Joseph. Winternitz
The Economic Journal, Vol. 58, No. 230, June 1948

NOTES AND MEMORANDA

Values and Prices: A Solution of the so-called Transformation Problem

What has been termed the " transformation problem " has had a place of some importance in the history of economic thought on the Continent, but has remained hitherto almost completely unknown in this country. It concerns the much-debated question whether the price structure under the conditions of industrial capitalism and perfect competition (based on cost price plus the prevailing rate of profit) can be logically derived from values determined by labour, as postulated by Ricardo and Marx.

Böhm-Bawerk, in his criticism of Marx's economic theory, alleged that the theory of " prices of production " (= cost price + the average rate of profit) of Marx stood in contradiction with the theory that exchange-values are determined by the quantity of labour socially necessary to produce a commodity. For when the ratio of capital invested in means of production to wages-capital differs in different industries, equal values correspond to equal prices only in those industries in which this ratio coincides with the average ratio.

The attention of English readers has recently been drawn to this problem by the discussion of this question in the interesting book of Paul Sweezy, *Theory of Capitalist Development* (Chapter 7). His discussion is mainly based on a solution offered by an Austrian economist, Bortkiewicz (in an article in *Jahrbücher für Nationalökonomie und Statistik*, July 1907). What follows is an attempt to provide an alternative solution, on different assumptions from those employed by Bortkiewicz and Sweezy.

When Marx in Volume I of *Capital* assumed that commodities are exchanged at values determined by labour, he was well aware that this can be regarded only as a first approximation. But the deviations of market prices from values are not relevant to the problem with which Volume I is concerned: the relation of the two main classes of capitalist society. In Volume III of *Capital* Marx explains how with a free flow of capital between different industries an average rate of profit is established.

Marx calls the capital spent on equipment, raw materials, etc., constant capital (c), the capital spent on wages, variable capital (v), the ratio between these two parts of capital (c/v) the organic composition of capital. Marx assumes that the ratio of surplus value (s) to variable capital, which he calls the rate of surplus value (s/v), tends to be the same in all branches of industry. The organic composition of capital varies: it is higher in those branches of industry which are technically more developed. The rate of profit, the ratio of surplus value to total capital $p = s/(c + v)$ would be smaller in technically higher-developed industries if commodities were exchanged at their values.

An equal rate of profit is established, according to Marx, by prices rising above value in industries with an over-average organic composition and prices falling below value in industries with an under-average organic composition. Surplus value is produced in equal ratio to labour employed, but it is distributed between the capitalists in equal ratio to their total capital. The ratio of the aggregate surplus value of society to the aggregate total capital is the average rate of profit (p). Then the " price of production " of a commodity produced with a capital $c + v$ will be equal to $(c + v)$ $(1 + p)$. Marx's law of value says that the exchange relation of commodities (and also the distribution of social labour among the various branches of production) is determined in the last analysis by the labour required to produce them. Under certain conditions which prevailed between independent small producers in pre-capitalist societies (what Marx calls " simple commodity production ") exchange of equal values was the rule. If under capitalist conditions there are other more complicated relations determining the quantitative exchange relations, this does not make an economic theory based on the determination of value by socially necessary labour inconsistent, provided there is a clear and consistent method of deriving prices from values.

Bortkiewicz and Sweezy base their analysis of the transformation problem on Marx's scheme of simple reproduction, *i.e.*, such relations between the main departments of production as will make a continuation of production on the same scale possible.

With Marx's method of transformation, the equilibrium of simple reproduction if it obtains with an exchange of equal values would not obtain with an exchange at prices of production. Sweezy finds this result logically unsatisfactory (*l.c.*, pp. 114 f.).

This objection seems to me not justified. Every change in the price structure normally disturbs an existing equilibrium. A change of prices may necessitate a changed distribution of social labour to restore the equilibrium.

But there is another objection. Marx calculates an equal rate of profit in relation to the *value* of the invested capital. But if generally in a capitalist society prices of production prevail, the rate of profit should be related to the prices of production of the elements of which capital consists. Marx was not unaware of this difficulty, but it seems that he regarded his method of calculation as a sufficient approximation.

