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GROWTH ACCOUNTING

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ABSTRACT

Incomes per capita have grown dramatically over the past two centuries, but the increase has been unevenly spread across time and across the world. Growth accounting is the principal quantitative tool for understanding this phenomenon, and for assessing the prospects for further increases in living standards. This paper sets out the general growth accounting model, with its methods and assumptions, and traces its evolution from a simple index-number technique that decomposes economic growth into capital-deepening and productivity components, to a more complex account of the growth process. In the more complex account, capital and productivity interact, both are endogenous, and quality change in inputs and output matters. New developments in micro-level productivity analysis are also reviewed, and the long-standing question of net versus gross output as the appropriate indicator of economic growth is addressed.

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GROWTH ACCOUNTING[1]

I. Introduction

World income per capita increased from $651 in 1820 to $5,145 in 1992, according to estimates by Maddison (1995). Though spectacular compared to the negligible 15 percent gain during the preceding three centuries, this eight fold increase was not shared uniformly. Income per capita in Western Europe and its offshoots grew by a factor of 15, while the rest of the world experienced only a six-fold increase. This general pattern continues to this day, with some notable exceptions in Asia. The unevenness of the growth experience is also evident within specific countries over time. Output per hour in the U.S. private business sector grew at a robust annual rate of 3.3 percent from 1948 to 1973, then slowed to 1.6 percent per year between 1973 and 1995, and then picked up to 2.6 percent from 1995 to 2007 (U.S. Bureau of Labor Statistics Multifactor Productivity Program).

[1]This review builds on, and expands, my 2001 survey of the topic. As with any survey, space considerations limit the material than can be covered and choices have been made with which others may quibble. A survey of the growth accounting field is particularly hard because of the nature of the field, which aims to provide a summary measure of the determinants of economic growth and therefore links a large number of other research areas. For example, the econometric side of productivity analysis has largely been omitted, and will be touched upon only in so far as it affects growth accounting. The same is true of growth theory, and the reader may wish to consult the general treatment of that subject by Barro and Sala-i-Martin (1995) and many of the articles in the Handbook of Economic Growth. Other areas, like the economics of R&D, the measurement of service sector output, the information technology revolution, and the determinants of the productivity slowdown of the 1970s and 1980s, do not get the full attention they would deserve in a longer treatise. Perhaps the greatest omission is my decision to focus on growth accounting in an economy that is closed to international trade. This choice is made because of the great complexity that trade adds to the problem, and not because international income flows are unimportant. The ultimate problem is that the proper treatment of trade flows requires a different national income accounting structure than the ones currently in place (see, for an example of the complexity involved, Reinsdorf and Slaughter (2006)). The interested reader is also referred to the important paper by Diewert and Morrison (1986).

Like its 2001 predecessor, this survey is a somewhat personal view of the field, stressing its core technical evolution rather than specific applications or numerical estimates (different theories give different numbers). The reader is directed to the surveys by Barro (1999), Griliches (1996, 2000), Jorgenson (2005), and the OECD manual on productivity measurement, for other recent treatments of the subject. Anyone wishing to delve further into the history of the field will be rewarded by reading Solow (1988), Maddison (1987), Jorgenson, Gollop, and Fraumeni (1987), Brown (1966), and Nadiri (1970).

Income per capita and the closely related output per worker are key determinants of national living standards, and the field of growth accounting evolved as an attempt to explain these historical patterns. It grew out of the convergence of national income accounting and growth theory, and, in its simplest national income form, it is a rather straightforward exercise in which the growth rate of real GDP per capita is decomposed into separate capital formation and productivity effects.[2] The unevenness of growth rates over time and across countries can then be traced to these two general sources, providing insights into the nature of the growth process.

This is the simple story of growth analysis. A more complex tale has emerged over time as data and computing power have improved, and economic theory has evolved. In the process, growth accounting has itself changed, and this evolution is the story told in this chapter. The essay is organized into three main sections: Section II devoted to the basic growth accounting framework; Section III covering the measurement of the key variables; and Section IV devoted the last to a critique of the growth accounting method. Several cross-cutting themes resurface throughout these sections: the role of economic theory in determining the appropriate form of the growth account and the related index number problem, and the issue of 'path independence.' Other issues also appear in multiple contexts: the question of whether capital formation and innovation can be treated as separate phenomena, the distinction between product and process innovation and the associated problem of output and input quality change, and the basic issue of whether growth accounting is supposed to measure changes in

[2] As with many ideas in economics, the dichotomy appears in a rudimentary form in Adam Smith. Kendrick starts his seminal work on U.S. productivity growth with the following lines from Smith: "The annual produce of the land and labour of any nation can be increased in its value by no other means, but by increasing either the number of its productive labourers, or the productive powers of those labourers who had before been employed …" (quoted from Kendrick (1961), page 3).

consumer welfare or changes in the supply-side constraints of an economy. Answers to these questions, however imperfect given the current state of the art, help illuminate the nature and boundaries of the growth accounting method and the interpretation of the results.

II. The Growth Accounting Model

A. The Basic Aggregate Model

1. Origins. Growth accounts are a natural byproduct of the basic national accounting identity which relates the aggregate value of the final goods and services produced in a country (gross domestic product (GDP)) to the total value of the labor and capital used to produce the output (gross domestic income (GDI)). Using more or less standard notation for output, labor and capital, and the corresponding prices, the accounting identity takes the following form:

(1) $$p_t Q_t = w_t L_t + c_t K_t \ .$$

The simplicity of this formulation conceals the complexity and effort actually involved in measuring and reconciling GDP and GDI, and the development of these estimates is one of the great achievements of economic science. Though measures of national income can be traced back to the late 17th century, the development of comprehensive national accounts is a relatively recent event and reflects the conceptual efforts of Simon Kuznets, Richard Stone, Richard Ruggles, and many others (Kendrick (1995)).[3]

[3] It is noteworthy that a group of economists, statisticians, and national accountants came together in the later 1930s to form the Conference on Research in Income and Wealth, with Simon Kuznets as the first chairman and Milton Friedman as secretary. The CRIW provided a forum in which many of the conceptual issues involved in setting up a national account were discussed. The conference proceedings were published in the volumes of the Studies in Income and Wealth Series, and they provide a valuable insight into the complexity of the national accounting undertaking (many of the issues on the table then are still there, in one form or another). The CRIW continues its work, and the conference proceedings now run to almost seventy volumes.

The U.S. national income accounts date from 1947, while the United Nation's System of National Accounts was published in 1953.

The study of economic growth requires estimates of GDP and GDI that control for price inflation. Most accounts therefore provide estimates of GDP in constant prices, but a parallel adding-up identity between real GDP and real GDI is only possible for the base (comparison) year in which all prices are normalized to one. If there is a change in the efficiency with which inputs are used, the real GDP identity cannot hold in subsequent years (if the same quantity of input produces ever more output, valuing output at fixed prices will break the identity). An additional term is needed to account for this possibility, which, in the simplest case takes the form

(2) $$p_0 Q_t = T_t [\, w_0 L_t + c_0 K_t \,] \ .$$

Rearranging this expression shows that the variable T_t is a scalar index that can be interpreted as the level of real output per unit of total input. The equation is also a rudimentary form of a growth account, since real output is decomposed into a real input effect $[w_0 L_t + c_0 K_t]$ and a productivity effect T_t. The index T_t calculated in this way is a residual that sweeps in many things, a feature that led Abramovitz (1956) to bestow on it the title "a measure of our ignorance."

This form of the proto-growth accounting model is based on simple linear index numbers and is very close to the underlying data. The formulation is largely atheoretical, except for the theory implicit in the assumptions that statisticians use in their measurement procedures. This atheoretical approach can be justified by an appeal to the axiomatic index number theory, that is, by the kind of rules and "tests" advocated by Irving Fisher (1927) and his followers.

However, the simplicity achieved by imposing a minimum of economic structure comes at a substantial cost. Why is a linear index number formulation desirable? What types of technical change are envisioned for the index T_t? What variables are appropriately included in the growth account and in what form? Without some theoretical foundation, there are no firm criteria for constructing any type of growth account. There is little help, in this regard, from axiomatic index numbers other than a set of "reasonable" (though somewhat arbitrary) "tests" and rules. It was against this backdrop that the 1957 paper by Robert Solow made its seminal contribution.

2. The Solow Residual. The seminal paper by Solow (1957) provided the economic structure missing from the axiomatic approach (Griliches (1996)). Rather than appealing to some implicit production function to interpret the index T_t, his model starts with an explicit function and derives the implied index. This involves several assumptions: that there is a stable functional relation between inputs and output at the economy-wide level of aggregation; that this function has neoclassical smoothness and curvature properties, that inputs are paid the value of their marginal product, that the function exhibits constant returns to scale, and that technical change has the Hicks'-neutral form:

(3) $$Q_t = A_t F(K_t, L_t) \ .$$

The variable A_t plays the same conceptual role as the index T_t, that is, as a measure of output per unit input, but it now has an explicit interpretation as a shift in the production function. Changes in output due to growth in inputs are interpreted as movements along the function $F(K_t,L_t)$.

T_t is an index number while A_t is a parameter of the production function. What Solow did was to show how to measure A_t as an index number, that is, using observable prices and quantities alone without imposing the assumptions needed for econometric analysis.[4] The first step in this derivation is to express the production function in growth rate form.

(4) $$\frac{\dot{Q}_t}{Q_t} = \frac{\partial Q}{\partial K}\frac{K_t}{Q_t}\frac{\dot{K}_t}{K_t} + \frac{\partial Q}{\partial L}\frac{L_t}{Q_t}\frac{\dot{L}_t}{L_t} + \frac{\dot{A}_t}{A_t} \ .$$

The dots denote time derivatives, so the corresponding ratios are rates of change. This form indicates that the rate of growth of output equals the growth rates of capital and labor, weighted by their output elasticities, plus the growth rate of the Hicksian shift parameter. These elasticities are equivalent to income shares s^K_t and s^L_t when inputs are paid the value of their marginal products ($\partial Q/\partial K = c/p$; $\partial Q/\partial L = w/p$), giving

(5) $$R_t = \frac{\dot{Q}_t}{Q_t} - s^K{}_t \frac{\dot{K}_t}{K_t} - s^L{}_t \frac{\dot{L}_t}{L_t} = \frac{\dot{A}_t}{A_t} \ .$$

The left-hand part of the equation defines the "residual" of R_t as the growth rate of output not explained by the share-weighted growth rates of the inputs (the residual is

[4] Solow was not the first to suggest estimating a production function with a time index. Tinbergen (1942) is usually credited with this advance (see, for example, Griliches (1996, 2000)). Solow's great contribution was to show how the time effect could be estimated directly from the data on prices and quantities presented in national income and product accounts.

also called “total factor productivity” (TFP) and “multifactor productivity” (MFP)).[5] The second equality shows that the residual equals the growth rate of the Hicksian efficiency parameter A_t. The residual can therefore be interpreted as the shift in the underlying production function and the weighted growth rates of capital and labor as movements along the function.[6]

Though linked to an underlying production function, the residual itself is a pure index number because it is based on prices and quantities alone (actually, (5) is a form of the Divisia index). By implication, the shift in the function can be measured without actually having to know its exact form. The trick, here, is that the slope of the production function along the growth path of the economy is measured by real factor prices ($\partial Q/\partial K = c/p$; $\partial Q/\partial L = w/p$). The price paid for this generality is that the estimates are “local” to the path actually followed by the economy and may therefore depend on the path, as we will see in the following section.[7]

5 Multifactor productivity (MFP) is the name given to the Solow residual in the BLS productivity program, replacing the term ‘total factor productivity’ (TFP) used in the earlier literature, and both terms continue in use (usually interchangeably). The “F” in both terms refers to the factor inputs K and L, and the “M” and “T” distinguish MFP/TFP from the single productivity indexes Q/L and Q/K (labor and capital productivity). The “M” is perhaps preferable to the “T” simply because the latter presumes that all the relevant K and L are counted, which is typically not the case. A problem also arises at the industry level of analysis, where inputs of energy, materials and purchased services are also used to produce output. “Multi-input productivity” (MIP) would be a more accurate term in this situation, but to avoid confusion, we will continue to use “MFP” in this paper.

6 Under constant returns to scale, the residual (5) can equally be written as the growth rate of labor productivity (Q/L) less the growth rate of the capital/labor ratio (K/L), weighted by capital’s income share, since the shares sum to one in this case.

7 The shift parameter A in the Hicksian production function might also be estimated using econometric techniques. The result would be “global,” in the sense that the estimated parameters would reveal the structure of production throughout the entire production space, not just along the growth path through the space. Choosing between the index number and the econometric approach depends on the choice of the biases that one is prepared to accept. Fortunately, this choice need not be made, since both approaches can be used on the same data (much of which come in the form of index numbers).

The implicit link between growth accounting and the aggregate production function has another implication: it places constraints on the variables included in the analysis.[8] This implication was drawn out by Jorgenson and Griliches (1967), who established the modern form of growth accounting (the form underpinning the empirical estimates of the BLS and the EU KLEMS productivity programs, and the recent OECD manual on productivity analysis). In the strict production function interpretation, real output is based on the number of units actually produced, and, by implication, it should be measured gross of depreciation.[9] However, capital stocks should be measured net of physical depreciation but the price of capital services should include depreciation cost. Jorgenson and Griliches also incorporated the educational dimension of labor input into growth analysis. These advances virtually define modern growth accounting, but it should be emphasized that they were controversial in their day -- witness the exchange between Denison (1972) and Jorgenson and Griliches (1972).

