Dynamics, Disequilibrium, and Marxian Economics: A Formal Analysis of Temporal Single-System Marxism

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Received July 27, 2003; accepted December 21, 2004

Abstract

This article analyzes the temporal single-system interpretation (TSSI) of Marx's economics. From a methodological viewpoint, the TSSI lacks both a clear definition of equilibrium and a rigorous analysis of disequilibrium dynamics, and the dynamic framework is incomplete. From a substantive viewpoint, temporal single-system (TSS) claims are trivially obtained by assuming that goods exchange at values, apart possibly from out-of-steady-state random deviations. Finally, the proof of the law of the tendential fall in the profit rate is tautologically true, but its theoretical relevance is unclear.

JEL classification: *E11; B24; B41*

Keywords: *disequilibrium; dynamics; Marxian economics; theory of value*

1. Introduction

The temporal single-system interpretation (TSSI; see, e.g., Kliman and McGlone 1988, 1999; Freeman and Carchedi 1996) aims to provide a rigorous and "*general* formalisation of Marx's theory of value—that is a different paradigm for economics in Marx's framework" (Freeman and Carchedi 1996: xiii). According to temporal single-system (TSS) authors, TSS models vindicate the internal consistency of Marx's theory and prove "in completely general form the propositions which Marx has been accused of getting wrong" (xviii): in the TSSI,

Author's Note: I am grateful to Meghnad Desai, Duncan Foley, Andrew Kliman, Simon Mohun, Ernesto Screpanti, and Ian Steedman for discussions on the TSSI and for valuable comments on an earlier draft of this article. I am greatly indebted with Gary Mongiovi, Ajit Sinha, and Gilbert Skillman for detailed comments and suggestions. Special thanks go to Alan Freeman for various helpful clarifications on the TSS model. The usual disclaimer applies.

Review of Radical Political Economics, Volume 37, No. 4, Fall 2005, 517-529
DOI: 10.1177/0486613405280809

> (a) all of Marx's aggregate value-price equalities hold; (b) values cannot be negative; (c) profit cannot be positive unless surplus-value is positive; (d) value production is no longer irrelevant to price and profit determination; (e) the profit rate is invariant to the distribution of profit; (f) productivity in luxury industries affects the general rate of profit; and (g) labor-saving technical change can cause the profit rate to fall. (Kliman and McGlone 1999: 55)

According to TSS proponents, claims a-g hold thanks to the *temporalist* emphasis on disequilibrium and on a sequential determination of values and prices based on historical costs—as opposed to *simultaneist* approaches based on current values and prices, and to a *single-system* approach in which values and prices are determined interdependently, instead of being derived from separate accounting systems as in traditional *dualistic* approaches. The ability to replicate *all* of Marx's results "clearly recommends [the TSSI] as a superior interpretation" (Kliman and McGlone 1999: 35), as opposed both to the standard approach, which leads to the "transformation problem" (see, e.g., Desai 1991 for a survey), and to the "New Interpretation" (Duménil 1980; Foley 1982), which provides a coherent model of Marx's value theory by choosing *only* a "subset of propositions to maintain . . . as the essential core of the theory" (Foley 1982: 40).

Given the scope and relevance of claims a-g, the TSSI has stimulated an intense debate. Several critiques have been expounded on the TSSI both from a broad philosophical perspective (e.g., Foley 2000; Laibman 2000) and on specific issues, such as the TSS approach to Marx's theory of exploitation (Mohun 2003) and to the theory of the falling profit rate (e.g., Foley 1999; Laibman 1999). In this article, a priori philosophical issues are left aside, whereas a thorough formal analysis of the TSS *quantitative* interpretation of Marx's theory is presented. Despite a critical stance, by focusing on the logic and implications of the TSS formalization of value theory and on the robustness of TSS methodological and substantive claims, this article aims to provide a positive contribution to the debate. The formal approach adopted here matches the TSS insistence on the need to compare alternative approaches mainly on their ability to replicate Marx's propositions.[1]

More specifically, unlike other formal analyses of the TSSI that focus on discrete time models (e.g., Skillman 2001; Mongiovi 2002; Veneziani 2004), this article analyzes Freeman's (1996, 1998) surprisingly neglected continuous time model,[2] which provides both the most general formalization of the TSSI, aiming to replace "the simultaneous equations approach . . . with an economically superior and more general formalism based on Marx's analysis" (Freeman 1996: 225), and the neatest representation of its main features.