Nor is it logically inconsistent. If we start from a state where equal values are exchanged and if prices then are changing in such a way that an equal rate of profit is established, Marx's method of calculation is valid. It is an ordinary occurrence that during a turn-over of capital, prices are changing and that, in calculating the rate of profit, prices at a different level at a later stage have to be compared with the value of capital (mainly fixed capital) put into production earlier when another level of prices prevailed.

This simple method of calculating the average rate of profit does, however, not correspond to capitalist reality. For as soon as capitalist production is established, the basis of price determination is not the exchange of equal values, but the realisation of equal profits. Engels pointed out that when capitalists began to invest money in industry there existed already a rate of profit which they were aiming at, the rate established by merchant capital.[1] So the rate of profit under normal conditions of industrial capitalism must be calculated under the assumption that capitalists have to pay prices of production for their input.

Bortkiewicz's method of transformation seems to me, however, unsatisfactory, because it makes unjustified and unnecessary assumptions.

Bortkiewicz bases his calculations on the equations of simple reproduction. In fact they are not relevant to this problem. A transformation which was valid only under this assumption would be insufficient. For the normal case is expanded reproduction when there is some "net investment."

Bortkiewicz distinguishes three main departments of industry: means of production, means of working-class consumption (wage goods) and capitalists' consumption goods. He assumes that gold, the money commodity, is one of the luxury goods so that prices in the third department are not affected by the change-over from values to prices of production, an arbitrary and unjustified assumption which makes the sum of prices deviate from the sum of values.

A straightforward algebraic analysis without any specific assumptions gives a satisfactory solution.

[1] "Supplement to *Capital*, Vol. III" in *Engels on Capital*, p. 109.

If we denote the deviations of prices from values in the scheme of three departments with x, y, z respectively, and the prices of production corresponding to the values c, v, s with the capital letters so that $C = cx$, $V = vy$, we get :

	Values.	Prices of production.
I.	$c_1 + v_1 + s_1 = a_1$	$c_1x + v_1y + S_1 = a_1x$
II.	$c_2 + v_2 + s_2 = a_2$	$c_2x + v_2y + S_2 = a_2y$
III.	$c_3 + v_3 + s_3 = a_3$	$c_3x + v_3y + S_3 = a_3z$

As the rate of profit must be equal in I and II, we have

$$1 + p = \frac{a_1x}{c_1x + v_1y} = \frac{a_2y}{c_2x + v_2y}$$

From this an equation of the second degree can easily be derived for $m = \frac{x}{y}$.

$$m = \frac{a_2c_1 - a_1v_2 + \sqrt{(a_2c_1 - a_1v_2)^2 + 4a_1a_2v_1c_2}}{2a_1c_2}$$

With m given, the average rate of profit is already determined :

$$p = \frac{a_1m}{c_1m + v_1} - 1$$

The formula shows that the rate of profit in department III which supplies consumption goods for the capitalist class and the amount of capital invested there has no influence on the average rate of profit. This is an interesting thesis to which Bortkiewicz already drew attention. It follows from the assumption that the deviation of prices from values affects equally input and output without the special assumptions Bortkiewicz made.

We need a further equation to find the prices of production. The equalisation of the rate of profit determines the price relations between the three departments ($x : y : z$), the price level for the system as a whole has still to be determined. The obvious proposition in the spirit of the Marxian system is that the sum of prices is equal to the sum of values. This is not a tautological or meaningless thesis. It says that the sum of all prices changes only if and in so far as the number of hours necessary to produce the aggregate output or the value of the money commodity changes. As a matter of fact, the price level goes up and down in the trade cycle at variance with the sum of values and the equation holds true only in the average over a whole cycle.

So we have :

$$a_1x + a_2y + a_3z = a_1 + a_2 + a_3 = a$$

Substituting $y = \frac{x}{m}$ and eliminating z by putting the rate of profit in III equal the rate of profit derived from I and II, we get :

$$x = \frac{am(c_1m + v_1)}{a_1m(c_3m + v_3) + (a_1m + a_2)(c_1m + v_1)}; \; z = \frac{a_1(c_3m + v_3)x}{a_3(c_1m + v_1)}$$

If we apply this transformation to the equations of simple reproduction, we find that they are invariant not only to this specific transformation, but to every transformation which affects the prices of input and output in the same way.

This transformation is, however, equally applicable to the conditions of expanded reproduction. These are essentially functional relations between the rates of accumulation in the various departments. These relations do not remain unchanged by the transformation.

J. WINTERNITZ