent of the form of value and the nature of exchange equivalence.[18] The quantification of the contribution of means of production to value is, however, different here, and therefore is the key conceptual step. The question is, given the processes assumed in both production and circulation, what quantity of social labor time associated with the consumed means of production represents the amount *socially necessary* to reproduce the commodity output as the product of capital. Since Marx is considering a social object in which the processes of circulation constitute effective preconditions for the process of production, we believe that the relevant magnitude must be the *price of production* of the consumed means of production and *not* the abstract labor time physically embodied in them. We interpret Marx's *Capital* III discussion in this way because, within the socioeconomic context there specified by Marx, it is only through the advance of money representing this price of production that the necessary means of production can be procured.

We may provisionally summarize our interpretation to this point. Under specifically capitalist exchange as well as production conditions, the value form—exchange value—is no longer quantitatively equal to value; to signify this transformation of value form, 'price of production' supplants 'exchange value.' Moreover, the transformation affects value itself: the quantity of labor time physically/technically embodied in the means of production is no longer identical to what is socially necessary for the reproduction of output. Given the equivalence of market exchange, the quantity of labor time in money form which each capitalist must actually advance to get his constant capital goods (their respective prices of production) becomes a constituent part of the *value* of the commodities produced with those constant capital goods. This is what Marx means when he asserts that the price of production is "a prerequisite of supply, of the reproduction of commodities to each individual sphere."[19] The form of

18. This is certainly the case at this level of Marx's discourse, as it was in *Capital* I. It should be noted, however, that the determination of abstract labor time is itself fully endogenous to the process of capital accumulation. Thus, a more developed discourse on the relationship between value and value form would have to analyze the role of altered circulation conditions in affecting the particular quantitative and qualitative determination of an abstract labor hour.

19. *Capital* III, p. 198.

value in exchange is a constituent element in determining the magnitude of commodity value. Insofar as the values of commodity outputs are transformed into prices of production in market exchanges (of equivalents), this must include those purchased commodities constituting the elements of constant capital. Their prices of production are then incorporated into the values of all commodity output, since they represent the magnitude of capital socially necessary for the production of that output. Equation (3) below states this central point formally.

Simultaneously, however, the concept of value remains crucial to the quantification of prices of production. Price of production, as an absolute magnitude of labor time, can be conceived only as a specific deviation from value, since the price of production of any individual commodity is constrained by the aggregate amount of unpaid labor time performed in the economy. The profit component of price of production is then simply a proportional share of the surplus value generated in the aggregate by all capitals.

This interpretation of the transformation theory in *Capital* III departs sharply from that built around the Ricardian concept of commodity value as necessarily and always the direct and indirect physically/technically embodied labor. The lines of our argument are implicit in *Capital*,[20] and Marx explicitly makes this point in *Theories of surplus value*:

> It is clear that what applies to the difference between the [price of production] and the value of the commodity as such—as a result of the production process—likewise applies to the commodity insofar as, in the form of constant capital, it becomes an ingredient, a precondition of the production process. Variable capital, whatever difference between value and [price of production] it may contain, is replaced by a certain quantity of labor which forms a constituent part of the value of the new commodity, irrespective of whether its price expresses its value correctly or stands above or below the value. On the other hand, the difference between [price of production] and value, insofar as it enters into the price of the new commodity independently of its own production process, is incorporated into the *value* of the new commodity as an antecedent element.[21]

This passage is simply not comprehensible within the traditional—what

20. As an example we cite the argument in *Capital* III, p. 206–8, in which Marx discusses the price of production of commodities of average composition. The passage implies that the 'average composition' of capital is an average of price of production magnitudes, not values ("80*c* may be greater or smaller than the value of *c*"). As a result, Marx's assertion that the price of production of the output of such a capital is "in practice . . . equal to the value of the commodity" can be understood as an application of the concept of commodity value in the sense we have presented.

