Revue de presse

à propos de :
Jean-Philippe Bouchaud, Marc Mézard
"Wealth condensation in a simple model of economy", *Physica A*, vol.282, 2000. Téléchargeable ici ou ici

NewScientist
That's the way the money goes

Mark Buchanan, *New Scientist magazine*, vol. 167 issue 2252, 19/08/2000

WHY do rich people have all the money? This may sound like the world's silliest question, but it's not. In every society, most of the wealth falls into the hands of a minority. People often write this off as a fact of life-something we can do nothing about-but economists have always struggled to explain why the wealthy take such a big slice of the pie.

If Jean-Philippe Bouchaud and Marc Mézard are right, it is more than a fact of life: it's a law of nature. These two scientists have discovered a link between the physics of materials and the movements of money, a link that explains why wealth is distributed in much the same way in all modern economies. Their theory holds out a scrap of hope to the poor of the world: there may be some surprising ways to make society a bit more equal. And it promises a new fundamental science of money. Economic theory is about to grow up.

In the 19th century, economists were certain that each society would have a unique distribution of wealth, depending on the details of its economic structure. But they were dumbfounded in 1897 by the claim of a Paris-born engineer named Vilfredo Pareto. The statistics, he insisted, prove otherwise. Not only do a filthy-rich minority always hog most of the wealth, but the mathematical form of the distribution is the same everywhere.

To get a feel for Pareto's law, suppose that in Germany or Japan or the US you count up how many people have, say, $10 000. Next, repeat the count for many other values of wealth (W), both large and small, and finally plot your results on a graph. You will find that there are only a few extremely rich people, and that the number of people increases as W gets smaller-at least until you get down to those with almost no wealth at all. This is exactly what Pareto found: the number of people having wealth W is proportional to 1/WE.

Pareto found that the exponent E was always between 2 and 3 (see Diagram) for every European country he looked at, from agrarian Russia to industrial England. And up-to-date statistics show the same thing.

This distribution means that most of the wealth gathers in the pockets of a small fraction of the people. In the US, for example, 20 per cent of the people own 80 per cent of the wealth. In Britain and in the nations of Western Europe the numbers are similar. The shape of the graph seems to be universal. For over a century, this universal law of wealth distribution has defied explanation, many economists simply putting it down to the inherent distribution of people's abilities. The truth may be simpler.

Pareto's law of the distribution of wealth across a population

Number of People / Wealth
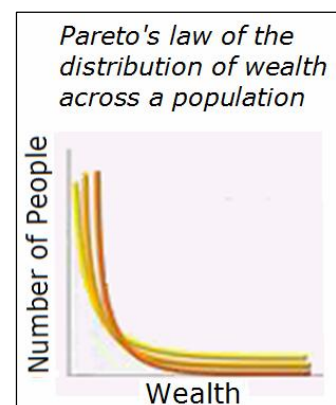
Economic theories have for years been founded on all sorts of dubious assumptions: that markets are in equilibrium, for example, or that people behave with perfect rationality. These assumptions simplify economists' intricate equations, but they often lead to rather peculiar conclusions. There is even a "no trades" theorem, says Bouchaud, that in an

economy where all the participants are perfectly rational, no trade should ever take place. So Bouchaud and Mézard are pioneering a totally different approach. They are going back to basics, trying to get by with an absolute minimum of assumptions.

"Ten years ago," says Mézard, speaking from his office at Paris-Sud University, "Jean-Philippe was one of the first physicists to get interested in finance." Questioning many traditional ideas, he at first met resistance. Since then, however, Bouchaud has built a company dealing in risk management that has won the attention and respect of the financial industry. Now at the Centre d'Etudes de Saclay in Paris, he and Mézard are setting their sights on a more ambitious goal: to build a theory of economics from the ground up.

An economy is just a large number of people who can trade with each other. Each individual has a certain amount of money he or she can invest or use to buy the services or goods of others. This is all beyond argument. Things get more contentious when you try to turn these words into precise equations. Who trades with whom? Which investments pay off and which do not?

Bouchaud and Mézard start from ground zero, with only one assumption: life is unpredictable. Buy some stocks and you might get a healthy return or a devastating loss; returns on investments are random. The trade network is also haphazard. Each person trades with a few others chosen at random from the population. "Our idea," says Bouchaud, "is to see how much we can explain on the basis of little more than pure noise."

With these few ingredients, the model seems to contain almost nothing at all. "In its basic points," says Mézard, "it's really trivial." There are many ways to build these basic elements into some equations, but fortunately the researchers had another clue.

A guiding principle of physics is the notion of invariance. Rotate a circle about its centre, and its shape remains unchanged. This is what makes a circle an especially simple and important shape in geometry. Similarly, the fundamental equations of physics are invariant under the action of certain mathematical operations, making them special cases in the space of all possible equations. Newton's laws of mechanics don't change if you alter the velocities of every body by an equal amount; otherwise, the physics you saw would change depending on how fast you were moving.

The economic equivalent of this is that a theory should produce the same results if you change the units of currency. "This is what we try to explain to our children," he says, "when they complain that their pocket money will go down when we shift to euros." Consequently, Bouchaud and Mézard wrote down the simplest equations they could find that were invariant with changes in currency.

Getting equations is one thing; solving them, another. There are millions of people in an economy, and that means millions of equations, which is why economists have tended to shun this "bottom up" approach. Bouchaud and Mézard made their task easier by keeping the ingredients of their model so simple, but they were still left with a daunting task. Then, earlier this year, they became the beneficiaries of a miraculous mathematical coincidence.

As "condensed matter" physicists, the pair have for two decades been investigating the properties of solids and liquids, substances in which the atoms or molecules are crammed together. The traditional subjects of this field are materials such as pure metals and water, whose particles settle into a well-defined state such as an ordered crystal. But since the 1970s, researchers have been increasingly intrigued by "ill-condensed" matter in which competing forces frustrate this condensation. In this class of materials-which includes glass, dirty alloys and polymers-the particles can end up in a vast number of disordered but more or less equivalent configurations.