3. The Potential Function Theorem. The Solow-Jorgenson-Griliches model is familiar to most students of economic growth. This model establishes sufficient conditions for deriving the Divisia index (5) from the production function (3). The question of necessary conditions is rather less familiar: if you start from Solow’s Divisia index in (5),

8 The production function (3) is the basis for the Solow-Jorgenson-Griliches residual, and under the maintained hypotheses of the model, constant returns to scale and perfect competition, the basic accounting identity (1) can be derived from (3) using Euler’s Theorem on homogeneous functions. Thus, the variables in the production function appear, *ipso facto*, in the accounting identity. Christensen and Jorgenson (1969, 1970) develop this idea into a detailed income and wealth accounting framework, and Jorgenson and Landefeld (2006) develop it into a “Blueprint for Expanded and Integrated U.S. Accounts.”

9 The net versus gross output debate involves a conceptual issue about the aims of growth accounting (production versus welfare), and thus goes beyond a technical question about economic measurement. It will be discussed in greater detain in Section IV.

is there an underlying production function to which it corresponds, and is the solution necessarily unique?

The answer is "not necessarily" (Hulten (1973)). Because the residual is a differential equation involving continuous growth rates between two points, finding a solution involves line integration. This, in turn, requires the existence of a vector-valued "potential" function, $\Phi(X)$ whose gradient is $\phi = \nabla\Phi$, to serve as an integrating factor. Line integration of ϕ over the path Γ followed by the vector X over the time interval [0,T] gives Φ.[10] Applied to growth accounting, the ϕ corresponds to the differential equation in (4), and the Γ to the path of inputs and output over time. The solution Φ is related to the production function, and the gradient $\nabla\Phi$ is related to the marginal products of the inputs. Intuitively speaking, the production function (or bits and pieces of the production function) serves as the potential function for integrating the Solow residual back to the original production function.[11]

Growth accounting deals with index numbers (its defining characteristic), and the property of uniqueness is important. If the economy starts at the point X_0 and ends at X_T, uniqueness requires that the path Γ followed by the X during intervening years should not affect the final value. In this situation, the index is said to be "path independent". The property of path independence is not guaranteed merely by the

[10] A more complete account is given in Hulten (1973, 2007). See also Richter (1966) for the invariance property of the Divisia index.

[11] It is important to emphasize that the application of potential function theory to growth accounting is not limited to production functions. In some variants of the problem, a factor price frontier or cost function is the appropriate potential function (see below). Moreover, there is no reason to exclude the use of utility functions as a possible integrating factor (Hulten (2001)). Indeed, in some versions of growth theory, utility and production functions are tangent along the growth path, implying that the Solow residual could be interpreted in both output and welfare terms (an idea developed in Basu and Fernald (2002)).

existence of the potential function Φ, it must also be homothetic or linearly homogenous, depending on the application (Hulten (1973), Samuelson-Swamy (1974)).[12]

These results imply that the Solow conditions -- the existence of an aggregate production function, competitive price, and constant returns --- are both sufficient *and* necessary for growth accounting. Together, they imply that some underlying economic structure is needed in order to "solve" the growth accounting equation (5), and, moreover, that *whatever belongs in the production function belongs in the growth account, and vice versa*. This establishes boundaries for the Solow growth accounting exercise, and it has a more general implication for the construction of index numbers: stringing together an arbitrary set of variables in an index number format without some underlying conceptual rationale does not necessary result in a valid economic index.[13] This is the basic difference between economic index numbers and axiomatic indexes.

[12] An index number is a one-dimensional indicator of an underlying phenomenon. While the data may allow an index number to be computed, the usefulness of the index is compromised if more than one possible value is associated with the same value of the underlying variable(s). The *a priori* assumption that the index is a reliable indicator carries with it the implicit assumption of uniqueness, and thus path independence. This condition may not be true, for there are many circumstances in which path dependence is an inherent attribute of the underlying production function (e.g., non-Hicksian technical change, non-separability of the production into capital or labor subaggregates). In some cases, like Harrod-neutral technical change, a correction can be made if the analyst has *a priori* information about the nature of the problem. In other cases, the analyst must use more complex econometric techniques or simply live with the suspicion of non-uniqueness.

[13] It is sometimes tempting to put together a list of disparate variables in order to inform some interesting issue. The attempt to construct an index of technological innovation is an example: elements like the number of patents issued, the number of engineers employed or in training, lists of citations to scientific research, and real R&D expenditures are all plausible elements of such an index. These elements can be arithmetically combined into a single number, but without a potential function to guide the construction of the index, how is the analyst to know which variables to include in the index, what form they should enter, and what weight they should be given? The intuitive approach to this problem risks double-counting and potentially confuses inputs and outputs. At a minimum, imposing a potential function on the problem forces the analyst to think about the nature of the innovation process (and its determinants) in a sufficiently precise way that the resulting index numbers have a meaningful interpretation.

4. Relaxing Some of the Assumptions. Potential function analysis has other implications for measurement. First, there is no particular reason to assume *a priori* that the shift in the production function has the Hicks'-neutral form. In Solow (1956) and Cass (1965)-Koopmans (1965) models of steady-state growth, technical change is assumed to have the Harrod-neutral form, i.e., $Q_t = F(a_tL_t,K_t)$ where a_t is now the shift parameter. This form of the production function no longer provides the necessary potential function for the Solow MFP residual in equation (5). It does, however, serve as a potential function for a variant of the MFP residual in which the original R_t is divided by labor's income share: R_t/s^L_t. A similar result holds for purely capital-augmenting technical change, but not for the more general factor augmenting model $Q_t = F(a_tL_t,b_tK_t)$, except in the case in which the production function has the Cobb-Douglas form.

The assumption of constant returns to scale can also be weakened. Suppose that the production function in (3), $Q_t = A_tF(L_t,K_t)$, is not restricted to the case of constant returns to scale, but also allows for the possibility of increasing or decreasing returns. Suppose, also, that it is possible to obtain an independent estimate of the user cost, c^*_t, that is equal the value of the marginal product of capital. In this situation, GDI is $w_tL_t+c^*_tK_t$, and is not equal to GDP. If a new set of cost shares, $s^{K^*}_t = c^*_tK_t / [w_tL_t+c^*_tK_t]$ and $s^{L^*}_t = w^*_tL_t / [w_tL_t+c^*_tK_t]$, are calculated and used in the residual equation (5), the resulting residual R^*_t is a path independent Divisia index of the growth rate of A_t. In other words, the equalities in equation (5) hold under non-constant retunes to scale, but at the price of violating the GDP/GDI identity.

Potential function theory also plays a useful role in determining the appropriate way to aggregate across various types of labor and capital. The labor variable in the

production function, L_t, is based on the assumption that labor is a homogenous input whose wage reflects the value of its marginal productivity. If there are N categories of workers, the original $\dot{L}_t / L_t$ must be expanded to allow for the heterogeneity. One way is to expand the Divisia index to allow separately for the hours worked ($H_{i,t}$) by each of the different categories, weighted by the relative share in the total wage bill:

$$(6) \qquad \frac{\dot{L}_t}{L_t} = \sum_{i=1}^{N} \frac{w_{i,t}\, H_{i,t}}{\Sigma_i\, w_{i,t}\, H_{i,t}} \frac{\dot{H}_{i,t}}{H_{i,t}} \,.$$

For this index to be path independent, the production function must be weakly separable into a sub-function of the N types of labor alone: i.e., that the function $Q_t = A_tF(H_{1,t},\ldots, H_{N,t},K_t)$ must be expressible as $Q_t = A_tF(L(H_{1,t},\ldots, H_{N,t}),K_t)$. This separability restriction requires that the marginal rate of substitution between each pair of elements in $L(H_{1,t},\ldots, H_{N,t})$ is independent of the level of all variables outside the sub-function (in this formulation, the level of K_t). This is a very restrictive condition, but if it holds, $L(H_{1,t},\ldots, H_{N,t})$ serves as the potential function for the labor index (6). A parallel result holds when different types of capital (or any other heterogeneous input) is combined into a single index (Hulten (1973)).

The separability of the production function also plays a role in sorting out the debate over the appropriate measure of output: net versus gross output at the aggregate level of economic activity, and the question of when real value added can be used as a measure of output at the industry level. Each is a question of the existence of the requisite potential function and will be taken up in a subsequent section.

5. Discrete Time Analysis. The theory of growth accounting reviewed in the preceding sections is formulated in terms of continuous time paths. This facilitates the use of mathematical analysis, but is not directly relevant for real-world national and financial accounting data, which are collected and reported in discrete-time increments (years, quarters, etc.). The continuous-time model can, however, be operationalized using discrete approximations (Trivedi (1981)). The Tornqvist (1936) index is perhaps the leading example:

$$(7) \qquad \ell n[\frac{A_t}{A_{t-1}}]=\ell n[\frac{Q_t}{Q_{t-1}}]-[\frac{s^K{}_t+s^K{}_{t-1}}{2}]\ell n[\frac{K_t}{K_{t-1}}]-[\frac{s^L{}_t+s^L{}_{t-1}}{2}]\ell n[\frac{L_t}{L_{t-1}}]$$

Here the continuous-time growth rates of the variables in (5) are replaced with the difference from one period to the next in the natural logarithms of the discrete-time variables, and the continuous shares by the corresponding average income shares.

There is no particular economic rationale for choosing this, or any other, mathematical approximation procedure. The critical step forward was made by Diewert (1976), whose path-breaking paper established the economic basis for the discrete-time form (7) with his theory of exact and superlative index numbers. Diewert showed that the Tornqvist index is an exact index when there exists an underlying production function of the translog form of Christensen et. al. (1973). By analogy to continuous-time theory, the translog function plays the role of the potential function for the discrete-time Tornqvist index (which is analogous to continuous-time for of the residual (5)). As in continuous time, the production function supplies the underlying economic structure for judging the accuracy of competing index numbers and for interpreting the index.

Moreover, because the translog form is a second order approximation to a more general production function, the Tornqvist index (7) is said to be “superlative” as well as exact.

Diewert’s theory of exact and superlative index numbers is, however, more than just a rationale for the Tornquist/translog discrete-time approximation to the Divisia formulation. It provides an alternative way to approach growth accounting that is more focused on the index numbers than the underlying structure of production. In Diewert and Morrison (1986), the underlying structure is represented by the feasible set S^t, which contains the output vectors y that can be produced from the primary input vector *x* given the state of technology in each time period.[14] GDP is, as before, the value of output at market prices *p*, or $p{\cdot}y$. This is equal to GDI, $w{\cdot}x$, where w is the vector of input prices. A maximum GDP function is defined as $g^t(p,x) \equiv max_y \{p{\cdot}y : (y,x) \in S^t\}$, which is a tangent line (supporting hyperplane) to the technology set S^t. A shift in the technology set holding inputs constant is thus a shift in the maximum GDP function:

$$(7') \qquad \tau(p,x,t) \equiv g^t(p,x)/g^{t-1}(p,x).$$

The index $\tau(p,x,t)$, and its variants, are a measure of MFP, and are closely related to the Solow-Jorgenson-Griliches-BLS formulation (5). However, this approach shifts the focus from the structure of production to the index number problem of measuring $g^t(p,x)$ and $\tau(p,x,t)$. The emphasis now is on flexible approximations as opposed to the retrieval of the technology parameters of S^t. A range of issues can be handled in the index-number approach, like separability, but problems of uniqueness in characterizing the underlying technology remain. From an operational standpoint, however, the

14 I am indebted to Erwin Diewert for his input to the formulation of this section.

computation of MFP estimates is not much affected (both approaches use the Tornquist-translog method), though the interpretation may be. For more on this strand of literature, see Diewert (1978), Diewert and Morrison (1986), Kohli (1990), and Morrison and Diewert (1990).

6. <u>Level Comparisons</u>. Traditional growth accounting evolved largely as an explanation of one country's growth rates over time. The analysis can also be used to explain why growth rates differ across countries, but a look at the growth rates alone can be misleading. Countries with relatively high rates of productivity growth may also have relatively low levels when compared to the richest countries of the world (China and India are recent examples). Indeed, high growth rates may even be associated with a low starting point, a process known as 'convergence' or 'catching-up'.

There is no reason why the growth analyst should have to choose between a comparison of levels versus growth rates, since both can generally be calculated from the same set of data. However, an additional difficulty arises when estimating the relative level of productivity across countries. For any individual country, the level of MFP is a pure index number with a value of one in the base-year of the analysis (the year in which price indexes are also normalized to one). If applied to a collection of individual countries, each would have the same level of MFP in the base year. This is a severe limitation, since cross-national differences in the level MFP in any given base year are a potential cause of the income gap noted in the introduction.