Based on Freeman (1996, 1998), an original interpretation of the TSSI is proposed.[3] From a methodological viewpoint, section 2 shows that in the TSSI, neither equilibrium nor disequilibrium is clearly defined; that the TSSI lacks a coherent (dis)equilibrium methodology; and that the dynamic framework is not rigorously specified. Hence, it is unclear whether the TSSI can provide relevant insights on disequilibrium and dynamics in Marx's theory, and the definition "non-equilibrium economics" (Freeman and Carchedi 1996) seems unjustified.

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1. For a forceful critique of the TSS "replication criterion," however, see Mohun (2003).
2. Even Duménil and Lévy (2000) considered only a discrete time version of Freeman's (1996) model.
3. This article generalizes Veneziani's (2004) analysis of TSS discrete time models.

Section 3 shows that neither the sequential determination of values (Duménil and Lévy 2000) nor the underdetermination of TSS models (Skillman 2001; Mongiovi 2002) is crucial to derive substantive claims a-f. The claims are trivially obtained by assuming that goods exchange at values, apart possibly from unexplained out-of-steady-state deviations, which are in turn assumed to cancel out in aggregate. That is, the traditional transformation problem is merely assumed away, whereas the TSS "transformation problem" is defined as focusing on the random deviations of market prices from TSS values and is solved by imposing an empirically unverified and analytically unwarranted restriction on the random shocks. "Disequilibrium," "dynamics," and the extra unknowns are necessary (but not sufficient) only to distinguish market prices and TSS values. Finally, section 4 shows that the TSS proof of the Marxian law of the tendential fall in the profit rate, claim g, is tautologically true within the TSS framework, but its theoretical relevance is unclear.

2. The TSS Continuous Time Model: Methodological Issues

For the sake of notational simplicity and conceptual clarity, a slightly modified version of Freeman's (1996, 1998) model is presented, in which at all t, K_t is the $n \times n$ matrix describing the sectoral distribution of the stocks of all produced goods (k_{ijt} is the stock of good i in sector j at t); C_t is the $n \times n$ matrix of constant capital employed; V_t is the $1 \times n$ vector of variable capital employed (in hours); $£\mathrm{L}_t$ is the $1 \times n$ vector of V's value-creating capacity, "the total new value added by labour per unit time or the monetary expression of hours worked per unit time" (Freeman 1998: 150);[4] X_t is the $n \times n$ diagonal output matrix; W_t is the $n \times n$ matrix of goods purchased by workers; B_t^c is the $n \times n$ matrix of goods consumed by capitalists; and B_t^w is the $n \times n$ matrix of goods consumed by workers. Unlike in Freeman (1996, 1998), stocks and flows are clearly distinguished, and all variables represent instantaneous flows, apart from K.[5] Finally, p_t is the $1 \times n$ price vector, and the scalar μ_t is the *value of money*, that is, "the quantity of value expressed in one unit of current money" (Freeman 1996: 273). As a first approximation, it is assumed that $\mu_t = 1$, all t, and that one unit of money "represents one hour of socially necessary labour time" (235).

For any $\mathrm{y} \in \Re^n$, let $\dot{\mathrm{y}}_t \equiv d\mathrm{y}/dt$, and let $\tilde{\mathrm{y}}_t \equiv \mathrm{y}_t 1$, where $1 = (1, \ldots, 1)'$. The *stock accounting identity* describes the motion of aggregate physical quantities.[6]

$$\dot{\tilde{\mathrm{K}}} = \tilde{\mathrm{X}}_t - \tilde{\mathrm{C}}_t - \tilde{\mathrm{B}}_t^c - \tilde{\mathrm{B}}_t^W. \tag{1}$$

For the sake of simplicity, it is assumed that workers do not save (Freeman 1996: 268), so that $\tilde{\mathrm{W}}_t = \tilde{\mathrm{B}}_t^w$, all t, and K_t can be interpreted as the matrix of stocks held by capitalists; and that "the value of variable capital is equal to the current price of wage goods" (269), that is, $\mathrm{V}_t = \mathrm{p}_t \mathrm{W}_t$, all t.

4. As argued below, no rigorous definition of $£L_t$ is provided, and thus its interpretation is unclear. Actually, the verbal definition is also rather confusing because hours *are* units of time.

5. This is consistent with Freeman's (1996, 1998) model, whose main equations contain only *changes* in stocks (apart from *K*).

6. In Freeman (1996), rather lengthy derivations of the main relations are provided. They do not, however, provide any relevant insights and are simply skipped here.