Two competing forces

To overcome some of the mathematical difficulties in the theories of these materials, physicists have invented a simple "toy model" called the directed polymer. Imagine a long wire (the polymer) lying on a landscape that undulates up and down at random (see Diagram). The wire is tethered at one end to a post. Gravity will tend to pull it down into the valleys, but as the landscape is random, the wire will have to bend to do so. So two forces-gravity and the wire's desire to stay straight-compete with one another. As a result, the wire has to compromise: running through the valleys, so long as that doesn't entail too much bending, and, whenever the path becomes too tortuous, arching up over a pass to seek a straighter route. There is no obvious "best path" for it to follow.

Model to show the flow of wealth between different people over time
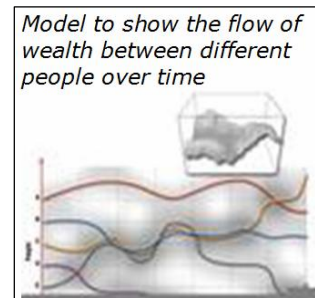
Many physical systems behave in a similar way. Take a magnetic field line, for example, as it tries to slip through a high-temperature superconductor. Left alone, it would follow a straight path. But these materials contain defects-analogous to the valleys and peaks-that attract or repel the lines. So the path they take is some compromise between going straight through and swinging by attractive defects.

In such real, physical problems, working out the details of the compromise is difficult. In 1988, however, physicists Bernard Derrida and Herbert Spohn of the Ecole Normale Supérieure in Paris solved a version of the directed-polymer problem exactly. There is one extra crucial element in the problem: temperature. In seeking some path across the random landscape, for example, the wire also puts up with a continual buffeting from air molecules, which knock it about from one path to another. The buffeting grows more vigorous with increasing temperature, and, as it turns out, the strength of this storm determines how the wire manages its compromise between going straight across and staying in the valleys.

"When the temperature is high," says Mézard, "the peaks and valleys have little effect." The buffeting is so violent that the polymer largely ignores the landscape, and flaps about all over the place. As the temperature falls, however, there comes a point at which the buffeting is no longer strong enough to drive the polymer over the landscape's peaks. Suddenly, the peaks and valleys become far more influential, and the polymer gets stuck in place, trapped along one irregular path. This sudden condensation is like the freezing of glass, or the pinning in place of a magnetic field line.

Mézard and Bouchaud have now discovered that the equations for this directed polymer model are identical to those for their economy. So to solve their equations they need do no more than pluck out some gems from the physics literature. And what these equations show is that under normal conditions, their economy follows Pareto's law.

To see how the model economy and a directed polymer are related takes a little imagination. Start with the irregular landscape, and throw a whole bunch of polymers across it. Let them settle down, and take a snapshot. This is now a picture of the economy over time.

Think of the people in the economy occupying positions on the y-axis of the landscape, and progressing to the right over time (see Diagram). A polymer plots the path of some quantity of money as it moves from person to person. So at any point, the wealth of a particular person is determined by the number of polymers that cross over their y-value.

The irregular returns on investments are reflected in the ruggedness of the landscape: deep valleys are places where there tends to be more money, the returns in investments being high; peaks are where investors fare badly and money is rare. The vigour of trade-how easily money flows between people-is analogous to the temperature. "The wealth follows a kind of random walk," says Bouchaud.

There is, however, more than one kind of random walk. Which kind wealth follows depends on how "hot" the economy is. When trading is easy, and the irregularity of returns on investment not too severe, the economy behaves like a polymer at high temperature. Just as the polymer flaps up and down with ease, adopting almost any configuration without being too strongly affected by the underlying landscape, so does vigorous trading enable wealth to flow easily from one person to another, tending to spread money more evenly.

But because the returns on investment are proportional to the amount invested, rich people tend to win or lose larger amounts than poorer. Over time, even if all changes are random, wealth ends up following Pareto's law with an exponent E between 2 and 3. How much money an individual has need have nothing to do with ability. Chop off the heads of the rich, and a new rich will soon take their place.

This is not to say that the distribution of wealth cannot be influenced. The model offers what might be the first lesson of economics to be firmly founded in mathematics: that the way to distribute wealth more fairly is to encourage its movement. Taxation, for example, tends to increase E. This is still a Pareto law, but with the wealth distributed somewhat more equitably, the rich own a smaller fraction of the overall pie. With an exponent of 3, for example, the richest 20 per cent would own 55 per cent of the wealth. It's still not fair, but it's better than the US today.

The model makes it clear, however, that taxes only work when they are redistributed evenly: if the rich get a disproportionate share-because of lucrative corporate contracts with the government, for example-then the social effect of the tax is wiped out. And according to economist Anthony Atkinson of Oxford University, economic texts have long assumed that there should be some kind of "trade-off" between equality and efficiency: that while spreading the wealth more evenly, higher taxes will also slow economic growth.

Fairer and freer trade

Yet there may be many ways besides taxes to help wealth move about, for instance by widening the number of people with which any one person tends to trade. In other words, Bouchaud and Mézard's model implies that a more equal society could come from encouraging fairer and freer trade, exchange and competition. Happily for economists, this idea dovetails with their experience and expectations, but the model gives these expectations a robust mathematical foundation.

If hot means vigorous trading and low volatility in investments, cold means restricted trading and highly irregular returns. As Bouchaud and Mézard reduce the ease of trade and increase the degree to which investments are random, they find a sudden change in their distribution of wealth: in a cold economy, wealth freezes.