This issue was resolved by Jorgenson and Nishimizu (1978) who developed a cross-country Divisia/Tornqvist index of comparative productivity levels. However, this

solution depends on which country is selected as the basis for comparison, and the result was generalized by Caves, Christensen, and Diewert (1982) to permit a country-invariant comparison. In this formulation, the levels of output and input for each country are expressed as logarithmic deviations from the corresponding average value across all countries, and the relative inputs are weighted with averaged income shares:

(8) $$\ell n[\frac{A_i}{A^D}] = \ell n[\frac{Q_i}{Q^D}] - [\frac{s^K{}_i + \bar{s}^K}{2}]\ell n[\frac{K_i}{K^D}] - [\frac{s^L{}_i + \bar{s}^L}{2}]\ell n[\frac{L_i}{L^D}]$$

Time subscripts have been omitted for clarity of exposition, the superscript D refers to the Divisia cross-country index, and the bar over the shares is an all-country average. Intuitively, this equation indicates that the gap between a country's MFP and the N-country average depends on the gap between the corresponding output and share-weighted inputs. By rearranging terms, the gap between output per worker in each country, Q_i/L_i, and the average level, Q^D/L^D, can be calculated.

Cross-national productivity comparisons also encounter a units-of-measurement problem. National accounting data are typically denominated in the currency units of each country, and have to be converted to a common price for a comparison with other countries to be meaningful. Official currency exchange rates are a poor choice for this conversion, since they may reflect non-market administrative or political decisions. The International Comparison Program seeks to correct for this potential bias by making direct price comparisons of similar items across countries (146 in the most recent 2005 ICP round). The result is a set of Purchasing Power Parity price indexes suitable for income and productivity comparisons.[15] The switch to PPPs can have major consequences: according to Deaton and Heston (2008), world GDP in 2005 was $54,975 billion under the new 2005 ICP estimates compared to $44,306 billion when world GDP is valued in dollars using official exchange rates.[16]

7. Price Duality. Traditional growth accounting links the *quantity* of output to the *quantities* of the inputs via the aggregate production. The emphasis on input and output quantities is warranted because it is ultimately the quantity of consumption (current and future) that determines the standard of living. However, the story of growth accounting can also be told using prices under the assumptions of the Solow-Jorgenson-Griliches model. Jorgenson-Griliches show that differentiation of the basic GDP/GDI identity, equation (1), gives the following equation.

$$\frac{\dot{Q}_t}{Q_t} - s^K_t \frac{\dot{K}_t}{K_t} - s^L_t \frac{\dot{L}_t}{L_t} = -\frac{\dot{p}_t}{p_t} + s^K_t \frac{\dot{c}_t}{c_t} + s^L_t \frac{\dot{w}_t}{w_t} = \frac{\dot{A}_t}{A_t}. \tag{9}$$

This result indicates that the residual estimate of the parameter of interest, $\dot{A}/A$, can equally be obtained from the growth rates of prices or quantities. Put differently, a growth account based on quantities implies a parallel and equivalent growth account based on prices.

[15] This view of the PPP is not universally held. Bosworth and Collins (2003) argue that national prices provide a better measure of the relative value of capital goods.

[16] Deaton and Heston also issue a "health" warning about using the ICP price data. The changes introduced in 2005 caused a substantial downward revision in world GDP relative to the methods of the previous round, which had put world GDP at $59,712 billion (significantly greater than new $54,975 billion estimate). The downward revision was particular large for the high-growth economies of China and India, both of which saw their GDP in dollar terms revised downward by around 40 percent. A change of this magnitude is a reminder about the evolving nature of international comparisons, even in a "gold standard" program like the ICP, which is one of the major achievements of economic data collection.

Quantity-based estimates of the residual are interpreted as a shift in the production function, but what is the interpretation of the price-based growth estimates? Intuitively, the answer is that under the conditions under which the production function $Q_t = A_tF(L_t,K_t)$ serves as a potential function -- constant returns to scale, strict quasi-concavity, Hicks'-neutrality, and marginal productivity pricing -- there is an associated "factor price frontier" that has the form: $p_t = (A_t)^{-1}\,\Psi(w_t,c_t)$.[17] This is the "price dual" to the production function, serves as the potential function for integrating the price-based form of the residual in (9).

The work of Hsieh (2002) illustrates one practical consequence of the dual approach. In some cases, developing countries for example, price data may be more reliable than published quantity estimates, so that the price side may lead to a more accurate growth account. This was the idea implemented by Hsieh in his critique of the papers of Young (1992,1995).[18]

The price side of growth analysis is also the 'port of entry' for introducing changes in product quality into growth accounting. The best example is the Hall (1968) model of quality improvements in capital goods. This seminal paper showed how capital-embodied technical progress could be incorporated into the price dual and how it could be measured using the hedonic price approach. This is an important subject by itself and will be discussed in subsequent sections, including the one that follows.

[17] The productive efficiency term enters the price dual in inverse form because an improvement in productive efficiency reduces output cost for a given level of input prices, and output price equals marginal (and average) cost. Because of the linear homogeneity property, the price dual can also be expressed as a relation between the level of MFP and real factor prices: $A_t = \Psi[(w_t/p_t),(c_t/p_t)]$. This form emphasizes the role of productive efficiency in increasing the real return to the factor inputs.

[18] It is important to emphasize, here, that a sources-of-growth table constructed using prices does not give different results than a table based on the quantity approach constructed from the same data set. It is the use of a different, and presumably more accurate, set of prices that makes the difference, and this implies a different set of quantity estimates.

8. Product Quality. The productivity residual has thus far been associated with a shift in the production function, inviting the view that it is due to improvements in the efficiency of the production process. Process-oriented technical change is certainly an important source of growth, but it is not the only source. Technical change also results in a profusion of new or substantially improved consumer and producer goods, and in many industries, this is just as important as process innovation (if not more). Mandel (2006) underscores this point with this comment "Where the gizmo is made is immaterial to its popularity. It is great design, technical innovation, and savvy marketing that have helped Apple Computer sell more than 40 million iPods." In other words, it is product development, not production *per se*, that counts here.

Bringing product quality into the sources of growth framework is more easily said than done. Differences in quality are often hard to detect, as Adam Smith observed in the early days of the Industrial Revolution: "Quality ... is so very disputable a matter, that I look upon all information of this kind as somewhat uncertain (page 195)." All solutions are likely to involve assumptions and approximations, and the solution most commonly used in growth accounting assumes that the arrival of a superior good in the market place is equivalent to having more units of its inferior predecessor. This "better is more" approach involves measuring the price differential between inferior and superior varieties to infer the corresponding "quantity" difference attributed to the new good.

Several methods are available for measuring the price differential. When there is a reliable overlap between the prices of the old and new goods, the gap can be

measured directly, and when this is not possible, the gap can be forecasted using price hedonic techniques.[19] Once prices have been adjusted for quality change, that is the P_t converted to quality-based price P^e_t, the efficiency-adjusted quantity Q^e_t is defined implicitly from the equation $V_t = P_tQ_t = P^e_tQ^e_t$, as $Q^e_t = V_t/P^e_t$. Since quality change is usually associated with product improvements due to technical change, the quality-adjusted P^e_t. is less than P_t, and the quality-adjusted quantity Q^e_t is larger than its counterpart, and "better" becomes "more."

In this case, the quality-adjusted Q^e_t will grow more rapidly than Q_t and will therefore give a different pattern of output growth to be explained by the growth accounting decompositions. What exactly does this mean for the Solow residual? The residual can now be computed in two ways: with and without the quality adjustment to output, and some simple algebra yields the following relation between the corresponding residuals:

$$(10) \qquad \frac{\dot{A}^e_t}{A^e_t} = \frac{\dot{A}_t}{A_t} + [\frac{\dot{p}_t}{p_t} - \frac{\dot{p}^e_t}{p^e_t}].$$

Multiple outputs can be accommodated by weighting the individual price terms in (10) by the corresponding output shares.[20]

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[19] Price hedonics is too a large topic to cover in a survey focused on growth accounting. However, a few general comments are in order. In the price hedonic model, a good is thought of as a bundle of underlying "characteristics," like the number of bathrooms and square footage of a house, or the processor speed and storage capacity of a computer. A change in the quality of a good is thought of as an increase in one or more characteristics, and the difference between superior and inferior varieties is defined in terms of the differences in the component characteristics. Regression analysis establishes the implicit price of each characteristic, and these prices can be used to put a price on the quality gap (how much of the price increase is due to quality change and how much to pure price inflation). This method was used by BEA to adjust the observed price changes of computers (Cole et. al. (1986), Cartwright (1986)). The hedonic method and its alternatives are discussed in greater detail in Triplett (1990, 1987).

[20] The term in square brackets in (10) reflects the fact that "better" has been converted to "more" via the price differential. Because the quality-corrected quantity grows at a more rapid rate than uncorrected quantity, the quality-corrected price grows at a *slower* rate than uncorrected price. The term in brackets thus makes a positive contribution to generalized productivity growth (it's a reflection of the "more").

This formulation can be interpreted as a decomposition of the quality-corrected rate of productivity change, on the left-hand side of the equation, to process-driven productivity growth (the ability to produce more units of the good from given inputs) and quality-driven productivity growth (the price correction for quality change) on the right-hand side. Unfortunately, the latter is rarely made explicit in growth accounting data, so this formulation is largely notional at this point.

IIB. Industry Growth Accounting

The GDP of the macro economy reflects the economic activity of the component industries and firms that make up the total economy. Some attention should therefore be given to how these components evolve and how their growth relates to the growth of the economy as a whole. There are two general ways of approaching this problem, one that proceeds from the top down, and another that proceeds from the bottom up, as in much of the recent work on micro-productivity data sets.

1. Disaggregation from the Top Down

a. Aggregate GDP is conceptually the sum of the contributions of each industry, adjusted for imports and exports. One way of moving from the top down is therefore to disaggregate the total back into its industry components and (in principle) continue all the way down to the shop floor. This is the so-called “unpeeling the onion” approach. A separate Solow residual could be calculated at each step along the way, and the

However, the quality change effect need not be positive. Companies may cut costs by reducing the quality of their products, through cheaper materials, a lesser degree of “workmanship,” or reductions in ancillary features. In this situation, the quality factor acts as a drag on productivity.

individual residuals linked back to the grand total. This is a conceptually straight-forward process if real value added were the only measure of output at each level of disaggregation. Unfortunately, it is not, because some firms make goods and services that are inputs to the production functions of other companies. These intermediate goods are both an input and an output of the economy, and this complicates the way the industry or firm-level residuals are linked to economy-wide measures of productivity.

The problem becomes apparent when examining how GDP and GDI are related at different levels of aggregation. On the GDP side of the aggregate accounting identity, aggregate output is the sum of deliveries to final demand from each sector, $D_{i,t}$, while on the GDI side, it is the sum of sectoral value added. The basic national accounting identity (1) can be expanded to show this detail:

$$(1') \quad GDP_t = \Sigma_i\, p_{i,t}D_{i,t} = \Sigma_i\, w_{i,t}L_{i,t} + \Sigma_I\, c_{i,t}K_{i,t} = GDI_t\ .$$

Intermediate inputs and outputs do not appear in aggregate GDP/GDI, because their totals are offsetting. However, this is not the case at lower levels of aggregation where there is no reason that the purchase of intermediate inputs from one set of industries should exactly match the value of the intermediate outputs delivered to another set of industries. This is reflected in the industry (or company) accounting identity:

$$(11) \quad p_{i,t}Q_{i,t} = p_{i,t}D_{i,t} + \Sigma_j\, p_{i,t}M_{i,j,t} = w_{i,t}L_{i,t} + c_{i,t}K_{i,t} + \Sigma_j\, p_{j,t}M_{j,i,t}\ .$$

The value of industry gross output sums to an amount that exceeds GDP, and total input cost exceeds GDI. Moreover, since the terms involving intermediate goods in (11) do not necessarily cancel, deliveries to final demand ($p_{i,t}D_{i,t}$) generally do not equal to industry value added ($w_{i,t}L_{i,t} + c_{i,t}K_{i,t}$) -- a point that sometimes gets lost when real value added is used as a measure of sectoral output.

The difference in these accounting identities reflects differences in the underlying structure of production. The constant returns technology that corresponds to the sectoral accounting identity (11) implies, via Euler's Theorem, a production function in which output is produced by a list of inputs that includes intermediate goods. The Hicks'-neutral form of this function is

(12) $Q_{i,t} = A_{i,t} F^i(L_{i,t}, K_{i,t}, M_{1,i,t}, \dots, M_{N,i,t})$.

Proceeding as before with the aggregate model, the industry version of the Solow residual is then

(13) $R_{i,t} = \frac{\dot{Q}_{i,t}}{Q_{i,t}} - s^K_{i,t}\frac{\dot{K}_{i,t}}{K_{i,t}} - s^L_{i,t}\frac{\dot{L}_{i,t}}{L_{i,t}} - \sum_j s^M_{j,i,t}\frac{\dot{M}_{j,i,t}}{M_{j,i,t}} = \frac{\dot{A}_{i,t}}{A_{i,t}}$,

for which (12) serves as the requisite potential function. The share-weights used in (13) are based on the value of industry gross output, not value added, and thus have a larger denominator that the share-weights in the aggregate Solow residual (5). The so-called "KLEMS" model is the most common form of (13), in which the list of intermediate goods includes energy, material, and purchased services, in addition to capital and labor.

b. How do the industry MFP residuals, $R_{i,t}$, map into the aggregate MFP residual, given the difference in scope of output and the corresponding difference in the share-weights?