A basic tenet of the TSS approach is that the total change in the price vector, $\dot{p}_t$, can be divided into the change due to production (value formation), $\dot{p}_t^p$, and the change due to circulation (yielding observed market prices), $\dot{p}_t^c$.

$$\dot{p}_t = \dot{p}_t^p + \dot{p}_t^c. \tag{2}$$

Let λ_t denote the $1 \times n$ vector of unit values. In the TSSI,

> shorn of the mystery with which generations have shrouded it, λ_t is another name for $[p_t + \dot{p}_t^p\, dt]$, the partial derivative of price with respect to production, and the celebrated transformation of values into prices is the partial derivative of price with respect to circulation, $[\lambda_t + \dot{p}_t^c\, dt]$. (Freeman 1998: 150–51)

Let $\hat{K}$ be the $n \times n$ diagonal matrix formed from $\tilde{K}$. The *value accounting identity* states that total new value created—new value produced, p_tX_t, plus revaluation of aggregate stocks, $\dot{p}_t^p\hat{K}_t$—is equal to value transmitted by constant capital, p_tC_t, plus value added by labor power, $£L_t$, in production:

$$\dot{p}_t^p\hat{K}_t + p_tX_t = p_tC_t + £L_t. \tag{3}$$

The *price accounting identity* can be obtained similarly as

$$\dot{p}_t\hat{K}_t + p_tX_t = p_tC_t + £L_t + \dot{p}_t^c\hat{K}_t. \tag{4}$$

In every sector, the *rate of surplus value generation* (Freeman 1998: 155) corresponds to new value produced, plus revaluation of stocks, $\dot{p}_t^pK_t$, minus value consumed in production, $p_tC_t + V_t$, so that $S_t = p_tX_t + \dot{p}_t^pK_t - p_tC_t - V_t$ or, using equation (3),

$$S_t = £L_t - V_t + \dot{p}_t^p(K_t - \hat{K}_t), \tag{5}$$

where $\dot{p}_t^p(K_t - \hat{K}_t)$ is value redistributed among sectors due to competition among capitalists. Taking into account circulation, the *rate of profit generation* (Π_t) is derived analogously as "the rate of surplus value generation plus the transfer vector, $[\dot{p}_t^c\hat{K}_t]$" (Freeman 1998: 155):

$$\Pi_t = £L_t - V_t + \dot{p}_t^p(K_t - \hat{K}_t) + \dot{p}_t^c\hat{K}_t. \tag{6}$$

The *value rate of accumulation* is $d(p_t\tilde{K}_t)/dt = p_t\dot{\tilde{K}}_t + \dot{p}_t\tilde{K}_t$. Therefore, by substituting for $\dot{\tilde{K}}_t$ and $\dot{p}_t\tilde{K}_t$ from equations (1) and (4), $d(p_t\tilde{K}_t)/dt = £\tilde{L}_t - p_t(\tilde{W}_t + \tilde{B}_t^c)$, or, equivalently, given equation (5) and $V_t = p_tW_t$,

$$\frac{d(p_t\tilde{K}_t)}{dt} = \tilde{S}_t - p_t\tilde{B}_t^c. \tag{7}$$

Finally, Marx's general profit rate is defined as the ratio of aggregate surplus value to the aggregate value of stocks, $r_t = \tilde{S}_t / p_t \tilde{K}_t$ and its motion is given by

$$\frac{\dot{r}_t}{r_t} = \frac{\dot{\tilde{S}}_t}{\tilde{S}_t} - \frac{\tilde{S}_t - p_t \tilde{B}_t^c}{p_t \tilde{K}_t}. \tag{8}$$

According to Freeman, equations (1-8) represent the TSS continuous time model of value theory or, more precisely, the TSS "formalism": it "is not a model. It does not yield predictions or 'solutions' from particular restrictive assumptions. It is an axiomatic system" (Freeman 1996: 225). Although this suggests interpreting equations (1-8) as a set of mere accounting identities, there is a degree of ambiguity about their logical status, because Freeman sometimes seemed to view them as a proper model determining prices and values: "Equations [3] and [4] are the basic dynamic relations of price and value. . . . Given only the observed data of the economy they are determinate and distinct vectors of values and prices" (269). This ambiguity cannot be resolved because some variables are undefined (e.g., $£L_t$ and the monetary expression of labor time, discussed below), whereas others have no obvious empirical interpretation, for example the price derivatives in equation (2). Nonetheless, several important methodological features of the TSSI can be highlighted.

