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WAGES AND INTEREST: A MODERN DISSECTION OF MARXIAN ECONOMIC MODELS

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Modern economic analysis can throw light on the ancient problems of Ricardo and Marx. Neither of these gave a logically complete description of factor and goods pricing in the simplest case where land is free and where labor and intermediate capital goods applied today produce output after one period of time according to a constant-returns-to-scale production function. I propose to analyze such a simple economy, and then compare it with their formulations.

Just as the utilitarian Bentham was called "Paley without hell-fire," Marx can be classified by the modern theorist as "Ricardo without diminishing returns." The present treatment is part of a longer study of Ricardo-like systems. It makes no attempt to do justice to the many noneconomic and imperfect-competition aspects of Marx's thought, but takes seriously his belief that he was baring the inner workings of competitive capitalism.

Technological Assumptions. Assume two industries. Industry I produces homogeneous physical machines or raw materials called K (for physical capital). Industry II produces homogeneous consumption goods called Y. Production in both industries requires homogeneous labor $L_1 + L_2 = L$ and physical capital $K_1 + K_2 = K$ today, with output appearing one period later. Or:

$$
(1) \qquad \begin{aligned} K^{t+1} &= F(L_1^t, K_1^t) \qquad & L_1^t + L_2^t &\leqq L^t \\ Y^{t+1} &= f(L_2^t, K_2^t) \qquad & K_1^t + K_2^t &\leqq K^t, \end{aligned}
$$

where the inequalities reflect the fact that one input may be redundant in supply.

Marx is supposed to have thought the production functions F and f in (1) to be of the fixed-coefficient type rather than of the smooth J. B. Clark type. So in this case we can[1] replace the functions of (1)

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[1] For this and other facts about linear programming and modern economic theory, see R. Dorfman, R. M. Solow, and P. A. Samuelson, *Linear Programming and Economic Analysis* (New York, 1957), particularly Ch. 11. It is shown there that the functions F and f can be written in the form:

$$\text{Minimum of } (L_i^t/a_i,\ K_i^t/b_i).$$

by the logically equivalent relations:

$$L_1^t \leqq a_1 K^{t+1} \qquad K_1^t \leqq b_1 K^{t+1}$$
$$L_2^t \leqq a_2 Y^{t+1} \qquad K_2^t \leqq b_2 Y^{t+1},$$

where $(a_1, b_1;\ a_2, b_2)$ are the positive technical production coefficients characterizing the fixed-proportion constant-returns-to-scale production functions.

The system's production possibilities can be summarized by

$$\begin{aligned} a_1 K^{t+1} + a_2 Y^{t+1} &\leqq L^t \\ b_1 K^{t+1} + b_2 Y^{t+1} &\leqq K^t. \end{aligned} \tag{2}$$

These relations are portrayed in Figures 1a and 1b. In Figure 1a, the straight lines correspond to the two equations of (2) with inputs L^t and K^t given. The corner A of the production-possibility locus will move northwest or southeast when one of the inputs is increased. Figure 1b shows the equations of (2), but with outputs K^{t+1} and Y^{t+1} specified: if an output rises, the corner A' of society's input-requirement locus $RA'S$ will move northeast.

The relative prices of outputs K^{t+1} and Y^{t+1}, $(p_2/p_1)^{t+1}$, must equal the absolute slope of the NAM locus at the production point actually observed. The relative prices of inputs L^t and K^t, $(w/p_1)^t$, where w is the wage of labor, can be any nonnegative number because the corner A' in Figure 1b can have a straight line of any slope tangent to it.

I. *Stationary Conditions*

Simple Reproduction. Under stationary conditions, or slowly chang-

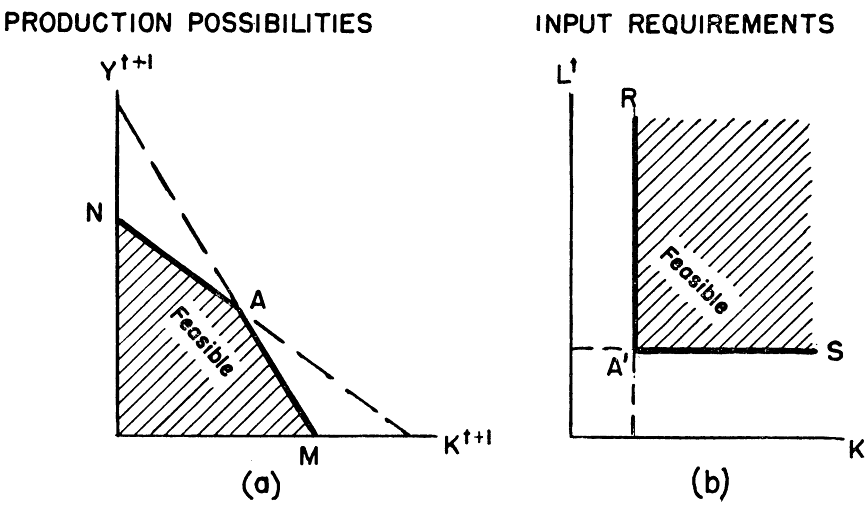

FIGURE 1. *NAM* shows goods producible with given inputs. *RA'S* shows inputs needed to produce specified outputs.

ing conditions, the capital stock K^t will accommodate itself to the supply of labor L^t, which is assumed to be fixed, so that we shall be at a corner A rather than at a point on NA or AM where one of the inputs would be redundant and therefore free. Hence, p_1, w, and p_2 will all be strictly positive. These prices, or their ratios, need not be constant through time but may be slowly changing—probably in a rather predictable way.

The model of "simple reproduction," in which all variables repeat themselves over time, is the natural starting place for an exact analysis. In this case we replace (2) by:

$$L^t = L^{t+1} = \cdots = L$$
$$K^t = K^{t+1} = \cdots = K$$
$$Y^t = Y^{t+1} = \cdots = Y$$

$$\begin{aligned} a_1K + a_2Y &= L \\ b_1K + b_2Y &= K; \end{aligned} \tag{3}$$

or solving, by:

$$\begin{aligned} Y &= \frac{1 - b_1}{a_2(1 - b_1) + a_1b_2} L \\ K &= \frac{b_2}{a_2(1 - b_1) + a_1b_2} L \end{aligned} \tag{4}$$

where labor supply L^t is taken as given at the L level. Being the only factor nonaugmentable in the long run, labor plays a pivotal role: all other magnitudes are proportional to it. The national product NP can be expressed in labor units simply as L; in consumption-good units NP is given by Y in the first equation of (4). Production of K goes into gross product; but K being an intermediate good needed to produce final consumption goods, it is not included in stationary NP.[2]

Prices, Wages, Interest. Though prices and wages are constant under repetitive stationary conditions, this does not mean that production is timeless or that intermediate products just now produced by labor and machines will exchange one for one against themselves when

[2] Ricardo made quite different assumptions about L. He assumed a Malthus-like subsistence wage level at which any number of workers would be produced and reproduced. Such subsistence wages he treated as intermediate product—like hay being fed to horses or coal to furnaces; hence Ricardo's net product would be mine minus wages. Marx assumed actual L used to be less than available L because of the existence of a "reserve army of the unemployed." He would interpret L in (4) then as actual L and would have to add this magnitude as a further unknown variable of the system. A new equation is then needed. The Marxian literature relates the size of the reserve army to labor-saving innovations, depressions, and migration but does not appear to contain a determinate quantitative equation to explain why it is as large as it is, why it is not larger than it is.

"ripened" one period from now—or one for one against finished goods produced today from last period's inputs. The fundamental factor relating unripened product today to ripened product one period from now is the market interest rate r (or what Ricardo and Marx would call the rate of profit, a pure percentage per period).

If the interest rate were $r = .05$ per period, then 100 finished units of Y (or of K) would today trade in the competitive market for 105 unfinished units of Y (or of K) just produced by current labor and capital goods. Free competition among producers, investors, owners of labor, and owners of capital goods will insure the following unit cost-of-production equations:

$$\begin{aligned} p_1 &= (wa_1 + p_1b_1)(1 + r) \\ p_2 &= (wa_2 + p_1b_2)(1 + r). \end{aligned} \tag{5}$$

The first of these equations is directly solvable for p_1/w; and substituting the result into the second, we get the following explicit solution to (5) in terms of $(a_1, b_1; a_2, b_2; r)$:

$$\begin{aligned} \frac{p_1}{w} &= \frac{a_1(1 + r)}{1 - b_1(1 + r)} \\ \frac{p_2}{w} &= \frac{a_2(1 + r)[1 - b_1(1 + r)] + a_1(1 + r)b_2(1 + r)}{1 - b_1(1 + r)}. \end{aligned} \tag{6}$$

The reciprocal of the last of these is the real wage expressed in terms of consumption goods. If interest were zero, this expression would equal the full productivity of labor in producing consumption goods, as given in the first equation of (4). But of course (4) refers only to steady states of output and input, paying no attention to the time lag between inputs and outputs. Only under special, and unrealistic, market assumptions can the competitive supply and demand relations be expected to ignore these timing relations: if supply and demand among investors and consumers yields a positive r, then workers will receive their "discounted" productivity. This means many things to many writers: exploitation to some, to others merely that workers (and machine-owners) receive their full *undiscounted* productivities in terms of the intermediate product that they *now* produce. Because of the workers' supply and demand for ripe and unripe products, and the corresponding supply and demand of those who own consumption or capital goods, the market rate of interest r is what it is. And being what it is, costs and prices and incomes are what they are.

