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The Transformation from Marx to Sraffa

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I. Introduction

Recent history has seen a tremendous revival of Marxist economic analysis. But this process has also produced its own specific problems, because as Marxist economics gain in respectibility, the temptation to represent itself in respectable terms grows accordingly. And these terms, in the end, are almost always the wrong ones.

There is no question but that Marxism must appropriate all modern developments. But to appropriate them involves much more than merely adopting them. It involves tearing them out of the bourgeois framework in which they appear, examining their hidden premises, and re-situating them (when and if possible) on a Marxist terrain—a terrain which cannot be derived merely by algebraic variation or sociological transformation of the premises of orthodox economics. We must, and indeed we do, have our own ground to stand upon.

It is my contention that the Sraffian, neo-Ricardian, tradition is by far too respectable. Its roots in left Keynesianism are easy to establish, and its refuge in mathematical economics is quite revealing. Nonetheless, the claims made by this school must be addressed, and its real contributions must be separated out from what is merely part of its cloak of respectability.

In this paper I do not intend to reproduce previous criticisms of the neo-Ricardians, nor even to reproduce my own arguments in favour of Marx's theory of value. Instead, in the discussion that follows I would like to show that even within the algebraic framework of which the neo-Ricardians are so proud, there are a host of issues which they do not, and cannot, face. These issues depend crucially on the difference between Marx's concepts and those of the neo-Ricardians. The very same algebra that they use, when asked different questions, will generate different answers. And these answers, it turns out, favour Marx much more than they do the neo-Ricardians.

In the discussion which follows, I will therefore examine in some detail the neo-Ricardian arguments concerning the redundancies and inconsistencies in Marx's theory of value. Since their treatment of both joint production and fixed capital are embellishments on their main argument, and since they are discussed by Emmanuel Farjoun in this volume, I shall ignore them here. An adequate treatment would in any case require a separate analysis.

Throughout this discussion, the difference between value and form-of-value is crucial. Thus all prices are distinct from values because price is always money price, the monetary expression of value within the sphere of circulation. From this point of view, the transformation brought about by the tendential equalization of profit rates is a transformation in the form-of-value: from direct prices, prices proportional to values, to prices of production. All price differences are thus differences between existing prices and direct prices. Nonetheless, in deference to traditional usage, I will frequently speak of 'price-value' and 'profit-surplus-value' deviations, when what is meant is respectively the deviations between prices and direct prices, and profits and direct profits (money profit proportional to surplus-value).

Lastly, I should mention that this paper is a prelude to a more general critique of the neo-Ricardians, the first thrust of which is a direct confrontation with their major claims. Ian Steedman's book *Marx after Sraffa* provides a welcome opportunity to take issue with the neo-Ricardians, which I do in a recently published paper entitled 'The Poverty of Algebra'.[1]

II. Production, Reproduction and Exchange

1. The Contradiction of Commodity Production

In all societies, the objects required to satisfy human needs imply a certain allocation of society's labour-time, its productive activities, in specific proportions and quantities. Otherwise the reproduction of society is impossible. The relationship of people to nature must be reproduced if society is to be reproduced. But in the case of commodity production, the products of labour which constitute the material basis of this reproduction process are produced without any direct connection to social needs. They are produced for exchange, as the products of private autonomous labours carried out independently of one another, but within and through the social division of labour. 'Hence, lacking any conscious assignment or distribution on the part of society, individual labour is not immediately an articulation of social labour; it acquires its character as a part . . . of aggregate labour only through the mediation of exchange relations or the market.'[2]

We know of course that commodity production is generalized only under capitalism, hence only when labour-power becomes a commodity. But the very fact that commodity production is generalized gives rise to a paradox. It rests on private autonomous labours carried out independently of one another with only exchange, generally exchange for profit, in mind. In order to be undertaken, each constituent labour must presuppose, must risk, the existence and reproduction of other such labours, along with the reproduction of their social basis. In other words, each such independent labour must be undertaken on the presupposition of the social division of labour.

In order actually to be reproduced, however, private and apparently anarchic labours must somehow end up being allocated in specific proportions and quantities consistent with the social division of labour. It is precisely through exchange that this presupposition is realized, that private independent labours are forcibly articulated into a social division of labour. Exchange is the process by which, as Marx puts it, the contradictions of commodity production are 'both exposed and resolved'.[3] And since the generalization of commodity production implies the generalization of exchange, at the same time it implies the generalization of the forcible articulation of private independent labour into a social division of labour. The necessity of this forcible articulation then appears to the individual agents as an 'inner law, . . . as a blind natural force . . .'[4] Thus the society comes to possess particular and peculiar laws of motion, which assert themselves in-and-through the collision of the producers in exchange.[5]

2. The Double Role of Exchange

Exchange now appears in a double role. On the one hand, because exchange is the mediating process, the outcome of exchange is the immediate regulation of reproduction. It is through the movements of wages, prices and profits that the immediate regulation of social production is accomplished. On the other hand, it is precisely because exchange functions to articulate private independent labours into the social division of labour that the necessity of the distribution of social labour asserts itself as the domination and regulation of wages, prices and profits by social labour-time. The sphere of exchange has a relative autonomy, but it is ruled, regulated and dominated by the conditions of production and reproduction. The operation of this double relation is what Marx means by the *law of value:* prices as the immediate regulators of reproduction, social labour-times as the intrinsic regulators of prices and hence of reproduction.

> 'Every child knows that a nation which ceased to work, I will not say for a year, but even for a few weeks, would perish. Every child knows, too, that the masses of products corresponding to the different needs require different and quantitatively determined masses of the total labour of society. That this necessity of the distribution of social labour in definite proportions cannot possibly be done away with by a particular form of social production but can only change the mode of its appearance, is self-evident. No natural laws can be done away with. What can change in historically different circumstances is only the form in which these laws assert themselves. And the form in which this proportional distribution of labour asserts itself, in a state of society where the interconnections of social labour are manifested in the private exchange of the individual products of labour, is precisely the exchange-value of these products.
>
> Science consists precisely in demonstrating how the law of value asserts itself.'[6]

3. *Money and Price*

The above understanding of capitalist exchange implies several things for a Marxist analysis of price phenomena. First of all, it implies that money is an absolutely necessary aspect of developed commodity production. Exchange is a process in which people must equalize different use-values, that is abstract from their differences as use-values. As the sphere of exchange grows, so too does the necessity for a universal equivalent in which this abstraction is expressed, and through which the articulation of independent labours is accomplished. Money is the medium of abstraction, and the means of forcible articulation.

Second, because money is a necessary aspect of exchange, the elementary relation of exchange is sale and purchase, not barter (C–M not C–C). This means that each commodity now has a price, a quantity of money which represents its quantitative worth. Conversely, it also implies that money itself has no price. It does not have to be sold, it is money.

Third, all price phenomena now appear in a double light. On the

one hand, as price magnitudes they are distinct from value magnitudes, and have a more complex determination. For instance, even in the case of exchange in proportion to value, the price of a commodity is a quantity of gold determined by the commodity's relative value, that is, value relative to the standard of price, say one ounce of gold, and is therefore already a form of the commodity's value. As such, the movements of prices need not parallel those of commodity values. A fall in a commodity's value, for example, can be manifested as a rise in its price if the value of gold happens to fall even faster.[7]

More generally, as the price-form is developed by Marx, so too is its relative complexity. In the first volume of *Capital*, price is generally treated as a simple money-form of value, but wages, as time-wages and piece-wages, are already more complex forms of the value of labour-power. In the second volume, costs of circulation and turnover add fresh determinations to the price-form. Lastly, in the third volume, the development of prices of production and of the splitting of surplus-value into profits, rents and interest further consolidate the price-form, while the distinction between individual value and average value consolidates the determination of value magnitudes, and through them, those of price magnitudes (individual, average and regulating prices of production, differential profitability, and rent, absolute and differential). It must be noted here that the increasing complexity of the price-value relationship is no defect. Since price magnitudes are the immediate regulators of reproduction, the law of value must contain within it a theory of the structure of price phenomena, right down to their most concrete determinations. Otherwise the law remains abstract, unable to grasp the real movements of the system.

On the other hand, because the price magnitudes are themselves regulated by the socially necessary distribution of labour, the various forms of price categories must be developed in relation to the quantities of socially necessary labour-time whose magnitude and movements dominate and regulate these price phenomena. We must be able to conceive not only of the relative autonomy of price magnitudes, as expressed in their variability and complexity relative to values, but also of limits to these variations and of the connection of these limits to social labour-time. It is significant that in his own development of the increasingly complex categories of price phenomena, Marx never loses sight of the domination of these phenomena by the law of value.

> 'In whatever way prices are determined, the following is the result:
>
> (1) The law of value governs their movement in so far as the reduction or

increase in the labour-time needed for their production makes the prices of production rise or fall . . .
(2) The average profit, which determines the prices of production, must always be approximately equal to the amount of surplus-value that accrues to a given capital as an aliquot part of the total social capital . . . Since it is the total value of the commodities that governs the total surplus-value, while this in turn governs the level of average profit and hence the general rate of profit—as a general law or as governing the fluctuations—it follows that the law of value regulates the prices of production.'[8]

In a highly modern vein, Marx goes on to note how meaningless it is—but also how very convenient it is—to treat the difference between price and value, that is the relation between the two, as a mere separation.

'The price of production includes the average profit. And what we call the price of production is in fact the same thing that Adam Smith calls "natural price", Ricardo "price of production", or "cost of production" and the Physiocrats "*prix necessaire*", though none of these people explained the difference between price of production and value. We call it price of production because in the long term it is the condition of supply, the condition for the reproduction of commodities, in each particular sphere of production. We can also understand why those very economists who oppose the determination of commodity value by labour-time, by the quantity of labour contained in the commodity, always speak of the prices of production as the centres around which market prices fluctuate. They can allow themselves this because the price of production is already a completely externalized and *prima facia* irrational form of commodity value, a form that appears in competition and is therefore present in the consciousness of the vulgar capitalist and consequently also in that of the vulgar economist.'[9]

I remind you that Marx is speaking of the economists who claim to ground themselves in classical economics—less the embarrassment of the labour theory of value, of course!

4. *Tendential Regulation*

It follows from the above that within the moving contradiction that is capitalist commodity production, the reproduction of society is necessarily a process of trial through error, in which discrepancies of one sort are constantly followed by those of an opposite nature. It is only in and through perpetual disorder that the necessary distribution of social labour-time asserts itself.[10] This is why Marx always speaks of a process of tendential regulation and not of some static equilibrium situation. Conversely, it is precisely the concept of

equilibrium which enables orthodox economics to abolish all the contradictions of the forcible articulation, thus abolishing both the necessity of money and the possibility of crises.[11]

'[The] determination of [market] price by [the price] of production is not to be understood in the sense of the economists. The economists say that the average price of commodities is equal to the [price] of production; that is a law. The anarchical movement, in which rise is compensated by fall and fall by rise, is regarded by them as chance . . . But it is solely these fluctuations, which, looked at more closely, bring with them the most fearful devastations and, like earthquakes, cause bourgeois society to tremble to its foundations—it is solely in the course of these fluctuations that [market] prices are determined by the [price] of production. The total movement of this disorder is its order.'[12]

III. The Aggregate Effects of Price–Value Deviations

In the preceding section I have been concerned to emphasize the distinctiveness of Marx's conception of the relation between production and exchange in the process of social reproduction. But these differences between Marx's conceptions and those of orthodox economics, be they classical or marginalist, need not, indeed cannot, be restricted to this level of abstraction. Every real difference in conception inevitably implies a difference in the questions to be asked, in the empirical phenomena to be examined, and ultimately in the conclusions to be drawn. Consequently, in the sections that follow I would like to demonstrate exactly how these differences manifest themselves in a set of problems which, according to some modern Marxists, have already been definitively resolved:[13] namely, the host of issues which have their origins in the debates around the so-called transformation problem.[14] Since the transformation problem is itself a special case of the general problem of price-value deviations (differential rent and market prices are two other equally important cases), I will often deal with the general case first and only then, where necessary, restrict the analysis to the consideration of prices of production alone.