21. *Theories of surplus value* III (Moscow, 1971), p. 167; emphasis altered. There are occasionally terminological differences in Marx's various works. *Theories* uses the term 'cost price' to refer to what Marx in *Capital* called 'price of production.' Identifying their

we call the Ricardian—interpretation of Marx's concept of value, because it implies a clear rejection of the traditional view of transformation as a one-way derivation of production prices *from* pre-given values. Marx here affirms the interdependence of value and value form (price of production), an interdependence which cannot be expressed by treating the relation between the two concepts as merely a functional relation between dependent and independent variables. Instead, Marx constructs each concept from and by means of the other. Marx's theoretical commitment to overdetermination implies his particular methodological approach, the step-by-step development of interdependent categories. The fact that surplus value is distributed via commodity circulation necessarily involves the circulation process in the determination of value. Surplus labor in its distinctively capitalist form is *unpaid* labor, labor time for which no equivalent is received; the magnitude of surplus value can therefore only be determined in conjunction with the capitalist structure of payment equivalence in market exchanges. But equally and conversely, the fact that surplus value is created only in production by the performance of labor places absolute constraints on production prices as magnitudes expressed in labor-time terms.

Marx conceives a social object in which both production and circulation processes participate in the overdetermination of quantitative outcomes at any point in time. Thus, in the mathematics of transformation (the construction of a particular outcome by *imposing* an equalized distribution between capitalists), both spheres contribute to the determination of both values and prices of production. Value and the form of value in exchange are different concepts, but they do not have different basic determinants.

This interpretation of Marx contrasts sharply with the view which finds a 'classical' continuity in his use of the concept of value. Value is not, for Marx, as it was for Ricardo, simply the amount of directly and indirectly embodied labor time, because that view of value is one which can be applied to *any* commodity-producing society, independently of the class relations which structure that society. Marx does, of course, use just this notion to begin his study of capitalism, since the circulation of objects and services as commodities is a basic condition of existence of capitalist class relations. But to interpret the transformation chapters of *Capital* III as Marx's attempt to derive specifically capitalist quantitative results, by mathematically manipulating a vector of values given independently of the particular circulation practices which make an economy capitalist, strikes us as imposing on Marx an essentialist and fundamentally Ricardian notion of the role of labor time in the theory of value. Steedman, for ex-

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meaning from the context, we have, for clarity, substituted 'price of production' where the text refers to 'cost price.'

ample, does this.[22] In our view, to conceive quantitatively the process of capitalist competition in both production and circulation, the concept of value itself must be transformed. The passage quoted above is merely the most explicit of many which suggest to us that Marx was doing just that. In *Capital* III he concretized the meaning of his general concepts at a level which incorporates new, specifically capitalist exchange determinations. To summarize: Marx used *Capital* I-level concepts of value and value form as necessary means to construct other key concepts (surplus value, competition, accumulation, super-profit, etc.); by means of these key concepts he then 'transformed' the *Capital* I-level concepts of value and value form by allowing for more complex determination of the 'social necessity' of labor time in capitalistically circulated commodities.

IV. *The Quantitative Relation Between Value and Price of Production*

Our interpretation of Marx's argument may be explained in general mathematical terms. We will here focus on the simplest general case, that of n single-product industries with no fixed capital. Our first step is to distinguish carefully between numéraire prices and prices of production. Numéraire prices express the exchange ratios between commodities in terms of some particular commodity whose price is arbitrarily set equal to 1. For Marx, these price ratios must be linked to the n prices of production, each of which is a definite quantity of social (abstract) labor time per unit of commodity. Thus, if the kth commodity is chosen as numéraire:

$$P_j = \frac{1}{\rho_k}\rho_j \quad (j = 1 \ldots n) \tag{1}$$

where P_j is any numéraire price and ρ_j is any price of production. The distinction is important; numéraire prices are relative magnitudes, but prices of production are, for Marx, absolutely defined, as will be seen.

Define:

$A = [a_{ij}]$,	matrix of physical commodity inputs per unit output;
$L = [L_j]$,	row vector of direct (living) labor inputs per unit output;[23]

22. *Marx after Sraffa*, pp. 39–42, 55–57, and passim. Virtually all of the citations from Marx which Steedman presents (pp. 208–13) to support his reading of Marx's value categories come from *Capital* I, where, as argued above, none of the crucial problems of conceptualizing 'socially necessary' labor time can arise.

23. The definitions of A and L may appear to imply an assumption of constant returns to scale; in fact, no such assumption is needed. With the scale and composition of gross output taken as given, all that is necessary is that A and L express the average conditions of production of commodities. Thus A and L are constructed from a weighted average of all the various techniques involved in producing the given output of each of the commodities in question. See *Capital* III, pp. 178–79.