Domar (1961) resolves this problem in the following way. Suppose that there are two industries, one that makes an intermediate good, $M_t = A_{M,t}F^M(L_{M,t}, K_{M,t})$, and another that makes a final good, using labor, capital and the intermediate good as an input, $D_t = A_{D,t}F^D(L_{D,t}, K_{D,t}, M_t)$. If both functions have the multiplicative Cobb-Douglas form, the first can be substituted into the latter to eliminate the intermediate good from the final demand function, which then becomes a quasi-aggregate production function. In this altered form, the Solow MFP residual (5) can be computed, but it is now the sum of two components: the growth rate of $A_{D,t}$ and the growth rate of $A_{M,t}$ weighted by the output elasticity of M in the production of D. This result shows that efficiency gains in the production of intermediate goods affect overall MFP, even though intermediate goods cancel out in the aggregate.

What happens when there is more than one final-demand producing industry, when multiple industries produce deliveries to both intermediate and final demand? Domar proposes a weighting scheme in which aggregate MFP is the weighted sum of the individual industry residuals, where the weights are equal to the value of industry gross output divided by total value across industries of deliveries to final demand. These weights sum to a quantity greater than one (recall equation (11), here), allowing for the leveraging effects of intermediate inputs on MFP.

Each element in the Domar MFP index has the usual Solow interpretation as a shift in an industry production function. But what interpretation can be given to the weighted average of the industry shifts, given that the Solow aggregate production function generally does not exist in this situation? What is the relevant potential function for the aggregate index? The production possibility frontier (PPF) is the natural choice for this role, since it is the basic supply-side constraint of an economy in which each sector has it own production function (11). The PPF is defined implicitly as

(14) $\Omega(D_{1,t}, \ldots, D_{N,t}; K_t, L_t; A_{1,t}, \ldots, A_{N,t})$;

The growth in the vector of real final demands can be decomposed into the contribution of the growth in aggregate inputs, on the one hand, and the growth in sectoral technology indexes, on the other. Hulten (1978) develops an aggregate index of MFP based on the latter, R_t^{PPF}, and shows that the shift in the PPF is equal to the weighted sum of the sectoral rates of MFP change, where the weights are those proposed by Domar:

(15) $$R_t^{PPF} = \sum_{i=1}^{N} \frac{p_{i,t} Q_{i,t}}{\Sigma_i p_{i,t} D_{i,t}} \frac{\dot{A}_{i,t}}{A_{i,t}} \neq \frac{\dot{A}_t}{A_t}.$$

In this formulation of the multi-sector MFP problem, the PPF (14) serves as a potential function for (15), but there is no guarantee of path independence. In general, R_t^{PPF} is not equal to the aggregate Solow residual obtained by imposing an aggregate production function like (5) across sectors.

c. The problem with the gross output approach is that growth results are not invariant to the degree of vertical integration in an industry. If a company merges with a supplier, what was counted as an intermediate flow becomes an internal flow and disappears. Another problem arises from a lack of reliable and timely input-output data on the price

and quantity of the intermediate flows among industries. Problems with the measurement of intermediate goods imply problems with industry final demand as well.

One popular solution is to abandon the gross output approach and work with value-added data instead. Industry value added excludes intermediate flows and therefore does not vary with the degree of vertical integration. Indeed, it has the property that it sums to GDP/GPI. On the other hand, industry value added, in current or constant prices, is basically a measure of primary input (the industry's contribution to GDI), and as we have already seen from the accounting identity (11), it does not necessarily equal industry final demand. Still, because of measurement problems, the growth accountant may opt to use real value-added as an indicator of industry output. The first step in this direction is to derive a price index with which to deflate nominal value added. This is often done using a "double deflation" technique in which the value of intermediate inputs is subtracted from the value of gross output in both current and constant prices to get an implicit deflator for the difference (which is value added). A Divisia procedure based on equation (13) can also be used, and is in fact recommended, since the next step is to modify (13) to derive the industry value-added residual:

(16) $$R_{i,t}^{v} = \frac{\dot{V}_{i,t}}{V_{i,t}} - v_{i,t}^{K} \frac{\dot{K}_{i,t}}{K_{i,t}} - v_{i,t}^{L} \frac{\dot{L}_{i,t}}{L_{i,t}},$$

where $V_{i,t}$ is industry real value added and $v_{i,t}^{K}$ and $v_{i,t}^{K}$ are the relative shares of capital and labor in $V_{i,t}$. Though calculated on a different basis than the gross output residual, (15), the value-added residual $R_{i,t}^{v}$ is equal to that residual divided by the value-added share of the value of gross output: $R_{i,t}/(s_{i,t}^{K} + s_{i,t}^{L})$.[21]

The valued added-weighted average of the industry $R_{i,t}^{v}$ sums to R_t^{PPF}, implying that the two approaches arrive at the same aggregate result via different paths. However, this nice aggregation property is deceptive. The problem lies at the industry level, where potential function theory implies that, in general, the industry residuals $R_{i,t}$ and $R_{i,t}^{v}$ cannot simultaneously be an exact index of the shift in the industry production function (12), $A_{i,t}$. Which is correct? That honor goes to $R_{i,t}$ when the technical change augments intermediate inputs as well as labor and capital, that is, if the efficiency term $A_{i,t}$ multiplies all inputs as in (12). In that case, what does $R_{i,t}^{v}$ measure? That index is "exact" for a restricted form of (12) in which the production function is separable into a value-added sub-aggregate and in which technical change augments only capital and labor:

$$(12') \qquad Q_{i,t} = F^{i}(a_{i,t}V(L_{i,t},K_{i,t}),M_{1,i,t}, \ldots ,M_{N,i,t}) .$$

If this is the correct specification of technology, then $R_{i,t}^{v}$ is the appropriate form of the MFP residual because it gets at the variable of interest, which now is $a_{i,t}$. Thus, from a theoretical standpoint, the choice between the value-added or gross-output approach to industry growth accounting comes down to a question of which specification of technical change is thought to be the more compelling. The value-added approach generally

[21] The analogy with Hicks' versus Harrod-neutral specifications of the aggregate production function is worth noting here. In the former, technical change augments both capital and labor, but it augments only labor in the latter. In the gross-output industry specification, technical change augments capital, labor, and intermediate inputs, while in the value-added approach, it augments only capital and labor. In both cases, the respective residuals are linked by an algebraic expression involving input shares.

loses this contest, since it implies (improbably) that efficiency-enhancing improvements in technology exclude materials and energy.

2. Disaggregation from the Bottom Up

Much of the recent growth in the field of productivity analysis has been generated by the development of panel data sets like the Longitudinal Research Database of the U.S. Census (Bartelsman and Doms (2000)), Foster, Haltiwanger, and Krizan (2001)). The LRD data set contains establishment-level data on inputs and outputs at a highly disaggregated level of industry detail. These data permit a close-in look at the growth dynamics of units in which the production actually occurs, and their uses go beyond growth accounting (the study of job loss and creation, for example). The panel nature of the data inclines the analysis more towards the econometric branch of productivity analysis, but growth accounting has also been greatly enriched by the capacity to study the effects of the entry and exit of establishments.

Industry MFP change has several sources in the bottoms-up approach: within establishment change in technology or organization, changes in the shares of incumbent establishments, and, discontinuous share changes due to entry and exit to and from the industry. One goal of industry-level growth accounting has been to incorporate this richness of detail into the analysis, and one index that captures at least some of these effects was developed by Baily-Hulten-Campbell (1992). This index has several forms, but for purposes of this review, we will focus on one in which the top-down residual R_t^{PPF} in (15) is generalized to include terms associated with the change in the shares.[22]

These shift-share terms allow for an increase in aggregate productivity even when industry (establishment) productivity is unchanged, if resources are transferred from lower to higher productivity units. However, Petrin and Levinsohn (2005, 2008) argue the assumptions used to derive R_t^{PPF} in (15) -- constant returns, perfect competition, and costless and immediate adjustments -- imply that the shift-share terms should be zero. Intuitively, this occurs because output price equals the same marginal cost for all firms within an industry, and competitive pricing insures that there are no marginal efficiency gains from transferring resources from one industry (or firm) to another. In this case, R_t^{BHC} and R_t^{PPF} are the same. However, the literature finds that reallocation effects do matter empirically, implying a disconnect between the two formulations.

The disconnect between the top-down and bottom-up approaches is illustrated in Figure 1, based on Basu and Fernald (2002). This diagram shows the production possibility frontier of a two-good economy as it evolves over time. The economy is located initially at the point B on the PPF bb, the point of utility maximization, and over time, the PPF expands outward along the path EE to the PPF aa and the optimal point A. The shift along EE is due to two factors: the growth in aggregate capital and labor, and the growth in MFP in the sectoral production function for goods X and Y.

[22] This is but one form of the BHC index. A fuller account is given in Foster et, al (2001), who provide an extensive discussion of the BHC index and other approaches. See, also, Petrin and Levinsohn (2005, 2008).

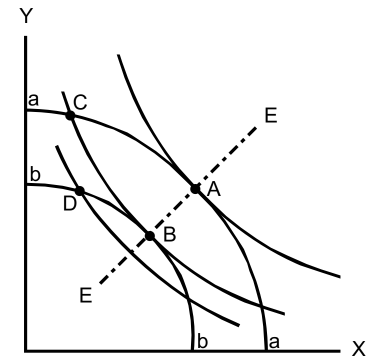

Figure 1

The aggregate measure R_t^{PPF} from (15) is the weighted sum of the latter, measured along the ray EE. There are no reallocation effects along this path. Thus, for reallocation effects to matter, the growth path of the economy must be located somewhere other than along EE.

Foster et. al. (2001) identify several reasons why this may happen: adjustment costs and diffusion lags in technology transmission, monopolistic pricing, and resource distorting policies (e.g., taxes and regulations). Foster, Haltiwanger, and Syverson (2008) highlight the role of price dispersion and product differentiation as a source of the reallocation. These distortionary effects push the economy off the optimal expansion path EE, to say, a path through points like C and D. At C, there is a distortion in output prices that results in too much of good Y and too little of X, but keeps the economy on the maximal PPF. A distortion in the use of inputs can also drive the PPF inward, back to the PPF bb, to, say, the point D. Growth of the economy can then occur through changes in the distortion (reallocation) or because the efficient frontier shifts. This is the basis for the Basu-Fernald distinction between aggregate productivity and aggregate technology.[23]

Reallocation effects are an important addition to the original Solow-Jorgenson-Griliches paradigm. However, there is potentially an issue of consistency. The individual industry or establishment MFP estimates that make up the direct-growth part of the decomposition are typically computed under the assumptions of the Solow residual, which envision distortion-free markets and efficient allocation. On the other hand, the reallocation model is based on the existence of distortions and frictions *across* industries. The level of industry aggregation is to some extent arbitrary (often determined by data availability), and it is not always clear why the factors that distort the allocation of resources should operate at one level of aggregation but not another.

[23] The PPF reflects the efficient operation of the component industry production functions. In a two sector model with given amounts of total labor and capital, the PPF is the locus of efficient input allocations in an appropriately drawn Edgeworth box. These give the output combinations at which the isoquants of the two technologies are tangent. Along the locus of tangencies, relative factor prices are the same in both industries. A distortion in these factor prices will tend to result in a point (like D in Figure 1) off the PPF at which the isoquants cross. Removing *all* input price distortions increase aggregate output by restoring the economy to the maximal PPF. This is the idea behind the reallocation effects estimated of Jorgenson et. al. (2007).

Modeling reallocation as a shift in the interior distorted PPF to the frontier is intuitively appealing, but the existence of the frontier as a stable concave function may require strong assumptions about separability, particularly in the presence of intermediate inputs produced in other industries (Basu and Fernald (2002)). Moreover, a sub-optimal PPF may also be due to the inefficiency in the industry technology itself (a suboptimal value of the Hicksian shift term "A" in the production function $Q = AF(L,K)$). Bloom and Van Reenan (2006), for example, have found a wide variation in management efficiency across companies and countries. The voluminous literature on cross-national differences in economic growth is also focused on this source of inefficiency. The large difference in output per worker between emerging-market and OECD economies is widely attributed to cultural, institutional, and environmental barriers to attaining the best practice technology frontier (see Bosworth and Collins (2003) and Hulten and Isaksson (2007) for recent reviews), and possibly to the lumpiness of infrastructure capital. Growth accounting comparisons suggest that effective PPF in many low income countries may be below the best-practice PPF by as much as a one-to-five ratio. Frontier estimation is another one way to get at this sort of problem, but is beyond the scope of this paper (see, for example, Fare et. al. (1994)).

3. The Company-Establishment Problem

Any analysis of industry productivity must also confront the company versus establishment problem. Many large companies have diversified product mixes (autos and auto insurance, pharmaceutical drugs and home care products, jet engines and refrigerators, etc.). These diverse products are often made in different "establishments" within the company, and it makes sense, from the production function perspective, to group similar establishments across companies when defining an industry.