One last point. Throughout what follows I will explicitly accept the mathematical formulations which are now so widely accepted in the post-Sraffian literature on these issues. These are exactly the tools and formulations which are the cornerstone of the most recent attacks on Marx's theory of value, and it is my intention to show that even on this terrain, Marx's answers are superior because Marx's questions are superior. Only at a later point will it be possible to show how the existing formulations are themselves inadequate—precisely because their very structure already embodies many conceptions of orthodox economics.

1. Calculation Versus Conception; The Redundancy Argument

It has always been a popular claim among Marx's critics that value categories are unnecessary in the analysis of capitalism because they are somehow less direct than price categories. Steedman, for instance, insists that given the physical flows of inputs and outputs, of the labour requirements for these outputs, and of the real wage of this labour, one can determine prices of production and the rate of profit without 'any reference to value magnitudes'. Indeed, Steedman goes on, since the 'physical data' which is required to determine values is also an element in the determination of prices of production, it would follow that values can 'play no essential role in the determination of the rate of profit or of prices of production.'[15]

Steedman's use of words is quite revealing. To begin with, the very use of the term 'physical data' is symptomatic of the whole neo-Ricardian approach to social reproduction. In Marx's analysis, 'relations between men within the process of creating and reproducing their material life' appear as a double relation, in which the people–nature relation exists in-and-through the people–people relation.[16] These are different aspects of the same set of human activities. In the neo-Ricardian conception, however, these double-edged relations are separated and alienated into 'physical data' and 'distribution'. The labour process, a fundamental social relation which involves the performance of labour and the forcible extraction of surplus labour, disappears from view. It is replaced instead by so-called given conditions of production.[17]

It is worth considering the various senses in which the conditions of production may be said to be 'given'. We begin by noting that the overall circuit of capital can be represented as M–C . . . P . . . C′–M′. In the first phase, capitalists invest money-capital M in the purchase of commodity-capital C—means of production and labour-power. At this point, therefore, we might say that they possess given conditions of production, but only as pre-conditions of production: as the necessary objective and subjective factors of the yet-to-be-performed labour process.[18] The capitalists must still unite these factors in the labour process itself, in the form of productive capital P, and only if this is done successfully will they be in the possession of the results of production: expanded commodity-capital C′.

Once the labour process has been completed, and input translated into output through the actual performance of labour, then, and only then, can we conceptually appropriate the results of the labour process in the form of input-output measurements—the so-called physical data to which Steedman constantly refers. But now this physical data is itself a conceptual summary of the real expenditures of social labour-time. In the real economy, the results of production on which the so-called physical data are based are themselves given only through the actual materialization of social labour-time, and hence only because value has been actually created. Values are, so to speak, built into the very fabric of this physical data.

As observers of the process, we can now extract from this data estimates of the value flows that were actually involved, just as we can also extract from it estimates of the prices of production that might correspond to such data (actual prices are of course market prices). We might then fall into the simple error of confusing our estimation process with the real determination of values. We might even naively believe that since we can calculate estimates of values and prices of production with almost equal facility from the physical data,[19] they are indeed co-equal in reality—ignoring completely how this so-called physical data comes into being. We might then, in this idealist fashion, arrive at the neo-Ricardian conception of production, in which input proceeds magically to output without the toil and misery of real labour, and in which values acquire a real existence only if we deign to consider them. The production of things by means of things.

2. The Sum of Values and the Sum of Surplus-Values

We noted earlier that for Marx price is itself always the monetary expression of value, the form necessarily taken by value in the sphere of exchange. The social labour process results in a given mass of commodities with given values: in circulation, these commodities acquire specific monetary expression in the form of prices. But it is obvious that in exchange these money prices can do no more than bring about the distribution of the social product among the individuals involved. They cannot in themselves change the mass of use-values so distributed. As such, neither can they change the mass of value and surplus-value represented by these commodities.

It follows from the above that different possible exchange relations among producers of a given mass of commodities involve only different possible distributions of the total mass of value and surplus-value contained in these commodities. This is precisely why Marx argues that price-value deviations cannot in themselves alter the sums of values and surplus-values involved. 'It needs no further elaboration here that, if a commodity is sold above or below its value, there is simply a different distribution of the surplus-value, and that this distribution, the altered ratio in which various individuals partake of the surplus-value, in no way affects either the magnitude or the character of the surplus-value itself.'[20]

It must be said, however, that just because different patterns of distribution cannot alter the total mass of surplus-value to be distributed, it by no means follows that the monetary expression of this total surplus-value (money profit) cannot—within certain strict limits—vary in magnitude. In what follows we shall show that Marx approaches the question of how and why a given mass of surplus-value materialized in a given surplus product can nonetheless have a variable monetary expression in circulation. How and why, in other words, profits can deviate from surplus-value and still remain determined by it.

3. *Profit and Surplus-Value*

The distinction between the sphere of production and the sphere of circulation is essential in Marx's analysis of reproduction. The production of social wealth (goods and services) occurs in the former, while in the latter the objects or performances produced are transferred via exchange from their owners to their consumers. Obviously, both production and circulation are absolutely necessary for capitalist reproduction. Nonetheless, their effects are quite distinct: the former sphere results in the creation of value and surplus-value, and the latter in their transfers.[21]

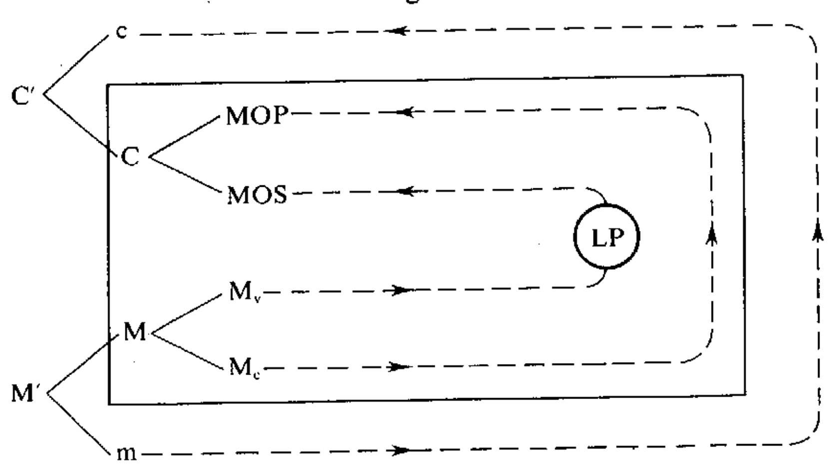
The essential mechanism for the transfer of value is the deviation of prices from proportionality to values. We will follow Marx in referring to these as price-value deviations with the understanding that, as in Marx, this always means deviations of prices from direct prices. For instance, when a commodity is sold at a price below its direct price, then the seller receives in money-form a value less than the value represented by the commodity sold. Conversely, the buyer receives in commodity-form a value greater than that which he or she handed over in the form of money. The surplus-value transferred out of the hands of the seller therefore directly reappears in the hands of the buyer. Something quite important follows from this. Suppose that

some sellers have prices below direct prices, and others have prices above direct prices, but that for the economy as a whole the sum of these prices is equal to the sum of direct prices. Then what some sellers lose in exchange is exactly offset by what other sellers gain, so that in their capacity as sellers the capitalist class as a whole receives money in proportion to the total value materialized in their commodity-capital. But note: the capitalist sellers who lose in value do so to their own buyers, while those who gain in value do so from their own buyers. The question then arises: who are these buyers and how do their gains and losses appear in the determination of total money profits?

To answer this, we need to look at the process of capitalist reproduction in greater detail. To keep the exposition simple, let us initially assume a system in simple reproduction in which all production takes one year, at the end of which capitalists and workers meet in the market-place to buy and to sell. Capitalists enter the market with commodities C′, and with money M′. Workers, having consumed their wages during the previous period of production, enter the market with only their labour-power LP which they hope to sell afresh so as to be able to consume once again. On the basis of their investment plans for the coming year, capitalists invest money-capital M to purchase the elements for next year's production. Of this money, M_c represents constant money-capital advanced for means of production MOP: it therefore buys back a portion of the overall commodity-product C′. The remaining portion of capitalist investment expenditures consists of variable-capital M_v, which is used to purchase labour-power LP for next year's production. The workers in turn spend this money on their means of subsistence MOS, thus buying back a second portion of the available commodity-product C′. Finally, capitalists must also buy a certain amount of goods for their own personal consumption. They therefore expend an amount of money-revenue m to buy back the remaining portion c of the total product C′. Figure 1 below summarizes money flows in the overall process. The flows remaining within the circuit of capital, which as we shall see shortly are crucial to the analysis, are contained within the rectangle drawn below.

It is evident from the above that the circuit of capital M–C (the rectangle in Figure 1) encompasses the purchase of the vast bulk of the social commodity-product C′: directly, through the exchange M_c–MOP, and indirectly through the circuit M_v–LP–MOS. It follows that any transfer of value arising from price-value deviations of means of production MOP and workers' means of subsistence

Figure 1



MOS remain internal to the circuit of capital: what one capitalist loses as capitalist-seller of MOP and MOS, another gains as capitalist-investor in MOP and LP.

The remaining circulation to consider is that encompassed by the capitalists' own circuit of revenue m – c. Here too, what the sellers of commodity-capital lose in value through a price below direct price is gained by the capitalists in the form of a lower price for their articles of consumption. But now a crucial difference arises. What the capitalists in this case lose as sellers will show up in business accounts as the amount by which actual profit is below direct profit (by which actual profit is below profit proportional to surplus value). But what they gain as consumers shows up only in their personal accounts, as a lower amount of money required to purchase the same articles of consumption. In other words, value is transferred out of the circuit of capital into the circuit of revenue, and in the business accounts this transfer manifests itself as profits lower than direct profits.

In most analyses of social reproduction, the circuit of capitalist revenue is not explicitly accounted for. Of course, under these circumstances it appears completely mysterious that as prices deviate from values a given surplus-product and hence a given mass of surplus-value can manifest itself as a variable mass of profit.[22] However, once the whole of social circulation is analysed, the mystery disappears. To the extent that price-value deviations give rise to transfers between the circuit of capital and the circuit of capitalist revenue, these transfers will manifest themselves as differences between actual profit and direct profits. Ironically, though this

phenomenon is evidently a mystery to most Marxist discussions of this issue, it was no mystery to Marx himself.[23] 'This phenomenon of the conversion of capital into revenue should be noted, because it creates the illusion that the amount of profit grows (or in the opposite case decreases) independently of the amount of surplus value.'[24]

None of this should come as any surprise once the difference between value and form-of-value has been grasped. Value and surplus-value are created in production, and expressed as money magnitudes in circulation. Since the circulation magnitudes are more concrete, they are necessarily more complexly determined than value magnitudes, for they express not only the conditions of production of value but also the conditions of its circulation. As such, the relative autonomy of the sphere of circulation necessarily expresses itself as the relative autonomy of price magnitude from value magnitudes. Profits, in other words, depend not only on the mass of surplus-value but also on its specific mode of circulation. The concept of the relative autonomy of circulation from production implies not only that profit can vary independently of surplus-value, but also that this independence is strictly limited. It is necessary, therefore, to show how value categories themselves provide the limits to the variations in their money expressions.