First, equations (1-8) show that, contrary to Freeman and Carchedi's (1996: xviii) suggestion, TSS claims a-f are not *proved*; they are essentially assumed, given a set of empirically unverified, analytically unwarranted, and sometimes plainly arbitrary postulates. Claims a and c trivially follow from equations (2-4) and (5-6) by setting $\dot{p}_t^c \tilde{K}_t = 0$, all t, which reflects the assumption that circulation can only redistribute value. This additional condition is simply imposed: it is neither shown to be empirically relevant nor proved to be analytically consistent with, let alone implied by, equations (1-8). Claims d-f follow by *defining* the price rate of profit as the average value rate of profit. Claim b can only be an axiom, given the lack of restrictions on equations (1-8). Finally, the robustness and the significance of all claims crucially depend on the arbitrary assumptions (discussed below) that p_t and the *undefined* $£L_t$ are nonnegative, and that price and labor magnitudes are commensurable—actually, even more strongly, equivalent.

Second, given the underdetermination of equations (1-8), and especially under Freeman's own interpretation of the "formalism," any conclusion on the path of any variable derived *only* from equations (1-8)—such as the TSS proof of the tendential fall in r_t (see section 4, below)—would necessarily be arbitrary or at most tautologically true but theoretically irrelevant. Actually, equations (1-8) contradict Freeman's own claim that alternative approaches to value theory "should be tested by the normal method of science: which best explains the observed facts" (Freeman 1996: 249). They do not *explain* facts—neither prices, nor values, the profit rate, wages, production, accumulation, and so on are *explained*—and they prove little; indeed, they tautologically escape evaluation and falsification.

Third, despite the emphasis on disequilibrium and dynamics, neither plays an essential role in the model. Unless one interprets equilibrium in the narrow and rather arbitrary sense of "an economy reproduced perfectly and identically" (Freeman 1996: 251), nothing in equations (1-8) is specific to a disequilibrium approach: they are not derived under the assumption of market clearing, but nor are they inconsistent with it. Actually, equations (1-8)

are compatible with virtually any economy, including one on a dynamic equilibrium path with markets clearing in every period. Besides, it is clearly untrue that changes in stocks are caused only by "the gap between supply and demand" (251) or, more generally, that the results derived from the model cannot be stated in an equilibrium framework (249).

In general, the equilibrium concept is not rigorously defined, and Freeman's informal discussion seems questionable. Not only simultaneous equation systems are automatically and arbitrarily associated with an equilibrium approach: "simultaneous equations impose an immediate identity of supply and demand" (Freeman 1996: 251); with a rather puzzling logical leap, every equilibrium concept is identified with *Walrasian* general equilibrium (225–27). Moreover, the very logical status of the equilibrium concept seems unclear, as shown by Freeman's claim that there are "features of the formalism which distinguish it from equilibrium" (225): it is not evident in what sense a *mathematical system* may be distinguished from a *solution concept*.[7]

Similarly, the dynamic dimension of equations (1-8) reduces to the fact that variables have time subscripts and can change throughout time in an unexplained fashion: nothing makes equations (1-8) inherently dynamic. Moreover, equations (1-8) are inconsistent with the TSS emphasis on sequentiality and with the claim that value is determined "by production *followed* by circulation" (Freeman 1998: 147), because production and circulation take place simultaneously in an infinitesimal amount of time; although Freeman (1996: 228) criticized the assumption of a fixed, identical period of reproduction in each production process—typical of discrete time models—he assumed that production and circulation periods "are not only of the same duration, but equal to zero. Freeman's model does not solve any of the problems concerning the strictly positive duration of production and circulation periods" (Duménil and Levy 2000: 125); they are basically assumed away.

Besides, if all stocks turn over during the period of production, price derivatives disappear from all equations (except from equation (2), whose significance becomes even more obscure), and the TSS model turns out to be "ultra-simultaneist." For instance, equation (3) becomes a "simultaneist" equation applied to an infinitesimal period of time (Freeman 1996: 269):

$$\mathbf{p}_t\mathbf{X}_t = \mathbf{p}_t\mathbf{C}_t + £\mathbf{L}_t \tag{3'}$$

Thus, contrary to TSS claims, sequentialism and differences between input and output prices are not an essential part of the continuous time model but at most a by-product of fixed capital. Instead, abstracting from fixed capital, the main methodological contribution of the TSS continuous time model seems to concern the importance of choosing a sufficiently short production/circulation period, such that differences in input and output prices can be assumed away.