However, something gets lost in this approach. The company, as a whole, is the legal and organizational entity that manages the various production establishments, as well as non-production activities like research and development, marketing, and finance. The latter are treated as overhead costs by accountants and fixed costs by economists, but they are activities that are essential for the ongoing success of a company. Unfortunately, there is no good way of attributing many of these costs to individual production establishments within the company -- they belong to the company as an entire business entity and, like any joint product, they cannot be uniquely partitioned.[24] On the other hand, to treat them as a separate establishment of their own is to miss the role they play in defining the overall business model and administrative functioning of the company (the Coase-Penrose firm is more than the sum of its parts). To exclude the product development and management activities of the firm entirely is to miss the

[24] In the case of the larger establishments with a company, some part of the "overhead" may actually be assigned to the establishment. But, there are establishments within establishments, and at some point this devolution stops.

vital synergies that determine much of the dynamism of the company as a whole and the changes observed at the establishment level.[25]

Technical change exacerbates the industrial classification problem. If establishments are grouped according to similarity of their products, product innovation may force a reclassification, as with computing devices which used to be mechanical machines and were classified accordingly, but now they are electrical. New management and production processes may also lead to changes in the composition of a company's establishments, as in the financial services industry. Care must therefore be taken when interpreting industry-level growth accounting estimates over long spans of time.[26]

III. The Individual Sources of Growth

The Koopmans (1947) injunction against measurement without theory has greatly influenced the evolution of growth accounting theory, from its national accounting origins through Solow, Jorgenson and Griliches, and beyond. The theoretical foundations are well-developed, and now the problem lies in the opposite direction: theory without measurement. In his 1994 Presidential Address to the American

[25] It is worth noting, here, that European accounting practice is more oriented to the company as the basis for industry classification, though problems still exist. This helps with the overhead cost problem, but comes at the cost of combining operating units with different products and technologies. Unfortunately, there is no one correct approach that addresses all the questions that are asked of the data.

[26] The periodic change in industrial classification is illustrated by the recent adoption of the North American Industrial Classification System (NAICS), which has replaced the older Standard Industrial Classification System (SIC). Once a new system is adopted, it is hard (and costly) to extend the newly reorganized data backward in time for more than a decade or two. This problem tends to be more acute at lower levels of industry aggregation, where the establishments are more prone to reclassification.

Economic Association, Zvi Griliches pointed to the propensity of academic economists to give priority to the former at the expense of the latter

> "We [economists] ourselves do not put enough emphasis on the value of data and data collection in our training of graduate students and in the reward structure of our profession. It is the preparation skill of the chef that catches the professional eye, not the quality of the materials in the meal, or the effort that went into procuring them (page 14)."

Ingredients matter a lot in growth accounting, and inadequate data can have a greater impact on the results than inadequate theory. Nordhaus (1997, pages 54-55) has argued "that official price and output data "may miss the most important revolutions in history," because they miss the really large ("tectonic") advances in technology. Part of the blame belongs to theory because, as Griliches also observed, "... it is not reasonable for us to expect the government to produce statistics in areas where concepts are mushy and where there is little professional agreement on what is to be measured and how (page 14)." This section is devoted to the principal ingredients of growth accounting: output, labor, and capital.

A. Output

1. Output is an intuitively simple concept when it is just the "Q" in a textbook production function. It is often called "widgets," which is short-hand for a product that can be measured in neat and tidy physical units. In the real world, however, there is a great diversity of "widgets", tangible and intangible, and even within relatively homogeneous product categories, differences in quality, variety, and location matter. The range of products in a modern economy is so diverse that it is virtually impossible for the statistician to capture the full richness of detail. Some degree of sampling and

aggregation is necessary before estimates are presented to the public, and this degree is rather high in the data typically used for growth accounting. In the process, the growth of real output no longer refers to specific products, but to synthetic constructs that represent broad groups like autos, pharmaceutical drugs, machine tools, and houses, where the exact units of measurement are somewhat fuzzy.

One way to attack the heterogeneity problem is from the price side. This is the strategy for dealing with changes in product quality, and a variant works for within-group diversity. The total value of sales or revenues for a given product group, is usually available for products for which there are active markets (e.g., auto sales), and a measure of average real output can be obtained by deflating product value by an index of average price. This approach implicitly assumes that there is a synthetic product Q_t whose notional price is P_t and whose value is $V_t = P_tQ_t = \Sigma_i P_{i,t}Q_{i,t}$. The implicit quantity index is then $Q_t = V_t/P_t$. In the case of the U.S. national accounts, the consumer and producer price indexes fill this role.[27] The units of measurement of Q_t are in price-adjusted units of currency, not in numbers of widgets. This ambiguity is not so much of a problem for most manufactured goods or agricultural commodities, where the connection between the constant-price quantity index and the underlying physical units is more intuitive, but it is a greater problem as the growth accountant moves to product types and sectors at the outer boundaries of accurate measurement.

[27] The use of an average price deflator in this context is encouraged by the Law of One Price. The price of a given item tends to be similar across outlets, whereas the quantities sold vary considerably. In the U.S. CPI program, agents visit stores and other outlets to observe the prices of representative items in a range of product groups. They can get many item prices “from the shelf” or the menu, etc., without having to estimate the associated product sales. One downside of relying on price indexes to back out an estimate real output is that price estimates are typically made using data sources and methods that differ from those used to estimate the nominal value of GDP and its components.

2. Boundary Issues.[28] The data underlying output statistics (e.g., national accounts and census survey data) are mainly based market-mediated transactions. This forms a loose boundary between what gets into the accounts and what does not. Since growth accounting results are sensitive to what gets included, a few remarks about the main ingredients are warranted.

a. The government sector is usually included in national accounting data, because it is a large draw on national resources and because the boundary between public and private sectors is often indistinct. The main measurement problem is that much of the output originating in the public sector is not distributed through markets (or is distributed at prices that do not reflect full costs), and there are thus no reliable valuation data. Moreover, there are a dearth of output price indexes with which to estimate real output even if the public P_tQ_t were available. As a result, the growth rate of real output is typically inferred from the growth rate of input, with productivity change assumed to be zero. For this reason, some growth analyses omit the public sector and focus only on the market sectors of the economy.

b. The household sector presents an even greater problem for growth accounting. Not only is most of the sector’s output not sold in markets, but here is also little reliable data on the value of inputs (the recent American Time Use Survey is an attempt to get at this

[28] The question of what to include in a national account has been debated since the beginning of the national accounting movement. There is currently a good deal of interest in the topic, generated, in part, by an increased concern about the environment, as well as a renewed focus on health and education issues. The papers by Nordhaus (2006) and Abraham and Mackie (2006) provide recent surveys of many of the central issues.

problem for the U.S.). The household sector is therefore (largely) excluded from national accounting data. According to estimates by Landefeld and McCulla (2000), the household production of consumption goods in the U.S. national accounts was 43 percent of 1946 GDP, falling to 24 percent in 1997.

Not only does the exclusion of most household output affect the level of measured GDP, it also changes the growth picture as well. For example, the shift in female labor force participation from the household sector to the market and government sectors is an important source of growth in measured labor input in the latter, but much of the reallocation is portrayed in the GDP statistics as a net expansion. Moreover, the household sector is where much of the investment in human capital occurs via the opportunity cost of time (Jorgenson and Fraumeni (1989, 1992)). Growth accounts that focus only on the market and near-market segments of an economy may thus present a distorted picture of the true sources of growth of the economy. A variant of this problem arises in countries with a large non-market component to their economies, and in countries with significant "informal" markets.

c. The service-producing sectors. There are data on the value of product transacted for these sectors, but here it is the real product that is notoriously hard to measure (Griliches (1994) refers to this part of the economy as the "unmeasurable sectors"). The heart of the problem is the lack of clarity on just what is meant by a "unit" of service output. Should a service be measured as a unit produced by its supplier, or as the 'outcome' obtained by the recipient? Doctors typically sell expertise, not health outcomes, because the latter depends on the initial condition of the patient and the

extent to which advice or treatment is followed. Education is similar, since the production of human capital involves student and family inputs as well as formal schooling. Indeed, many, if not most, services involve some form of contingent outcome that drives a wedge between the resource cost of the service and the value ultimately captured by the consumer. Sorting out the P's and Q's is hard if there is no clear idea of what a unit of Q actually is.[29]

4. R&D and other Business Intangibles. Until very recently, expenditures for intangibles like R&D were treated as intermediate goods and not counted as part of GDP or GDI. This treatment was due, in part, to the fact that business intangibles are largely produced and used within firms, without any market transaction to provide a dollar metric of the volume produced, and without any visible real product to measure. However, as previously noted, expenditures for R&D, marketing, and worker training are the source of much of the product and process innovation that drive the future profitability and productivity of companies. These expenditures usually operate with lags (often with long lags), and the benefits spill out over a number of years, and they are thus more appropriately seen as capital investments than intermediate goods.

Various aspects of this omission have received a fair amount of attention in recent years. [30] Nakamura (1999, 2001) was the first to develop comprehensive

[29] There is quite a large literature on the problem of measuring service sector output, stimulated, in part, by Baumol's (1967) hypothesis that the labor intensity of that sector makes productivity gains inherently harder to achieve. This possibility, along with the difficulty in measuring service sector output, was a leading explanation of the U.S. productivity slowdown of the 1970s and 1980s (Griliches (1994)). However, the recent productivity boom in the service sector, associated with the IT revolution, puts these explanations in a different light (Triplett and Bosworth (2003)).

[30] Lev (2001), Blair and Wallman (2001), and Sichel (2007) provide overviews of the economic and accounting issues involved in capitalizing intangibles. For the links to technology, and information technology in particular, see Basu et. al (2004) and Brynjolfsson and Hitt (2005).

expenditure estimates and Corrado, Hulten, and Sichel (2005, 2006)) the first to incorporate intangibles into a growth accounting framework (Sichel (2007)). This line of research finds that intangible investments by U.S. companies have grown rapidly in recent decades and are now larger than investment in plant and equipment (adding around 10% or more to GDP if fully counted), and that the sources-of-growth accounts for U.S are rather different when intangibles are included as both inputs *and* outputs of the non-farm business sector. The BEA has launched a program to include scientific R&D investments in U.S. GDP as a satellite account to the main national accounts (Robbins and Moylan (2007)), and the BLS plans to incorporate these estimates into their productivity estimates.

B. Labor Input.

1. Labor services. The flow of labor services is generally seen as the appropriate concept of labor input for the production function framework of growth accounting and hours worked as a reasonable measure of the flow. However, hours paid or employment are often the only available measures.[31] Other complications include the likelihood that some labor input and compensation are misclassified as the capital income of sole proprietors, and that unpaid family workers and other undocumented workers go uncounted. Accounting for fringe benefits and equity participation (e.g., stock options) also poses problems for measurement. Finally, some workers (e.g.,

[31] A supplementary form of growth accounting is available when there are data on both hours paid (H) and employment (L). In this case, the growth rate of output (Q) can be decomposed into the growth rate of output per worker (Q/L) and the growth rate of hours per worker (and further when hours worked are also available). This sort of decomposition is often of interest in cross-national comparisons, as, for example, studies that examine the recent growth experience of the EU countries and the U.S. (Van Ark et. al (2008)).

managers) are essentially quasi-fixed inputs whose direct impact on production does not vary with the flow of output.

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2. Labor Composition. In the production-function approach, different types of labor input should be grouped according to differences in their marginal products. In analytical terms, if there are N types (cohorts) of labor, the production function would contain a separate variable for each type: $Q_t = A_t F(K_t, H_{1,t}, \ldots, H_{N,t})$, where $H_{i,t}$ is the number of hours worked in the i^{th} cohort. Following Jorgenson and Griliches (1967), a Divisia index of labor input can be computed by weighting growth rates of the $H_{i,t}$ with their shares of total labor income. This index is decomposed into two parts, the first equal to the growth rate of total hours across all cohorts, $H_{i,t}, = \Sigma_I H_{i,t}$, and the second a compositional term which measures the weighted contribution of each cohort's hours to the growth of total hours:

$$(17) \qquad \frac{\dot{L}_t}{L_t} = \frac{\dot{H}_t}{H_t} + \sum_{i=1}^{N} \frac{w_{i,t} H_{i,t}}{\Sigma_i w_{i,t} H_{i,t}} \left[\frac{\dot{H}_{i,t}}{H_{i,t}} - \frac{\dot{H}_t}{H_t} \right] .$$

The compositional part of (17) is sometimes called the "labor quality" effect, because it is positive when the composition of the labor force shifts toward cohorts with higher wages, which are assumed to equal the corresponding value of marginal products (an assumption that is frequently challenged). The compositional effect is zero when wages are equal or when there is no relative shift in the make-up of the work force.

The index (17) is path independent when the production function is separable, i.e. $Q_t = A_t F(K_t, L(H_{1,t}, \ldots, H_{N,t}))$, in which case there is a well-defined labor aggregate of the form $L(H_{1,t}, \ldots, H_{N,t})$ to act as the potential function. This is a strong assumption, since it requires the marginal rate of technical substitution between different types of labor (e.g., skilled versus unskilled) does not depend on the amount of capital available. A parallel formulation applies to the existence of a capital subaggregate.

The labor cohorts are typically constructed using such characteristics as education, age, gender, and age (or experience). Because of the multiplicity of dimensions and the resulting large number of cohorts, much of the cohort data has to be estimated using techniques like the method of bi-proportional matrices. This imputation introduces an additional source of measurement error into the growth account, but the benefits from this formulation are substantial, since the empirical studies that use this method tend to show a large positive contribution of increased educational attainment to economic growth.