Intuitively, it is evident from the preceding discussion that the overall deviation of actual profits from direct profits is the combined result of two factors. First, it depends on the extent to which the prices of capitalists' articles of consumption deviate from the values of these articles—that is, it depends on the manner in which surplus-value is distributed among capitalists, and on the resultant pattern of individual price–value deviations. And second, it depends on the extent to which this surplus-value is consumed by capitalists as revenue—that is, on the distribution of this surplus-value between capital and revenue. Even when prices deviate from values, the size of any transfer from the circuit of capital to the circuit of revenue will also depend on the relative size of the circuit of revenue. Where all surplus-value is consumed (as in simple reproduction), then the relative deviation of actual profits from direct profits will be at its maximum. When, on the other hand, all surplus-value is re-invested (as in maximum expanded reproduction), then there is no circuit of capitalist revenue and consequently no transfer at all. Total actual profits must, in this case, equal total direct profits, regardless of the size and nature of individual price–value deviations.[25]

Let π° = direct profits (money profits proportional to surplus-value), π = actual money profits, b = the fraction of actual profits which goes towards capitalist consumption, $\bar{g}$ = the average growth rate of the economy, and $\bar{\delta}_F$ = the average percentage price–value deviation of articles consumed by capitalists. Then, as derived in appendix A, it can be shown that the percentage deviation of profits from surplus-value (from direct profits) is a fraction $b(1/1+\bar{g})$ of the average percentage price-value deviation of capitalist consumption goods.

$$\frac{\pi-\pi^{\circ}}{\pi} = \frac{b}{1+\bar{g}}\bar{\delta}_F \tag{1}$$

where $0 \leqslant b \leqslant 1$, $(1/1+r) \leqslant (1/1+\bar{g}) \leqslant 1$, r = the uniform rate of profit and

$$\bar{\delta}_F \equiv \sum_{i=1}^{n}\left(\frac{p_i^{\circ}-p_i}{p_i}\right)\frac{F_i}{F}$$

in which p_i, p_i° refer to actual and direct prices of the i-th good, F_i to the capitalist expenditures on these goods, and $F \equiv \sum_{i=1}^{n} F_i$ to the total consumption expenditure of capitalists. $\bar{\delta}_F$ is therefore a weighted average of individual negative and positive deviations.

It should be noted at this point that this result holds for arbitrary prices, the only restriction being that aggregate money-value of the social product be held constant, so that the purchasing power of money is held constant. The latter condition of course implies that the average price-value deviation for the total product is exactly zero. Insofar as capitalist consumption goods encompass a wide variety of objects produced in industries having a wide range of production conditions, then their average price-value deviation will be the weighted average of many positive and negative individual deviations. In general, therefore, the average price–value deviation ($\bar{\delta}_F$) of capitalist consumption goods is likely to be quite small. Further discussion on this issue will have to be reserved for section IV of this paper, where the determinants of individual price–value deviations will be analysed.

To get an idea of the magnitudes actually involved, it is useful to recognize that $(1-b)$ is the fraction of profits invested by capitalists. It follows therefore that it is also the ratio of total investment to total profits, or, what is the same thing, the ratio of the average growth rate $\bar{g}$ to the average profit rate $\bar{r}$. This means that equation (1) can also be written as

$$\frac{\Delta\pi}{\pi} \equiv \frac{\pi-\pi^{\circ}}{\pi} = \left(\frac{\bar{r}-\bar{g}}{\bar{r}}\right)\left(\frac{1}{1+\bar{g}}\right)\bar{\delta}_F. \tag{2}$$

For the US economy over the postwar period, the average rate of profits (before taxes) was roughly 12%, and the average growth rate roughly 4%.[26] For these orders of magnitude the resulting profit–surplus deviation would be roughly 64% of $\bar{\delta}_F$, the average price–value deviation of capitalist consumption goods. If the latter deviations were of the order of −10% (given the definition of $\bar{\delta}_F$, this means that capitalist consumption goods sell at prices roughly $(0.1/1.10) \cong 9\%$ lower than values), the direct profits would differ from actual profits by roughly −6%.

$$\frac{\Delta\pi}{\pi} \equiv \frac{\pi-\pi^{\circ}}{\pi} \cong -0.064.$$

It is worth remembering, incidentally, that the above formula abstracts from fixed capital and differences in turnover time. A proper treatment of these factors is beyond the scope of the present paper, but their inclusion would imply an even lower profit–surplus-value deviation.

With only a little more effort we can extend the preceding results on the mass of profit to the case of the rate of profit. Let M, W, P stand for the money values of production used up, the total wage bill, and the aggregate sum of prices, respectively, all at arbitrarily given relative prices. Now let M°, W°, P° stand for the corresponding money aggregates when relative prices equal relative values (when prices 'equal' values). Then

$$P \equiv M + W + \pi \tag{3}$$

$$P^{\circ} \equiv M^{\circ} + W^{\circ} + \pi^{\circ}. \tag{3a}$$

Since we are abstracting from turnover and fixed capital, the actual average rate of profit $\bar{r}$ is simply the ratio of profit π to cost-price (= capital advanced) M + W. Hence

$$\bar{r} = \frac{\pi}{M+W} \quad \text{whence} \quad \pi = \bar{r}(M+W)$$

whence

$$\frac{\pi}{P} = \frac{\pi}{M+W+\pi} = \frac{\bar{r}}{1+\bar{r}} \tag{4}$$

$$\frac{\pi^\circ}{P^\circ} = \frac{r^\circ}{1+r^\circ} \tag{4a}$$

where $\bar{r}$ = the average money rate of profit with actual prices and r° = the average money rate of profit with prices proportional to values = the average value rate of profit.

Finally, since the sum of prices is held constant, $P = P^\circ$. Dividing (2) by P and applying (4), we can, after a little manipulation (see appendix A), write:

$$\frac{\Delta\bar{r}}{\bar{r}} \equiv \frac{\bar{r}-r^\circ}{\bar{r}} = \frac{\bar{r}\left(\frac{\Delta\pi}{\pi}\right)+\frac{\Delta\pi}{\pi}}{\bar{r}\left(\frac{\Delta\pi}{\pi}\right)+1}. \tag{5}$$

Intuitively, given that the sum of prices is held constant, if price-value deviations cause π to be below π°, they must also cause $(M+W)$ to be above $(M^\circ+W^\circ)$ (see equation (3)). This means that the average rate of profit will be lower than the value rate because its numerator (π) is lower and also because its denominator $(M+W)$ is higher, which in turn implies that profit rate deviations will tend to be a bit larger than profit mass deviations $\Delta\pi/\pi$. This is exactly what (5) tells us, and if we use the previously calculated magnitudes of $\Delta\pi/\pi \cong -0.064$ along with the previously given value of $\bar{r} \cong 0.12$, we get

$$\frac{\Delta\bar{r}}{\bar{r}} \equiv \frac{\bar{r}-r^\circ}{\bar{r}} \cong -0.07 > \frac{\Delta\pi}{\pi} = -0.064.$$

It is important to understand what this numerical result implies: given that $\bar{r} \cong 0.12$, (5) implies that $r^\circ \cong 0.13$! Such a difference, incidentally is considerably less than the probable error in any empirical measurement of $\bar{r}$, and we may as well say that for empirical purposes $\bar{r}$ and r° (as well as π and π°) are virtually indistinguishable—providing, of course, that our estimate of price—value deviations is of the correct order of magnitude. Before we come to that, however, we

need to clarify a bit further the inner relation between value rate of profit and its monetary expression.

4. *Prices of Production: The Profit Rate*

The preceding discussion was based on more or less arbitrary prices. In order to derive more precise results, we must now restrict ourselves specifically to prices of production. In this regard, since we have already established in (5) that even in the general case there exists an intrinsic connection between profit mass deviations and profit rate deviations, it is sufficient to deal with the latter alone.

We begin by noting that for given conditions of the labour process, the value rate of profit r° can always be expressed as a steadily increasing function of the rate of surplus-value:

$$r^\circ = \frac{S}{C+V} \tag{6}$$

where S = surplus-value, V = value of labour-power. Let $L \equiv V+S$ = value added by living labour (if N = the number of workers employed, and h = the length of the working day in hours, $L = Nh$). Let $k \equiv C/L$ = the ratio of dead to living labour. Then

$$r^\circ = \frac{\left(\frac{S}{V}\right)}{\frac{C}{L}\left(\frac{L}{V}\right)+1} = \frac{\left(\frac{S}{V}\right)}{k\left(1+\frac{S}{V}\right)+1}. \tag{7}$$

Since k depends only on the technology and the length of the working day h, when these conditions of the labour process are given r° will vary directly with the rate of surplus-value. Thus the value rate of profit is a monotonic increasing function of the rate of surplus-value.

In recent years, several authors have shown that when direct prices are transformed into prices of production, though the transformed money rate of profit r will in general deviate from the value rate (we have explained how and why in the preceding section of this paper), nonetheless this transformed rate is also a monotonic increasing function of the rate of surplus-value.[27] But once it is recognized that the value rate of profit r° and the transformed rate r both increase as S/V increases, it follows at once that they must move together: when the value rate of profit rises (or falls) its reflection in the sphere of circulation, the transformed rate of profit, also rises (or falls).

We can be even more specific. In general, the average value rate of profit r° is a weighted average of individual industry value rates of profit, the weights being all positive and summing to 1 (this is known as a convex combination of the individual industry value rates of profit). Let us suppose that the actual system is growing at a rate g, $0 \leqslant g \leqslant r$ (this includes simple reproduction). The level of this actual rate of growth g will of course depend on b, the proportion of profits consumed by the capitalist class. By way of comparison with the actual economy, let us now consider what would happen to the system if capitalists progressively consumed less and less out of profits ($b \to 0$). As this happened, the growth rate would rise, and the fraction of the social product destined for capitalist consumption would fall. In the limit, capitalists would consume nothing, all profits would be invested, and the growth rate g would equal the transformed rate of profit r. Moreover, as indicated in section III.3, when $g = r$ the average value rate of profit under these hypothetical circumstances would itself equal the transformed rate r.

The situation pictured above is one of maximum expanded reproduction (MER). Since there is no capitalist consumption under these circumstances, it follows that of the industries which exist under the actual rate of growth, a small subset—industries whose products are consumed only by capitalists (yachts?)—would not be in operation in MER. This in turn implies that the average value rate of profit in MER is a weighted average of all industry value rates of profit except those industries producing pure luxury goods, the weights being strictly positive fractions determined by the output proportions necessary for MER.

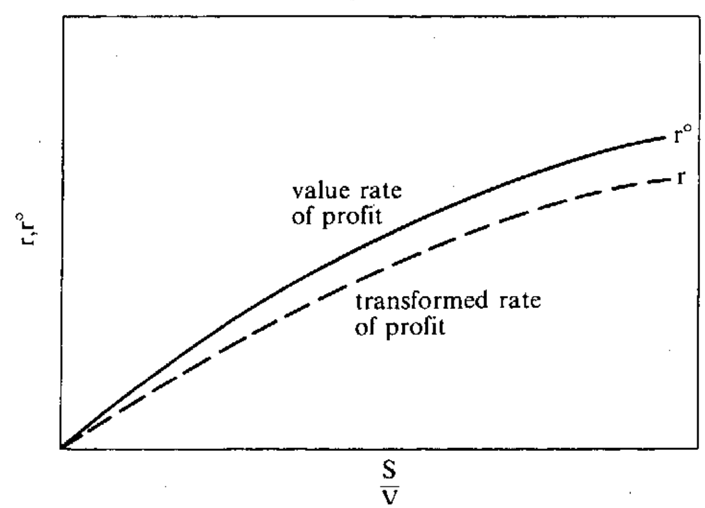
But since this average value rate in MER is exactly equal to the transformed rate of profit r, we can immediately say that the transformed rate of profit is itself a weighted average of individual value rates of profit, the weights and the industry coverage being determined by the MER output proportions. Though we arrived at these MER weights by considering what would happen as $g = r$, we can equally well consider them to be weights which define a sort of 'composite industry' in the actual system. This composite industry, which I will call the central industry, is invariant to the transformation process since its transformed rate of profit is equal to its value rate. As such, it corresponds to what Marx calls 'spheres of mean composition, whether these correspond exactly or only approximately to the social average', for it is to the rate in 'those average spheres of production where the average composition of capital prevails' that the rate of profit is adjusted among industries.[28]

The preceding result is quite powerful, for it tells us that the average value rate of profit r° and the transformed rate of profit r are merely different kinds of weighted averages of a common set of individual industry value rates of profit. The former of course corresponds to the value rate of profit for capital of what Marx calls the 'social average' composition, while the latter corresponds to the central composition (what Marx simply calls the 'average' composition), a composition which, as we have seen, he correctly perceives to be 'only approximately the same as the social average'. The sole difference between the two types of averages arises from the fact that the industry coverage differs somewhat, and from the fact that though each set of weights is composed of positive fractions which add up to one, the individual weights in the two sets will not exactly correspond to each other. As is expected, therefore, these two types of averages behave in essentially the same way, and in a real economy even their respective magnitudes are likely to be virtually the same.