Note too that the price ratio between any two goods, such as $p_2/w \div p_1/w$ in (6), or between either of these and any third good, will *not* be proportional to their embodied labor contents as given in the first equation of (4) and the corresponding equation derivable for

K in terms of L_1 alone.[3] Exchange values would precisely be given by such labor contents if interest or profit were zero. (Remember we have also conveniently banished all land rents from existence.) This mathematical fact will not be of comfort to one looking for a labor theory of value as a base point for a theory of labor exploitation; the proportionality of market price to labor content applies validly only when surplus value is zero and not worth talking about!

When interest is positive, a change in its magnitude will change all relative prices, a hard fact that Ricardo never could square with his desire to find an absolute measure of value based upon labor. And even had Marx lived to write a fourth or fortieth volume of *Capital,* he could not have altered this arithmetic obstacle to the relevance of his labor theory of value.

The Tableau Économique. For each stationary state based on L and r, we can combine the prices of (6) and the quantities of (4) to get the Quesnay-Marx-Leontief money-flow matrix. Of course, we must reverse the Marxian emphasis, beginning with market exchange values rather than labor values because that is what the market that determines people's incomes and goods' prices begins (and ends!) with. We get:

$$\begin{aligned} p_1K &= (wL_1 + p_1K_1)(1 + r) \\ p_2Y &= (wL_2 + p_1K_2)(1 + r). \end{aligned} \tag{7}$$

Write p_1K_1 as the Marxian "constant capital" C_1, wL_1 as "variable capital" V_1, and the difference between Industry-I receipts and the sum of these as "surplus value" S_1. Define C_2, V_2, S_2 for the second industry likewise. Then by definition (7) can be rewritten:

$$\begin{aligned} p_1K &= C_1 + V_1 + S_1 \\ p_2Y &= C_2 + V_2 + S_2. \end{aligned} \tag{8}$$

Such a relation would be valid even if positive accumulation were taking place, with $\Delta K = K^{t+1} - K^t > O$, and (7)'s $K = K_1 + K_2 + \Delta K$. If simple reproduction is assumed, with $K = K_1 + K_2$, then it is easy to derive the Marxian condition for simple reproduction.[4]

$$C_2 = V_1 + S_1 \tag{9}$$

However, the supposition made in *Capital,* Vol. I, of equal rates of surplus value in different industries, $S_1/V_1 = S_2/V_2$, is seen to be gen-

[3] If we write $\Delta K = K^{t+1} - K^t$ as the net production of physical capital, over and above what is used up as intermediate product in production ("depreciation"), then the steady-state production-possibility equation of final goods producible for each L may be shown to be given by:

$$a_1(1 - b_1)^{-1}\Delta K + [a_2 + a_1(1 + b_1)^{-1}b_2]Y = L.$$

[4] P. M. Sweezy, *The Theory of Capitalist Development* (New York, 1942), p. 77. This seems by all odds the best book on Marxian economics.

erally untrue. By (6)-(8), we find:

$$
\begin{aligned}
\frac{S_1}{V_1} &= \frac{r(wa_1 + p_1 b_1)}{wa_1} = r + r\frac{p_1}{w}\frac{b_1}{a_1} = \frac{r}{1 - b_1(1+r)} \\
\frac{S_2}{V_2} &= \frac{r(wa_2 + p_1 b_2)}{wa_2} = r + r\frac{p_1}{w}\frac{b_2}{a_2} = \frac{S_1}{V_1} + r\frac{p_1}{w}\left(\frac{b_2}{a_2} - \frac{b_1}{a_1}\right).
\end{aligned} \tag{10}
$$

It would be a fortuitous selection of $(a_1, b_1; a_2, b_2)$—namely that for which $b_1/a_1 = b_2/a_2$—that would make these equal when both are not zero. However, the situation is a little better than Marx's critics have realized: for if the "organic composition of capital" happened to be the same for different industries at one interest rate, then it would have to be the same for all values of r.

Table I shows the simple reproduction model in the Leontief tableau form of input-output money flows. Each industry is listed in rows and in columns. Thus, the column of Industry I gives the dollar production costs it pays out. The row indicates where Industry I sells its products. Above and to the left of the broken lines are the intermediate-goods flows; then on the right comes the value of final output, and below come the value-added cost items (excluding, of course, all depreciation). The starred quantities represent national product, as final commodity flow or equivalent factor costs. The sums of rows or columns are indicated by Σ, and the $\Sigma\Sigma$ checks the identity of all the table items to the gross sum of column sums and to the gross sum of row sums. As a condition of stationariness, $\Delta K = O$ in row I's third column: hence (9)'s identity between p_1K_2 and the value-added items of column I.

TABLE I.—SIMPLE REPRODUCTION, LEONTIEF-STYLE

Industries	I	II	Final Products	Gross Product Totals
I	p_1K_1	p_1K_2	0	Σ
II	0	0	p_2Y^*	Σ^*
Value Added: Wages	wL_1	wL_2		Σ } Σ^*
Value Added: Interest	$r(wL_1+p_1K_1)$	$r(wL_2+p_1K_2)$		Σ
Gross Costs	Σ	Σ^*	Σ^*	$\Sigma\Sigma$

To be stressed is the fact that our table is limited by more than the tautological accounting identities: having committed ourselves to equations (1)-(6), we must make each entry in the table directly proportional to total labor L, with a proportionality coefficient that is an easily determined function of $(a_1, b_1; a_2, b_2; r)$ and nothing else. I leave the working out of such coefficients to the reader, since they are important only for Marx's special two-industry circular model. Later we shall see how the coefficients vary for each percentage rate of growth of the system.

A Digression on the "Transformation" Problem. Marx seems never to have quite mastered the purely technological implications of his simplest models. It is idle to speculate whether his Volume II analysis of circular flows might not have been more fruitful if he had not misled himself by Volume I's attempted labor theory. After all, we don't expect in 1860 to find 1960 models. But later scholars surely would have made progress faster in this field if they had subjected the labor theory to careful analysis rather than spent so much time in what must seem to a critic as sterile apologetics.

One honest attempt to analyze the relations between exchange values and labor values beyond the unsatisfactory state left by the posthumous Volume III is associated with the names of Bortkiewicz, Sweezy, and Winternitz.[5] Yet the present *exact* analysis of this model suggests that this so-called "transformation problem" is rather pointless. Equations (6)-(7) determine all market magnitudes in terms of $(a_1, b_1; a_2, b_2; r; L)$. Using the definitions implicit in (8), we can then evaluate all the Marxian expressions as functions of these same variables. Logically this transformation goes from exchange values to Marxian-defined values—not vice versa! This is because exchange values are solidly based on equations (5)-(6), as Ricardo, Smith, and all modern economists would agree. There is no similar solid ground to be found in the Marxian labor theory of value; a model based on equal rates of surplus value is like a made-up nursery tale, of no particular relevance to the ascertainable facts of the simple competitive model (nor to the facts, for that matter, of the Chamberlin monopolistic competition models or the models of developing and oscillating capitalism).