C. Capital input.

1. Owner-utilized Capital. In a world in which the flow of input services from capital is priced in an active rental market, the formation of the Divisia index of the capital aggregate would be almost as straight-forward as the labor counterpart (17). Unfortunately for the statistician, the measurement problem is greatly complicated by the fact that most capital is owner-operated and some of it (intangibles) is owner-produced as well. As a result, there are no market transactions for this type of capital, so the price of capital services must be imputed using indirect methods. The absence of market data also means that the quantity of capital services must be imputed. The imputation procedures in common use are discussed in some detail in the following

subsections, because of their importance to practical growth accounting and because there are still areas of controversy.[32]

2. The "perpetual inventory method". Capital is both an input and an output of the production process. It is also a durable good (by definition) in which there may be different vintages of past investment goods in operation at any point in time. Together, these characteristics suggest an aggregate production function like $Q_t = C_t + I_t = A_tF(L_t, I_t, \ldots, I_{t-T})$, with T being the age in which the oldest vintage is removed from service, and C_t the amount of output used for consumption. This is an extension of the Solow production function of the preceding sections, which is based on the stock of capital, K_t, rather than on its component vector $(I_t, \ldots, I_{t-T})$. The problem for growth accounting is to connect the two. One possibility is to work directly with the vector form and interpret the results in the stock context. There is, however, a problem: while the price and quantity of new investment goods is readily observable, the price and quantity of the older vintages is not so easily obtained. The other way to proceed is to convert the vector $(I_t, \ldots, I_{t-T})$ into the implied stock K_t. This is the rationale for the perpetual inventory method.[33]

The perpetual inventory stock of capital is the sum of current and past investment goods, weighted by the productive efficiency of those investments:

[32] General treatments of the capital measurement problem can be found in Diewert (1980), Hulten (1990), and the recent OECD manual.

[33] Capital stock can also be estimated directly from historical book value accounting data, but this is problematic because of the difficulty in adjusting for price level changes, and because somewhat arbitrary methods of depreciating capital are often used. Moreover, book value data are usually presented at a highly aggregated level of asset detail, and the shift in the composition of capital is also of interest to growth accountants.

(18) $$K_t = \Phi_0 I_{t-0} + \Phi_1 I_{t-1} + \ldots + \Phi_T I_{t-T}.$$

The weighting index, Φ_s, is the efficiency of an s-year old asset relative to a new asset, defined as the ratio of the marginal product of an s-year old asset to the marginal product of a new asset. The term $\Phi_s I_{t-s}$ is therefore the amount of investment put in place s years previously, measured in units of productive efficiency. In this formulation, K_t is the total amount of effective capital denominated in units of new capital, that is, the equivalent amount of new capital needed to replace the capacity of the actual stock with its various layers of vintage capital.

The Φ_s are generally treated as fixed parameters in the perpetual inventory approach. This is a strong assumption, given that they are the ratios of marginal products that can vary over time according to economic conditions, and because the age-efficiency path of an asset will generally depend on intensity of use and maintenance.[34] Since they cannot be observed directly, the Φ's must be obtained by indirect procedures. A number of methods have been proposed, but all involve a high ratio of assumption to fact. BEA uses a procedure derived from the price-based study of depreciation rates of Hulten and Wykoff (1981), who apply price-hedonic procedures to samples of used investment good prices for a variety of assets and find that the depreciation pattern (and rate) is approximately geometric. Other methods involve estimation of the average service life, T in (18), combined with assumptions about the appropriate pattern of efficiency decay (one-hoss shay, hyperbolic, and straight-line are the common alternatives).

The problem is complicated by a fallacy of composition: each individual asset in a group of similar assets may follow one Φ pattern based on its own useful life, T_i, but the average group Φ can have an entirely different pattern when the T_i vary among the assets in the group (the usual case unless all are retired from service at the same point in time). This fallacy is strengthened when assets are grouped into broad assets types (e.g., machine tool, factory buildings), as they must be in order to keep the measurement problem statistically manageable. But, as the heterogeneity of assets increases, the group Φ tends toward a convex pattern that is usually well-approximated by the simple geometric form.

The perpetual inventory model can also accommodate the situation in which the productivity ("quality") of new assets improves over time (Hall (1968, 1971)). In this case, the weighting index of a new asset, Φ_0, is allowed to drift upward over time to reflect the increase in the marginal productivity of new assets due to improvements in design and technology. In the notation of capital accounting, the index of a new asset in year t, $\Phi_{t,0}$, exceeds that of a new asset in the preceding year, $\Phi_{t-1,0}$. The index of relative efficiency now has two subscripts, one denotes the year and the other the

[34] The marginal product problem is usually dealt with by assuming strong separability of the production function, in which case the ratio is invariant to changes in the variables outside the capital aggregate. The second problem is usually dealt with by assuming that the Φ's reflect optimal utilization and maintenance programs that are relatively stable when averaged across individual assets. These assumptions are used more from necessity than conviction, because the data needed to improve on them are unobtainable for a wide range of assets. The model of Diewert and Lawrence (2000) permits a more flexible approach in which different vintages of investment are not assumed to be perfect substitutes, although this approach gives the same results as the perpetual inventory method when depreciation has the geometric form.

Another important implication of the exogenous Φ approach is that periods of exceptional retirement are not allowed for. Exceptional retirements may arise from a natural disaster, a large hurricane or earthquake, from wars, or from technological sources. The Y2K problem is an example of the latter. As the millennium year 2000 approached, there was a concern that existing software could not handle the transition from "99" to "00," and this triggered a wave of IT purchases to replace older systems. This led to a parallel wave of retirements not captured by the fixed-life perpetual inventory method. Bartelsman and Beaulieu (2007) estimate that the failure to account for these Y2K retirements had a small, but material, effect on measured MFP in the nonfarm private business sector.

asset's age: in general, $\Phi_{t,s}$. The efficiency profile of any single vintage of asset is therefore $[\Phi_{t,0}, \Phi_{t+1,1}, \Phi_{t+2,s+2}, \ldots, \Phi_{t+T,s+T}]$, and the capital stock in year t is made up of investments from past vintages. Thus:

$$(18') \qquad K_t = \Phi_{t,0} I_{t-0} + \Phi_{t,1} I_{t-1} + \ldots + \Phi_{t,T} I_{t-T}.$$

This form of (18) is also measured in efficiency units based on relative marginal products. This amended form of the capital accumulation equation is closely related to Solow's "jelly" stock of capital that will be discussed in the section on capital-embodied technical change. The "dual" side of this formulation links the rise in the efficiency of new asset, $\Phi_{t,0}$, to the induced decline in the value of preceding vintages of capital, a process known as obsolescence.

3. The Price of Capital Services. Once the capital stocks have been estimated, the unobserved price of capital services can be imputed. For a single type of capital and labor, the basic GDP/GDI identity can be rearranged to obtain an implicit estimate of the capital service price: $c_t = (p_t Q_t - w_t L_t)/ K_t$. This is the reverse of the usual procedure in which an independent estimate of the price deflator is used to compute quantity. The difficulty with this solution is the feature that long-lived assets like buildings are lumped together with short lived goods like autos. Since the shift in the composition of capital is a potentially important factor influencing economic growth, as with the composition of labor input, the number of capital goods in the analysis is ideally greater than one.[35]

[35] Indeed, much of the recent literature on growth accounting has focused on the role played by the rapid growth of information and communications equipment relative to other types of capital (see, for example, the survey by Oliner and Sichel (2000) and the early contribution by Baily and Gordon (1988))

One solution to the composition problem was developed by Jorgenson and Griliches (1967), based on the path-breaking work by Jorgenson (1963) on the service price of capital (also called the "user cost" or "rental price"). The investment price of a capital good (p^I_t) is assumed to equal the expected present value of the annual service prices/user costs/rents (c_t) generated by the asset over its life. Jorgenson (1963) showed that an explicit formula for the service price can be obtained by solving the present value equation for c_t, as a function of the rate of return per dollar of investment (r_t), the asset price (p^I_t), the asset price revaluation ($\rho_t = dp^I_t/p^I_t$), and the rate of depreciation (δ):

$$(19) \qquad c_t = (r_t - \rho_t + \delta)\, p^I_t.$$

Taxes were added to the model by Hall and Jorgenson (1967). Estimates of the individual components on the right-hand side provide an estimate of the user cost, which is, itself, assumed to equal the value of the marginal product of capital in competitive equilibrium.

The estimation of each component of the user cost presents its own difficulties and has its own literature. However, the rate of return component deserves special mention because of its history and theoretical implications for growth accounting. The main candidates are to use the endogenous *ex post* approach developed by Jorgenson and Griliches (1967), or to use an exogenous *ex ante* rate of return based on the finance decision underlying the investment. The former is typically used in empirical growth accounting because it preserves the adding-up property of the basic GDP/GDI.

In this approach, the adding-up property of the GDP/GDI identity is expanded to allow for multiple types of capital and labor and solved for total income accruing to capital:

$$(20) \qquad \Pi_t = p_t Q_t - \Sigma_j w_{j,t} L_{j,t} = \Sigma_I c_{i,t} K_{i,t} \ .$$

The formula for the service price of capital can be substituted into this equation, and solved for r_t, under the rather strong assumption that the rate of return is the same for all assets regardless of risk. In this formulation, r_t is simply the residual average rate of return that insures that GDP equals GDI.

A case can also be made for the alternative procedure of using an independent *ex ante* estimate for rates of return in (19) and (20) (for example, Schreyer (2004)). Under uncertainty, investment decisions are made on the basis of the rate of return expected (*ex ante*) at the time that the investment is made, not the return actually realized *ex post*. Moreover, investors are likely to apply different *ex ante* rates according to the degree of risk associated with each investment. The *ex ante* approach thus leads to a user cost that is presumably closer to the price/marginal product linkage envisioned in the neoclassical theory on which the Jorgensonian capital pricing model is based. It should also be noted that the *ex ante* procedure gives an estimate of the service price that does not presume that the GDP/GDI adding-up condition necessarily holds, so growth accounting with non-constant returns to scale can be accommodated (at the expense of the GDP/GDI identity). The *ex ante* approach is also the important option when implementing the pricing-duality version of the growth accounting model.

The Hsieh-Young debate is a reminder how important the choice can be for growth accounting. [36]

4. Capital utilization. The perpetual inventory method produces a measure of capital stock, not of capital services. The latter will depend on the intensity of use of the former, and this will vary over the business cycle and may even show a secular drift over time as management practices change. To the extent that this stock-flow problem is ignored, the discrepancies between the two will be suppressed into the MFP residual. Field (2003) shows how big an impact the utilization problem can have on the observed pattern of growth.

One way to correct for the stock-flow problem is to introduce an explicit utilization term into the production function. However, this runs afoul of the pricing problem associated with unused capacity. It also ignores the fact the some capital is continuously in service, as with a building that keeps out rain and thieves, or capital for which the demand is stochastic (telephones).

Berndt and Fuss (1986) offer a different solution. They show that the stock approach (I.e., one without an explicit utilization adjustment) does, in fact, correct for variations in capital utilization, at least in the case of a single capital good.[37] In this

[36] There are also some important issues with the other elements of the service price formula (19). For example, during periods of asset price inflation, the service prices imputed under the perfect foresight assumption implicit in (19) can be negative (an implausible outcome for a variable that proxies for the marginal product of the underlying capital). Doing away with the perfect foresight assumption ameliorates this problem, but raises the question of how expectations are formed and how they (along with risk) should be incorporated into the model.

[37] When there is more than one type of capital good, a separate Berndt-Fuss utilization correction does not apply to each type. However, one does apply to the entire collection of capital goods, and when the production function is separable into a capital subaggregate, the Berndt-Fuss correction applies to the subaggregate.

framework, capital is a Marshallian quasi-fixed input, and the degree of utilization is determined by the amount of variable input applied to the stock. The correction for this form of utilization is captured by the *ex post* service price as computed using the Jorgenson-Griliches procedure underlying equation (20). But, apart from this implicit correction, the stock-based perpetual-inventory-method estimates used in growth accounting are not adjusted for fluctuations in the flow of service over time, and this introduces a pro-cyclical bias in the estimate of MFP.[38] This pro-cyclical pattern makes it hard to interpret the size and timing of MFP movements, and is one of the reasons that the U.S. productivity slowdown of the 1970s and the 1980s is still something of a mystery.

5. Some Final Caveats on Capital. The methods used for imputing the price and quantity of capital input are strongly neoclassical. Capital is treated as a malleable homogenous entity in which the different efficiency-adjusted investment goods produced in different years (vintages) are generally assumed to be perfect substitutes, and for which markets function smoothly and competitively. These assumptions were a source of great controversy in the 1950s, and while the controversy has largely receded, the basic assumption that capital goods of greatly differing characteristics can be lumped into a single entity and substituted freely against labor must be recognized as a strong assumption (e.g., it asserts that many workers with abacus' are equivalent to one worker with a personal computer). Fisher (1965, 1969) shows just how restrictive these assumptions really are.

[38] The economics of procyclical productivity are discussed in Basu and Fernald (2001).