Figure 2 below summarizes the results of the preceding discussion. For the sake of illustration it is assumed that r° is larger than r, though of course it could equally well be the other way around.[29] Their actual relation to each other will in general depend on the relation between the social average composition of capital (which determines r°) and the central composition (which determines r).

It is interesting to note that although Marx insists that the equalization of the rate of profit and the formation of individual

Figure 2



prices of production are of great importance for individual capitals or subsets of capitals, he at the same time also insists that for the system as a whole the previously derived laws are basically unaltered. In a letter to Engels, after having developed the basic phenomena arising from the transformation process, Marx goes on to summarize the remaining tasks. 'Further: the changed outward form of the law of value and surplus-value—which were previously set forth and which are still valid—after the transformation of value into price of production.'[30]

At all times and in all places, price is the outward form of value, the reflection of value in the sphere of circulation. What the transformation does, Marx argues, is to transform this outward form, to introduce into it certain fresh determinations and new sources of variation, but to do so exactly in such a way as to leave the intrinsic connections unchanged. Look again at Figure 2: it illustrates this conception perfectly. In the relatively autonomous mirror of circulation the transformed rate of profit appears as a displaced image of the value rate of profit, essentially the same in determination but somewhat different in exact magnitude. The autonomy of the sphere of circulation expresses itself in this displacement of magnitude. On the other hand, the limited nature of this autonomy manifests itself precisely through the fact that it is the structure of value categories (the pattern of organic compositions, and the proportion of surplus-value which is converted into revenue) which provides the limits to this displacement effect. The variations in the form of value are thus shown to be conditioned and limited by the very structure of value itself.

IV. Individual Price–Value Deviations

The notion of the duality of the exchange process is central to Marx's analysis. On the one hand, it is through the movements of market prices that the day to day regulation of capitalism is brought about. But, on the other hand, it is the structure and distribution of social labour-time which in the end regulates and dominates these day-to-day price fluctations. Thus it is the tendential regulation of price by value which transforms this daily disorder into some kind of order—not by abolishing the disorder, but rather by imposing tendential movements upon it. As Marx puts it, the law of value is a 'law governing fluctuations'.

From this point of view, prices of production are important

because they mediate the relation between values and market prices. The competition of capitals tends to equalize rates of profits across industries, and in so doing tends to reduce market prices towards prices of production. Prices of production are therefore the regulating prices of market price, 'the centre around which the daily market-prices revolve, and at which they are balanced out in definite periods.'[31] Values then in turn regulate these regulating prices of production, and thereby through them dominate the movements of market prices. It is for this reason that the relation between individual values and individual prices of production, the transformation process, plays such an important role in Marx's analysis.

As we have seen, at the level of the whole the individual price-value differences brought about by the transformation process do not substantially alter previously derived laws. But once we move to a more concrete analysis, then these differences, and the transfers of value which they give rise to, become important in their own right. When we examine the relation of one firm to another, of agriculture versus industry, of North versus South, of developed versus under-developed capitalist countries, then knowledge of individual price-value deviations is of great importance. The current debate on unequal exchange is an excellent example of this sort of problem even though I have argued elsewhere against the unequal exchange thesis itself.[32]

Once we consider these issues, then two questions immediately arise. First, what are the relative magnitudes of these deviations and how do they affect the regulation of individual prices of production by individual values? And second, what are the determinants of the directions of these deviations and how do they bring about transfers of value between capitals?

The first question can be answered by analysing the determinants of the size of the typical individual price–value deviation. Of course, if the sum of prices is held constant, the average deviation is zero, since it is the sum of positive and negative deviations. But if we look at the absolute size of these deviations, regardless of their signs, then we can get an idea of the typical deviation and its effects.

The second question is much harder, however, because it requires us to specify both the size and the direction of all individual deviations. Marx of course does just this, but the difficulty arises in generalizing Marx's results. In the traditional case of three 'departments', Francis Seton has already established that completely transformed prices of production deviate from values in the same directions as do the prices of production derived by Marx—that is, according to the relation of the individual department's organic composition to the social average composition. But in the more general case of a given number of industries the problem remains unsolved. Therefore, in what follows I will focus on the first problem alone: namely, on the regulation of individual prices by individual values.

1. The Significance of Individual Price–Value Deviations

The notion that variations in prices are dominated by variations in values can be expressed formally through the notion that the correlation between prices and values is high. And this notion of correlation can in turn be applied to two distinct questions concerning the price–value relation. First of all, as we move across industries during any given period of time, how do the inter-industry price variations compare to the corresponding variations in values? In other words, how close is the cross-sectional correlation between prices and values? Second, how do variations in prices over time compare to the corresponding variations in values? In other words, how strong is the inter-temporal correlation between prices and values?

It is worth recalling that neither Marx nor Ricardo argue that cross-sectional variations are negligible. Indeed, they both emphasize that at any moment of time prices of production may significantly differ from values. Still, it is interesting to note that even in their own examples on the importance of this difference, the actual deviations involved are themselves quite moderate: Ricardo's numerical examples concerning this problem in fact yield relative prices which deviate by only 10% from relative values, whereas Marx's famous transformation tables yield a typical deviation on the order of only ±12%. Even the infamous von Bortkiewicz example, around which so much debate has swirled over the years, yields a typical deviation of only about ±10%.[33]

Granted that particular price–value deviations can be quite large (in Marx's tables, they range from a low of +2.2% to a high of +85%), it is nonetheless important for two reasons to establish what determines the typical deviation. First of all, we have already seen that for the economy as a whole the percentage deviation of the transformed rate of profit from the value rate is itself a fraction of the net price–value deviations of the goods consumed by capitalists. A similar statement applies to the transformed mass of profits. If, for instance, the typical deviation is on the order of ±20% of values, then

the net deviation of any bundle of commodities (such as those consumed by capitalists) is likely to be much smaller than this because positive and negative deviations will tend to offset each other, so that the earlier assumption that $\bar{\delta}_F \cong 0.10$ is fully justified. This in turn would imply that for the economy as a whole the corresponding profit-rate and profit-mass deviations would be very small indeed.

A second reason for examining cross-sectional correlations is that they can provide us with a clue to the inter-temporal correlation between prices and values. The closer that prices are to values at any one moment, the greater is the likelihood that their variations over time will be highly correlated. The reverse is not true, however, since it is perfectly possible to have prices differing significantly from values at any moment, and still have the two moving at roughly the same speeds. This latter outcome is the one Marx emphasizes when he argues (along with Ricardo) that notwithstanding the possibility of large price-value deviations at any moment, over time the significant variations in prices of production are brought about 'by changes in the value of commodities, that is [by] changes in the quantity of labour employed in their production (Ricardo is far from expressing this truth in these adequate terms)'.[34]

All of the preceding discussion has concerned the relation between values and prices of production. But prices of production, it will be recalled, are important primarily because they mediate the relationship between values and market prices, and it is this latter relation which a Marxist analysis ultimately seeks to grasp. Consequently, this latter connection will also be analysed in the sections which follow.

2. The Determinants of Individual Price–Value Deviations

By definition, price is simply the sum of wage costs, material costs, and some arbitrary amount of profit. Let us suppose that the wage rate is uniform, so that the wage cost is wL, where w = the uniform wage per hour, and L = the number of hours worked (the value added by living labour). If M = materials costs and π = (arbitrary) profits, then any arbitrary price P can be written as

$$p = wL + \pi + M. \tag{8}$$

In this expression, the term M represents the price of the material inputs (including depreciation) used up in the process of production. But this price in turn can be thought of as itself being composed of wages, profits and material costs of the industries which produced these means of production. Designating these by $wL^{(1)}$, $\pi^{(1)}$ and $M^{(1)}$ (the superscript (1) tells us that they refer to a production cycle which is one conceptual stage behind the current stage), we can write $M = wL^{(1)} + \pi^{(1)} + M^{(1)}$, or

$$p = wL + \pi + wL^{(1)} + \pi^{(1)} + M^{(1)}. \tag{9}$$

Clearly, the new (residual) material cost $M^{(1)}$ is smaller than the original material cost M. What is more, if we repeat the above process we can reduce $M^{(1)}$ to its wages, profits and material costs, so that $M^{(1)} = wL^{(2)} + \pi^{(2)} + M^{(2)}$, and then in turn reduce this remaining material cost to its components, and so on, until in the limit there is no residual material cost at all. In this way, no matter how the price is actually determined, we can always express it as an infinite series of wages and profits in conceptually receding stages of production.

$$p = W^T + \pi^T \tag{10}$$

where

$$W^T = wL^T \equiv w(L + L^{(1)} + L^{(2)} + L^{(3)} + \cdots)$$

and

$$\pi = \pi + \pi^{(1)} + \pi^{(2)} + \pi^{(2)} + \cdots$$

In the above expression, the term π^T represents the sum of the direct profits π actually received by the sellers of this commodity, plus all the indirect profits $\pi^{(1)}$, $\pi^{(2)}$, $\pi^{(3)}$, ..., each of which represents a prior stage of production. We will call this sum π^T the integrated profits of this commodity.[35]

The same thing applies to L^T. It is the integrated labour-time of this commodity, the sum of the direct labour-time expended in the production of this commodity, and of all the indirect labour-times required to produce its means of production, and the means of production of these means of production. Thus the term $W^T = wL^T$ is the integrated wage bill. But L^T, the integrated labour-time, has another interpretation also: it is simply the (labour) value of the commodity, the sum of direct labour-time L (the value added by living labour), and all indirect labour-times $L^{(1)} + L^{(2)} + L^{(3)} + \cdots$ (the latter sum being C, the value transferred to the product through the means of production used up). Thus:

$$\Lambda = \text{value} = L^T = \text{integrated labour-time.} \tag{11}$$

In preparation for the next step, let us rewrite the price expression in (10) using (11)

$$p = w\Lambda(1 + Z) \tag{12}$$

where

$$Z \equiv \frac{\pi^T}{W^T} \equiv \text{the integrated profit–wage ratio.}$$

Now let us use the above expression to write the relative prices of any two commodities i and j. Denote the price of i by p_i, its integrated labour-time by λ_i, and its integrated profit–wage ratio by z_i. Since the wage rate w cancels out of numerators and denominator, we get

$$p_{ij} = \lambda_{ij} z_{ij} \tag{13}$$

where

$$p_{ij} \equiv \frac{p_i}{p_j}, \quad \lambda_{ij} \equiv \frac{\lambda_i}{\lambda_j}, \quad z_{ij} \equiv \frac{(1+z_i)}{(1+z_j)}.$$

Equation (13) tells us that for any arbitrary prices, the deviations of relative prices from relative values depend on the extent to which the integrated profit–wage ratios of the two commodities differ from each other (where z_{ij} differs from 1). But these immediately gives us a very powerful analytical explanation of the limits to individual price–value deviations. To see why, let us write out the expression for a given integrated profit–wage ratio:

$$Z \equiv \frac{\pi^T}{W^T} \equiv \frac{\pi + \pi^{(1)} + \pi^{(2)} + \pi^{(3)} + \cdots}{wL^T}$$
$$= \frac{\pi}{wL^T} \cdot \frac{w}{w} + \frac{\pi^{(1)}}{wL^T} \cdot \frac{w^{(1)}}{w^{(1)}} + \cdots$$

$$= \frac{\pi}{W}\frac{wL}{wL^T} + \frac{\pi^{(1)}}{W^{(1)}}\frac{wL^{(1)}}{wL^T} + \frac{\pi^{(2)}}{W^{(2)}}\frac{wL^{(2)}}{wL^T} + \cdots$$

$$Z \equiv \left(\frac{\pi}{W}\right)^T = \left(\frac{\pi}{W}\right)\frac{L}{L^T} + \left(\frac{\pi}{W}\right)^{(1)}\frac{L^{(1)}}{L^T} + \left(\frac{\pi}{W}\right)^{(2)}\frac{L^{(2)}}{L^T} + \cdots \qquad (14)$$

We see from the above that the integrated profit–wage ratio $(\pi/W)^T$ is a weighted average of the direct profit–wage ratio (π/W) and of all the profit–wage ratios of commodities which enter either directly, via this commodity's means of production, or indirectly, via the means of production of its means of production, into its production. Moreover, since $L^T \equiv L + L^{(1)} + L^{(2)} + \cdots$, the weights themselves are strictly positive and sum to one. Thus $(\pi/W)^T$ is a convex combination of the direct and indirect profit–wage ratios of this commodity.