Just as the polymer gets trapped into one irregular valley, and so follows a path dictated almost entirely by the random landscape, so wealth finds itself unable to flow easily between people. In this case, the natural diffusion of wealth provided by trading is overwhelmed by the disparities kicked up by random returns on investment. In Bouchaud and Mézard's model, the economy falls out of the Pareto phase into something much nastier. Now wealth becomes even less fairly distributed, condensing into the pockets of a handful of super-rich "robber barons".

Might this be the case today in some developing or troubled nations? It has been estimated, for example, that the richest 40 people in Mexico have nearly 30 per cent of the money. According to economist Thomas Lux of the University of Kiel in Germany, "most economists would anticipate that wealth concentration will be higher in economies with limited exchange opportunities-such as Russia, for example". Unfortunately, he adds, "these are usually the economies for which we have poor data or no data at all." Mézard suspects that such societies may have been more common in the past, but again the economic data are sparse. So testing this prediction of the model won't be easy.

Having illuminated Pareto's law of wealth, Bouchaud and Mézard's approach is pointing towards a deeper theoretical perspective on economics. They hope to build more realistic models by moving away from the assumptions of complete randomness, and that every economic agent is identical. Their work offers the promise of understanding not only how the economy behaves now, but also how things might conceivably change.

Could political instability throw an economy out of the Pareto regime? Or might wealth condensation be a generic risk if there is too much central planning in an economy? And are there any other hidden variables, changes that might tumble an economy over the precipice and into the depths of inequality? With the global economy becoming more and more tightly knit, these are questions that should concern the whole planet.

FINANCIAL REVIEW
The Mathematics of Inequality
Mark Buchanan, *The Australian Financial Review*, September 2002

Why is wealth so unevenly distributed among individuals? This is perhaps the most controversial and inflammatory of all topics in economics. As JK Galbraith noted, the attempt to explain and rationalize inequality "has commanded some of the greatest, or in any case some of the most ingenious, talent in the economics profession".

We all know that a few people are very rich and that most of us have far less. But inequality in the distribution of wealth has a surprisingly universal character. You might expect the distribution to vary widely from country to country, depending not only on politics and culture but also, for example, on whether a nation relies on agriculture or heavy industry. Towards the end of the 19th century, however, an Italian engineer-turned-economist named Vilfredo Pareto discovered a pattern in the distribution of wealth that appears to be every bit as universal as the laws of thermodynamics or chemistry.

Suppose that, in Britain, China, the US or any other country, you count the number of people worth, say, $10,000. Suppose you then count the number worth $20,000, $30,000 and so on, and finally plot the results on a graph. You would find, as Pareto did, many individuals at the poorer end of the scale and progressively fewer at the wealthy end. This is hardly surprising. But Pareto discovered that the numbers dwindle in a very special way: towards the wealthy end, each time you double the amount of wealth, the number of people falls by a constant factor.

Big deal? It is. Mathematically, a "Pareto distribution" implies that a small fraction of the wealthiest people always possess a lion's share of a country's riches. It is quite easy to imagine a country where the bulk of people in the middle of the distribution would own most of the wealth. But that is never so. In the US, something approaching 80 per cent of the wealth is held by 20 per cent of the people, and the numbers are similar in Chile, Bolivia, Japan, South Africa and the nations of western Europe. It may be 10 per cent owning 90 per cent, 5 per cent owning 85 per cent, or 3 per cent owning 96 per cent, but in all cases, wealth seems to migrate naturally into the hands of the few. Indeed, although good data are sadly lacking, studies in the mid-1970s, based on interviews with Soviet emigrants, suggested that wealth inequality in the Soviet Union was then comparable to that in Britain.

What causes this striking regularity across nations? The question is all the more urgent now that inequality seems to be growing. In the US, according to the economist Paul Krugman: "The standard of living of the poorest 10 per cent of American families is significantly lower today than it was a generation ago. Families in the middle are, at best, slightly better off. Only the wealthiest 20 per cent of Americans have achieved

income growth anything like the rates nearly everyone experienced between the 1940s and early 1970s. Meanwhile the income of families high in the distribution has risen dramatically, with something like a doubling of real incomes of the top 1 per cent."

Something similar is taking place on the global stage. Globalisation is frequently touted - especially by those with vested economic interests, such as multinational corporations and investment banks - as a process that will inevitably help the poor of the world. To be sure, greater technological and economic global integration ought to have the potential to do so. Yet as Joseph Stiglitz, the former chief economist of the World Bank, notes in his recent book Globalisation and Its Discontents: "Despite repeated promises of poverty reduction made over the last decade of the 20th century, the actual number of people living in poverty has actually increased by almost 100 million. This occurred at the same time that total world income actually increased by an average of 2.5 per cent annually."

What is the origin of these distinct but seemingly related trends: the greater inequality within nations (which applies to Britain, and many other countries, especially in eastern Europe, as well as to the US) and the greater inequality between them? We can blame tax cuts, liberalization of capital markets, new communication technologies, the policies of the International Monetary Fund and so on. But might there be a general science that could illuminate the basic forces that lead to wealth inequity?

Conventional economic theory has never before managed to explain the origin of Pareto's universal pattern.

But two physicists, Jean-Philippe Bouchaud and Marc Mézard of the University of Paris, venturing across the lines between academic disciplines, have recently done so.

Forget for the moment about ingenuity, intelligence, entrepreneurial skills and other factors that might influence an individual's economic destiny. Instead, take a step into the abstract, think of an economy as a network of interacting people, and focus on how wealth flows about in this network.

It will flow - causing individuals' wealth to go up or down - in one of two fundamental ways. The first is through the bread-and-butter transactions of our daily economic lives: your employer pays you for your work; you buy groceries; you build a fence to keep in the dog; you take a holiday. The second is though rises and falls in asset values: houses and shares, for example. The physicists have shown how the interplay of these two basic forces largely determines how wealth is distributed.

