THE PURE THEORY OF PUBLIC EXPENDITURE

Paul A. Samuelson

1. *Assumptions.* Except for Sax, Wicksell, Lindahl, Musgrave, and Bowen, economists have rather neglected the theory of optimal public expenditure, spending most of their energy on the theory of taxation. Therefore, I explicitly assume two categories of goods: ordinary *private consumption goods* $(X_1, \cdots, X_n)$ which can be parcelled out among different individuals $(1, 2, \cdots, i, \cdots, s)$ according to the relations $X_j = \sum_1^s X^i_j$; and *collective consumption goods* $(X_{n+1}, \cdots, X_{n+m})$ which all enjoy in common in the sense that each individual's consumption of such a good leads to no subtraction from any other individual's consumption of that good, so that $X_{n+j} = X^i_{n+j}$ simultaneously for each and every ith individual and each collective consumptive good. I assume no mystical collective mind that enjoys collective consumption goods; instead I assume each individual has a consistent set of *ordinal preferences* with respect to his consumption of all goods (collective as well as private) which can be summarized by a regularly smooth and convex utility index $u^i = u^i(X^i_1, \cdots, X^i_{n+m})$ (any monotonic stretching of the utility index is of course also an admissible cardinal index of preference). I shall throughout follow the convention of writing the partial derivative of any function with respect to its jth argument by a j subscript, so that $u^i_j = \partial u^i/\partial X^i_j$, etc. Provided economic quantities can be divided into two groups, (1) *outputs* or goods which everyone always wants to maximize and (2) inputs or factors which everyone always wants to minimize, we are free to change the algebraic signs of the latter category and from then on to work only with "goods," knowing that the case of factor inputs is covered as well. Hence by this convention we are sure that $u^i_j > 0$ always.

To keep production assumptions at the minimum level of simplicity, I assume a regularly convex and smooth production-possibility schedule relating totals of all outputs, private and collective; or $F(X_1, \cdots, X_{n+m}) = 0$, with $F_j > 0$ and ratios F_j/F_n determinate and subject to the generalized laws of diminishing returns.

Feasibility considerations disregarded, there is a *maximal* (ordinal) *utility frontier* representing the Pareto-optimal points — of which there are an $(s-1)$fold infinity — with the property that from such a frontier point you can make one person better off only by making some other person worse off. If we wish to make normative judgments concerning the relative ethical desirability of different configurations involving some individuals being on a higher level of indifference and some on a lower, we must be presented with a set of ordinal interpersonal norms or with a *social welfare function* representing a consistent set of ethical preferences among all the possible states of the system. It is not a "scientific" task of the economist to "deduce" the form of this function; this can have as many forms as there are possible ethical views; for the present purpose, the only restriction placed on the social welfare function is that it shall always increase or decrease when any one person's ordinal preference increases or decreases, all others staying on their same indifference levels: mathematically, we narrow it to the class that any one of its indexes can be written $U = U(u^1, \cdots, u^s)$ with $U_j > 0$.

2. *Optimal Conditions.* In terms of these norms, there is a "best state of the world" which is defined mathematically in simple regular cases by the marginal conditions

$$\frac{u^i_j}{u^i_r} = \frac{F_j}{F_r} \qquad \begin{array}{l}(i = 1, 2, \cdots, s;\ r, j = 1, \cdots, n) \text{ or} \\ (i = 1, 2, \cdots, s;\ r = 1;\ j = 2, \cdots, n)\end{array} \qquad (1)$$

$$\sum_{i=1}^{s} \frac{u^i_{n+j}}{u^i_r} = \frac{F_{n+j}}{F_r} \qquad \begin{array}{l}(j = 1, \cdots, m;\ r = 1, \cdots, n) \text{ or} \\ (j = 1, \cdots, m;\ r = 1)\end{array} \qquad (2)$$

$$\frac{U_i u^i_k}{U_q u^q_k} = 1 \qquad \begin{array}{l}(i, q = 1, \cdots, s;\ k = 1, \cdots, n) \text{ or} \\ (q = 1;\ i = 2, \cdots, s;\ k = 1).\end{array} \qquad (3)$$

Equations (1) and (3) are essentially those given in the chapter on welfare economics in my *Foundations of Economic Analysis*. They constitute my version of the "new welfare economics." Alone (1) represents that subset of relations which defines the Pareto-optimal utility frontier and which by itself represents what I regard as the unnecessarily narrow version of what once was called the "new welfare economics."

The new element added here is the set (2), which constitutes a pure theory of government expenditure on collective consumption goods. By themselves (1) and (2) define the $(s - 1)$-fold infinity of utility frontier points; only when a set of interpersonal normative conditions equivalent to (3) is supplied are we able to define an unambiguously "best" state.