But it turns out that as long as the economy is connected, i.e. is composed of basic goods in the sense of Sraffa, then all industries will enter either directly or indirectly into the production of any given industry,[36] which in turn implies that the integrated profit–wage ratio of any commodity is a weighted average of all the direct profit–wage ratios in the economy. But if that is so, then it follows from equation (13) that the deviations of relative prices from relative values depend on the extent to which different weighted averages (convex combinations) of the same set of direct profit–wage ratios differ from each other. In an actual economy with its extensive network of industrial interconnections, it becomes quite clear why even large variations in direct profit–wage ratios $(\pi/W)_i$ can be reduced to relatively moderate variations in integrated profit–wage ratios $Z_i \equiv (\pi/W)_i^T$. The influence of the variations in z_i is then further reduced by the fact that for price–value deviations it is the variations in $(1 + z_i)$ which are the relevant ones, these latter variations being always smaller than the former ones. For direct, and hence integrated, profit–wage ratios which are generally less than one, which is the case in all the major capitalist economies, this latter effect is important in its own right.

All of the above applies to any arbitrary prices. It therefore also applies to prices of production. But here we can specify the argument somewhat more by noting that in the case of prices of production the mass of profit equals the uniform rate of profit r times the (transformed) money value of the capital advanced K. But then integrated profits must be equal to r times the integrated capital advanced K^T. Thus for prices of production:

$$\pi = rK$$

$$\pi^T = rK^T$$

$$z_i = \left(\frac{\pi}{W}\right)_i^T = \frac{r}{w}\left(\frac{K^T}{L^T}\right)_i \qquad (15)$$

$$p_{ij} = \lambda_{ij} \cdot z_{ij} \qquad (16)$$

where now

$$z_{ij} \equiv \frac{1 + \frac{r}{w}k_i^T}{1 + \frac{r}{w}k_j^T}$$

and

$$k_i^T \equiv \left(\frac{K^T}{L^T}\right)_i = \text{the integrated capital–labour ratio.}$$

In this case we see that the variations in integrated profit–wage ratios are proportional to the variations in the integrated capital–labour ratios. The previous analysis for profit–wage ratios then applies also to capital–labour ratios: namely, even large variations in direct capital–labour ratios $(K/L)_i$ can be reduced to relatively small variations in integrated ratios $k_i^T \equiv (K/L)_i^T$, and these in turn are further reduced in their influence on price–value deviations because it is the variations in $[1 + (r/w)k_i^T]$ which matter. In the end, the resulting deviations of prices of production from direct prices can be quite moderate even though the variations in direct capital–labour ratios are quite large.

Equation (16) applies to cross-sectional variations in price–value deviations. If we now consider observations at two different periods t and t_0, then we can write an expression for the determinants in inter-temporal variations in relative prices and relative values.

$$(p_{ij})_{\Delta t} = (\lambda_{ij})_{\Delta t} \cdot (z_{ij})_{\Delta t} \qquad (17)$$

where

$$(p_{ij})_{\Delta t} \equiv \frac{(p_{ij})_t}{(p_{ij})_{t_0}}, \qquad (\lambda_{ij})_{\Delta t} \equiv \frac{(\lambda_{ij})_t}{(\lambda_{ij})_{t_0}}$$

and

$$(z_{ij})_{\Delta t} \equiv \frac{(z_{ij})_t}{(z_{ij})_{t_0}}.$$

Equation (17) tells us that the change over time in relative prices will differ from changes over time in relative values to the extent that the relative integrated capital–labour ratios of the two commodities themselves change over time. What this means is that if over some period of time the different elements in the constellation of integrated capital–labour ratios all rise at roughly the same rate, so that their relative positions are not altered terribly much, then the changes in relative prices over time will correspond fairly closely to changes in relative values. As Ricardo and Marx foresaw, this is clearly possible even when the individual integrated capital–labour ratios differ quite a bit at any one moment of time.

Lastly, the nature of the expressions for cross-sectional and inter-temporal correlations of relative prices and relative values (equations (16) and (17) respectively) suggests that we can rewrite them in the following useful forms:

$$\ln p_{ij} = \ln \lambda_{ij} + \ln z_{ij} \tag{18}$$

$$\ln (p_{ij})_{\Delta t} = \ln (\lambda_{ij})_{\Delta t} + \ln (z_{ij})_{\Delta t}. \tag{19}$$

When written in the above form, we can see that the relation between relative prices and relative values is a log-linear one, in which the terms $\ln z_{ij}$ and $\ln (z_{ij})_{\Delta t}$ play the parts of a 'disturbance' term. This in turn suggests that we can picture the extent of price–value deviations by drawing up a scatter diagram of the log of relative prices versus the log of relative values. Moreover, it also suggests that a natural form for cross-sectional and inter-temporal hypotheses is that empirical correlation between relative prices and relative values is log-linear.

Cross-Sectional Hypothesis H_0:

$$\ln p_{ij} = \alpha + \beta \ln \lambda_{ij} + u_{ij} \tag{20}$$

Inter-Temporal Hypothesis H_0:

$$\ln (p_{ij})_{\Delta t} = \alpha + \beta \ln (\lambda_{ij})_{\Delta t} + u_{ij}. \tag{21}$$

It is evident that we cannot develop this argument much further

without resort to some evidence on actual dispersions of integrated capital–labour ratios, and, where possible, on the dispersions of price–value deviations themselves. We turn to that next.

3. *Empirical Evidence*

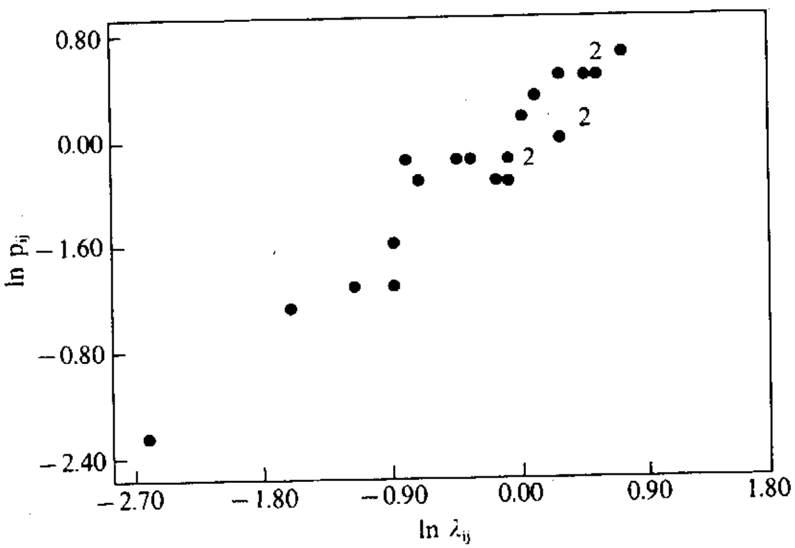
The line of reasoning I have adopted in the preceding sections is no accident. On the contrary, the very nature of Marx's conception of the relation between production and exchange forces us to pose not only the question of the differences between prices and values, but also the question of their inter-connections, their correlations. On this latter issue, it is interesting to note that most of the empirical evidence which I will draw upon in the discussion that follows has been available for quite some time. In a sense, the answers have been there all along. It is the questions, however, which have been missing.

A. Marzi and Varri Data

Let me begin with the evidence on prices of production. Suppose we ask the following question: given an actual economy, what would the prices of production for this economy look like, and how would they compare to direct prices? We could answer this question by using an actual input–output table to calculate prices of production corresponding to different possible rates of profit, and then comparing these hypothetical prices of production to estimates of direct prices. Such experimental data, it turns out, already exists in the form of a study published in 1977 by Graziella Marzi and Paolo Varri (see appendix B). These authors take the 1959 and 1967 25-order input–output tables for the Italian economy, and for each year they calculate prices of production relative to the money wage, for profit rates ranging from $r=0$ to $r=0.80$, the maximum rate of profit. The basis of their calculations in Sraffa's circulating capital model which Steedman, for example, also uses in his numerical examples. I should point out, incidentally, that because this model abstracts from fixed capital the rates of profit it generates are higher than they would be otherwise. Since price–value deviations increase as profit rates increase, this means that such a model actually tends to exaggerate the extent of these deviations.

At $r=0$, capitalists are assumed to make no profits, the calculated prices are proportional to values, and their ratios therefore equal relative values. At the other extreme, at $r=0.80$, workers are assumed to receive no wages, so that labour does not enter at all into the costs of production, and the calculated prices in turn therefore bear no relation to labour-times. Clearly, neither extreme can be meaningfully said to represent prices of production. The relevant range has to be somewhere in between, and for the sake of illustration I will utilize the Marzi–Varri data for r = 0.40, the midpoint between the two extremes (see appendix B for the actual data). In figure 3 below, the vertical axis represents the natural logarithm of the ratios of individual prices of production to the average price of production, at r = 0.40. The

Figure 3



horizontal axis, on the other hand, represents the natural log of the ratios of individual values to the average value, which as I explained above can be calculated from the prices of production at r = 0. Lastly, this particular data refers to 1967. The corresponding data for 1959 gives virtually the same picture, though, with only a slightly lower correlation (see equation (22) below).

Since this sort of data is cross-sectional we can test the correlation between relative prices of production and relative values using the log-linear hypothesis of equation (20). The results of both the 1967 and 1959 tests are summarized in equation (22) below (t-ratios are given in parentheses below each coefficient).

For this data, we find that the typical percentage deviation (the absolute value of the average deviation as a percentage of the average price) is about 17% for 1967 and 19% for 1959.