Although the previous arguments raise several doubts about TSS claims, from a methodological viewpoint, as noted above, the most puzzling feature of the model is its incompleteness, which makes the logical status of the main relations unclear. For instance, given that $£\mathrm{L}_t$ is undefined and it is not evident how $\dot{\mathrm{p}}_t^p$ and $\dot{\mathrm{p}}_t^c$ can be retrieved, it is arbitrary to say that "given only the observed data of the economy . . . determinate and distinct vectors

7. See Vercelli (1991) for a thorough analysis of the concept of equilibrium in economics and philosophy; for a discussion in the context of Marx's theory, see Mongiovi (2002).

of values and prices" (Freeman 1996: 269) can be derived from equations (2-4). In particular, it is unclear how equation (3) can be interpreted as a relation describing value formation, which states that "new value enters the economy at the rate £L_t" (269).

Freeman rather casually suggested that "the new value entering the economy—the value product—is proportional to the time worked by employed labourers" (Freeman 1996: 262); but because he did not define, or derive, the proportionality factor, £L_t remains unexplained. A natural option would be to set £$L_t = \varepsilon_t L_t$ (see, e.g., Kliman and McGlone 1999), where L_t is the $1 \times n$ vector of direct labor and ε_t is the *monetary expression of labor time* (MELT), which expresses "the social equivalence of money and labor time which is inherent in commodity production" (Foley 1982: 39). Setting £$L_t = \varepsilon_t L_t$, however, raises more questions than it solves, because, as noted by Foley, TSS authors have failed "to put forward a single, consistent definition of [MELT]" (2000: 33) or of an analogous variable that allows us "to move back and forth between money and labor accounts" (7).

Actually, unlike in TSS discrete time models, which contain an *undefined* MELT (Veneziani 2004), in the continuous time model, MELT does not appear at all. At the same time, the theoretical and analytical relevance of the *value of money*, μ_t—which denotes "the quantity of value expressed in one unit of current money" (Freeman 1996: 273) and should convert value and price magnitudes—is unclear. First, neither Freeman's verbal discussion (240–41) nor his formal analysis of μ_t (272–75) convincingly show that in the TSSI, "money at all times has a known and calculable value" (273). According to Freeman,

> [W]e may take the value of money at some given initial starting point as the standard of value (and hence price). Thus, if in 1980 the total assets of the economy were priced at £1000 billion and in 1981 the same goods would have been priced at £1250 billion, then a 1981 pound is worth 1.25 times a 1980 pound; the value of £1 has thus fallen to £0.8 measured in 1980 pounds. (273)

But because no definition of μ_t at the initial period is provided, Freeman's argument can at most account for *changes* in μ_t relative to an arbitrary initial μ_0.

Second, assume, for the sake of the argument, that only changes in μ_t relative to an arbitrary μ_0 matter, and consider Freeman's analysis of the "money rate of profit generation" (Freeman 1996: 274), Π_t^m, which is defined as the money value of profits, $\mu_t\Pi_t$, "adjusted for the *rate of change of the value of money*, multiplied by the price of capital stock" (274), $\dot{\mu}_t \mathbf{p}_t \mathbf{K}_t$. More formally,

$$\Pi_t^m = \mu_t \Pi_t + \dot{\mu}_t \mathbf{p}_t \mathbf{K}_t, \tag{6'}$$

which implies $\Pi_t^m = \Pi_t$, all t, if $\mu_t = 1$, all t. By equation (5), it also follows that

$$\Pi_t^m = \mu_t \mathbf{S}_t + \mu_t \dot{\mathbf{p}}_t^c \hat{\mathbf{K}}_t + \dot{\mu}_t \mathbf{p}_t \mathbf{K}_t. \tag{5'}$$

Thus, μ_t seems nothing more than an index of price inflation[8] that converts *both* a price magnitude, Π_t, *and* a value magnitude, S_t, into a "money" magnitude, Π_t^m: its theoretical relevance is unclear both per se and in the context of Marx's value theory, and from an analyti-

8. The interpretation of μ_t as a sort of price inflation index was hinted by Freeman (1996: 239).

cal viewpoint, it plays no role either in the derivation of claims a-f or, more importantly, in the conversion of labor and money magnitudes. Therefore, without a definition of MELT, not only does £L_t remain unexplained—which calls into question its presumed nonnegativity—but it is also completely arbitrary to assume that $V_t = p_t W_t$, all t, and in general that money and labor magnitudes are commensurable, let alone equivalent.

