

Piketty's Elasticity of Substitution: A Critique

Gregor Semieniuk*

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Abstract

This note examines Thomas Piketty's (2014) explanation and prediction of simultaneously rising capital income ratio and profit share by an elasticity of substitution, σ , greater than one between labor and capital in an aggregate production function. I review Piketty's elasticity argument, which relies on a non-standard capital definition. In light of the theory of land rent, I discuss why the non-standard capital definition is problematic for estimating elasticities. For lack of existing results, I make a simple estimate of σ in the class of constant elasticity of substitution functions for Piketty's data as well as for a subset of his capital measure that comes closer to the standard capital definition. The estimation results cast doubt on Piketty's hypothesis of a σ greater than one.

*Department of Economics, New School for Social Research, semig068@newschool.edu. This note arose out of discussions with Duncan Foley, Lance Taylor and Thomas Michl; their and Isabella Weber's comments improved earlier versions. By making their data available on the web, Thomas Piketty and Gabriel Zucman enabled the empirical estimates in this note. The research is supported by a grant from the Institute for New Economic Thinking to the Schwartz Center for Economic Policy Analysis, New School for Social Research.

“In our view, it is natural to imagine that [the elasticity of substitution between labor and capital in a two-factor, one-commodity neoclassical growth model of the economies of rich countries] was possibly much less than 1 in the 18th-19th centuries and became significantly larger than 1 in the 20th-21st centuries. One expects a higher elasticity of substitution in more diversified economies where capital can take many forms.” (Piketty and Zucman 2013, p. 36 – all page numbers refer to their working paper version not the QJE version)

In his important book, Thomas Piketty (2014) explains the simultaneously rising capital income ratio and profit share that he observes in his dataset with an elasticity of substitution greater than one between labor and capital in an aggregate production function.¹ Piketty deserves great credit for fueling the debate about the pressing issues of income and wealth inequality. But, criticisms of using aggregate production functions aside, his elasticity argument still has a problem. Previous elasticity estimates have tended to be below one.

Piketty goes against this evidence because he uses a new definition of capital. By capital he means “all non-human assets” existing in the economy, whether used for production or not. This contrasts with the “cumulated investment expenditures” used in production, that has been the definition otherwise. The sum of non-human assets valued at market prices fluctuates more over time than cumulated investment valued at cost and the elasticity of substitution, which links these assets values to the

¹An elasticity of substitution above one implies that an increase in one input factor’s quantity relative to the other input factors, leads to a rise in this increased input factor’s share in output. Then a rising capital relative to labor input increases the capital share, α .

return on capital has to be bigger.² It is not clear how big because Piketty appears not to report quantitative estimation results. From inspection of the data, however, he mentions a range of 1.3 to 1.6 for the elasticity of substitution between capital and labor in present times high income economies (Piketty 2014, p. 221.) Any value above one allows him to explain the simultaneous increase in capital/income ratio and capital share with neoclassical growth theory. Since, as Robert Rowthorn (2014) points out, the high value of the elasticity is the pivot on which Piketty's theory of the rising in capital share hinges, it is interesting to understand the argument and data better.

In this note I will review Piketty's elasticity argument and then discuss it critically in light of his new capital definition by drawing on insights from the theory of land rent. Then, assuming a production function as Piketty does, I will make a simple quantitative estimate of the elasticity of substitution for the data used by Piketty as well as for a subset of his capital measure that comes closer to the traditional definition. The estimation results cast doubt on the hypothesis of an elasticity of substitution greater than one.

Piketty's Elasticity Argument

Piketty's (2014) discussion of the elasticity of substitution is based on the paper by Piketty and Gabriel Zucman (2013.) In Section 7 of Piketty and Zucman they observe that both the capital/income ratio, K/X or β in their notation, and the

²See James Galbraith (2014a) on the problem of using a financial measure of capital and Galbraith (2014b) on how the valuation of financial assets influences Piketty's accounting.

capital share in income, α , have risen in high income countries, while the rate of return on capital, $r = \alpha/\beta$, has fallen slightly.³ Then they write “*Of course, this decline [in r] is what one would expect in any model: when there is more capital, the rate of return to capital must go down. The interesting question is whether it falls more or less than the quantity of capital. According to our data it has fallen less, implying a rising capital share.*” (ibid. p. 34 – same wording also in their QJE version)

They must mean whether the rate of return falls more than proportionally to the rise in the capital/output ratio, β , since $\alpha = r\beta$.

Then they assert that this observation can be explained by a two factor, one commodity production function with an elasticity of substitution greater than one, which implies precisely that a rise in capital leads to an increase in its share in output. This is because it can be substituted for labor so well, i.e. the additional units of capital lead to almost as much additional output per unit capital as for the previously existing units of capital (what economists call the marginal product of capital.) Since by assumption the rate of return on capital equals the marginal product, the additional units of capital add more to capital’s income than the diminished return per capital unit reduces it and the total remuneration capital increases relative to that of labor.