$b = [b_i]$,	column vector of commodities advanced per unit direct labor (the 'real wage bundle');
$X = [X_i]$,	column vector of gross output levels in physical units;
$Y = [Y_i]$,	column vector of net output levels in physical units, such that $Y = [I - A]X$;
$V = [V_j]$,	row vector of commodity values (value per unit commodity);
$\rho = [\rho_j]$,	row vector of prices of production per unit commodity;
$P = [P_j]$,	row vector of numéraire prices, with $P_k = 1$;
r,	the general rate of profit.

A, L, b, X, and Y are historically determined, the result of capitalists' choices, accumulation, class struggle, etc., and are taken as given for purposes of transformation. P, ρ, V, and r are the variables to be determined.

The basic transformation system is expressed in the following equations:

$$\rho = [\rho A + \rho bL](1 + r) \tag{2}$$

$$V = \rho A + L \tag{3}$$

$$r = \frac{LX - \rho bLX}{\rho AX + \rho bLX} \tag{4}$$

Equation (2) expresses price of production as the sum of the advances made for means of production and labor power (the cost price, in Marx's terms) marked up by the general rate of profit. Though the equation is structurally identical to a Sraffian price equation, all its terms are magnitudes of labor time per unit output. Equation (3) expresses the value of each commodity, in the sense argued above, as the sum of the prices of production of its means of production plus the living labor required. In our opinion, this is the only possible equational statement of value which is consistent with Marx's argument in the extended passage quoted in Section III above. Equation (4) expresses the general rate of profit, a pure number, as a ratio of aggregate quantities of labor time: the ratio of total unpaid labor time to total paid labor time. The numerator of this fraction, total living labor minus the aggregate wage bill evaluated in price-of-production terms, is the total surplus value realized in the economy. Since, as Marx argues, "the value of labor power is determined by the production price of the means of subsistence,"[24] the vector product, ρbLX, is the aggregate expression for the total necessary labor performed in the econ-

24. Ibid., p. 868.

omy. Any labor performed in excess of this amount is surplus (unpaid) labor, creative of surplus value. The denominator is the total advanced capital expressed as a magnitude of paid labor time by a sum of price of production magnitudes. Thus, the profit-rate equation directly satisfies Marx's condition that total profit is equal to total surplus value (unpaid labor time). Moreover, the system simultaneously fulfills Marx's other condition, that total value and total price of production are equal. Adding 1 to each side of equation (4) yields

$$(1 + r) = \frac{LX + \rho AX}{\rho AX + \rho bLX}$$

which, by simple multiplication, implies

$$(\rho AX + \rho bLX)(1 + r) = LX + \rho AX$$

Applying equations (2) and (3) yields

$$\rho X = VX \tag{5}$$

which states Marx's second aggregate equality.

The system defined by equations (2), (3), and (4) is presented in this form to emphasize the conceptual definitions involved, but under certain well-known conditions, equation (2) can be solved independently for the profit rate via the Perron-Frobenius theorems. Equations (3) and (4) then contribute to the definition of a unique normalization. From equation (5):

$$\rho X = \rho AX + LX$$

$$\rho[X - AX] = LX$$

$$\rho Y = LX \tag{6}$$

This equation expresses what for Marx is a necessary equality between, on the one hand, the direct labor-time expression of the net product (LX), and on the other hand the expression in labor-time terms for the revenues which are realized by the two classes together when that net product is distributed between them through the circulation process (ρY).[25] Equation (6), while not necessarily complete for all prices of production (since some commodities may not appear in the net product), is nevertheless sufficient to determine a unique normalization.

The solutions which emerge for the rate of profit and numéraire prices are identical to those which emerge from the Sraffa physical-quantities model. That numerical similarity notwithstanding, the Sraffian approach is quite different, since it makes no use, analytic or conceptual, of either value or value form as labor-time magnitudes. A Sraffian approach uses