Cross-Sectional (r = 0.40)

$$\textit{1967}:\quad \ln p_{ij} = \underset{(0.23)}{0.0095} + \underset{(16.60)}{0.8470} \ln \lambda_{ij} \tag{22}$$

$R^2 = 0.920$ (adjusted for degrees of freedom)

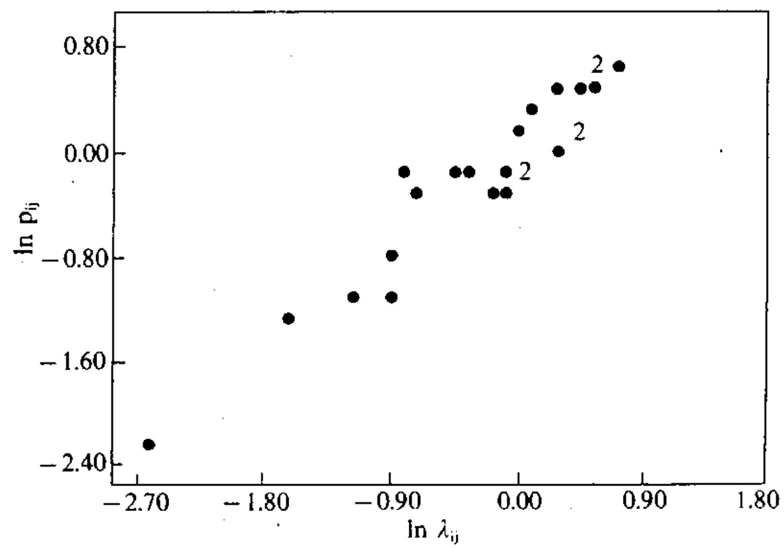
$$\textit{1959}:\quad \ln p_{ij} = \underset{(-0.20)}{-0.0096} + \underset{(12.48)}{0.8717} \ln \lambda_{ij}$$

$R^2 = 0.866$ (adjusted for degrees of freedom).

The above graph and regression results are unambiguous. The cross-sectional variations in the calculated prices of production are entirely dominated by the corresponding variations in relative values, with between 87% and 92% of the former being explained by the latter.

Because the data covers two different time periods, we can also use it to test the inter-temporal correlation between changes in relative prices and changes in relative values. Figure 4 below pictures $\ln(p_{ij})_{\Delta t}$ and $\ln(\lambda_{ij})_{\Delta t}$ on the vertical and horizontal axes, respectively, where both are in terms of 1959 prices relative to 1967 prices.

Figure 4



Using the log-linear inter-temporal hypothesis of equation (21) above, we get:

Inter-Temporal (r = 0.40)

$$1959/1967: \quad \ln (p_{ij})_{\Delta t} = -0.0298 + 1.008 \ln (\lambda_{ij})_{\Delta t} \qquad (23)$$
$$(-1.90) \qquad (16.08)$$

$$R^2 = 0.915 \text{ (adjusted for degrees of freedom)}$$

In the light of the closeness of the cross-sectional correlation in each period, the closeness of the inter-temporal correlation is not surprising. Nonetheless, the above result tells us that almost 92% of the changes in calculated prices of production are explained by changes in calculated values. This is Ricardo with a vengeance—the very Ricardo scorned for over a century for having a so-called '93% theory' of prices of production! Of course, this particular aspect of Ricardo's analysis is carefully avoided by the neo-Ricardians.

B. *The Leontief Data*

The Marzi-Varri data pertains to prices of production and values calculated from a 25-order input–output table. But for the relation of market prices to values, even more detailed data is available in some earlier work by Leontief. In his now famous 1953 article on the empirical relevance of the Heckscher-Ohlin Theorem, Leontief lists various calculations made on the 1947 input–output table for the United States at 190-order. Among these he includes what he calls each sector's direct and total (direct plus indirect) labour and capital requirements, per million dollars of that sector's output (see appendix C).

Let us suppose some sector's total value is 200 worker-years of labour-time, which sells for a price of 10 million dollars. Then its value/market price ratio (its integrated labour/market price ratio) would be 20 worker-years per million dollars worth of output. This tells us that Leontief's total labour requirements per million dollars of output really represent the value/(market) price ratios of the various industries. Similarly, his total capital requirements measure integrated capital/(market) price ratios in various industries, and his direct labour and capital requirements measure direct labour/(market) price and direct capital/(market) price ratios.[37]

In my discussion of the determinants of price–value deviations, I

had argued on theoretical grounds that the integration process by which one moves from direct capital–labour (and profit–wage) ratios to the corresponding integrated ratios will greatly reduce the variations involved. Leontief's data enables us to test this proposition, since has direct and total labour and capital requirements data enable us to compute direct and integrated capital–labour ratios. We then find that although the coefficient of variation (the ratio of the standard deviation to the mean) of the direct ratios $(K/L)_i$ is 1.14, that of the integrated ratios $(K/L)_i^T$ is only 0.60. The integration process, in other words, cuts the degree of variation by almost 50%.

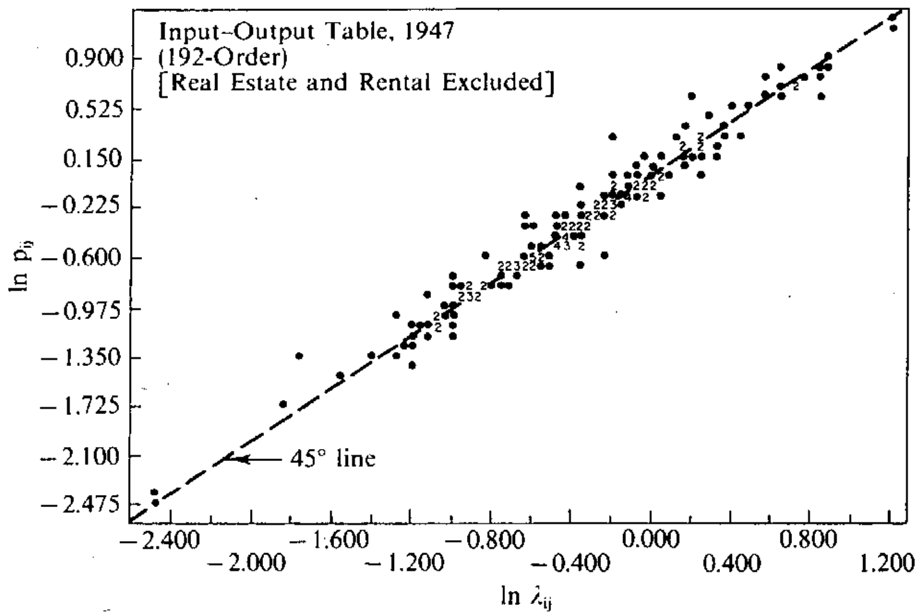
Leontief's data does not provide us with data on integrated profit–wage ratios. Nonetheless, we can approximate these by assuming that the integration process more or less averages out whatever variations exist in market profit rates and wage rates, so the ratio of the integrated profit rate to the integrated wage rate tends to be equal across industries.[38] Let $\bar{r}$ = the average profit rate in the economy as a whole, and $\bar{w}$ = the average money wage per worker-year. Then $(\pi/W)_i^T \cong (\bar{r}/\bar{w})(K/L)_i^T$. Since the coefficient of variation is unchanged when the variable is multiplied by a constant, this means that the coefficient of variation of $(\pi/W)_i^T$ is roughly 0.60 also.

Lastly, we saw earlier that it is the variations in $[1 + (\pi/W)_i^T]$ that are crucial for the deviations of market prices from values. For the US in 1947, $\bar{r} \cong 0.14$, and $\bar{w}$ = $2612 per worker-year.[39] Using this data to estimate the term in brackets above, we get a coefficient of variation of about 0.20. We see therefore that in the end the disturbance term has only about 18% of the variability of direct capital–labour ratios. This is exactly the kind of result anticipated by the theoretical analysis in section IV.2.

Leontief's data enables us to do even more than this, however. Because his total labour requirements represent the ratios of total values to total sales for each of 190 sectors, we can use industry sales data to derive total values for each industry, and by using the average value–price ratio as the value of the dollar, we can derive direct prices from the values. These in turn can then be compared directly with market prices (sales). Figure 5 below is a graph of the natural log of relative market prices versus that of relative direct prices, for 190 sectors (the real estate and rental sector is excluded on theoretical grounds, since differential rent, though determined by surplus-value, is not expected to be proportional either to prices or to values).

The closeness of the correlation between market prices and direct prices is obvious. For this data, the typical deviation is about ±20%, and, as indicated below, a log-linear regression yields excellent results

Figure 5



(standard errors are in parentheses below the coefficients. A parametric test indicates no significant heteroskedasticity in the data):[40]

Cross-Sectional: 1947

$$\ln(p_{ij}) = \underset{(0.0106)}{-0.00095} + \underset{(0.01498)}{0.96809} \ln(\lambda_{ij}) \tag{24}$$

$$R^2 = 0.95814$$

On the basis of data made available by Edward Wolff of New York University, I was able to repeat the preceding experiment for the 1967 input–output table, on 83-order data. The results are virtually identical to those for Leontief's data:

Cross-Sectional: 1963

$$\ln(p_{ij}) = \underset{(0.01457)}{0.01380} + \underset{(0.02602)}{0.99078} \ln(\lambda_{ij}) \tag{25}$$

$$R^2 = 0.94894$$

Both the preceding results attempt to test the relation between market prices and values directly. But we also have on hand indirect evidence on this very same issue, in the form of a very clever statistical test performed on business-cycle data by the US mathematician Jacob Schwartz. To understand the rationale of this test, let us look again at equation (13):

$$p_{ij} = \lambda_{ij} \cdot z_{ij}$$

where

$$z_{ij} \equiv \frac{1+z_i}{1+z_j}, \qquad z_i \equiv \left(\frac{\pi}{W}\right)_i^T$$

This quite general relation tells us that relative prices equal relative values times a disturbance term z_{ij}, a term whose elements are dependent on the integrated profit–wage ratios of the two commodities involved.

In the course of a business cycle, the movement from peak to trough can be very rapid, usually taking less than a year. Both because of the phase of the cycle and the short length of time involved, there is little change in the structure of production under these circumstances but there are large fluctuations in outputs and profits. Since λ_{ij} reflects the (input–output) structure of production and z_{ij} the conditions of profitability, the relative prices in this phase of a business cycle are bound to primarily reflect the variations in the disturbance term z_{ij}: variations which are themselves likely to be abnormally high because of the very turbulent conditions under which they are examined.

Reasoning in a similar way, Schwartz proceeds to examine relative price movements for the average of four business cycles from 1919–1938 (one of these 'business cycles' is the Great Depression!). His results, summarized below, once again reveal that even under these extreme circumstances the average relative price variation is about 7%.

It is interesting that a brilliant mathematician like Jacob Schwartz should so strikingly parallel Ricardo's famous argument while the many grey eminences who populate mathematical economics should so confidently dismiss it as being unrigorous. But then, no doubt this is in good part because much of the so-called mathematics in mathematical economics is merely bourgeois economics in thin disguise.

Figure 6

Peak to Trough Average Prices, Relative to the Wholesale Price Level, For 4 Cycles 1919–1938

	% Variation
Wholesale Prices of Semi-Manufactured Goods	0.07
Raw Materials	0.09
Wholesale Foods	0.02
Retail Foods	0.04
Pig Iron	0.12
Farm Prices	0.10
(Simple) Average	0.07

4. Summary of the Empirical Evidence

The results of the previous section can now be briefly summarized. In general, for both prices of production and for market prices, the typical percentage deviation (the sum of the absolute values of deviations divided by the sum of prices) is moderate: for the price of production data it is of the order of ±17–19%; and for the market price data of the order of ±20–25%. The fact that for an individual commodity a typical deviation is on the order of ±20% means that when we consider a bundle of commodities such as those consumed by capitalists, then the net deviation $\bar{\delta}_F$ of this bundle is likely to be much smaller than ±20% because negative and positive deviations will tend to offset each other. This justifies the assumption that $\bar{\delta}_F \cong 10\%$, which I used earlier (see p. 65) to estimate aggregate profit and profit-rate deviations from their corresponding value categories.

A typical deviation of ±20% of course implies that the typical non-deviation is on the order of ±80%. In other words, it implies that the variations in prices are likely to be highly correlated with corresponding variations in values. And we find that this is just the case. For price of production data, the cross-sectional regression yields an $R^2 = 0.92$ for 1967 and $R^2 = 0.87$ for 1959, while the inter-temporal regression yields an $R^2 = 0.92$. For market price data, we get a cross-sectional $R^2 = 0.96$ for 1947 and $R^2 = 0.95$ for 1963. Finally, on the basis of the data utilized by Jacob Schwartz, we find that even under the turbulent conditions of business cycle downturns, relative price variations are small enough (about 7%) for us to conclude that by far the major source of variations in relative prices over a period of several years will be the variations in the corresponding relative values. Ricardo, it seems, had a vastly superior grasp of these issues than the neo-Ricardians.