Many Marxians have thought it a virtue of the labor theory of value that it "explains its deviations" from the market-price theory. If so it shares this virtue with every theory, however nonsensical: for

[5] See Sweezy, *op. cit.*, Ch. 7 for discussion and references. Also, L. von Bortkiewicz, "On the Correction of Marx's Fundamental Theoretical Construction in the Third Volume of *Capital*," transl. by Sweezy from the July 1907 *Jahrbücher für Nationalökonomie und Statistik* and given as an appendix in Sweezy's English edition of Böhm-Bawerk's critique of Marx and Hilferding's rejoinder: *Karl Marx and the Close of His System* (New York, 1949). J. Winternitz, "Value and Prices: A Solution of the so-called Transformation Problem," *Econ. Jour.*, June 1948, LVIII, 276-80. R. L. Meek, *Studies in the Labour Theory of Value* (London, 1956), pp. 189-200, discusses this problem and gives reference to later *Econ. Jour.* writings.

truth always equals "error plus a deviation"; and while I should prefer to say that Euclid's geometry explains the deviations between it and my daughter's geometry rather than vice versa, I would not go to the guillotine over such a semantic issue. A quite different defence of the Volume I detour is the historical argument that prices *once* were in accord with Volume I's labor theory, but just as Volume III evolved from Volume I so did the capitalistic system outgrow the simple labor theory: ontogeny repeating phylogeny may be accurate biology, but a respect for the facts of history and anthropology stands in the way of this hypothesis. There is finally Marx's own view that the labor theory of Volume I is needed to "determine" or "explain" the *aggregate* of surplus value, with the bourgeois theories of Volume III having the mundane task of settling the details of how the determined aggregate is to be *allocated* among the different industries. Actually, in the competitive Marxian model defined by equations (1) and the following, there can be no prior determination of the aggregate: the whole is the sum of its (admittedly nonindependent) parts and all the pricing relations are simultaneously determined.[6]

I have not the space to deal with the defensive argument that Volume I's labor theory is a (needed or unneeded?) simplifying first approximation. Modern science and economics abound with simplifying first approximations, but one readily admits their inferiority to second approximations and drops them when challenged. Moreover, to my mind, the only legitimate first approximation would be that of Smith

[6] Maurice Dobb, *On Economic Theory and Socialism* (London, 1955), Chapter 17, deals with the transformation problem. Dobb, as does Sweezy, seems to feel that Bortkiewicz came to criticize Marx but in effect ended up justifying him by showing that labor's wage was determined after a "deduction" and by arguing as follows: "If . . . the rate of profit in no way depends on the condition of production of those goods which do not enter into real wages, then the origin of profit must clearly be sought in the wage-relationships and not in the ability of capital to increase production." (L. von Bortkiewicz, "Value and Price in the Marxian System," English transl. in *International Economic Papers No. 2* [1952], p. 33). I do not see that the Bortkiewicz "deduction" or "withholding" theory of wages differs essentially from the conventional "discounted" productivity theories here analyzed and subscribed to by Taussig, Wicksell, Böhm-Bawerk, and non-Austrians. Adding a nonwage-good sector with its new (a,b) coefficients and adhering to horizontal labor-supply conditions which fix the real wage, we may find it true that all three industries can come into stationary equilibrium and with r determinable from (6) or (11) quite independently of the new (a,b) coefficients. But how does this make anyone prefer Volume I to Volume III or to any modern bourgeois theory?

Without going into the social relations of the past or future, any economist can see these implications of competitive market prices. (He can also see that the (b_1,b_2) coefficients reflecting the productivity of capital *do* affect r; and he can envisage a case where Industry III alone, by virtue of having $a_3 = 0$ and $b_3 < 1$ will determine its own-rate of profit by itself, and he will realize that if this new r differs from that of (11) what must give is not bourgeois economic theory or the capitalistic institutional economy but rather the assumption of stationary relative prices!)

and Ricardo in which the labor theory is first introduced with zero surplus value or profits (as in Ricardian comparative advantage examples) but is then to be dropped as unrealistic. Volume I's first approximation of equal positive rates of surplus value, S_i/V_i, is not a simplifying assumption but rather—to the extent it contradicts equal profits rates $S_i/(V_i + C_i)$—a complicating detour. Marxolaters, to use Shaw's term, should heed the basic economic precept valid in all societies: Cut your losses!

II. *Incompatibility of Falling Profit and Falling Real Wage*

Falling Real Wage or Falling Rate of Profit? We now have the equipment to answer an unresolved problem of the Marxian literature. Is there a law of the declining rate of profit as time goes on? Ricardo and Sir Edward West in 1815 showed that the answer is, Definitely yes, if you assume Malthusian reproduction of labor matches the capital accumulation that is applied to scarce land. The law of diminishing returns applied to land then guarantees that profit, or interest, should fall.

Marx, having in most of his work ruled out such rising rent considerations, explicitly rejects this explanation of falling profits. Moreover, Marx was like Malthus and older economists in not bothering to distinguish between technological changes and changes within a given production function. This does not mean that for him a postulated secular econometric law meant that literally what it prophesied would indeed happen; for, like Malthus and others, he often spoke of "tendencies," and in such a way that we hardly know how to decide when he was wrong—and hence when he was right!

From a tautology relating the profit rate r to society's rate of surplus value $\Sigma S/\Sigma V$ and its organic composition of capital $\Sigma C/\Sigma V$, Marx deduced the tautology that higher values of the latter, the former being held constant, would necessarily mean that r falls. Sweezy, Joan Robinson, and most analysts of Marx have rightly, I think, criticized this arbitrary *ceteris paribus* type of argument. The rate of surplus value is a purely derived concept about which little can be said in advance until we already know what is happening to the (a, b) technological coefficients and the supply-demand relations for labor and interest loans. Instead therefore we must tackle directly the question of what accumulation will tend to do to r, basing ourselves on the actual behavior equations of competitive capitalism.

First though, we should note a contradiction in Marx's thinking that analysts have pointed out. Along with the "law of the falling rate of profit," Marxian economists often speak of the "law of the falling (or constant) real wage of labor." Some Marxians have even thought that the important fruit of *Capital's* peculiar definitions has been this law of the "immiseration" of the working classes, with the rich getting richer the poor poorer, and with nothing to be done about it until capitalism becomes so senile and cycle-ridden as to lead inevitably to a revolutionary transformation into socialism or communism. The facts of economic history have, of course, not dealt kindly with this law. And Marx himself did not adhere to it at all times. But he perhaps didn't fully realize the inconsistency of his two inevitable laws. As Joan Robinson points out: "Marx can only demonstrate a falling tendency in profits by abandoning his argument that real wages tend to be constant."[7] Our model is well-designed to show this.

Specifically, with specified (a, b) coefficients if attempts to accumulate did succeed in bringing profit r down to a lower plateau, the real wage would have to be higher—and by a quantitative amount to be predicted from our second formula of (6), namely

$$\frac{w}{p_2} = \frac{1 - b_1(1+r)}{a_2(1+r)[1 - b_1(1+r)] + a_1(1+r)b_2(1+r)}. \tag{11}$$

This rational function grows as the interest or profit rate falls, reaching its maximum when r reaches its zero level.

A Theorem about Technological Change under Perfect Competition. This wage-profit relation is derived, not from the orthodox model involving smooth marginal productivities, but from the simplest fixed-coefficients model that Marx seems often to have had in mind.[8] It does rest though on fixed technology as given by the (a, b) coefficients. Since Marx admits technological change into his system, doesn't *my*

[7] Joan Robinson, *An Essay on Marxian Economics* (London, 1942), p. 42. Also, Sweezy, *op. cit.*, Ch. 6.

[8] J. Robinson, *op. cit.*, p. 43 demonstrates the orthodox case, making implicit use of a smooth two-factor homogeneous production function. Her next page's numerical example, suggesting that with a fixed real wage r *might* fall, is inconsistent with such a model, no matter how "very sharply" the marginal productivity of capital is assumed to fall; forgotten is the fact that when increased capital to labor leaves the real wage constant, decreased labor to capital *must* leave the profit rate constant too; actually, for all changes within a smooth or unsmooth homogeneous production function, Δ(real wage) equals $-\lambda\Delta$(profit rate), where λ is an intermediate positive capital/labor ratio.

Recently William Fellner, "Marxian Hypotheses and Observable Trends under Capitalism: A 'Modernized' Interpretation," *Econ. Jour.*, Mar. 1957, LXVII, 16-25, argues that a two-factor, homogeneous production function, zero-monopoly world can have its real-wage marginal productivity and its profit marginal productivity simultaneously fall—provided a sufficiently labor-saving invention has intervened. Fellner's conclusion is inconsistent with my theorem: competition would keep the invention he envisages from ever becoming exclusively dominant. The rest of Fellner's excellent paper is quite unaffected by his pp. 20-21 discussion of this point, which in any case no longer represents his opinion on the subject. Since writing this paper, I note H. D. Dickinson, "The Falling Rate of Profit in Marxian Economics," *Rev. Econ. Stud.*, Feb. 1957, XXIV, 120-31, deals with a similar topic, attempting to use the Marxian *C, V, S,* categories. The sharp contrast with the present treatment is worthy of note.

argument that falling r with given (a, b) coefficients implies rising real wage w/p_2 become irrelevant? In the competitive model, I believe not completely.