Bouchaud and Mézard formulated a set of equations that could follow wealth as it shifts from person to person, and as each person makes random gains or losses from his or her investments. They also included one further feature to reflect how the value of wealth is relative. A poor single parent might face near-ruin over the loss of a $50 note; in contrast, a very rich person wouldn't flinch after losing a few thousand. In other words, the value of a little more or less wealth depends on how much one already has. This implies that when it comes to investing, wealthy people will tend to invest proportionally more than the less wealthy.

The equations that capture these basic economic processes are quite simple. However, there is a catch.

For a network of many people - say, a thousand or more - the number of equations is similarly large. A model of this sort, therefore, lies well beyond anyone's mathematical abilities to construct (and this explains why it has not appeared in conventional economics). But the philosopher Daniel Dennett has for good reason called the digital computer "the most important epistemological advance in scientific method since the invention of accurate timekeeping devices". The work of Bouchaud and Mézard falls into

a rapidly growing area known as "computational economics", which uses the computer to discover principles of economics that one might otherwise never identify.

Bouchard and Mézard explored their model in an exhaustive series of simulations. And in every run, they found the same result - after wealth flows around the network for some time, it falls into a steady pattern in which the basic shape of wealth distribution follows the form discovered by Pareto. Indeed, this happens even when every person starts with exactly the same amount of money and exactly the same money-making skills.

Why? Transactions between people should spread wealth around. If one person becomes terrifically wealthy, he or she may start businesses, build houses and consume more products; in each case, wealth will tend to flow out to others in the network. Likewise, if one person becomes terrifically poor, less wealth will flow through links going away from him, as he will tend to purchase fewer products. Overall, the flow of funds along links in the network should wash away wealth disparities.

But it seems that this washing-out effect never manages to gain hold, because the random returns on investment drive a counterbalancing "rich-get-richer" phenomenon. Even if everyone starts out equally, and they remain equally adept at choosing investments, differences in investment luck will cause some people to accumulate more wealth than others. Those who are lucky will tend to invest more, and so have a chance to make greater gains still. Hence, a string of positive returns builds a person's wealth not merely by addition but by multiplication, as each subsequent gain grows ever bigger. This is enough, even in a world of equals where returns on investment are entirely random, to stir up huge disparities of wealth in the population.

This finding suggests that the basic inequality in wealth distribution seen in most societies - and globally as well, among nations - may have little to do with differences in the backgrounds and talents of individuals or countries. Rather, the disparity appears as a law of economic life that emerges naturally as an organizational feature of a network.

Does this mean that it is impossible to mitigate inequities in wealth? Pareto found (as many other researchers found later) that the basic mathematical form of wealth distribution is always the same. You find that, each time you double the amount of wealth, the number of people having that much falls by a constant factor. This is the pattern that always leads to a small fraction of the wealth possessing a large fraction of everything.

Nevertheless, the "constant factor" can vary: there is a huge difference between the richest 5 per cent owning 40 per cent of the wealth, and their owning 95 per cent. An additional strength of the Bouchaud-Mézard network model is that it shows how this degree of inequity can be altered.

The physicists found two general rules. First, the greater the volume of wealth flowing through the economy the greater the "vigour" of trading, if you will - then the greater the equality. Conversely, the more volatile the investment returns, the greater the inequity. This has some curious practical implications, some obvious and some not so obvious.

Take taxes, for instance. The model confirms the assumption that income taxes will tend to erode differences in wealth, as long as those taxes are redistributed across the society in a more or less equal way. After all, taxation represents the artifical addition of extra transactional links into the network, along which wealth can flow from the rich towards the poor. Similarly, a rise in capital gains taxes will tend to ameliorate disparities in wealth, both by discouraging speculation and by decreasing the returns from it. On the other hand, the model suggests that sales taxes, even those targeted at luxury goods, might well exaggerate differences in wealth by leading to fewer sales

(thus reducing the number of transactional links) and through encouraging people to invest more of their money.

The model also offers an excellent test of some arguments that politicians commonly use. For example, the pro-free market policies of Britain and the US in the 1980s and 1990s were defended on the grounds that wealth would "trickle down" to the poor. Everything was done to encourage investment activity, regardless of the risks involved. As we know, the wealth did not trickle down and wealth in both countries is now significantly less equally distributed than it was three decades ago. Under the network model, this is just what one would expect - a dramatic increase in investment activity, unmatched by measures to boost the flow of funds between people (such as higher taxes), ought to kick up an increase in wealth inequality.

What about globalisation? Our model suggests that, as international trade grows, it should create a better balance between richer and poorer nations: Western corporations setting up manufacturing plants in developing nations and exporting their computing and accounting to places such as India and the Philippines should help wealth flow in to these countries. But, as Stiglitz notes, Western countries have pushed poor nations to eliminate trade barriers, while keeping up their own barriers, thus ensuring that they garner a disproportionate share of the benefits. As the Bouchaud-Mézard model illustrates, free trade could be a good thing for everyone, but only if it enables wealth to flow in both directions without bias.

If we go back to the model, it reveals another, rather alarming prospect. Bouchaud and Mézard found that if the volatility of investment returns becomes sufficiently great, the differences in wealth it churns up can completely overwhelm the natural diffusion of wealth generated by transactions. In such a case, an economy whether within one nation, or across the globe - can undergo a transition wherein its wealth, instead of being held by a small minority, condenses into the pockets of a mere handful of super-rich "robber barons". Some countries, particularly developing nations, may already by in this state. It has been estimated, for example, that the richest 40 people in Mexico own nearly 30 per cent of the wealth. It could also be that many societies went through this phase in the past.

In Russia, following the collapse of the USSR, wealth has become spectacularly concentrated; inequality there is dramatically higher than in any country in the West. The model would suggest that both the increased volatility of investment and lack of opportunities for wealth redistribution might be at work. In the social vacuum created by the end of the Soviet era, economic activity is less restricted than in the West, as there are few regulations to protect the environment or to provide safety for workers. This not only leads to pollution and human exploitation, but also generates extraordinary profits for a few companies (the politically well-connected, especially; a popular pun in Russia equates privatization with the "grabbing of state assets"). Economists have also pointed out that Russia has been slow to implement income taxes that would help to redistribute wealth.