V. Summary and Conclusions

Throughout this paper, I have tried to emphasize that Marx's conception of capitalist production and reproduction is quite distinct from that underlying the work of many modern Marxists (such as Steedman). I have particularly stressed Marx's concept of the relative autonomy of the sphere of circulation, because it is only thus that it becomes possible to understand why and how prices can differ systematically from values and yet at the same time be regulated by them. Moreover, the preceding conceptions enable us to examine the status of arguments concerning so-called redundancies and inconsistencies between values and prices. Even accepting the conventional mathematical formulations on these subjects, it becomes possible to show that these formulations exhibit a set of properties which remain hidden to the neo-Ricardians because they lack (or refuse) the conceptions necessary to uncover them. These properties are, moreover, by and large exactly those anticipated by Marx.

To take an example, it is a well-known mathematical result that the transformation from direct prices (prices proportional to values) to prices of production will in general cause the transformed rate of profit to deviate from the overall value rate of profit. To the critics of Marx, this difference implies a break, a complete divorce of any inner connection. But the notion of relative autonomy requires us to show not only how and why such a difference can exist, but also how and why its effects are strictly limited. This approach then enables us to show that the value rate of profit and its transformed rate necessarily move together: in the mirror of circulation, the value rate of profit appears as a displaced image, somewhat different in magnitude but essentially the same in determination. Further consideration enables us to argue that even the displacement effect is likely to be quite small, with typical differences in magnitude of the order of 8–10%.

These results for the economy as a whole are then extended to individual price-value deviations, which are important in their own right because they mediate the transfer of value between capitals, between regions and even between nations. Here too, it becomes possible to argue on both theoretical and empirical grounds that these deviations are strictly limited in magnitude ($\pm 20\%$ for the absolute value of the typical deviation) and even more limited in scope since deviations of this magnitude necessarily imply a high co-variation of prices and values. This latter concept of co-variation is very important because Marx's argument (and Ricardo's also) that the variations in prices are dominated by variations in values can be expressed in terms of the correlation between the two. Theoretical considerations developed in this paper provide strong support for Marx's argument, and what is more, a variety of empirical tests of the relations involved fully bear out the theoretical expectations. As a typical result, for both prices of production and market prices, roughly 93% of both cross-sectional and inter-temporal variations in these prices can be explained by the corresponding variations in values.

As I noted earlier, these are results which can be derived from the very same framework that the neo-Ricardians themselves use to criticize Marx. It is a great irony that this so-called Ricardo–Marx tradition is so adamant in its opposition to these fundamental theses of Ricardo and Marx, while at the same time its own ties to orthodox economics are seldom explicitly acknowledged.[42]

In ending, I might note that the issues I have analysed here are only a small part of those that could be treated in a similar manner. I have not treated fixed capital or joint production, for example, nor indeed the striking absence of money in an algebraic framework which claims to represent the formation of prices. Each of these issues can and must be addressed, and when they are, even the algebra behind which the neo-Ricardians hide will become increasingly transparent.

Appendix A

In the case of a circulating capital model, prices reflecting arbitrary positive profits can be written as:

$$\mathbf{p} = \mathbf{p}(\mathbf{A}+\mathbf{bl})+\boldsymbol{\pi} \tag{1}$$

where $\mathbf{p}$ = row vector of unit prices
$\mathbf{A}$ = input–output coefficients matrix
$\mathbf{b}$ = column vector of wage-goods per worker
$\mathbf{l}$ = row vector of labour coefficients
$\boldsymbol{\pi}$ = row vector of profits per unit output.

By definition, direct prices are prices proportional to value. These can be expressed as:

$$\mathbf{p}^0 = \mathbf{p}^0(\mathbf{A}+\mathbf{bl})+\boldsymbol{\pi}^0 \tag{2}$$

where $\mathbf{p}^0$ = row vector of unit direct prices
$\mathbf{p}^0(\mathbf{A}+\mathbf{bl})$ = row vector of unit direct cost-prices
$\boldsymbol{\pi}^0$ = row vector of unit direct profits.

Lastly, outputs in reproduction can be written as:

$$\mathbf{x} = (\mathbf{A}+\mathbf{bl})\mathbf{x}\cdot(1+g)+\mathbf{f} \tag{3}$$

where $\mathbf{x}$ = column vector of industry outputs
$\mathbf{f}$ = column vector of commodities consumed by the capitalist class
g = the rate of growth.

In simple reproduction, $\mathbf{f}$ absorbs the whole surplus product (i.e. $\mathbf{f}=\mathbf{x}-(\mathbf{A}+\mathbf{bl})\mathbf{x}$), whereas at the other extreme of maximum expanded reproduction, $\mathbf{f}=\boldsymbol{\Phi}$ (where $\boldsymbol{\Phi}$ is a null vector).
If we hold the sum of prices (the purchasing power of money) as constant, then:

$$\mathbf{p}^0\mathbf{x} = \mathbf{p}\mathbf{x} \tag{4}$$

Multiplying (3) by $\mathbf{p}$ and $\mathbf{p}^0$, respectively, subtracting the latter from the former, and recalling (4), we get:

$$(\mathbf{p}-\mathbf{p}^0)(\mathbf{A}+\mathbf{bl})\mathbf{x} = \left[\frac{1}{1+g}\right](\mathbf{p}-\mathbf{p}^0)\mathbf{f} \tag{5}$$

On the other hand, multiplying (1) and (2) by x, subtracting, and recalling (4), we get:

$$(\mathbf{p}-\mathbf{p}^0)(\mathbf{A}+\mathbf{bl})\mathbf{x} = \boldsymbol{\pi}^0\mathbf{x}-\boldsymbol{\pi}\mathbf{x} \tag{6}$$

The first term on the right-hand side of (6) is the mass of direct profits and the second is the mass of actual profits. Designating these scalars by Π^0 and Π, respectively, and combining (5) and (6):

$$\Pi^0-\Pi = \frac{1}{1+g}(\mathbf{p}-\mathbf{p}^0)\mathbf{f} \tag{7}$$

Let p_i, p_i^0, and f_i represent the i-th components of $\mathbf{p}$, $\mathbf{p}^0$, and $\mathbf{f}$, respectively, for $i=1,\ldots,n$. Then:

$$\Pi - \Pi^0 = \frac{1}{1+g} \sum_{i=1}^{n} (p_i^0 - p_i) f_i$$

Let **pf** $= F =$ the money value of the goods consumed by the capitalist class, and $p_i f_i = F_i =$ their expenditure on the i-th good. Then:

$$\frac{\Pi^0 - \Pi}{\Pi} = \frac{1}{1+g} \frac{F}{\Pi} \sum_{i=1}^{N} \frac{(p_i^0 - p_i)}{p_i} \frac{F_i^0}{F}$$

The term in the summation sign is a weighted average of the individual price/direct price percentage deviations, the weights being determined by the pattern of capitalist expenditures on various commodities. Since $F_i = 0$ for all goods which are not consumed by the capitalist class, the term in the summation sign clearly represents the average price-value deviation of capitalist consumption goods. This deviation, it should be noted, is likely to be much smaller than a typical individual deviation because negative and positive deviations will tend to offset each other.

Appendix B

(Graziella Marzi and Paolo Varri, *Variazioni de Produttivita Nell' Economia Italiana: 1959–1967*, Bologna 1977)

In Marzi-Varri's notation, ${}_i w_t$ represents the reciprocal of the i-th price of production relative to the money wage (the wage-price), for the year t ($t = 1959, 1967$). These are listed for rates of profit from $r = 0$ to $r = 0.85$. The actual maximum rate of profit is $r = 0.80$, however. For reasons explained in the text I select the midpoint, $r = 0.40$.

Cross-sectional relative prices of production are formed for year t, $r = 0.40$, by expressing the i-th wage-price relative to the average wage-price, the latter calculated as a simple average of the individual wage-prices. Cross-sectional relative values are formed in the same way, by using the $r = 0$ data.

Inter-temporal data is formed by dividing 1959 relative prices of production by the corresponding 1967 data, and by dividing 1959 relative values by the 1967 ones.

Techniques of Calculation

1. In theory, an input–output matrix **A** and the corresponding row vector of direct labour-coefficients **L** are:

$$\mathbf{A} \equiv [a_{ij}] \equiv [x_{ij}/x_j]$$

$$\mathbf{L} \equiv [l_j] \equiv [l_j/x_j]$$

where x_j = amount of commodity j, produced in a given year
x_{ij} = amount of commodity i used in the production of commodity j, in a given year
l_j = worker-years of direct labour employed in the production of commodity j in a given year.

From this we may derive the vector of total labour coefficients:

$$\lambda = \mathbf{L} \cdot [\mathbf{I} - \mathbf{A}]^{-1}$$

2. In practice, however, input–output coefficients are measured in terms of the dollar cost of the i-th input per dollar of the j-th output. If we let **A*** be the matrix whose coefficients are costs per dollar of output, and **L***, the vector of direct labour requirements per dollar of output in each sector, then:

$$\mathbf{A^*} \equiv [a_{ij}^*] \equiv [(p_i x_{ij})/(p_j x_j)]$$

$$\mathbf{L^*} \equiv [L_j^*] \equiv [l_j/(p_j x_j)]$$

where $p_j =$ the money price of the commodity. From this, we may define the vector λ^* as:

$$\lambda^* \equiv \mathbf{L^*}[\mathbf{I} - \mathbf{A^*}]^{-1}.$$

The question is, what does λ^* represent and what is its relation to λ?

3. We begin by noting that we can relate $(\mathbf{A}, \mathbf{L})$ to $(\mathbf{A^*}, \mathbf{L^*})$ through a diagonal matrix $\langle \mathbf{P}_i \rangle$ whose elements are the unit prices p_i:

$$\mathbf{A^*} = \langle \mathbf{P}_i \rangle \mathbf{A} \langle \mathbf{P}_i \rangle^{-1}$$

$$\mathbf{L^*} = \mathbf{l} \langle P_i \rangle^{-1}$$

It follows, therefore, that:

$$\lambda^* = \mathbf{L^*}[\mathbf{I} - \mathbf{A^*}]^{-1} = \mathbf{L}\langle \mathbf{P} \rangle^{-1}[\mathbf{I} - \langle \mathbf{P}_i \rangle \mathbf{A} \langle \mathbf{P} \rangle^{-1}]^{-1}$$

Since, $\mathbf{I} = \langle \mathbf{P}_i \rangle \langle \mathbf{P}_i \rangle^{-1}$, we may write:

$$\lambda^* = \mathbf{L}\langle \mathbf{P}_i \rangle^{-1}[\langle \mathbf{P}_i \rangle \langle \mathbf{P}_i \rangle^{-1} - \langle \mathbf{P}_i \rangle \mathbf{A} \langle \mathbf{P}_i \rangle^{-1}]^{-1}$$

$$\lambda^* = \mathbf{L}\langle \mathbf{P}_i \rangle^{-1}[\langle \mathbf{P}_i \rangle (\mathbf{I} - \mathbf{A}) \langle \mathbf{P}_i \rangle^{-1}]^{-1}$$

The term in square brackets is the product of three matrices; its inverse is therefore the product of their inverses, in reverse order: $(\mathbf{ABC})^{-1} = \mathbf{C}^{-1}\mathbf{B}^{-1}\mathbf{A}^{-1}$:

$$\lambda^* = \mathbf{L}\langle \mathbf{P}_i \rangle^{-1} \langle \mathbf{P}_i \rangle (\mathbf{I}-\mathbf{A})^{-1} \langle \mathbf{P}_i \rangle^{-1}$$
$$\lambda^* = \{\mathbf{L}(\mathbf{I}-\mathbf{A})^{-1}\} \langle \mathbf{P}_i \rangle^{-1}$$
$$\lambda^* = \lambda \langle \mathbf{P}_i \rangle^{-1}$$

Thus, the j-th element $\lambda_j^* = \lambda_j/p_j$. That is, each element of the row vector λ^* is in fact the ratio of total labour requirements per unit output. Clearly, this ratio is independent of any choice of the unit of output (lbs., tons, etc.)