For technological change is itself subject to *some* laws. A technical improvement must be an improvement or it will not be introduced in a perfect-competition market economy: Marx cannot repeal the valid part of Adam Smith's law of the Invisible Hand, for its validity depends only on the existence of numerous avaricious competitors. To illustrate, imagine an old set of coefficients $(a_1, b_1; a_2, b_2; r)$ and a new possible set $(a_1', b_1'; a_2', b_2'; r')$. Then if $r' < r$ and if the new technology will actually win its way in a competitive market over the old, I assert the theorem that *the new steady-state real wage* $(w/p_2)'$ *must be greater than the old real wage.*[9]

This is straightforwardly provable by the mathematics of linear programming. It will become intuitively clear if one considers the special Ricardian case where $b_1 = 0$ and no circular complications can arise from the fact that it takes machines (K_1) to make machines (K). Remember that in a perfectly competitive market it really doesn't matter who hires whom: so have labor hire "capital," paying the new market interest rate $r' < r$; then labor could always use the old technology and paying less than r get better than the old real wage. If labor does not do this, it must be because it can now do even better than better.[10]

If my result or my argument seems paradoxical, remember that perfect competition—like Christianity—will be found to be very paradoxical if ever it is universally tried. And remember too that Marx has made the unrealistic assumption that everything except labor is reproducible in the long run. If he had abandoned his labor-theory-of-value concepts and from the beginning built on the patent fact that natural resources too are productive (in the unemotive sense that if the U.S.A. or U.S.S.R. didn't have them, its product would be less), then the possibility of having profit and wages both fall would have to

[9] Rewriting (11) as $w/p_2 = \Phi(r;a,b)$, and now letting (a,b) be variable as a result of technological change, the competitive Invisible Hand can be proved to select (a,b) so that $w/p_2 = \Phi(r) =$ maximum of $\Phi(r;a,b)$ with respect to (a,b). Similarly, $r = \Phi^{-1}(w/p_2)$ $=$ maximum of $\Phi^{-1}(w/p_2;a,b)$ with respect to (a,b). Always $\Phi'(r) < 0$. I believe this to be a new theorem. Of course, it is a prosaic mathematical fact not a Dr. Pangloss teleology.

[10] The argument holds even if capitalists do all the hiring, provided only that workers go where they get highest w and competing capitalists do what gives highest profits. If $b_1 > 0$, the argument needs some amplification because workers have to hire some of the old-type K_1 to carry through the old-type activities and for quite a while the rents of the K's might be adverse to labor; also we could not be sure of being able to settle down to a steady state in two periods when $b_1 > 0$. The stated theorem remains valid though. (Note that with $b_1 > 0$, there *must* have been other ways of producing or getting K, else the system could never have gotten started and could never recreate any K if it were all bombed out—or if, like passenger pigeons or dodo birds, K once became extinct.)

be admitted. He would also have been in a better position to explain why some people are very rich indeed and why some countries are more prosperous than others.

Causality and History. Faced with two contradictory dogmas, what are we to do? Decide that the capitalistic system is doomed to contradiction, and that when the irresistible force meets the immovable object there will ensue an inconceivable disturbance—with communism peeking up through the revolution's ruins? This is the "pathetic fallacy" —in which the observer imputes to Nature his mental states—with a vengeance.

Instead, of course, we jettison one (at least!) of the dogmas. Which one? I nominate the law of the declining (or constant) real wage for the junk pile, and note with interest that modern Marxians increasingly turn to that part of the sacred writings more consistent with last century's tremendous rise in workers' real wage rates.[11]

It would be unsafe to predict an actual secular decline in interest or profit rates in that most economists—notably Schumpeter and Irving Fisher—have emphasized how technological change may raise sagging interest rates, just as plucking a violin string restores its dissipating energies. Moreover, interest rates have historically oscillated in such a way as to lead many economists to the view that there is a fundamental law of constancy of the interest rate. (Taussig, *e.g.*, tried to frame a theory of a horizontal savings schedule to explain this alleged constancy.)

None the less it is of some import to know what would be the effect of attempts to accumulate capital at a rate greater than labor supply increases, *on the assumption of unchanged technology*. For such an inquiry can throw light on the tendencies upon which technological changes of a labor-saving, capital-saving, or neutral character have to be superimposed. Within the framework of my simple two-sector fixed-coefficients model, the resulting analysis will be seen to be at least a little like the despised wage-fund doctrines of Smith, McCulloch, and the Mills.

III. *Steady Growth*

The Expanded-Reproduction Model. Apparently Marx did not have the time to perfect his "expanded reproduction" model in which investment and growth take place. Modern techniques make such analysis a simple task. I retain the fixed-proportions assumption and take up the natural case where, instead of being geared to a stationary level, the economic system is geared to steady growth. This necessarily

[11] See for example, discussion of this topic in *Econ. Rev.* (Tokyo), Jan. 1957, VIII, particularly 21-25.

means steady geometric or exponential growth at uniform percentage rates: no other time-path is possible if many variables and their rates of change are to remain in constant proportions. Such a geometric progression has the further property that relative contemporaneous prices and relative intertemporal prices can be constant along it.

Our production conditions (1) and (2) remain applicable. So do our cost-of-production conditions (5)-(6). But now our simple-reproduction equations (3)-(4) must be replaced by their equivalent relations corresponding to each percentage rate of growth m per period. Now:

$$
\begin{aligned}
K^{t+1} &= (1+m)K^t = \cdots = (1+m)^t K^o \\
L^{t+1} &= (1+m)L^t = \cdots = (1+m)^t L^o \\
a_1(1+m)K^t + a_2 Y^{t+1} &= L^t \\
b_1(1+m)K^t + b_2 Y^{t+1} &= K^t,
\end{aligned}
\tag{12}
$$

where I have substituted for K^{t+1} its indicated value in terms of K^t and have omitted all inequalities by virtue of the assumption that the system is geared to its rate of growth with no excess capacities of men or machines. Just as we solved the static (3) for (4), we can solve the last two equations of (12) explicitly to get

$$
\begin{aligned}
Y^{t+1} &= \frac{1 - b_1(1+m)}{a_2[1 - b_1(1+m)] + a_1 b_2(1+m)} \\
K^t &= \frac{b_2}{a_2[1 - b_1(1+m)] + a_1 b_2(1+m)}.
\end{aligned}
\tag{13}
$$

The first of these coefficients has a slight similarity to the expression for the real wage in (11) or (6). In (11) and (6) the positive interest factor r acted to blow up, so to speak, every input requirement a_i or b_i into $a_i(1+r)$ and $b_i(1+r)$. Here the positive growth rate m acts to blow up b_1 and a_1 into $b_1(1+m)$ and $a_1(1+m)$, but b_2 and a_2 are quite unaffected.[12]

Table II presents the moving equilibrium. Except for $p_1\Delta K$, which is equal to $mp_1(K_1+K_2)$, it looks like the earlier Table I. National product is now given by fewer starred sums Σ^*, and this must equal the sum of all the value-added items. No longer does the condition for simple reproduction, $p_1K_2 = wL_1 + r(wL_1 + p_1K_1)$ as in (9), hold. Also the precise dollar magnitudes are now definitely weighted toward more importance to Industry I, since we now spend more of our available final incomes on capital growth: the exact quantitative magni-

[12] In the closed von Neumann model of dynamic equilibrium, characterized by constant-returns-to-scale and everything plowed back into the system, m and r turn out to be identical. This is not such a system and the possible relations are $m \gtreqless r$.

TABLE II.—STEADY-GROWTH EXPANDED-REPRODUCTION, LEONTIEF-STYLE

Industries	I	II	Final Products	Gross Product Totals
I	p_1K_1	p_1K_2	$p_1\Delta K$	Σ
II	0	0	p_2Y	Σ
Value Added: Wages	wL_1	wL_2		Σ } Σ^*
Value Added: Interest	$r(wL_1+p_1K_1)$	$r(wL_2+p_1K_2)$		Σ
Gross Costs	Σ	Σ	Σ^*	$\Sigma\Sigma$

tudes are given by functions of the $(a_1, b_1; a_2, b_2; r; m)$ coefficients and are easily computed from equations (6) and (13).

In the next period our tableau would look like that of this period, but with all magnitudes blown up by the common factor $(1 + m)$; and so forth with each succeeding period. Hence, such a steady-growth progression *could* go on forever if only the same behavior rules continue to prevail. (The only restriction on the possible rate of growth is that $1 - b_1(1 + m) > 0$ or $0 \leqq m < (1 - b_1)/b_1$ so that all indicated ratios shall exist and keep all our variables positive. A similar restriction $1 - b_1(1 + r) > 0$ had to hold for r. Otherwise production of capital goods K could never have paid.)

I have said nothing about the saving habits of wage or interest earners that would give rise to the analyzed growth rate m. Certainly if each group saved a constant proportion of its income at all times, say σ_w for workers and σ_r for interest receivers, we could solve for the only "warranted rate of growth" m that is compatible with these properties. (Of course, to assume that L^t is always available at the resulting geometric rate is tantamount to postulating a "natural rate of growth" equal to whatever warranted rate results.)[13]

The solution for m in terms of σ_w and σ_r is more complicated than one might at first think. Obviously, the distribution of income depends upon the interest rate r, postulated to go along with the given $(a_1, b_1; a_2, b_2)$ technical coefficients. Call the fractions of income going to wages and interest k_w and $k_r = 1 - k_w$. Then the community's average propensity to save must be

$$\sigma = k_w\sigma_w + k_r\sigma_r = k_w(\sigma_w - \sigma_r) + \sigma_r;$$

and we see that this will be the higher the higher is the income of the relatively more thrifty interest receivers.