3. Values and Prices

At first sight, the TSS claim that *all* of Marx's results hold independently from the definition of a variable that converts labor and price magnitudes is rather surprising. This is a crucial theoretical difference with respect, for instance, to the "New Interpretation," whose specific definitions of MELT (net national product at current prices over aggregate labor expended) and of the value of labor power (nominal wage over MELT) retain "the central ideas of the labor theory of value . . . [but] cannot and do not retain all of the results that hold when prices are proportional to labor values" (Foley 1982: 42–43). Despite the convoluted arguments, an often obscure notation, and the hypothetical distinction between $\dot{p}_t^c$ and $\dot{p}_t^p$, however, it can be shown that in the TSSI, all of Marx's propositions hold because rather than being determined interdependently, TSS values and prices are essentially indistinguishable.

Consider a steady state, where $\dot{p}_t = \dot{p}_t^p = \dot{p}_t^c = 0$. Because TSS values are defined as $\lambda_t = p_t + \dot{p}_t^p\, dt$, then by equations (2) and (3), TSS values and prices—albeit undetermined, due to the undefined £L—are identical *by definition* with $p = \lambda = £L(X - C)^{-1}$. Thus, provided one also *assumes* $£L(X - C)^{-1} \geq 0$, the transformation of TSS values and prices is solved by fiat.[9] Moreover, if £L_t is proportional to L_t, then in a steady state, goods are assumed to exchange at embodied labor values.[10] One hundred years of controversies are thus simply assumed away by postulating that the traditional transformation problem does not exist.

If $\dot{p}_t^c = 0$ and $\dot{p}_t = \dot{p}_t^p \neq 0$, all t, then by equations (2-4), TSS values and prices (and, by equations (5-6), surplus value and profits) are identical by definition *even out of a steady state*. Hence, although the undefined variables do make the model underdetermined, and out of a steady state indeed "anything goes," Kliman and McGlone's claim that the TSSI does not eliminate "the inconsistency in Marx's value theory by supplying extra unknowns, in effect by modeling a perpetual disequilibrium in which 'anything goes,'" (1999: 50) seems correct. The extra unknowns, "dynamics," and "disequilibrium" are necessary (but, as shown below, not sufficient) to distinguish TSS values (a sort of money-cost-based prices) from market prices and thus to define the TSS "transformation problem," that is, the out-of-steady-state random deviation of market prices from TSS values. All inconsistencies are removed by assuming that goods exchange at TSS values *at all t* and that any unexplained random deviations cancel out in aggregate.

Hence, it is not surprising that in the TSSI, Marx's results hold "for arbitrary market prices" (Freeman 1996: 227) and independently from a definition of MELT: from a mere analytical viewpoint, ε_t plays no role in the derivation of TSS claims; at most, as noted

9. The nonnegativity of the undefined £L can only be assumed ad hoc. Notice also that if joint production is allowed, then Steedman's (1977) critique can be extended to the TSSI.

10. The actual definition of the proportionality factor is irrelevant from a mere analytical viewpoint.

above, if $£L_t = \varepsilon_t L_t$, then MELT is just a proportionality factor between prices and embodied labor values in a steady state. Actually, assuming a steady state to exist, TSS value theory seems to reduce to the analysis of the convergence of market prices to $p = £L(X - C)^{-1}$, that is, to embodied labor values if £L is proportional to L.

Even if $\dot{p}_t^c \neq 0$, however, the theoretical and empirical relevance of the TSS price/value distinction is unclear. To begin with, if stocks turn over during the production period, there is no clear relation between $\dot{p}_t^p$—and thus TSS values λ_t—and any other variable in equations (1-8): if $K_t = 0$, $\dot{p}_t^p$ and $\dot{p}_t^c$ disappear from all equations except for equation (2). Hence, without any clear empirical interpretation of the derivatives in equation (2), the definition of TSS values and the TSS value/price distinction are unclear. Actually, if $K_t = 0$, all t, then by equation (3'), $p_t = £L_t(X_t - C_t)^{-1}$, all t, so that if $£L_t$ is proportional to L_t, goods exchange at embodied labor values *at every single instant of time*, even out of a steady state. Even if TSS values λ_t could be computed, it is unclear what additional insights they might offer on this economy.