On the other hand, Jorgenson (1966) made the important point that measurement errors are offsetting along the Golden Rule steady-state growth path, because capital is, in general, both an input and an output of the economy. The basic Solow residual in equation (5) can be modified by replacing the growth rate of output, Q_t, with the share-weighted growth rates of its components, consumption, C_t, and investment, I_t, and a rearrangement of terms gives:

$$(21) \qquad R_t = s^C{}_t \frac{\dot{C}_t}{C_t} + [\, s^I{}_t \frac{\dot{I}_t}{I_t} - s^K{}_t \frac{\dot{K}_t}{K_t}\,] - s^L{}_t \frac{\dot{L}_t}{L_t} = \frac{\dot{A}_t}{A_t} .$$

The terms involving capital goods are shown in brackets, and in optimal steady state growth they are exactly offsetting, because investment and capital stock grow at the same rate and the corresponding shares are equal. This result "works" because of the nature of investment. Investment arises from the act of saving, which is the attempt to shift consumption from the current period to the future, and today's optimal investment is tomorrow's dissaving. However, errors in measurement do matter outside of optimal growth.

IV. Critique of the Growth Accounting Model

An examination of specific innovations reveals that the arrival of a new technology involves much more than a simple shift in the production function. Studies of the Corliss steam engine by Rosenberg and Trajtenberg (2004) and the computer numerically-controlled machine tools by Bartel, Ichniowski, and Shaw (2007) bear witness to this. Culture and institutions also determine how much innovation an economy (or company) can achieve (see, for example, Landes (1998), for a recent examination of this point). No aggregate model can hope to capture all the complexity of the innovation process, but the growth accounting model does provide something that the detailed studies do not: a bottom-line metric of the importance of innovation in the process of economy-wide growth.

The benefits of this generalization come at the cost of accuracy. Some of the flaws and limitations of the growth accounting model have already been put on display. There are others, and three issues will be highlighted: whether the contribution of capital accumulation can be separated from that of technical change; whether the MFP model should be modified to reflect changes in consumer welfare; and how imperfect competition affects the interpretation of the standard results.

A. Capital versus Technology: A Clear Division?

The decomposition of the growth rate of output (or output per worker) into its basic sources, inputs and MFP, is the defining feature of the growth accounting approach. The two sources arise from different processes: the accumulation of capital per worker is linked to the propensity to save, while MFP is linked to the accumulation of knowledge and the propensity to innovate. Under the usual assumptions about the production function, the former may be subject to diminishing marginal returns while the latter is not so burdened (although advances of knowledge tend to be sporadic and come in waves), so sorting out the separate contributions is potentially important for the question of sustained growth.

Unfortunately, the dichotomy between the two effects is not so clearly drawn in reality. First, R&D expenditures are a form of capital formation, yet they are also the source of much technical change. Second, there are mutual feedback effects in which an increase in the MFP residual causes the amount of capital to increase, and the increase in capital leads to spillovers that increase MFP. And, third, improvements in technology are often embodied in the design of new capital goods. Each effect has implications for growth accounting and will be discussed briefly.

1. R&D and its Coinvestments. It is well established that R&D expenditures have a positive rate of return and that they are the source of much product and process innovation (see Griliches (2000) for survey) and company valuation (B. Hall (1993)). Since R&D has a positive marginal product, it is hard to avoid the conclusion that it deserves to be treated as an input to production, and, in this vein, Griliches (1973) showed how R&D capital might be incorporated into the growth accounting model on the input side. R&D as an output in this model appeared in Corrado, Hulten, and Sichel (2006) (CHS). The definition of output is expanded to include investments in R&D and other intangibles, H_t, giving $Q_t = C_t, +I_t, +H_t = A_t F(L_t, K_t, R_t)$, where R_t is the stock of R&D input. The corresponding GDP accounting identity is also expanded to include the value of R&D output; $p^C{}_t C_t + p^I{}_t I_t + p^H{}_t H_t$.[39]

The growth accounting model that accompanies this expanded production framework is a variant of the residual in (12). In optimal steady state growth, the R&D terms cancel and could have been ignored in calculating MFP (the Jorgenson (1966)

[39] According to the estimates by CHS, adding a broad list of intangibles ($p^H{}_t H_t$) expanded U.S. GDP by around 10 percent relative to its conventional counterpart in 2003. This list includes brand equity and organizational capital, in addition to R&D.

result in equation (21)), but then so do all the other capital terms. However, while steady-state growth is a useful theoretical parable, it is not observed in real world data, where capital terms generally do not cancel. In the CHS study, for example, the effect of adding intangibles to a growth account of the U.S. non-farm business sector for the period 1995-2003 is to reduce measured MFP from 1.42 percent to 1.08 percent. In other words, the intangible capital terms of an expanded version of (21) do not cancel, and their omission presents a rather different picture of the forces driving growth.

2. R&D Spillovers and Endogenous Growth. Growth accounting formulated in this way still provides for a dichotomy between capital formation and productivity growth, but the nature of the dichotomy has changed. It is no longer a story about technology versus capital formation, but a story about *costless* advances in technology versus different types of capital formation, including those that promote technical change. Costless MFP growth arises from serendipity, inspiration, or the diffusion of technical knowledge from the originator who bears the development cost to other users. [40]

The costless diffusion of knowledge leads to feedbacks effects that are central to the endogenous growth model of Romer (1986) and Lucas (1988). This model has an important implication for growth accounting, where diffusion appears as a spillover from the stock K_t (one that includes R&D and human capital) to the level of MFP and thus appears as a component of the Solow residual (Barro (1999) and Hulten (2001)). The R&D spillover is illustrated in the production function $Q_t = Ae^{\lambda t}L^{\alpha}_tK^{\beta}_tK^{\gamma}_t\mu_t$, where γ is the

[40] Kendrick (1980) observes that "Informal inventive and innovative activity, including the myriad small technological improvements devised by plant managers and workers, was the chief source of technological progress in the nineteenth century, and is still significant (page 25)." The MFP residual was a factor in growth before formal R&D programs were invented (as, for example, the case of the Corliss steam engine noted above), and thus before R&D spillovers.

externality associated with K, and α and β are the direct output elasticities of labor and capital, the parameter λ is the autonomous rate of productivity change, and μ is an "Abramovitz" term summarizing the other factors (errors and omissions) that affect production. There are constant returns to scale to the direct inputs, $\alpha+\beta=1$, so the overall production function exhibits increasing returns. However, producers do not capture the externality and are in a constant-returns perfect-competition equilibrium with respect to K and L, and the GDP/GDI identity thus holds. Under these conditions, the standard Solow residual calculated as per equation (5) is equal to

(5')
$$R_t = \lambda + \gamma\frac{\dot{K}_t}{K_t} + \frac{\dot{\mu}_t}{\mu_t}.$$

This makes the Solow residual an endogenous function of capital and adds structure to the original Solow model (5). However, the presence of the regular "Solow" λ and the "Abramovtiz" μ in (5') indicates that the endogenous growth view does not invalidate or replace growth accounting, it enriches it. Sorting out the relative importance of the various effects is another matter, since all that growth accounting produces is the residual R_t.

2. Simultaneity Bias and the Problem of Causality. The Lucas-Romer growth model makes the residual an endogenous function of the growth in capital. Neoclasssical growth theory points in the opposite direction: investment is a function of income, and the growth rate of capital is therefore endogenous and depends, among other factors, on the rate of technical change. An autonomous increase in the latter (e.g., via λ in (5')) will lead to an *induced accumulation effect* on capital that could plausibly be counted as

part of the MFP effect in assessing the importance of technology as a cause of growth. (Hulten (1975)).

One solution to the capital-endogeneity part of the problem is to use the Harrodian version of the MFP residual rather than the conventional Solow-Hicks model (recall that the former is algebraically equal to the latter divided by labor's share of income) (see Rymes (1971), Hulten (1975,1979)). The ability to handle capital-endogeneity is, indeed, the rationale for the Harrodian approach and the reason for its use in neoclassical steady-state growth models. In growth accounting terms, the Harrodian approach attributes the induced accumulation effect to the Harrodian version of the MFP residual.[41]

3. Capital-Embodied Technical Change. A third problem with the simple dichotomy between technology and capital formation arises when product-oriented technical change occurs in capital goods. In this case, the rate at which new technology is introduced depends on the rate of investment, and a clear division between the two as independent sources of growth is, again, impossible. However, growth accounting can accommodate capital-embodied technical change, as in the other cases in which the capital-technology dichotomy is broken.

The potential function in the capital-embodiment model is based on a specification of production in which each vintage, ν, of a capital good has its own technology: $Q^{\nu} = f^{\nu}(K^{\nu},L^{\nu})$. This formulation allows output and labor to be vintage

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[40] It is perhaps worth noting that the Harrodian version of the residual can be computed without actually having to impose Harrod-neutrality on the problem, though path independence may be a problem in this case. Hulten and Isaksson (2007) provide a more detailed discussion of this issue, as well as a comparison of the Harrodian and Hicksian residuals for a panel of high- and low-income countries. See also the results of Hall and Jones (1999) and Klenow and Rodriguez-Claire (1997).

specific as well as capital (e.g., the case of computer numerically-controlled machines tools above). Unfortunately, a model of this generality is usually not empirically feasible because of its data requirements, since, with a few exceptions, input and output data are not collected by vintage. This has led to three empirically tractable variations on the general embodiment model.

The first is Salter (1960), which directly compares plants of different vintages, and tries to detect embodied technical change via differences in plant productivity. This experiment is cleanest in cases like electricity generation in which the plant itself is strongly vintage dependent, but as Gort and Boddy (1967) pointed out, technological embodiment generally applies to capital that comprises only a fraction of the total capital stock of a plant, so plant-based studies in most industries may not detect the true size of the embodiment effect.

The Johannsen (1959) putty-clay framework provides an alternative approach. Different vintages are treated as though they were different techniques in the putty-clay mode but, in practice, all putty-clay models are hard to work with, for the same reasons as the Salter model but also because locking into any technique within the envelope of choices requires expectations about future technology and prices that must be included in the model.

The Solow (1960) jelly-capital variant of the capital-embodiment model is the closest to the growth accounting framework described in this survey. Solow derived a form of the aggregate production function $Q = AF(J,L)$ from the individual $Q^{\nu} = f^{\nu}(K^{\nu},L^{\nu})$ by assuming that each has the Cobb-Douglas form, and defining the 'jelly' stock of capital J as the weighted sum of the K^{ν}. This formulation of the jelly-stock is essentially the same as the perpetual inventory formulation of the capital stock measurement with quality change in capital goods, as represented by equation (18') in which "better" capital is equivalent to more units of capital.

The link between the product quality literature and embodied technical change is used in Hulten (1992b) to estimate the capital-embodiment effect in the growth accounting context.[42] The key conceptual issue that surfaces in this work is that there are actually two versions of the embodiment model that could be used: the pure Solow model in which embodiment only affects the capital stock but not the output of investment goods (costless quality change in capital goods), and the Jorgenson (1966) model in which capital-embodied technical change requires resources to obtain (i.e., R&D expenditures). Hulten (1992b) adapts the residual in (21) to accommodate both views as special cases, by introducing an additional parameter into the term that is zero for the Solow model and one for the Jorgenson model, but which can be sorted out empirically.

Some mention should also be made of the distinction between a change in technology that affects investment via capital-embodiment (the production of "better" investment goods) and disembodied technical change in the investment-goods producing sectors that leads to cheaper investment goods of the same quality. Both phenomena are at work in a modern economy and both are important, but they need to be kept separate. The picture has become somewhat ambiguous with the paper on

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[42] Estimates by Gordon (1990) were combined with a variant of the MFP residual in (21) to get at vintage quality effects in the capital equipment used by the U.S. manufacturing industry. The quality-adjusted equipment stock was found to grow at a significantly more rapid rate than its unadjusted counterpart (7.28 versus 4.37 percent) over the period 1949-1983, and the contribution of embodiment effects to the MFP residual was about 20 percent. Wolff (1996) reports a somewhat larger number for the U.S. economy as a whole, using different techniques, but concludes that his estimates are comparable when put on the same basis.

what Greenwood, Hercowitz, and Krussel (1997) term "investment-specific technical change" and the critical reaction by Oulton (2007).[43]

B. Production Versus Welfare-Based Growth Accounting

1. The model set out in Section II is built on the assumption that output gross of depreciation, and the corresponding production function, is the appropriate basis for growth accounting. However, early contributors, like Denison (1962), based their estimates on a welfare concept of product in which output is measured net of depreciation. As Denison puts it: "The proper goal of society and objective of policy clearly is the maximization of net product, from which this duplication [i.e., the depreciation of capital] has been eliminated, rather than gross product (1962, page 24)." The case for net output received a boost from Weitzman (1976), who linked annual national *income* (not net *output*!) to the time-discounted flow of future consumption. The net concept of output continues to be a viewed as a viable competitor to the traditional GDP concept (e.g., Sefton and Weale (2006)).

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[43] There are a number of other important contributions to the embodiment literature that merit a much longer treatment. There is the Nelson (1964) average-age model, in which output, labor, and technology are treated as aggregates, but capital enters the aggregate production function as individual vintages, not as a jelly aggregate. In this formulation, embodiment effects enter via a variable that measures the average ages of the stock. Variants of this model have subsequently appeared (Wolff (1996)), but they tend to rely on econometric estimation rather than on pure growth accounting techniques.