What we may not realize is that the distribution of income coeffi-

[13] These terminologies will be recognized as those of the modern Harrod-Domar growth models.

cients, besides being functions of the interest rate r, are also functions of the unknown m growth rate as well; indeed the ratio of total capital asset value to income, the so-called "accelerator" coefficient β, which is needed along with σ to define the warranted rate of growth, is itself a function of m (as well as of r). So the equation defining the warranted rate of growth:

$$m = \frac{\sigma}{\beta} \qquad \text{or } \beta m - \sigma = 0$$

must, even for given (a, b) coefficients, be written in the implicit-equation form:

$$m = \frac{\sigma(r; m)}{\beta(r; m)}, \qquad \text{or } \beta(r, m)m - \sigma(r, m) = 0. \tag{14}$$

Why do the accelerator and the distribution-of-income coefficients depend on m as well as on r? First, because the relative share of wages will differ generally in Industries I and II, and each different growth rate gives a different relative importance to the capital-goods and consumption industries. Our equations permit us to compute the exact effects for each $(a_1, b_1; a_2, b_2; r; m)$ coefficients. Second, and related to the above, each different r will change the dollar (or consumption-good or labor-hour) total of asset value to which the yield r is applied. The equation:

$$\begin{aligned} \text{Total interest return} &= r \text{ (total asset value)} \\ &= r(wL_1 + wL_2 + p_1K_1 + p_1K_2) \\ &= r[A(a_1, b_1; a_2, b_2; r; m)wL], \end{aligned} \tag{15}$$

where A is a function determinable from our earlier equations and where the bracketed expression represents total asset value.

Our whole problem then has a determinate solution quite free of any of the dilemmas of "capital metaphysics." All is grounded in hard technological fact and hard competitive-market fact: there are circular relations between interest and asset value, but they are virtuous circles not vicious ones.[14]

IV. *Changing Factor Proportions and Prices*

The Law of the Rising Rate of Profit. So long as labor and the sys-

[14] The case where profit receivers have $\sigma_r = 1$ and workers have $\sigma_w = 0$, however econometrically unrealistic, is a special case of the above analysis. Were $\sigma_w > \sigma_r$, the logic of the system would be little changed. Of course, with $\sigma_w = \sigma_r$, the distribution of income would become irrelevant and the analysis slightly simplified. Also, in the singular case earlier mentioned, where $a_1/b_1 = a_2/b_2$ and labor-values are proportional to prices, k_w and k_r are independent of m and the analysis becomes even more simple; but to assume away differences in the organic composition of capital is to ignore one relevant factor in the distribution of income.

tem are geared to grow at the same rate, there is no need for profit or interest to change. But if labor grows at a faster percentage rate than does "capital," our equilibrium conditions become inconsistent. Something has to give. What?

One definite possibility is for labor to become redundant and—if it has no reservation price or real cost of staying fit to work—its wage will have to fall. Fall how far? Adhering to the extreme assumption of fixed-coefficient production functions as given in (1) and what follows, we recognize that the real wage becomes literally zero. Kill off one of the now superfluous man-hours and you have outputs unchanged: so the competitive market will impute a zero wage to all man-hours. Mathematically, the inequality will now hold in the first relation of (2); and since all subsequent equations were based on the equality in this relationship, all must now be replaced by new relations. *E.g.*, cost-of-production now requires:

$$\begin{aligned} p_1^{t+1} &= b_1 p_1^t(1 + r^t) + a_1 0 \\ p_2^{t+1} &= b_2 p_1^t(1 + r^t) + a_2 0; \end{aligned} \tag{16}$$

and if prices are to be constant through time with $p_i^{t+1} = p_i^t$, we must have

$$\begin{aligned} 1 + r &= \frac{1}{b_1} \\ \frac{p_2}{p_1} &= b_2(1 + r) = \frac{b_2}{b_1}\,. \end{aligned} \tag{17}$$

These show that the interest rate, which is now interpretable as the own-rate and net-reproductive-rate of machines, must, so long as any of them are being produced, be determinable by technology alone quite independently of all time preferences; and that the terms of trade between consumer goods and machines now depends only on technology, and more specifically only on machine requirements as given by the b's with the a requirements of free labor now being irrelevant.

We can now reckon the national product from the first equation of (12). The following must all hold:

$$\begin{aligned} b_1 K^{t+1} + b_2 Y^{t+1} &= K^t \\ b_1 \Delta K + b_2 Y^{t+1} &= (1 - b_1) K^t \\ \frac{b_1}{1 - b_1} \Delta K + \frac{b_2}{1 - b_1} Y^{t+1} &= 1 \cdot K^t \\ \frac{p_1}{p_2} \Delta K + 1 \cdot Y^{t+1} &= r\left(\frac{p_1}{p_2} K^t\right). \end{aligned} \tag{18}$$

The next-to-the-last of these shows the total value of final products expressed in machine *numeraire* units. The last equation shows on the left side the total value of final products expressed in consumer-good *numeraire* units. The right side, which was derived by using the relations (17), shows that the national product is equal from the cost side to interest on value of machines alone. This is natural enough since wages are zero and must have a zero share of total income.[15]

In this case where capital goods have ceased growing as fast as labor, the rate of profit has risen to become all of the product. So bizarre a result came from the bizarre assumption of fixed coefficients. If there were many alternative techniques, a faster growth of labor than capital would imply rising interest or profit rates and falling real wages, but not a zero wage with profits getting all.[16]

Even in the extreme case of fixed-proportions technology, a zero wage is one possibility: indeed a quite likely one. But it is not the only possibility. As long as the organic compositions of the two industries differ, by shifting demand toward that industry with relatively high labor requirements—as measured by higher a_i/b_i—we could put off the evil day of labor redundancy and zero wage. There is no Invisible Hand, though, which inevitably leads the system to this demand shift: the reduction in the relative price of the labor-intensive good need not coax out much more physical demand for it. In any case, if labor really grows at a faster geometric rate than capital, labor must inevitably become more plentiful relative to capital than either industry could employ and must ultimately become free.

How Profits Fall. The case where capital grows more rapidly than labor is perhaps more true to Western life. In order to see what happens when people try to accumulate faster than the labor supply, consider the special instance where labor is completely stationary and yet savers would like to accumulate. This special case, where the natural rate of growth of the system is given by $m = 0$, does not differ in its qualitative features from any case where m is positive but less than the warranted percentage rate at which capitalists would like to have the system grow.

[15] If capitalists saved all, with $\sigma_r = 1$, and if they received all the income, with $k_r = 1$, then the system's actual rate of growth would be $m = r = (1 - b_1)^{-1}$, which would prevail so long as available labor grew even more rapidly and stayed freely available. It would involve a certain amount of implicit theorizing to argue that this actually would happen in a model in which laborer's-consumption was tied to subsistence and had already been included by convention in the b (rather than a) coefficients; but such a mode of arguing would not be logically wrong, however unrealistic these econometric assumptions might be regarded.

[16] The simplest neoclassical model is one where $Y + (dK/dt) = Q(K,L)$, Q being a homogeneous function of the first degree with partial derivatives ("marginal productivities") Q_L and Q_K. The diminishing-returns condition $\partial^2Q/\partial L^2 = Q_{LL} < 0$ implies that a rising trend in L/K entails a rising trend in $r = Q_K$ and a falling trend in $w = Q_L$.

The Marxian model with fixed coefficients presents some quite pathological features. For if the attempt to accumulate were to cause physical machines K to grow relative to fixed labor L, the machines would become redundant in supply and their rents would fall immediately to zero.[17] The most obvious case in which this would have to happen instantaneously is that in which the organic compositions of capital are equal: $b_1/a_1 = b_2/a_2 = b/a$. The instant K/L exceeded b/a, K would become free, with $(p_1/w)^t = 0 = (p_1/p_2)^t$. We should then have:

$$p_2^{t+1} = w^t a_2(1 + r^t). \tag{19}$$

No production of future K would take place unless it covered its production costs; so only so much would take place as could match the b/a machine-labor ratio. Industry I would therefore contract so as no longer to produce K^{t+1} in excess of La/b. Industry II would temporarily produce more consumption goods: whether these would end up consumed by workers or capitalists would depend on the interest rate and price configuration prevailing at the end of the next period.