The Bouchaud-Mézard model is not the last word in explaining the distribution of wealth, or how best to manage it. But it offers basic lessons. Though wealth inequity may indeed be inevitable, its degree can be adjusted. With laws to protect the environment and workers' rights, free trade and globisation should be forces for good, offering better economic opportunity for all. But we will do this only if global integration is carried out sensibly, carefully and, most important of all, honestly.

Le principe de Pareto

ou pourquoi la société crée 80 % de pauvres et 20 % de riches

Pierre Barthélémy, *Le Monde*, 1er septembre 2000

A la fin du siècle dernier, l'ingénieur et économiste italien Vilfredo Pareto fit une étonnante découverte : quelle que fût la société qu'il étudiât, de l'Angleterre industrielle à la Russie agraire, en passant par le jeune royaume d'Italie, la répartition de la richesse y était toujours aussi inégale. Comme par un principe immuable et naturel, 20 % de la population détenait 80 % de la fortune. Même si les chiffres peuvent avoir varié quelque peu, la situation générale n'a pas changé. Le principe dit de Pareto, qui décrit une situation inversement proportionnelle, où une minorité concentre une majorité, se retrouve curieusement dans bien d'autres secteurs que l'économie et c'est un jeu amusant que d'en déceler de nouvelles applications : 80 % des coups de téléphone s'adressent à 20 % de votre répertoire ; 20 % des routes supportent 80 % du trafic ; 80 % des informations sont contenues dans 20 % d'un journal…

Jusqu'ici, la concentration d'une grande part des richesses entre les mains d'une petite minorité était considérée comme un fait accompli. Le principe de Pareto ne s'expliquait pas. Grâce à un article de deux physiciens français, Jean-Philippe Bouchaud, du Commissariat à l'énergie atomique, et Marc Mézard, de l'université d'Orsay, publié par la revue scientifique néerlandaise Physica A et repris à la « une » de l'hebdomadaire britannique *New Scientist* du 19 août, un début de lumière éclaire cette inégalité de la société, fondamentale d'un point de vue historique. « Nous avons essayé de construire un modèle extrêmement simple pour décrire la façon dont évolue la fortune d'un individu donné avec le temps, explique M. Bouchaud. Or, qu'est-ce qui fait évoluer la fortune ? L'échange avec les autres membres de la société économique et le placement spéculatif. »

Les échanges contre les inégalités

En partant de ces seuls paramètres, le duo de chercheurs a écrit une équation simple et s'est appuyé, pour la résoudre, sur une analogie avec une équation identique présente en physique des systèmes désordonnés. Ils ont attribué la même fortune à tous leurs individus et ont laissé faire le temps et les choses sans se soucier des motivations extrêmement diverses et imprévisibles des acteurs d'un jeu économique lui aussi hasardeux. Et, comme par magie, la courbe de répartition de la richesse s'est mise à suivre le principe de Pareto. Comme si l'inégalité économique était inscrite dans la nature… « Cette répartition est universelle, peu sensible à la spéculation, assure le physicien du CEA . Quelles que soient les conditions initiales, elle finit par s'établir. »

Pour M. Bouchaud, la principale explication est à chercher du côté de la notion de rentabilité proportionnelle : « L'inégalité est due au fait que, quand on a 100 francs et qu'on les investit, on gagne dix fois moins d'argent que lorsqu'on a 1 000 francs. Mais comment pourrait-on faire pour rémunérer l'argent autrement que proportionnellement ? C'est impossible : si l'on décidait de rémunérer davantage 100 francs que 1 000 francs, celui qui posséderait 1 000 francs couperait son avoir en dix tranches de 100 francs et cela reviendrait au même. Pour contrecarrer cela, on arrive donc très vite à l'idée d'impôt progressif sur le revenu. »

Même si le principe de Pareto est universel, le modèle des deux physiciens français ouvre quelques pistes sur la manière de jouer les Robin des bois, sachant que les riches, telles les têtes de l'hydre de Lerne, « repoussent » automatiquement lorsqu'on les supprime, remplacés par d'autres nantis. La partie le plus intéressante de leurs travaux consiste à essayer de comprendre comment la répartition de l'argent peut être modifiée. Les points avancés ne sont pas révolutionnaires mais, dorénavant, les économistes disposeront d'une assise mathématique à leurs idées.

Primo, favoriser les échanges car, dans ce contexte, le contraste entre riches et pauvres diminue. En revanche, dans une économie plus fermée, dominée par les intermédiaires, la hiérarchisation des fortunes s'accentue. Secundo, et cette solution n'est pas nouvelle, les impôts. « Notre modèle montre que l'impôt sur le revenu a tendance à réduire les inégalités, ce d'autant plus si une partie de cet impôt est redistribuée. Mais nous avons obtenu un résultat auquel nous ne nous attendions pas : si l'impôt sur le capital est prélevé en plus de l'impôt sur le revenu et qu'il n'est pas redistribué de manière équitable, la fracture sociale s'élargit. » Dans ce cas particulier, l'argent prélevé par l'Etat finit par retourner dans la poche des plus riches. Une idée à creuser à l'heure d'une éventuelle réforme fiscale ?

Les mathématiques de la fortune
Fabien Gruhier, *Le Nouvel Observateur* n°1875, octobre 2000

Deux physiciens ont trouvé une équation qui décrit très exactement la répartition des richesses dans une population. Leur conclusion : pour devenir riche, le hasard suffit.