More strongly, let $\hat{k}_{it}$ be the ith diagonal entry of $\hat{K}_t$: the basic features of the TSSI depend on the assumption that $\hat{k}_{it} > 0$, *for all sectors i and all t*, and thus, again, they seem a by-product of fixed capital rather than an inherent property of the theory. Let X_{it} and C_{it} be the ith column of X_t and C_t, respectively, and let $£L_{it}$ be the ith element of $£L_t$. Then, equation (3) can be written as

$$\dot{\mathbf{p}}_{it}^{p}\hat{k}_{it} + \mathbf{p}_t\mathbf{X}_{it} = \mathbf{p}_t\mathbf{C}_{it} + £\mathbf{L}_{it}, \text{ for all sectors } i. \tag{9}$$

Hence, if $\hat{k}_{it} = 0$, some i, equations (1-8) place no restriction on $\dot{p}_{it}^p$, so that without any clear empirical interpretation of equation (2), the TSS value of good i, λ_{it}, is undefined. Moreover, because there is no relation between $\dot{p}_{it}^p$ and (undefined) value creating capacity, $£L_{it}$—and, a fortiori, productive labor—$\dot{p}_{it}^p$ seems to play no special role in the definition of the value of i. Finally, if $\hat{k}_{it} = 0$, some i, then the condition $\dot{p}_t^c\tilde{K}_t = \dot{p}_{1t}^c\hat{k}_{1t} + \ldots + \dot{p}_{nt}^c\hat{k}_{nt} = 0$ places no constraint on $\dot{p}_{it}^c$, and thus it is unclear that it implies that circulation only redistributes value.[11] But then, based on equations (1-8), $\dot{p}_{it}^p$ and $\dot{p}_{it}^c$ are indistinguishable, and it is difficult to identify them as the changes in price due to production and to circulation, respectively. In other words, a purely empirical property, the existence of fixed capital *in all sectors*, is a logical prerequisite for the essential features of the TSSI, including the very definition of values and the price/value distinction.

Even if $\hat{k}_{it} > 0$, all i, t, the relevance of TSS values and of the TSS price/value distinction is unclear. At a theoretical level, equations (1-8) are puzzling "and a considerable deviation from Marx's labor theory of value. Sequential values are consubstantial with prices, within a *labor-market price* theory of value" (Duménil and Lévy 2000: 127). Moreover, unlike in discrete time models, equations (1-8) establish only the dependence of values on prices, but not "the other single-system interdependence . . . of output prices on value magnitudes" (Kliman and McGlone 1999: 37); the dynamic path of market prices seems to be best understood simply in terms of past and present market prices. Actually, assuming $£L_t$ to be defined, if equations (1-8) are interpreted as a model rather than a set of identities, they assign primacy to observed market prices; given the path of physical magnitudes, the sys-

11. For instance, if $\hat{k}_{it} = 0$, some i, all t, then $\dot{p}_t^p = 0$, all t, is consistent with $\dot{p}_t^c\tilde{K}_t = 0$, $\dot{p}_{it}^c \neq 0$, and $\dot{p}_{it} \neq 0$, all t, that is permanent growth (or decline) of price, and TSS value, in sector i.

tem is entirely determined by the unexplained sequence $\{p_t, \dot{p}_t\}_{t=0,1,\ldots}$. Indeed, it is quite telling that, to avoid a problem of infinite regress, at $t = 0$, Freeman (1996: 230) directly *defined* values as observed market prices. Moreover, at the empirical level, production and circulation are indistinguishable and only one variable is observed, because "prices appear with circulation" (Freeman 1996: 234). But then, given that everything happens in the sphere of circulation, it is legitimate to wonder why, even outside a steady state, one should be interested in TSS values as distinct from market prices in the first place.

4. The Falling Rate of Profit

The law of the tendential fall in the profit rate (FPR) is among the most debated issues in Marxian economics, together with its refutation by Okishio's theorem (OT).[12] In the received view, by proving that "if technical change is introduced by capitalists only when it is cost reducing at current prices, then the equilibrium rate of profit will rise" (Roemer 1981: 97), OT settles "in a fundamental way, the Marxian conjecture of [FPR]" (98). Given Okishio's restrictive assumptions on technical progress and the unrealistic description of the perfectly competitive economy with identical firms, TSS authors may be right in stressing that such an interpretation stretches the result too far. TSS models do not, however, refute OT "in the strict logico-mathematical sense" (Kliman and McGlone 1999: 53), because they are based on patent violations of the formal assumptions of the theorem. The aim of this section is to show that the TSSI does not provide a robust analytical framework to evaluate the claim that the profit rate must, or even *might*, fall due to accumulation and technical change.