A similar but slightly more complicated analysis would handle the case where $b_1/a_1 \neq b_2/a_2$. In every case should the attempt to save cause a disproportionate temporary growth in K, K would become free. This does not imply euthanasia of the capitalist class, not even temporarily. For as (19) shows, interest would still be received on "advances" to workers. Machines are only one type of capital asset. Goods in process are another.[18]

Had the attempt to save forced K rents to zero, it could only be the result of a miscalculation: competitive future prices could not have been correctly quoted in the market place. To be sure, competitive capitalists have no crystal ball picturing the exact future and mistakes have often been made. But once K had become free, it could never stay

[17] There is the possibility, mentioned in the last section, that shifts in product-demand-mix toward the industry using more of the excessively-supplied factor might absorb its extra supply—at least for a while. Thus the cheapening of the machine-intensive good might meet a sufficiently elastic demand for that good to keep both factors nonredundant. But note that this shift could not carry us back to the stationary-state simple-reproduction configuration of Table I with the same price ratios and interest rate prevailing and the same zero net investment prevailing, because our hypothesis is that people are no longer content to refrain from saving in that situation. And growth of K at ever so small an exponential rate faster than labor's growth rate would inevitably make it a free good in finite time.

In this pathological model labor might collusively wipe out all K rents by producing one redundant unit of K. But only temporarily. Production of K will subsequently contract. In this model, collusion of all owners of K could limit its supply and wipe out wages. However, if any one unit of K escaped from the cartel, it and collusive labor could eventually reproduce any needed K *outside* the cartel.

[18] Such intermediate goods are probably a better description of capital than the old view of capital as the historic, now gone, food that was advanced to workers. The latter double-counts if we add it to the former; by itself, the latter undercounts in that interest is also earned on outlays for factors other than labor.

free and continue to be produced. Curtailment of its production by Industry I would undoubtedly take place. One could even try to construct a cobweb-like business cycle theory of intermittent over- and underproduction of capital goods; certainly, though, a two-sector fixed-coefficients model has such special features as to make the result rather unrealistic.

What then is the equilibrium time-path that is consistent with stationary L and attempts to accumulate? The fixed-coefficient Marxian model makes all "real" accumulation quite impossible: there can be *no* technical "deepening of capital" in it. Does this mean that the profit rate r cannot fall? No. Why should it mean this? If I wish to save, for my old age or to enhance my power, why should I be led to desist from trying to do so by the consideration that the system is incapable of using new investment? Rather will I continue to try to save, to try to buy up existing assets.

Thus, suppose I earn income from K rents, or from interest return on goods in process, or from selling goods for more than I paid in wages and rent in producing them, or for that matter merely from my wages. Then instead of spending all this income on current consumption goods Y, I may *try* to hire labor or machines for next period's production, giving up so to speak my consumption allotment to owners of those factors.

Now what is it which guarantees that there will be owners of such factors willing to hire them out in the amount that investors wish to employ them? Of course, it is the competitive pricing mechanism that causes all markets to be cleared.[18a] Crudely, you can say that the interest rate r^t falls enough to eliminate any excess in the value of what people want to save and invest over the value of factors available to them; contrariwise, if the wish to save and invest is lagging, the present factor prices $p_1{}^t$ and w^t will be depressed relative to future goods' prices $p_1{}^{t+1}$ and $p_2{}^t$ and the competitive rate of interest (or of profit) will be bid up very high. It is crude to speak of the interest rate r^t as alone providing equilibration: actually it is the whole pattern of present and future prices $(p_1{}^t, p_2{}^t, w^t; p_1{}^{t+1}, p_2{}^{t+1}, w^{t+1})$.

In the special case where the urge to accumulate is modest and steady, the profit rate r^t could be steadily falling as a result of this process, but at so slow a rate as to permit relative prices $(p_1/w)^t$ and $(p_2/w)^t$ to remain practically constant over time.[19] Then our cost-of-

[18a] See later sections for some qualifying remarks concerning "effective demand."

[19] I make a point of considering a slow change in r^t because the actual interest change in each period will cause changes in (p_2/p_1) and (p_2/w) and create revaluations and money windfalls. With relative prices changing, we no longer have equality of "own-interest-rates" and (5)-(6) need obvious modifications. By assuming $(r^{t+1}-r^t)$ always very small, we make these revaluation-effects small and ignorable.

production equations (5)-(6) would still be valid but are to be written with a slowly falling r^t in them. The steady attempt to accumulate leads to no physical accumulation of K or anything else; rather it causes an upward valuation of existing input prices relative to output prices, which is the same thing as a reduction in the profit rate r^t. Some savers may now succeed in hiring additional inputs ($K_1{}^t$, $K_2{}^t$, $L_1{}^t$, $L_2{}^t$) but, if they do, it is because other capitalists become content at the new interest rate and price pattern to hire less. If all capitalists are exactly alike, they merely bid up factor prices and bid down profit rates.

What has all this attempted accumulation done to real wages? With lower r^t in equations (5)-(6), and in particular in the last line of (6), we see that less is being "discounted" from labor's ("gross") productivity. Real wages have been rising. If, at the lower interest rate, net accumulation should now cease, the real wage going to the unchanged labor supply will not fall back to its previous level but will stay at the higher plateau forever.

Each capitalist in trying to save and increase his own profits ends up killing off the total of profits in favor of the workers. This extreme phenomenon results from the extreme assumption of fixed-coefficients with implied zero marginal-productivity to all further machines or changes in the roundaboutness of production. Yet something of what happens in this case will also hold in a more realistic case of multiple production techniques. As attempted saving lowers interest rates it lowers the discounting of real wages; but in the more neoclassical case, employers will not lose all that workers gain, the difference coming from the extra product producible from "deepening of capital" (*i.e.*, producible from the new complex of physical capital goods brought into existence by the pricing changes induced by the attempt to save).[20]

All this makes clear that the technical (a, b) coefficients and the competitive cost-of-production equations are insufficient to determine all our variables: we need further equations of supply and demand, as *e.g.* ordinal utility conditions showing how workers and interest receivers allocate their consumption expenditures among different goods. But even the latter consumption demand equations are not enough: the rate of interest r^t would still not be determined.[21] We need saving-investment propensities, and propensities to hold and add to earning assets to complete the system.

[20] See Figures 2b and 2c for elucidation of the many-techniques case.

[21] If labor is assumed always to be on a horizontal long-run supply schedule at a "subsistence real wage w/p_2," then (6) or (11) would alone determine r. But prescribing employment L leaves r and w/p_2 still to be determined.

The next sections show the wage-fundlike character of this competitive process.

V. *Wage-Fund Notions*

Perhaps the expression "wage fund" should be avoided altogether as conjuring up too many ghosts and as being too hopelessly ambiguous. Sometimes the wage fund meant merely sums of money "destined" for wage payments, whatever the word "destined" is supposed to mean. Sometimes it meant inventories of finished consumption goods "destined" for workers, and to some writers supposedly consisting of different consumption items than more elegant capitalists would deign to consume. Sometimes it meant a numerator of "all capital," which in some ill-described fashion got divided by the denominator of population number to give as an arithmetic quotient the real wage per capita. Finally in F. W. Taussig's resurrection, *Wages and Capital* (1896), the wage-fund doctrine merely becomes a reminder that production does take time and that men do not consume unfinished goods, with the implication of a certain short-run inexpansibility in the consumption goods available to the community (to nonworkers as well as workers).[22]

In connection with the present two-sector model, it is superficial to split consumption Y^t into two parts, Y^* "destined" for workers and Y^{**} destined for capitalists, and then to write down the trivial identities:

$$(1-\sigma_w)w^tL^t = p_2^t(Y^t - Y^{**}) = p_2^tY^*$$
$$\left(\frac{w}{p_2}\right)^t = \frac{(Y^t - Y^{**})/(1-\sigma_w)}{L^t}. \tag{20}$$

Except possibly for L^t, none of the right-hand variables are given constants. In the shortest run itself, when we are realistic enough to introduce inventories into our model, we see that not even total consumption Y^t is unilaterally given. And suppose it were: still, in anything but the shortest run, decisions could be made to cause it to change.

What does need emphasizing is the fact that in every run the supply-demand decisions of workers, of old capitalists, of new investors are

[22] In its most rigid form, the wage-fund doctrine implied that unionized or ununionized workers face a short-run aggregate demand schedule of exactly unitary elasticity. This neglects the short-run possibility of using up finished-goods inventories faster than the usual rate, and tells nothing about the longer-run demand elasticity, which could be on either side of unity. In its weakest form, it suggests that the demand for labor is not perfectly inelastic and that the demand curve's rightward and upward shift induced by accumulation *may* be slowed down by concerted measures to raise present wage levels at the expense of thrifty capitalists.

needed to give us determinate equations for our set of present and future prices (p_1^t, p_2^t, w^t, p_1^{t+1}, p_2^{t+1}, r^t, . . . etc.). Taussig was quite right in pointing out that the Malthus red herring of a (very-long-run) horizontal supply schedule of labor at the "[conventional] subsistence level" kept Ricardo, J. S. Mill, and most of the Classicals—but not the aging Malthus!—from perceiving how undetermined and implicit was their theory of current wage determination and pricing. Marx's reserve army is in some ways an even redder herring that deflects attention from the missing supply-demand relations.