Dans la nature comme dans les activités humaines, dans la physique fondamentale comme dans l'économie de marché, bref, partout, on trouve en gros deux types de distribution statistique : 1) la courbe de Gauss, dite «en cloche», avec une très forte majorité autour de la moyenne ; 2) la courbe en asymptote, où les valeurs extrêmes conservent une probabilité non négligeable. Exemple du premier cas: la taille des individus adultes, qui doit se situer en moyenne aux alentours de 1,70 mètre. Or au-dessous de 80 centimètres comme au-dessus de 2,5 mètres on ne trouve vraiment personne. Exemple du second cas : si le salaire moyen est, disons, de 8 000 francs, on compte quand même pas mal d'exclus qui gagnent zéro franc, et un nombre non négligeable de riches qui empochent des millions, ou des milliards.

Une relative justice sociale voudrait au contraire que la distribution des revenus adopte un profil en courbe de Gauss, mais rien à faire : déjà, voici plus de cent ans, l'ingénieur italien Vilfredo Pareto avait remarqué que dans tous les pays, à toutes les époques, 20% des individus possédaient environ 80% de la richesse. Il ne s'agissait que d'une simple observation, un constat aussi inéluctable qu'inexplicable. On ne savait pas pourquoi il en allait ainsi, mais on crut devoir s'y résigner : sous le nom de «principe de Pareto», l'injustice sociale fut promue au rang de loi naturelle, aussi inviolable que la gravitation universelle ou la vitesse de la lumière.

Mais (rêvons un peu) les choses pourraient changer. Ceci grâce à Jean-Philippe Bouchaud, physicien spécialiste de l'état condensé au CEA, et à Marc Mézard, du laboratoire de physique théorique de l'Ecole normale supérieure. Ces deux scientifiques viennent en effet de trouver l'explication du principe de Pareto. Ceci en épinglant une certaine équation, très commune en physique théorique, qui régit aussi bien les champs de vitesse dans les écoulements turbulents, la distribution des polymères dans un gel que l'influence des impuretés dans un supraconducteur. Or, à leur grande surprise, Bouchaud et Mézard ont réalisé que cette même équation

- «très simple», disent-ils - régule aussi la distribution des richesses dans les sociétés humaines, et son degré d'inégalité. Sur cette base, ils ont conçu un modèle mathématique assez rudimentaire, l'ont fait tourner sur ordinateur. Et ont aussitôt retrouvé la distribution de Pareto : dans une population d'individus qui possédaient chacun la même somme au départ, la plus grande partie des biens matériels tombe toujours entre les mains d'une minorité.

Pourtant leur équation, très simple encore une fois, ne fait intervenir que le hasard. Elle ne laisse aucune place à un éventuel talent de l'individu pour s'enrichir. «Cela n'est pas nécessaire, puisqu'on arrive au même résultat», dit Bouchaud, ajoutant : «Mettez cinq cents singes qui gesticulent dans une salle des marchés. Aucun d'entre eux n'écrira jamais un poème digne d'Eluard, mais au bout d'un certain temps, il y en aura un aussi riche que George Soros. »

L'équation de Bouchaud et Mézard a fait l'objet d'une publication dans la revue spécialisée néerlandaise «Physica A», puis eut les honneurs de la une du célèbre hebdomadaire scientifique britannique «New Scientist». Pour décrire les fluctuations de fortune de l'individu i, elle ne fait appel qu'à deux termes: la «fréquence des échanges» (chaque individu achète et vend, même si ce n'est que sa force de travail), et l'«écart-type des placements spéculatifs» (comment il emprunte ou place son argent). La courbe asymtotique globale, qui combine tous les i pour décrire la répartition globale des fortunes, est fonction d'un certain facteur, collectif, exponentiel: E. Lequel E, selon le principe de Pareto, varie entre 2 et 3 (c'est un fait d'observation: on n'a jamais relevé, dans aucune société, un E inférieur à 2 ni supérieur à 3). Plus E est proche de 2, et plus la société est sauvagement inégalitaire (du genre 5% possèdent 95%). Plus E s'approche de 3, et plus la société est relativement juste (30% possèdent 70%).

Un objectif de justice sociale consiste donc à hisser le coefficient E le plus près possible de 3 (un E supérieur à 3 étant considéré comme utopique). Or il se trouve que, pour ce faire, il convient de multiplier les transactions. Plus les individus ont des échanges nombreux et variés avec des partenaires diversifiés librement choisis, et plus la richesse se dilue. On voit donc tout de suite que la libre concurrence, à l'inverse des monopoles et des intermédiaires obligés, joue en faveur de la justice sociale. Du côté des emprunts et placements, il n'y a pas grand-chose à faire, dans la mesure où la fortune permet de multiplier les mises, donc les gains. Et c'est ici que le hasard pèse le plus lourdement, car, «dans les placements boursiers, par exemple, il suffit d'avoir eu raison plusieurs fois par hasard pour s'enrichir vraiment beaucoup». D'où le nécessaire rôle correcteur de l'impôt progressif sur le revenu et sur la fortune, dont nos deux scientifiques justifient mathématiquement le rôle socialement bienfaiteur.

Mais attention! il faut encore que l'impôt soit vraiment redistribué, sinon, même très fortement progressif, il ne fait que creuser les inégalités. Ainsi, lorsque le produit de l'impôt sert à rembourser des emprunts d'Etat, ou à financer de gros contrats, il retourne aussitôt - considérablement augmenté par la contribution des moins riches dans les poches des plus riches.