Solow et. al. (1966) develop an ingenious vintage model without capital-labor substitution (see also Solow (1987). Separate fixed-proportion functions, $Q = f^v(K^v,L)$, are permitted, as are separate capital goods, and an equilibrium is obtained that is similar to the aggregate solution. The obsolescence and retirement of older, less-productive, types of capital due to competition from newer, more productive, capital goods is the mechanism by which technology change takes place. This alternative formulation addresses the common complaint levied against the neoclassical model (implausible substitution and aggregation assumptions), and resonates with the mechanisms through which high-technology capital like computers affect growth. It also leads to the conclusion that the equilibrium age structure of the capital stock will not change even though superior new capital goods are appearing at a steady rate. See also the papers by Harper (2007) and Diewert (2009).

Since net income is a contemporaneous reflection of the future stream of consumption and thus of consumer welfare, a concept of output net of depreciation (net product) seems like a reasonable candidate for use in growth accounting. But what would such a model look like? How would it compare or coexist with the Solow-Jorgenson-Griliches gross output model? This issue can be explored by inserting the terms on the Jorgensonian user cost (19) into the GDP/GDI accounting identity:

$$(22) \qquad p^C_t C_t + p^I_t I_t = w_t L_t + [(r_t - \rho_t)\, p^I_t K_t + \delta p^I_t K_t].$$

The terms of the right-hand side of this equation are the components of gross value added: labor income, the return to capital, and capital depreciation. Net product on the left-hand side is obtained by subtracting the depreciation term from both sides of the equation (Hulten (1992a), Hulten and Schreyer (2009)). Under geometric depreciation, the perpetual inventory equation in (18) can be expressed as $\Delta K_t = I_t - \delta K_t$, with the result that (22) becomes

$$(22') \qquad p^C_t [C_t + (p^I_t/p^C_t)\, \Delta K_t] = w_t L_t + (r_t - \rho_t)\, p^I_t K_t.$$

The right-hand of (22') is value added net of depreciation, or net income. The term in square brackets, $N_t = C_t + (p^I_t/p^C_t)\, \Delta K_t$, is then defined as *net real output* measured in consumption units. It is also the concept proposed for a welfare-based MFP residual based on (22'). But, following the rules of Divisia growth rates, there also needs to be a potential function in order to obtain an index of the welfare-based MFP index, A^N_t. Some form of the utility function is one possibility, but a net production function of the form $N_t = A^N_t G(L_t, K_t)$ seems closer to what Denison and others have had in mind. This formulation says that labor and capital produce net units of output, and that the index A^N_t is the residual net output not generated by those inputs.

The problems with the net output approach are similar to the problems encountered in choosing between industry output gross of *intermediate goods* and industry gross (of depreciation) value added.[44] This analogy raises a parallel question of interpretation. Does the interpretation of A^N_t make intuitive sense? To answer this in the affirmative is to say that technical change and R&D spillovers augment output only net of depreciation. That interpretation seems to be highly implausible given the nature of technology.

Indeed, what is net *output* as an economic entity, as opposed to net *income*? The price and quantity of gross output, Q_t, and its price can be observed from market transactions, but net output N_t and its price cannot because that is not how transactions are structured.[45] Moreover, since capital and intermediate goods are produced within the economic system, are they too to be treated as net outputs and therefore as net inputs to production? What does a net input-output table look like?

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[44] Under one interpretation of net output, the depreciation is really an intermediate good that should be netted out against gross production just like any other intermediate good. In this view, capital is seen as a bundle of productive services that are given up bit-by-bit over time as the capital good is used up in production. The problem is that capital input comes in the form of services, not as bits of the physical capital stock. Capital services may decline over the life of the asset, and this causes a loss in asset value but not necessarily in its quantity. The case of the "one-hoss" shay (or "light bulb") asset is a useful example. The one-hoss shay retains its full productive capacity to generate services until it is retired from service, with no part of the physical asset being "used up" along the way and thus no "intermediate good" to deduct when figuring net product. The one-hoss shay asset does, on the other hand, lose its value over time as the date of its retirement approaches. This value depreciation is a charge against income, not against the physical amount of capital, and to equate the two is to commit the Triplett (1996) fallacy (Hulten and Schreyer (2009)).

[45] The output of the auto industry, for example, is observed to be the gross volume of the vehicles that emerge from the assembly line over a particular span of time (or their constant-price value). Fractions of autos are nowhere in evidence (the fraction left over after a deduction for depreciation), nor are their net prices and quantities evident in the show rooms. Subtracting depreciation from the value of the auto yields a measure of the net income associated with auto production, not the volume of autos produced.

An issue also arises when the rate of depreciation is not constant at a geometric rate δ (for example, when the pattern of depreciation has the "one-hoss shay" form). The geometric case is convenient for this analysis because δ is both the rate of depreciation of the value of existing capital, and equally the rate at which this capital loses its productive capacity. In the non-geometric case, these two processes follow different time paths (Jorgenson (1973)), with the result that $\Delta K_t \neq I_t - \delta K_t$, and the $\delta p^I_t K_t$ term on the right-hand side of (22) does not cancel out the corresponding term on the left-hand side (Hulten and Schreyer (2009)). In this situation, net product is different from net income, raising questions of interpretation and specification.

If net output were the only way to introduce welfare considerations into growth accounting, the problems might be tolerated (after all, the conventional gross output residual is not without its own flaws). However, there is another way to bring the consumption side of the economy into the growth accounts, a way that complements the existing Solow-Jorgenson-Griliches gross output residual. A consumption-based residual can be obtained from an analysis similar to that in Weitzman (1976), by regarding capital as an intertemporal intermediate good (Hulten (1979)). In this approach, the results of the Domar inter-industry aggregation can be applied to the intertemporal welfare problem to yield an intertemporal residual that measures the shift in the intertemporal production possibility frontier. Since the shift is measured along the utility-maximizing path of the economy, it captures the change in consumption wealth, and thus welfare, associated with costless technical change.[46] Because this new

[46] Basu and Fernald (2002) make a similar point in an inter-industry context. They point out that the shift in the production possibility frontier from the curve bb to aa in Figure 1 can be read as a change in utility, as the economy makes the transition from one equilibrium point to another.

residual is the weighted sum of the standard MFP gross output residuals, where the weights are equivalent to the Domar weights of inter-industry analysis, the two types of residual are complements rather than substitutes (indeed, when technical change is Harrod neutral and constant over time, the two concepts converge). By implication, there is no need to replace the conventional gross-output MFP residual with one based on net output in order to capture welfare effects.

2. The dividing line between the supply-side of economic growth and consumer welfare is complicated further by the fact that product innovation sometimes leads to goods that are entirely new to the market (the internet is a prime example). New goods cannot be handled with the hedonic and price-overlap techniques used to adjust output for changes in quality, because those techniques presume improvements in existing models. When a new good appears, it typically brings with it an increase in consumer surplus (see, for example, Hausman (1997)). Ignoring this consumer surplus (the usual case) may end up understating the true magnitude of efficiency-adjusted output growth, thus understating the role played by technical change as a source of growth during periods of rapid innovation.

C. Beyond Perfectly Competitive Markets

The non-parametric (non-econometric) nature of growth accounting is made possible by the assumption of competitive markets in which prices are equal to marginal cost. In this case, cost shares are equal to the corresponding output elasticities. This equality does not hold in non-competitive markets where prices are likely to deviate from marginal cost. The marginal-cost markup developed by Hall (1988) explores this issue in the context of monopolist competition, as does Basu and Fernald (2002). Some degree of monopolistic pricing is also an intrinsic part of the product variety and quality ladder models described in Barro (1999), the Schumpeterian framework in Aghion and Howitt (1992, 2007), and the models described in Greenwood and Jovanovic (2001). These (and other) models introduce a degree of complexity and realism that goes beyond the simple competitive-market framework.

However, the added dose of realism comes at a cost. These models generally require econometric solutions to get at the complexity of the resulting models (this is apparent in equation (5') which captures endogenous growth effects). The econometric approach may correct for some of the biases in the non-parametric Solow MFP residual, but it can introduce estimation and specification biases of its own (see, for example, Nadiri and Prucha (2001)).[47] Moreover, an added dose of realism is not the same thing as realism itself. It has been known for a long time that the conditions under which capital and technology can be functionally aggregated are extremely unrealistic (again, Fisher (1965, 1969)).[48] There are also many omitted, mismeasured, or unquantifiable variables that affect growth but are not captured by existing models or are not in the data needed to test the models. The real world is a very messy place, and fundamental

[47] "We note that any misspecification of the underlying technology of the firm will typically lead to inconsistent estimates of technical change and the determinants of the investment decisions. A simple illustration of misspecification is the case where the true technology is translog but the hypothesized model is Cobb-Douglas, or the case where the input adjustments involve considerable time lags but are ignored, or where the expectation process is not taken into account or not formulated properly" (Nadiri and Prucha (2001), page 104). The Cobb-Douglas functional form is widely used in macroeconomic models of economic growth (but typically not in growth accounting)

[48] In the first sentence of his 1957 article, Solow leaves no doubt on this point: " … it takes something more than a "willing suspension of disbelief" to talk seriously of the aggregate production function." This remark applies equally to non-parametric growth accounting and to production function estimation. By extension, it requires a further leap of faith to believe that adding theoretical bells and whistles to the aggregate production function will significantly increase the realism of the analysis. The role of the production function is to organize the data in an internally consistent way, and to serve as a framework for interpreting the results.

technological innovation is often idiosyncratic and episodic (Weinberg (2006)). In this context, different empirical and theoretical growth models (including the simple growth accounting model) are better seen as complements that offer different insights into the growth process rather than as competing descriptions of reality.

V. Conclusion

The Solow residual is now some 50 years old. It has evolved over time and has become the work horse of empirical growth analysis. As Solow (2001) puts it: "Like my children, it has aged well, and has produced many grandpapers (page 173)." The residual is now part of the official statistical *repertoire* of many countries, through the productivity program of the U.S. BLS, the newly developed EU-KLEMS productivity data base, and the program at the OECD. What ever else its flaws may be, fecundity is not among them.

Because of this success, it seems fitting to end this survey with another remark made by Robert Solow, this one in his 1987 Nobel Lecture:

> "… I would like to remind my colleagues and their readers that every piece of empirical economics rests on a substructure of background assumptions that are probably not quite true. For instance, these total-factor-productivity calculations require not only that market prices can serve as a rough-and-ready approximation of marginal products, but that aggregation does not hopelessly distort these relationships. Under those circumstances, robustness should be the supreme econometric virtue, and overinterpretation the endemic econometric vice. So I would be happy if you were to accept that [growth accounting results] point to a qualitative truth and give perhaps some guide to orders of magnitude" Solow (1988), page xxii.

What qualitative truth does growth accounting reveal? This, of course, depends on the country, the sector, and the time period of the analysis. For the U.S., BLS estimates for the U.S. private business sector show that output per unit labor grew at an average annual rate of 2.5 percent per year over the period 1948 to 2007. At this rate, the level of output per worker more than quadrupled, a stellar performance considering the length of the period involved and the fact that output per worker is one of the key factors that determine the standard of living. What accounts for this success? BLS estimates indicate that somewhat more than half (58 percent) of the increase was due to the growth in MFP and the balance to input growth. Within the latter, there was a shift in the composition of capital toward information and communication technology (ITC) equipment.

Growth accounting also reveals that the growth rate of Europe over recent years was only half that of the U.S. This result comes from the analysis of the EU-KLEMS data set for the period 1995 to 2005 by van Ark et. al. (2008), which reveals that output per hour worked in the market economies of the 15 countries in the European Union grew at an average annual rate of 1.5 percent, while the corresponding rate in the U.S. was 3.0 percent. Moreover, the drivers of growth were quite different: MFP explained about one-half of the U.S. growth rate, but only one-fifth of the EU rate. EU growth relied more heavily on the growth of capital per hour worked, and within capital, more heavily on non-ITC capital.

These two comparisons, BLS and EU/US, are based on a concept of capital that excludes intangible assets like R&D, brand equity, and organizational capital. As noted in Section III, adding these intangibles to the growth account for the U.S. changes the picture substantially. Corrado, Hulten, Sichel (2006) report that the inclusion of intangibles increases the growth rates of output per hour in the U.S. non-farm business sector by 10 percent for the 1995-2003 period. This is a small overall effect, but the role

of MFP as a driver of growth changes significantly, moving from 50 percent without intangibles to 35 percent when they are included. The role of ITC capital is also diminished, and intangible capital is found to account for more than a quarter of growth. A similar pattern is found in the U.K. during roughly the same period, though the contribution of MFP is smaller both with and without intangibles (Haskell and Marrano (2007)). Fukao et. al. (2007) find that the introduction of intangibles also matters in Japan's growth accounts, though tangible capital is by far the most important source of growth, and the contribution of MFP growth is quite low. As with the EU versus US comparison, different countries exhibit different patterns of growth.

The same is true of broader cross-national studies that include developing economies, though problems of data quality and availability make even qualitative comparisons problematic. Hulten and Isaksson (2007) show that the dichotomy between MFP and capital formation depends heavily on what assumptions are made about labor's share in income: when the labor share as actually reported in the data is used, MFP has a negative growth rate in the low and middle-income countries of the world (excluding the rapidly growing Asian Tiger economies); when a common labor share of two-thirds is imposed on the analysis, MFP growth rates are higher for all country groups (an average increase of 0.80 percentage points to an average growth rate of 1.05 percent per year), and only the low-income economies exhibit negative MFP growth. The main contribution of growth accounting in this situation lies in its ability to identify a glaring need for better data. Growth accounting is, first and last, a diagnostic technique that relies more on good data than on high theory.

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