25. Ibid., p. 877. Cf. also, pp. 840, 852–53.

the concept of 'surplus' only in the restricted sense of a physical surplus *product*, the distribution of which is dependent on a uniform profit rate. In contrast, Marx's focus on class relations as his object of discourse requires him constantly to link the existence within capitalism of a physical surplus to the parallel necessity for there to be surplus *labor* which creates surplus *value*.[26] The system presented here quantitatively specifies that surplus labor, and has the additional property that Marx's two aggregate equalities hold simultaneously. Indeed, they hold as identities, as properties of the meanings of the concepts employed. The rate of profit is expressed as a ratio of aggregate quantities of labor time, as a relation between paid and unpaid labor, just as Marx insisted it could be understood; in this system there is no difference between 'money' and 'value' rates of profit. Moreover, equation (4) expresses the rate of profit as a ratio of labor-time magnitudes summed across *all* industries. Application of the strong Frobenius theorems of course implicitly utilizes the Sraffian distinction between basic and non-basic goods, but the solutions so derived for r and ρ are consistent with equation (4), which makes no such distinction. We emphasize the aggregate nature of equation (4) as a response to those who have argued that Marx's approach is inherently 'wrong' because he fails to make the distinction explicit. In our conception of the categories that Marx uses at this stage of his discourse, the conditions of production in non-basic industries are indeed relevant to the expression of the profit rate as a ratio of labor-time magnitudes, whatever the mathematical technique used to derive these magnitudes.

The key to the behavior of the system is the fact that both price of production and value are here conceived as varying with all the givens—the real wage and the size and composition of output as well as technology. This might seem to conflict with various statements by Marx to the effect that changes in wages do not affect the values of commodities. In terms of the simultaneous system presented above, where advanced capital is evaluated at the prices of production implied by the full set of givens, a change in b will affect both values and prices of production. However, we would emphasize that Marx often chooses, for expository purposes, to treat the advances made for means of production as given prior to the production period in question, so that the contribution of means of production to value is taken as fixed and independent of the 'replacement prices' implied by the current conditions of production. With this assumption, it is true that changes in the wage will leave the value of commodities unaffected. Marx makes this sort of assumption quite frequently; for example, in the numerical illustrations with which he introduces the concept

26. See, for example, ibid., p. 819.

of price of production, he takes the 'values' of constant and variable capital in each of the five spheres of production as given.[27]

In contrast to the traditional interpretation which regards these numerical examples as embodying Marx's full theory of transformation, we understand his procedure in these examples as qualitatively different from that required for a fully simultaneous solution to the problem of transformation. His text makes clear that Marx knew this: he never attempted to formulate a mathematically complete transformation system, as we have done above, preferring instead to treat the difficult conceptual issues verbally. The section entitled "Appreciation, Depreciation, Release and Tie-up of Capital"[28] demonstrates that he was fully aware of the reevaluation of capital implied by changes in prices. Marx here argues that factors which alter the prices (or, equally, the prices of production) of commodities, e.g., a change in the real wage, will cause simultaneous changes not only in the valuation of capital in those industries which use such commodities as means of production but also in the value of the commodity output in these industries.[29] Thus, while changes in the real wage have no direct impact on values (the direct labor component of value is independent of wages), Marx was quite aware of the possible indirect effects on value via changes in the replacement cost of means of production.

V. *Conclusion*

The problem of transformation, as of any textual matter, is ultimately one of interpretation. The argument made here stems from our reading of Marx's work as unique in its method of and entry point into theoretical construction, and as particularly different in just these respects from the Ricardian theoretical approach. To continue to accept the traditional posing of the transformation problem, as a problem 'derived from Ricardo,' is implicitly to accept the premise that Marx's value theory postulates an essentialist structure of causality. That traditional posing views Marx as arguing (i) that the abstract labor physically embodied in commodities is something so fundamental and essential that it causally determines social outcomes independently of social relationships, and (ii) that the class relations of capitalism affect only the phenomenal form taken by the underlying value causation. These are precisely the points at which the Sraffa-based literature has correctly criticized both the consistency and the use-

27. Ibid., pp. 154–57.
28. Ibid., pp. 110–24.
29. See especially ibid., pp. 112–13, where Marx considers the impact of price fluctuations on value. Although Marx has not, at this point in his argument, introduced the concept of price of production, he does argue that "the present statements apply equally if prices rise or fall under the influence of the credit system, competition, etc., and not on account of fluctuations in value" (p. 113).

fulness of (its reading of) Marx's labor-time categories. But neither of these points is acceptable to an interpretation of Marx which premises overdetermination. More important, from our perspective, neither point need be understood as a necessary characteristic of Marx's value theory.

Some readers will no doubt view this study as an attempt to 'rescue' Marx from his own inconsistency by defining the difficulties out of existence. We can only respond that, in our view, many of those difficulties were defined into existence by a tendency—no longer acceptable—to collapse the different Marxian and Ricardian sciences into one all-encompassing 'classical' framework. It is time to break the near monopoly in the literature held by that framework's conceptualization of the 'transformation problem' and of Marxian economics generally.

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