Tel que, mathématiquement très simplifié et presque caricatural, le modèle proposé par Bouchaud et Mézard constitue déjà un outil qui pourrait être précieux pour la saine gestion de l'économie. Les deux scientifiques - à l'origine physiciens purs, et qui poursuivent d'ailleurs leurs recherches au CEA et à Normale sup - ont créé une société de services ([Science Finance](#)), que l'on peut consulter sur internet, et qui, grâce à des travaux d'analyse de risques commandés par les banques, réussit à financer les recherches fondamentales de plusieurs thésards. Jean-Philippe Bouchaud regrette toutefois le peu d'empressement des acteurs de l'économie à utiliser leurs modèles, venus de la physique théorique et des écoulements turbulents, des acteurs qui ont l'air de dire:

«De quoi se mêlent-ils?». «Parmi les économistes, dit Bouchaud, on n'aime pas la simulation, on préfère toujours les vieilles méthodes, jugées plus nobles, de la déduction pure et de la démonstration de théorèmes. » Alors qu'on pourrait faire tourner des modèles empiriques, même simplifiés, puis les perfectionner, et enfin s'en inspirer pour prendre les bonnes décisions dictées par l'intérêt général. Ah! si Karl Marx avait été moins nul en maths, on n'en serait pas là…

Jean-Philippe Bouchaud and Marc Mézard, two French econophysicists, believe that wealth falling into the hands of a minority is not just a fact of life, but a law of nature. They have discovered a connection between the physics of materials and the movements of money, a link that could revolutionise the way we think about the distribution of wealth in society. It could also provide a mathematical basis to theories of free trade and competition.

That wealth is unevenly distributed is now widely accepted: in the US, 20 per cent of the people own 80 per cent of the wealth, and similar figures apply throughout western Europe. Economists have always struggled to explain this seemingly universal trend, with most simply putting it down to the distribution of people's abilities. Bouchaud and Mézard started their analysis from the simplest possible basis, ignoring any economic theories and assumptions of previous years. Their one assumption was that life is unpredictable: returns on investments are random, trading is haphazard, profit and loss could come at any moment.

While investigating the simple equations that make up an economy (of which there are millions, because there are millions of people), Bouchaud and Mézard discovered that they could bring their knowledge of 'condensed matter' physics into play. For there is a model used in this area of physics concerned with

'ill-condensed' matter which is called a directed polymer. This model involves the imagining of a long wire lying on a landscape that undulates at random. The wire, which represents the polymer, is tethered at one end. Two forces then compete with one another: gravity wishing to pull the wire into the valleys of the landscape and the wire's desire to stay straight. The resultant path that the wire takes is a compromise between the two, and there is no 'best path' for it to take. Added in to this model is a third key factor of temperature: the hotter the temperature, the more the wire is buffeted by air molecules. The level of the temperature (and therefore the level of the buffeting) determines exactly how the wire manages its compromise between going straight across Bouchaud and Mézard have now discovered that this physics model corresponds exactly to their economic model. The wire now represents the path of a quantity of money as it moves from person to person (with the number of wires crossing over a particular point representing the wealth of a particular person at that point). The irregularity of returns from investments is represented in the landscape: deep valleys correspond to places of more money, while peaks are where investors lose money and fare badly. And temperature is analogous to the vigour of trade, or how easily money moves between people.

This shows, for example, that when trading is easy the economy is 'hot' and is not very strongly affected by the landscape below. Thus vigorous trading enables wealth to flow easily from person to person which tends to spread money more easily. By contrast, if an economy is 'cold', by which is meant restricted trading and highly irregular returns, then wealth flows far less easily between people as it follows a path dictated almost entirely by the landscape. Wealth becomes even less fairly distributed, as its natural diffusion is overcome by the disparities of the economic landscape.

It follows, therefore, that a more equal society can not only be achieved through taxation and redistribution, but through the encouragement of freer and fairer trade, competition and exchange. The hotter the temperature of an economy, the more vigorous the rate and number of people trading, the more equal a society will become. This fits with most

economists' experiences and expectations, but has now been given a mathematical basis. Also, the econophysicists' theory should mean that internet trading proves to be an enemy of inequality, as more and more people take part in the random flows and movements of wealth.

worldlink

Trade Routes To Equality

Robert Matthews, *World Link*, May 22, 2002

Surely a disappointment to anti-globalists, a theory that combines economics and theoretical physics confirms global trade can help spread wealth. Robert Matthews explains

You may not have noticed International Buy Nothing Day in late November. As campaigns go, it stood about as much chance of success as a warthog in a singles bar. Its plea late last year for consumers to put down their credit cards and chequebooks for just one day went pretty much unheeded.

As with many of these "world days", it did at least give renewed publicity to some striking statistics. According to the campaigners, "only 20% of the world population is consuming over 80% of the earth's natural resources." Does that statistic sound familiar? It should. For the 20/80-split rule rears its head in many fields with almost suspicious regularity.

Around 20% of Americans possess 80% of the country's wealth, for example, and a similar split applies to many other developed economies. Insurance companies have long known that around 20% of the claims in an insurance portfolio are responsible for over 80% of the total value of claims paid. In large companies, around 20% of customers often turn out to be responsible for 80% of sales. It even pops up in warehouses, where around 20% of the stock is often found to account for 80% of the usage.

It's known as the Pareto Law, after the Italian economist Vilfredo Pareto. In 1895 he claimed to have found a mathematical formula that governed the distribution of wealth. Essentially an inverse power law, it takes the basic form of $1/x^n$. Pareto found that when he applied his formula to the spread of wealth across the population, the power n is typically in the range of one to two. In the case of his own country, Pareto found that it implied that 20% of Italians owned 80% of the country's total wealth.

Pareto believed his discovery backed his belief that the distribution of wealth in society follows a universal law. It was a bold and controversial claim. Yet Pareto

was able to amass an impressive amount of supporting evidence. He showed that his power law applied to economies as diverse as those of industrialised Britain and agrarian Russia, and across the centuries, from Victorian societies back to the Roman Empire.

Nevertheless, Pareto's claim to have found a universal law of economics was derided by his contemporaries. Not until 12 years after his death did any support for it begin to emerge.