Since formulating the conditions (2) some years ago, I have learned from the published and unpublished writings of Richard Musgrave that their essential logic is contained in the "voluntary-exchange" theories of public finance of the Sax-Wicksell-Lindahl-Musgrave type, and I have also noted Howard Bowen's independent discovery of them in Bowen's writings of a decade ago. A graphical interpretation of these conditions in terms of *vertical* rather than *horizontal* addition of different individuals' marginal-rate-of-substitution schedules can be given; but what I must emphasize is that there is a different such schedule for each individual at each of the $(s - 1)$fold infinity of different distributions of relative welfare along the utility frontier.

3. *Impossibility of decentralized spontaneous solution.* So much for the involved optimizing equations that an omniscient calculating machine could theoretically solve if fed the postulated functions. No such machine now exists. But it is well known that an "analogue calculating machine" can be provided by competitive market pricing, (a) so long as the production functions satisfy the neoclassical assumptions of constant returns to scale and generalized diminishing returns and (b) so long as the individuals' indifference contours have regular convexity and, we may add, (c) so long as all goods are private. We can then insert between the right- and left-hand sides of (1) the equality with uniform market prices p_j/p_r and adjoin the budget equations for each individual

$$p_1X^i_1 + p_2X^i_2 + \cdots + p_nX^i_n = L^i \quad (i = 1, 2, \cdots, s), \qquad (1)'$$

where L^i is a lump-sum tax for each individual so selected in algebraic value as to lead to the "best" state of the world. Now note, if there were no collective consumption goods, then (1) and (1)′ can have their solution enormously simplified. Why? Because on the one hand perfect competition among productive enterprises would ensure that goods are produced at minimum costs and are sold at proper marginal costs, with all factors receiving their proper marginal productivities; and on the other hand, each individual, in seeking as a competitive buyer to get to the highest level of indifference subject to given prices and tax, would be led as if by an Invisible Hand to the grand solution of the social maximum position. Of course the institutional framework of competition would have to be maintained, and political decision-making would still be necessary, but of a computationally minimum type: namely, algebraic taxes and transfers $(L^1, \ldots L^s)$ would have to be varied until society is swung to the ethical observer's optimum. The servant of the ethical observer would not have to make explicit decisions about each person's detailed consumption and work; he need only decide about generalized purchasing power, knowing that each person can be counted on to allocate it optimally. In terms of communication theory and game terminology, each person is motivated to do the signalling of his tastes needed to define and reach the attainable-bliss point.

Now all of the above remains valid even if collective consumption is not zero but is instead *explicitly set* at its optimum values as determined by (1), (2), and (3). *However no decentralized pricing system can serve to determine optimally these levels of collective consumption.* Other kinds of "voting" or "signalling" would have to be tried. But, and this is the point sensed by Wicksell but perhaps not fully appreciated by Lindahl, now it is in the selfish interest of each person to give *false* signals, to pretend to have less interest in a given collective consumption activity than he really has, etc. I must emphasize this: taxing according to a benefit theory of taxation can not at all solve the computational problem in the decentralized manner possible for the first category of "private" goods to which the ordinary market pricing applies and which do not have the "external effects" basic to the very notion of collective consumption goods. Of course, utopian voting and signalling schemes can be imagined. ("Scandinavian consensus," Kant's "categorical imperative," and other devices meaningful only under conditions of "symmetry," etc.) The failure of market catallactics in no way denies the following truth: given sufficient knowledge the optimal decisions can always be found by scanning over all the attainable states of the world and selecting the one which according to the postulated ethical welfare function is best. The solution "exists"; the problem is how to "find" it.

One could imagine every person in the community being indoctrinated to behave like a "parametric decentralized bureaucrat" who *reveals* his preferences by signalling in response to price parameters or Lagrangean multipliers, to questionnaires, or to other devices. But there is still this fundamental technical difference going to the heart of the whole problem of *social* economy: by departing from his indoctrinated rules, any one person can hope to snatch some selfish benefit in a way not possible under the self-policing competitive pricing of private goods; and the "external economies" or "jointness of demand" intrinsic to the very concept of collective goods and governmental activities makes it impossible for the grand ensemble of optimizing equations to have that special pattern of zeros which makes *laissez-faire* competition even *theoretically* possible as an analogue computer.

4. *Conclusion.* To explore further the problem raised by public expenditure would take us into the mathematical domain of "sociology" or "welfare politics," which Arrow, Duncan Black, and others have just begun to investigate. Political economy can be regarded as one special sector of this general domain, and it may turn out to be pure luck that within the general domain there happened to be a subsector with the "simple" properties of traditional economics.