In continuous time, the TSS proof of claim g is based on equation (8): if aggregate variable capital and the aggregate rate of value creation are constant, so is aggregate surplus value; hence, if $d(p_t\tilde{K}_t)/dt = \tilde{S}_t - p_t\tilde{B}_t^c > 0$, then r_t falls, proving that FPR is "not merely valid, but scientifically and rigorously exact" (Freeman 1996: 272). This conclusion is rather surprising: equation (8) is trivially true within the TSS framework, but it says nothing about investment and technical progress. The relations among technical change, the undefined $£L_t$, and V_t are unexplained. Similarly, because the value rate of accumulation is the change in the value of all sorts of stocks of all commodities in the economy, investment does not play a significant role in the proof; in principle, the FPR might be entirely due to workers' accumulation of consumption goods. Far from making the result "absolutely general," the lack of any "special assumptions concerning wage rates, supply and demand, capitalist behaviour or the structure of production" (271)[13] seriously undermines its significance. It is even difficult to characterize such a result as specifically Marxian: a neoclassical FPR might easily be proved along the same lines by noting that if aggregate capital increases, then the profit rate falls, ceteris paribus, by the law of diminishing returns. In general, without any "special assumptions," it is legitimate to doubt that the TSSI can specify a relevant ceteris paribus scenario in which the denominator of the profit rate changes but the numerator does not.

12. See Okishio (1961); for a survey see, for example, Groll and Orzech (1989) and the essays in Caravale (1991).

13. Notice also that profit maximization and profit rate equalization play no role in the model.

The main problem of Freeman's argument is the assumption that to refute OT, it is sufficient to find an arbitrary path such that a suitably defined profit rate falls, a methodological principle that underlies all the ad hoc algebraic examples of FPR set up by TSS authors.[14] But then, the emphasis on temporalism versus simultaneism seems misplaced and TSS value theory is not necessary to refute OT; it should suffice to note that a sufficiently high deflation rate leads to a fall in monetary profit rates (as, e.g., in Freeman and Kliman 2000). More importantly, the theoretical and empirical relevance of arbitrary algebraic exercises such as Freeman's (1996, 1998) is not evident, and thus it seems legitimate to doubt that they might be suitable to analyze OT, let alone refute it.

5. Conclusion

The temporal single-system approach has stimulated an intense debate in value theory. The analysis of TSS models suggests, however, that former debates are not "superseded" (Freeman 1996: 225). If the TSSI is adopted, arguably, no major insights are gained on either methodological or substantive issues in Marx's theory, but much is lost in terms of analytical rigor and clarity with respect to alternative approaches, due to various ad hoc assumptions and undefined variables, and to the lack of a rigorous (dis)equilibrium framework. Moreover, Marx's claims are trivially obtained by assuming that goods exchange at values, apart possibly for random deviations out of the steady state; what is defined as a "justification of the circular, Hegelian type, [by which] what was initially taken as a premise has . . . been substantiated as a result" (Kliman and McGlone 1999: 44), is a straightforward assumption of the desired result. Therefore, *as an interpretation of Marx's theory*, the TSSI is not particularly persuasive, even though it obviously "corresponds to the original in a way that others do not" (43).

By focusing on existing TSS models, this article does not prove that a more robust formalization of the TSSI cannot be set up. The results presented suggest, however, that a coherent (dis)equilibrium methodology, an explicit definition of all variables, and a clear distinction of values and prices would imply that not *all* Marx's results obtain, as is well known in Marxian economics. In abandoning the assumption that goods exchange at values, one has "to choose some subset of propositions to maintain. This choice is an important one in any theoretical generalization, because the propositions which are maintained in the generalization act as the essential core of the theory in question" (Foley 1982: 40). Contrary to TSS (and Marx's critics') claims, this only leads one to question the literal interpretations of Marx, such as the TSSI, and the idea that Marx's theory is a "package deal" (Kliman 2001: 110), rather than to the conclusion that "not much is left" of Marx's economics. Indeed, much remains to be done in a Marxian perspective both at the theoretical and at the empirical level.

14. See Veneziani's (2004) analysis of the discrete time TSS refutations of OT (e.g., Kliman 1996, 1997; Kliman and McGlone 1999; Freeman and Kliman 2000).

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