In 1935, French mathematician Paul Lévy showed that Pareto's simple power laws tend to pop up in situations where extreme values aren't extremely rare. For example, the heights of people follow the well-known bell-shaped Normal curve, where extremely tall or short people are extremely rare. But when such extremes aren't all that rare – for example, immensely rich and utterly destitute people – the familiar bell-shape takes on a distinctly squatter, heavy-tailed shape.

Lévy showed that such curves tend to follow simple Pareto-like power laws, with values of the power n lying between zero and two. With such non-Normal curves cropping up so often, Lévy's proof helped explain the apparent ubiquity of Pareto laws (though some supposed examples, such as "80% of business success comes from 20% of your effort," sound suspiciously like business school sound bites). What Lévy did not do, however, was to explain why factors such as wealth tend to have heavy-tailed distributions. In other words, how does it happen that so much ends up in the hands of so few ?

Some insights into the cause of such inequality are now starting to emerge from ground-breaking research that combines economics with powerful ideas from theoretical physics.

At the heart of such research is the idea that an economy is ultimately made up of a host of agents – people, companies or whole nations – which interact with each other. It's a dauntingly complex situation with which physicists are completely comfortable, having spent most of the last century dealing with models of materials as hosts of atoms interacting with one another.

Real economies are of course more complex than any lump of glass or metal. But physicists have never let themselves be fazed by complexity. They consider themselves masters of the telling approximation that captures the essence of any problem. Among scientists, one joke tells of how physicists start guesstimating the amount of leather available from a cow with the words: "Consider a spherical bull."

The rules of the game

Two French physicists, Jean-Philippe Bouchaud of the Centre d'Etudes de Saclay, Paris, and Marc Mézard of the Université de Paris Sud, have applied this philosophy to the question of wealth distribution. To do it, they have started from what they argue are the simplest ground rules capable of generating a model economy.

The first is so obvious it seems hardly worth stating: that the equations governing wealth should be the same regardless of the currency used. Clearly, any useful economic model does this, giving the same basic behaviour whether one is using dollars or dinars. But such features have special meaning to theoretical physicists, telling them that the equations must possess a symmetry. That is, the equations must remain unchanged if they are multiplied by the same number – specifically, the exchange rate between, say, dollars and dinars.

The second ground rule is hardly more controversial: that wealth is the result of exchange of money and goods among the agents making up the model economy, plus random gains and losses due to market speculation, property investment and the like.

Unless some capital tax is redistributed, inequality will persist. Governments that believe that income tax alone can reduce or eliminate wealth inequalities are likely to be disappointed

Combining these two basic ideas, Bouchaud and Mézard came up with an equation that governs the ebb and flow of wealth through the model economy as agents trade with one another. To find out what light it casts on the spread of wealth among those agents, Bouchaud and Mézard applied a trick used by physicists studying the behaviour of interacting atoms. Called the mean field approximation, it glosses over the messy details of each interaction and instead assumes that they're all the same. In economic terms, that means that every agent trades equally with every other agent.

While it may not be realistic, it does allow the complex wealth-exchange equation to be solved, with intriguing results.

The average wealth within the model economy turns out to follow precisely the kind of $1/x^n$ power law found by Pareto for real economies. But there's a twist

– a nugget of insight of the sort that so often emerges from even rough approximations in physics. Pareto's data led him to suspect that the power n was always in the range of one to two and that wealth would thus typically follow 20/80-type rules. Bouchaud and Mézard's results reveal, however, that n is not a universal constant, fixed for all economies. Instead, it turns out to be dependent on the level of trade activity among the various agents. The model shows that n grows with increasing trade activity and higher values of n lead to Pareto laws giving a more equable distribution of wealth.

Thus this simple model suggests that an efficient way to reduce wealth inequalities in an economy is to encourage higher levels of trade. But that assumes everyone can trade with everyone else. This clearly isn't realistic. So Bouchaud and Mézard added a little extra sophistication to their model, allowing trade only among a random number of most-favoured agents. Again, they found that the power n could vary, this time according to the size of the trading network. The larger it is, the bigger n becomes and the better the spread of wealth.

Again, this simple model suggests that a good way to reduce wealth inequalities is to encourage as much trading activity, and over as wide a trading network as

possible. It's a conclusion that will no doubt be welcomed by advocates of globalisation. But how does it compare with taxation, that most familiar means of redistribution of wealth?

Bouchaud and Mézard again extended their model to include two forms of tax: income tax, which mops up a proportion of each person's wealth as it comes in, and capital tax, which takes a fraction of their accumulated wealth.

The results showed that, as expected, higher income tax leads to higher values of n, and thus to a more equitable spread of wealth, especially if the income tax is redistributed among the less wealthy. But again there's a twist. Bouchaud and Mézard's model revealed that unless a minimum fraction of capital tax is also redistributed, inequality will persist. In other words, governments that believe that punitive income tax alone has the power to reduce or eliminate wealth inequalities are likely to be disappointed.

While their model is hardly the last word in sophistication, Bouchaud and Mézard seem to have achieved the physicist's aim of extracting new insights from a minimum of assumptions. Other researchers are now extending the model, studying its predictions in more detail. Hideaki Aoyama of Kyoto University and colleagues have been combining the model with the so-called Small World Theory, which captures the nature of the trading links.

At one extreme, such links could be only among nearest neighbours; at the other, they could be spread randomly over the entire globe. In reality, trading links tend to be somewhere in between. Small World Theory shows that one needs only a few random long-range links to give an amazingly high degree of interconnectedness.

Recent research by Aoyama and colleagues has shown that this "small world effect" means that assuming – as Bouchaud and Mézard did – that everyone is linked to everyone else isn't hopelessly misleading after all. It also means that relatively small expansions in trading networks can bring about big reductions in wealth inequality.

The lesson from this heady combination of economics and theoretical physics is clear. By stepping up levels of trade across as wide a field as possible, the inequality that disfigures the global economy can be narrowed. What price, then, is there for International Buy Nothing Day?