4. The preceding results point to a simple way of deriving the data necessary for our calculations. Beginning with the empirical input–output matrix $\mathbf{A}^*$ and the corresponding vector $\mathbf{L}^*$ direct labour requirements per dollar of output, we can immediately calculate λ^*, total labour requirements per dollar of each sector's output. These correspond to the data we used from Leontief. The elements of λ^* are λ_j/p_j. Hence if we know the gross sales p_j/x_j for each sector, we can immediately derive the total labour requirements $\lambda_j x_j$ which correspond to these sales (even though we do not at any time actually define any units of output x_j).

$$\lambda_j x_j = \frac{\lambda_j}{p_j} p_j x_j = \lambda_j^*(p_j x_j)$$

The last operation gives us total labour requirements $\lambda_j x_j$ in worker-years and total prices (gross sales) $p_j x_j$ in dollars.

5. Two data sets were used, in which $\Lambda_j = \lambda_j x_j$ and $p_j = p_j x_j$ are derived in manner indicated in 4 above. Defining the average value of the dollar as $\alpha \equiv (\sum \Lambda_j)/(\sum P_j)$, we can then use this to define total direct prices $P_j^0 = (1/\alpha)\Lambda_j$. Finally, both P_j^0 and P_j are expressed as prices relative to their respective average prices $\bar{P}^0 = (\sum P_j^0/N)$ and $\bar{P} = (\sum P_j/N)$. Note that by construction, $\bar{P}^0 = \bar{P}$.

The first data set is based on Leontief's 1947 data, from W. Leontief, *Input–Output Economics*, Oxford, New York, 1966, appendix III, pp. 129–133. Total Sales P_j were taken from US 1947 input–output table, 192-order.

The second set was provided by Edward Wolff of New York University. In this data, the direct labour requirements vector was computed in two ways: first, in worker-years of undifferentiated labour requirements; and second, in a skill-weighted index of worker-years where relative wages were used as weights (for lack of better indexes). The latter data are the ones actually shown, but the regression results are substantially the same with either set.

Lastly, both the graphs and regressions leave out the real estate and rental sector, since on theoretical grounds within both the Ricardian and Marxist theories of rent, though the magnitude of rent can be derived from value relations it is not related to any labour-time expended in the collection of rent (in the real estate and rental sector). Once again, however, this makes little difference to the log-regression results.

Chapter Three

1. For a survey of previous criticisms, see Ben Fine and Laurence Harris, 'Controversial Issues in Marxist Economic Theory', *Socialist Register*, 1977. For my earlier arguments on Marx's theory of value, see 'Marx's Theory of Value and the Transformation Problem', in Jesse Schwartz, (ed.), *The Subtle Anatomy of Capitalism*, Santa Monica, California 1977, pp. 106–137. Finally, for 'The Poverty of Algebra', see *The Value Controversy*, London 1981, pp. 266–300.
2. L. Colletti, *From Rousseau to Lenin*, New York 1972, p. 83.
3. Karl Marx, *A Contribution to the Critique of Political Economy*, London, 1972, p. 86.
4. Karl Marx, *Capital*, Volume 3, Harmondsworth 1981, p. 1020.
5. Karl Marx, *Grundrisse*, Harmondsworth 1973, pp. 196–197.
6. Karl Marx, letter to Kugelmann, 11 July 1868.
7. *Capital*, Volume 1, Harmondsworth, 1976, p. 193.
8. *Capital*, Volume 3, pp. 280–281.
9. Ibid., p. 300.
10. Marx notes that this necessary distribution of social labour-time has two distinct aspects, which in turn give rise to two different concepts of socially necessary labour-time. There is in the first place the (abstract) labour-time which under given social conditions is necessary for the production of a given amount of a commodity. This quantity of socially necessary labour-time defines the total value of the commodity-product. It arises from the nature of a commodity as a value, as a bearer of exchange-value.

Second, from the nature of a commodity as use-value, as an object of social need, there is the question of the correspondence between the total quantity of the commodity-product and the social need for this product. This correspondence is expressed as a quantity of labour-time which is socially necessary to produce the appropriate amount of the product, that is an amount of product which at the regulating price fulfils the effective demand for it. In the first two volumes of *Capital*, Marx assumes that this regulating price is a direct expression of value; in the third volume he develops it into a transformed expression — the price of production.

The first type of socially necessary labour-time thus determines the unit value of a commodity, and through it the regulating price. The second type of socially necessary labour-time then determines the discrepancy between actual supply and effective demand: it therefore determines the discrepancy between market price and regulating price. (*Capital*, Volume 3, p. 774).

11. Karl Marx, *Theories of Surplus Value*, Moscow 1968, Part II, Ch. XVII, section 8 (pp. 499–507).
12. Karl Marx, 'Wage Labour and Capital', in Robert C. Tucker, ed., *The Marx-Engels Reader*, New York, 1972, pp. 174–175.
13. Steedman, pp. 13–14.

14. In my earlier paper (see note 1), I treat the derivation and structure of this problem in great detail. ('Marx's Theory of Value and the Transformation Problem', pp. 106–137).
15. Steedman, pp. 14–15. Steedman's position is not new of course, since it has always been a highly fashionable argument. Both Paul Samuelson and Joan Robinson, for instance, have long held this position.
16. *Capital*, Volume 1, p. 173.
17. For a related criticism of the neo-Ricardian conception of production, see my 'Political Economy and Capitalism: Notes on Dobb's Theory of Crisis', *Cambridge Journal of Economics*, no. 2, 1978, pp. 233–251, and the subsequent debate on these issues in the same journal, no. 4, 1980. For a detailed critique of Steedman and others, see 'The Poverty of Algebra', pp. 266–300.
18. *Capital* Volume 1, p. 291.
19. Even the facility of calculation is not at all equal. Estimation of values requires knowledge of labour flows and flows of means of production used up. Prices of production require in addition knowledge of the real wage bundle and of the stocks of capital advanced. These latter two pieces of information imply a much more detailed knowledge of the structure of the economy.
20. *Capital* Volume 3, p. 134.
21. *Capital* Volume 2, pp. 222–223.
22. M. Dobb, 'Mr Sraffa and the Rehabilitation of Classical Economics', p. 1.
23. It is interesting to note that Marx addresses this problem in connection with the theory of differential rent, not that of prices of production. It is often forgotten by Marxists that the former theory also implies price-value deviations, since it is the marginal conditions which rule price but the average conditions which determine value. As such, all the general phenomena involving price-value deviations appear here too.
24. *Theories of Surplus Value*, Part III, pp. 345.
25. Anwar Shaikh, 'Theories of Value and Theories of Distribution', unpublished PhD dissertation, Columbia University 1973, ch. IV, section4: and Michio Morishima, *Marx's Economics*, Cambridge 1973, p. 142.

When the economy is along the von Neumann ray, the rate of profit (in a circulating capital model) is independent of relative prices. But the rate of profit is the ratio of profits to cost-prices. If the sum of prices is constant, and the ratio of profits to cost-price is the same for direct prices and prices of production, then it follows that direct profits equal transformed profits and direct cost-price equals transformed cost-price.

26. The average rate of profit is from T. E. Weisskopf, 'Marxian Crisis Theory and the Rate of Profit in the Post-War US Economy', *Cambridge Journal of Economics*, no. 3, 1979, table 2 (Full Period), p. 351. The average ratio of growth is from 'Long Term Economic Growth, 1860–1970', US Department of Commerce, 1973, chart A, fig. 3, p. 8.
27. 'Theories of Value and Theories of Distribution', ch. IV, section 4; and *Marx's Economics*, p. 64.
28. *Capital* Volume 3, p. 273. Alfredo Medio also argues in favour of viewing what I called the central industry as the industry which satisfies Marx's definition of the sphere of average composition. See Alfredo Medio, 'Profits and Surplus Value: Appearance and Reality in Capitalist Production', in E. K. Hunt and J. G. Schwartz, eds., *A Critique of Economic Theory*, Harmondsworth, 1972.
29. The relation of r to S/V can be derived graphically from *Marx's Economics*, p. 64, fig. 2.
30. Karl Marx to Friedrich Engels, 30 April 1868.
31. *Capital* Volume 3, p. 280. Marx emphasizes that this process takes place over

periods of time defined by the conditions of production in different industries. The equalization process is therefore not a 'short-run' phenomenon.

32. Anwar Shaikh, 'Foreign Trade and the Law of Value', *Science and Society*, Autumn 1979 (part 1) and Spring 1980 (part 2).

33. *The Works and Correspondence of David Ricardo*, Piero Sraffa, ed., Cambridge 1962, vol. 1, p. 34; *Capital* Volume 3, p. 356; and Paul Sweezy, *The Theory of Capitalist Development*, Oxford 1942, ch. 7.

34. *Theories of Surplus Value*, Part II, pp. 193–194. See also *Capital* Volume 3, pp. 266 and 280.

35. L. Pasinetti, *Lectures on the Theory of Production*, London and New York 1977. Pasinetti calls this process 'vertical integration'.

36. Piero Sraffa, *Production of Commodities by Means of Commodities*, Cambridge 1960, p. 8.

37. See appendix B for a formal proof of this.

38. By definition π^T and K^T are the integrated profits and capital advanced, respectively. Define the integrated profit rate as $r^T \equiv \pi^T/K^T$. Then from equation (10) in the text:

$$\pi^T \equiv \pi + \pi^{(1)} + \pi^{(2)} + \dots$$

and

$$r^T = \frac{\pi}{K}\frac{K}{K^T} + \frac{\pi^{(1)}}{K^{(1)}}\cdot\frac{K^{(1)}}{K^T} + \frac{\pi^{(2)}}{K^{(2)}}\frac{K^{(2)}}{K^T}\dots$$

$$= r\frac{K}{K^T} + r^{(1)}\frac{K^{(1)}}{K^T} + r^{(2)}\frac{K^{(2)}}{K^T} + \dots$$

Thus each integrated profit rate is a convex combination of individual industry profit rates at various stages in the integration process. Insofar as the competition of capitals tends to equalize industry rates of profit, it will tend to result in individual rates r_i fairly close to each other at any one moment. This in turn means that the integrated rates are likely to be very close indeed. A similar argument can be constructed for integrated wage rates.

39. $\bar{r}$ is from 'Marxian Crisis Theory and the Rate of Profit in the Post-War US Economy', fig. 1, p. 349, and $\bar{w}$ from *National Income and Product Accounts, 1929–1974*, US Department of Commerce, supplement to *Survey of Current Business*, January 1976, p. 211.

It should be noted that Leontief's total capital requirement is in units of $10 000. Converted to these units, $\bar{w} = 0.2612$.

40. The Goldfield-Quardt test for heteroskedasticity was performed by ranking the observations by the size of the independent variable, running separate log-regressions on the first 69 and the last 69 observations, and then performing an F-test on the ratio of the respective sums of the squared residuals (s_1, s_2) to see if the ratio was significantly different from 1. (J. Johnston, *Economic Methods*, New York 1969, p. 219). The test indicates that there is no significant heteroskedasticity in the data.

$$s_1 = 0.84671,\ s_2 = 0.83568,\ \frac{s_1}{s_2} = 0.98698 < F_{0.95} = 1.53 \text{ (60 degrees of freedom)}$$

41. Jacob T. Schwartz, *Lectures on the Mathematical Method in Analytical Economics*, New York 1961, table IIIb, p. 43.

42. For further comments on this issue, see the debate as cited in note 17 above.