Here I shall simply sketch in a superficial way the process determining wages, surplus values or interest, and goods pricing. We start out with a given K^t owned by its owners, with a given L^t perhaps to be taken as a demographic parameter. Today's Y^t we suppose to be given by past decisions, and we overlook changes in short-term inventories of consumer goods. The system has a history of prices and wages. This period's market must determine decisions on how much of (K_1^t, L_1^t, K_2^t, L_2^t) are to be hired to produce next period's (K^{t+1}, Y^{t+1}). The competitive market does this through determining now (p_1^t, p_2^t, w^t; r^t). Simultaneously a set of notions about future prices (p_1^{t+1}, p_2^{t+1}) are formed and in terms of these relative prices, employers make decisions. If goods were homogeneous, undoubtedly a futures market would spring up to register and resolve differences of expectations about future prices; but if this did not happen, our theory would still be valid after certain easy alterations.

The "profits" of employers are, retroactively reckoned, determined by comparing $p_1^tK^t$ and $p_2^tY^t$ with their past wage and machine costs. The profits resulting from today's decisions will similarly be known in the next period. In tranquil times, the *ex ante* hopes for profit and *ex post* realized profits will not differ too much; but differences that do develop will be noted in the market and will influence later decisions in the obvious direction.

"Net or excess demands" for (Y^t, K^t; K_1^t, L_1^t, K_2^t, L_2^t) will be determinate interdependent functions of (w^t, p_1^t, p_2^t; p_1^{t+1}, p_2^{t+1}; r^t; . . . etc.). Our number of independent equations is equal to the number of unknowns, with only price ratios being determinable until we specify enough about the supply and demand conditions for a circulating medium (*e.g.*, given gold coins; or minable gold; or paper currency issued by the State according to specified behavior rules; or stipulated banking practices).

My fixed-coefficient Marxian model, in the absence of technical innovations altering the (a, b) coefficients, would probably be characterized by attempted accumulation whenever r^t is high. As we have seen, this would cause r^t to be falling; with no physical deepening of capital possible, capitalists would lose in income what workers gain, which might slow up the accumulation process and which later could even cause it to cease. (If we assume that interest and profit rates are quite high, we can perhaps avoid some of the effective-demand problems that arise from the temptation to hoard money when interest rates are very low.)

Where alternative (a, b) techniques exist, lower r^t will induce adaptations in technique. These adaptations can be expected usually to slow down the drop in total interest income. Does this mean that the real wage will grow less rapidly? If lower r^t induces irreversible (a, b) changes of a so-called "labor-saving" type, the rise in real wage could indeed be slowed down or even be wiped out; and if this were to happen, the fall in r^t would have been converted into a subsequent rise in r^t, interest rising more than the drop in total wages. However, any change to a new (a, b), which now pays only because r^t is lower, will produce a higher real wage for each r^t than would the old (a, b); but if the demand for "capital" is sufficiently elastic or sufficiently little inelastic, induced technical changes might slow up the rate of fall of r^t so much as to cause the real wage to rise more slowly than it would under a single technique. I suspect, but cannot prove conclusively, that a Marxian who takes seriously the fixed-coefficient single-technique case is selecting the very model in which improvement of labor's share of the total income would be easiest within the framework of an unchanged-technology capitalism.

Life's Libretto: One Technique or Many? The case of a single fixed-coefficient technique is a very peculiar one indeed. Increase labor by epsilon and its share of the product may go from 100 per cent to zero! The later neoclassical economists would consider this as the extreme case of a marginal product curve for labor that is infinitely steep over a wide range: confront so steep a curve with a coinciding infinitely-steep supply curve of labor, and you have indeed created an indeterminate equilibrium wage with all the scope for collective bargaining and class power struggles that you could want.

Perhaps Karl Marx really had such a technology in mind. Perhaps not. It may be reasonable to believe that Marx, like Ricardo and other early writers, and unlike modern neoclassicists, never explicitly thought about what properties of the production function (a concept not yet explicitly defined or named) he wished to posit. It would be reading into him things that he would not recognize to claim a smooth production function with infinite substitution possibilities. On the other hand, he speaks again and again of alternative techniques. While many of these clearly depict technological change in the production function rather than movement within one function, the fact that the old methods are still known along with the new shows that Marx and Ricardo definitely envisage the existence of more than one technique. (Both Ricardo and Marx write of technical changes induced by price changes and adapted to changed price ratios; neither rules out the possibility that if the old price ratios were restored, the old technique might again become more economical.)

Whether or not Marx would resent being interpreted as a believer in a fixed-coefficient single-technique world, I should resent on behalf of the real world any such description. Go into any machine plant, pick up any engineering catalogue, study the books of physics and the histories of industrial processes, and you will see the variety of different ways of doing anything. If fixed Leontief coefficients (a_i, b_i) had characterized the world, it could never have got started. If the world has changed, the old processes are still remembered. Changing prices will induce accommodating changes in techniques. Perhaps the bookish economist will reply, "Foul! You are bringing in nonstatical, nonreversible changes." To this the realistic observer of the world will shrug his shoulders and answer, "So much the worse for a statical one-technique theory, or for that matter for any statical theory of production: but if we are to approximate reality by quasistatical tools, the more realistic production function to use is one with numerous alternative techniques, quite different in their input combinations and intensities."

We must not be put off by the bogey-man query: "Do you think that God created the earth with smooth Wicksteed homogeneous production functions involving a few aggregative factors, Socially Necessary Labor, Efficiency-unit Land, and Catch-all Dollar Capital?" To deny such a belief is not to confirm a belief in fixed-coefficients. A more realistic interpretation of actuality will recognize the existence of a large, perhaps finite, number of alternative techniques. The modern theory of linear programming permits the economist to handle these analytically; but even if we ivory-tower observers could not easily handle the analysis of many techniques, it would be another case of the Pathetic Fallacy to think that the actors in the real world will desist from making jig-saw puzzle substitutions because we economists can't easily analyze them.

John Jay Chapman once said that a visitor to this world would find its people behaving more like the people in a Verdi opera than in an Emerson essay. So if a visitor from Mars insisted upon a grandiose simplification of the economic system—instead of using the less dramatic methods of Walras, Chamberlin, and Keynes—I think he'd be safer in positing an aggregative production function of the Clark-Wicksteed type than one of the Leontief-Walras type.[23]

VI. *The Reserve Army of the Unemployed*

I shall conclude my dissection by investigating whether the existence of a reserve army of the unemployed can do the powerful things Marxians have claimed for it. Can it lower real wages to subsistence? Can it lower real wages below the marginal product of the last man when all the unemployed are put to work? Can it lower real wages below the marginal product of the first man of the reserve army when put to work?

Such questions must not be answered in simple terms. First, we shall have to specify exactly what monetary assumptions we are making; what institutional assumptions with respect to unionism, labor mobility, interpersonal differentials in skills and zealousness; what microeconomic assumptions about the mix of demand; etc. I shall not attempt to deal with these intricacies but will for the sake of the argument walk along the road with the simple Marxian aggregative models, making drastically simplifying assumptions.

Thus I assume two industries: Industry I producing capital goods and Industry II producing consumption goods. I go along with the simplifying assumption that machines and chocolates are produced with the same organic compositions of labor and capital goods; and that all capital is used up in one period so that the Marxian "constant capital" concept is easiest to handle. I assume the unemployed workers are as zealous and able as the employed. I assume away monopsony and monopoly to see where cruel competition will lead.

How do the unemployed depress real wages? If the unemployed are away at a distance and unable to offer their services, they will have no effect on money or real wages. It is by offering to work for less, and only by so doing, that they can depress money wages. The employer cannot get his workers to accept a cut merely by talking about the threat of replacing them by the unemployed; he will get the cut only if experience has taught the workers that this is not an empty threat. If men out of work do offer to work for less, the money wage cannot remain stationary in a perfectly competitive labor market. The money wage will fall and continue to fall until no more excluded men bid it down. I stress these banalities because so much Marxian literature seems to regard the mere *existence* of the unemployed (or of the

[23] I speak here of the first-edition Walras. In his second-edition *Éléments*, Walras had the system select among a number of different techniques to minimize costs; and in his third edition, he considered the infinite-substitution homogeneous production function case. Leontief, it must be said, never meant that his fixed coefficients be applied to gross aggregates.

"disguised unemployed") as itself a reason for competitive wages to fall. The natural question to ask then is this: "What is the effect on wages after the unemployed have been employed? How much have they depressed money and real wages?" Today, thanks to Keynes and others, we know that this is a complicated question. Falling money wages need not mean falling real wages if prices are made to fall as much. Indeed, waiving favorable Pigou-Keynes effects resulting from increased real balances induced by the price-wage decline, we can construct models of hyperdeflation in which money wages push down prices indefinitely with unemployment never disappearing and real wages not necessarily changing. Had Marx used a reserve army of the unemployed as a reason for falling *money* wages, one could better understand the logic of his system.

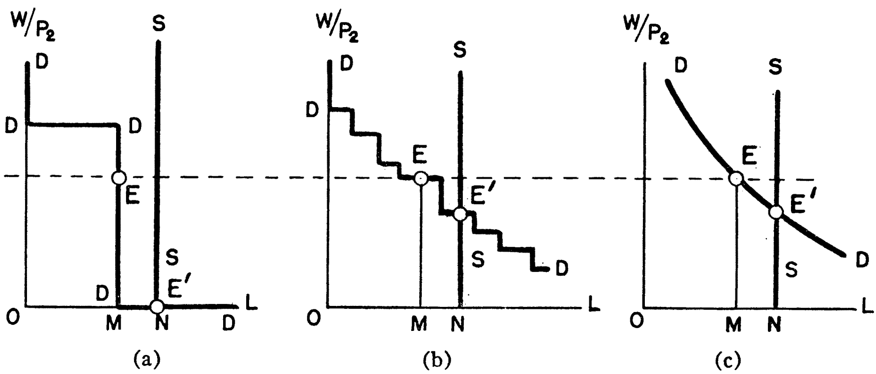

FIG. 2. In every case DD is "aggregate real demand for labor," SS is total labor force available for work, MN is the "reserve army of the unemployed," and E' the real wage when reserve army has disappeared.

To isolate the effects the unemployed have on real as against money wages, let's make the unrealistic supposition that they can bargain institutionally in terms of real wages—in terms of consumer goods or Ricardian corn. Then under the equal-organic-composition assumptions of our two-sector model, the "aggregative demand curve for labor in terms of wage goods" would be given by the discounted-marginal-physical-product curve of labor for either industry, the consumer-goods curve being exactly the same as the discounted-marginal-product curve in the capital goods industry once we have scaled the products so that they are 1-to-1 producible with the same labor and machine inputs.[24]

[24] The reader may make his own effective-demand assumptions to make this compatible with his theory of income determination. Thus, a good Keynesian will probably prefer to assume that aggressive government fiscal policy operates to offset any incipient deflationary or inflationary gaps threatened at full employment by nonintersecting saving and investment schedules. Some may give an active offsetting role to the central bank. Still others may be unaware or may deny that a problem could arise.

Figures 2a, 2b, and 2c show the resulting aggregative real demand for labor in the single-technique case, the many-technique case, and the infinitely-many-techniques neoclassical case. In every case, the unemployed reserve army of *NM* is made about 10 per cent of the labor force. Depending upon the technical elasticity of the marginal-product curve, the reserve army could reduce real wages by different amounts—but in Figures 2b and 2c wages can be reduced only by the reserve army's shrinking in size. The wage level *E′* in the three diagrams represents the lowest that real wages could fall when the reserve army had done its worst and become indistinguishable from the army of the employed. Would any competent observer of U.S.A., U.K., or U.S.S.R. technology believe that 10 per cent more men could not in any way be employed without making the last man incapable of adding much to product?[25]

The question is not whether in the shortest run, before employers knew they were to employ more and had made the necessary adjustments, marginal products might not fall greatly. Of course, they might fall. To get me to hire more workers in the next minute or day might require a great reduction in real wages. But let this happen for a few days or for months and years. Spurred by the ridiculously low real wages, employers will make needed adjustments and if we insist upon letting the real wage fall to absorb the unemployed in the long run, the equilibrium long-run wage will be at *E′* along the long-run marginal product curve *after* adjustments are made.[26]

I conclude from this way of looking at the problem that the strongest competition among the unemployed, the employed, and the employers will—when it has done its worst and depressed real wages enough to wipe out the unemployed—fail in modern western societies to depress real wages to anything like the subsistence level, instead bringing it down at worst to the (quite high) discounted marginal

[25] Writing in the 1860's, Marx could with some excuse think that real wages might fall to a subsistence level. A Marxian acquainted with the statistics of real wages in modern Western economies has no such excuse.

[26] A simple set of mathematical equations describing the content of Fig. 2c would be:

$$Y + (dK/dt) = Q(L,K),\ dK/dt = \sigma_w(LQ_L) + \sigma_r(KQ_K),$$

with government expenditure or aggressive central bank policy assuring that (dK/dt) is always such as to take up the resources not required for consumption. With fixed K, we can compute the reduction in real wage resulting from ΔL of the unemployed becoming employed, as follows: new real wage $= w + \Delta w = \partial Q(L + \Delta L, K)/\partial L$, and with $\Delta w/w$ equal to $[Q_{LL}L/Q](\Delta L/L)$, where the bracketed expression is the "reciprocal of the elasticity" of the marginal product curve at some intermediate point. Note that for given K and L, w is here quite independent of σ_w and σ_r. If we drop Marx's equal-organic-composition-of-capital assumption, this will no longer be true and the analysis has to be expanded.

product of labor at the level of employment equal to 100 per cent of the available labor force. Such a wage-floor is not only very high in the most advanced capitalistic society, but the bulk of the statistical evidence of economic history and the qualitative evidence concerning scientific invention and capital formation suggest as well that this wage-floor is advancing dynamically from year to year, decade to decade, at a rate that doubles perhaps about every 30 years.

VII. *Some Conclusions*

I have dealt with Karl Marx the economist, not Marx the philosopher of history and revolution. A minor Post-Ricardian, Marx was an autodidact cut off in his lifetime from competent criticism and stimulus. In applying to the models of Ricardo and Marx modern tools of analysis, I hope we are violating no rules of etiquette and in no way trying to suggest we are cleverer than they were!

What then is the verdict of the present dissection? Our post-mortem suggests the following conclusions:

1. Marx did do original work in analyzing patterns of circular interdependency among industries. Such work gains few converts and is not very helpful in promoting revolution or counterreactions. But like all pioneering effort it deserves the commendation of later craftsmen, and it deserves further development. There is half-truth in Schumpeter's adaptation of Clemenceau: "Marxian economics is too hard to be left to the Marxians." Only half, because the present paper is seen to involve little worse than school algebra and to be well within the frontier of modern economic theory.

2. Marx's labor theory of value of *Capital,* Volume I, does appear to have been a detour and an unnecessary one for the understanding of the behavior of competitive capitalism. The admittedly important analysis of imperfect or monopolistic competition is helped little or none at all by the "surplus-value" approach. That Böhm-Bawerk, Wicksteed, and Pareto were essentially right in their critiques of Marx seems borne out by the present investigation of the Marxian model.

3. I have concentrated, however, not on the problem raised for the pricing of many different goods by the unnecessary Marx-Ricardo labor-value assumptions. Instead I have concentrated on the more-neglected implications for relative goods-factor pricing of the Marxian surplus-value notations and notions. The present logical analysis suggests that the Marxian notions do not achieve the desired goal of "explaining the laws of motion or of development of the capitalistic system."

If it were true that the rich get richer the poor poorer, the distribution of income more skewed against labor and in favor of profit,[27] the two-sector models here analyzed would provide no particular hint of this. Indeed, writing in 1860 and being aware of the Industrial Revolution going on, an economist who took those models seriously should have (i) expected technological change to lower the (a, b) coefficients, (ii) should have expected the odds to favor a strong increase in real wages, the only exception arising from an extreme "bias" of inventions toward the extreme labor-saving type (a phenomenon *not* particularly sugested by the pre-1860 data known to financial journalists or men-of-affairs, nor particularly suggested by any a priori reasonings about the model or about the nature of technology).

I blame no one for failing to foresee the trends in the century after his death. But one can be forgiven for insisting upon the established fact that real wages in Germany, England, and America did rise more or less proportionately with total product from 1857 to 1957. To have been judged lucky by economic historians, Marx should have phrased a theory to explain the approximate constancy of wage's relative share of the national product, not the secular decline of this relative share. His actual models, we have seen, were perhaps better than he: for gifted with hindsight, we see that they contain in them no tendency for real wages to fall or to lag particularly behind the growth of output.

Nor do such models throw much light on the secular trends in the degree of imperfection of competition or on the propensity of the system to oscillate or stagnate. But all that is another story.

[27] We know little about the secular trends of the inequality of the personal distribution of income, as measured by Pareto's coefficient or by Gini's parameter describing the Lorenz curve. Pareto himself thought he had established a natural law of constancy of income inequality, independent of all public policies and institutional frameworks. The empirical basis for this generalization was never very impressive. The bulk of the available evidence, in fact, suggests that as capitalism has developed the Pareto coefficient has moved towards greater equality: whereas underdeveloped countries did, and do, show Pareto coefficients around 1.3, we find in developed countries Pareto coefficients of 2.0 for income before taxes and 2.2 after taxes. See J. Tinbergen, "On the Theory of Income Distribution," *Weltwirtschaftliches Archiv*, 1956, LXXVII, 156-57. Modern economics has no grandiose explanations to offer, but it can contribute to an analysis of the relevant forces at work.