Piketty and Zucman highlight that their explanation is more parsimonious than alternative models with imperfect competition or with three input factors: skilled and unskilled labor and capital (ibid. p. 35). Stressing that their “*discussion of*

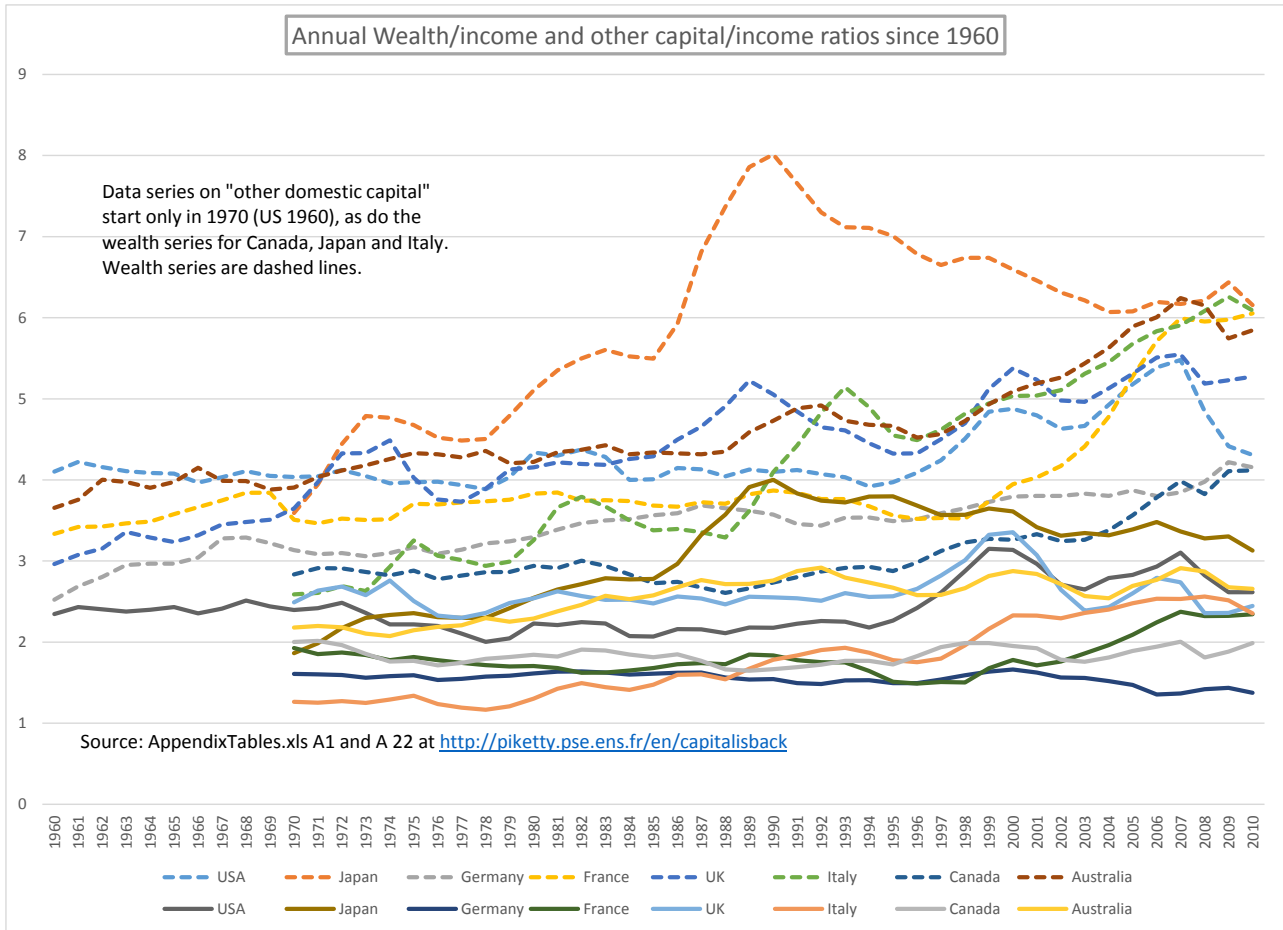
³Their data appendix includes files on eight countries: Australia, Japan, Canada, USA, France, Germany, Italy and UK.

capital shares and production functions should be viewed as merely exploratory and illustrative” (p. 36) they nevertheless predict that it is likely that α will rise to above 40% because of a strong growth in β as it returns to its value of the 18-19th centuries combined with an elasticity of substitution greater than one. Leaving aside for the moment that such a theory rests entirely on a technological explanation of substitutability between capital and labor as the cause of increasing inequality (but see Barbosa-Filho (2014) for a discussion of how social factors may impinge on the elasticity magnitudes, and Rowthorn (2014) and Taylor (2014) who offer alternative macroeconomic theories of inequality) I will discuss next the non-standard definition of capital employed by the authors and its potential to produce different results.

The Definition of Capital

An elasticity greater one between input factors into production stands in contrast with previous empirical work (Rowthorn, 2014; Chirinko, 2008), but is likely an artifact of a non-standard definition of capital. Piketty and Zucman prefer the all non-human asset definition for capital because it is consistent with balance sheet estimates of wealth (2013, p. 6–7), because housing assets make up half of national wealth and some housing may be used for business purposes (ibid. p. 12 fn. 17) and because it is what “*eighteenth and nineteenth century economists aimed to capture.*” (ibid. p. 7) In particular, according to them the lack of balance sheet data was a major reason why economists from Cambridge, U.K., took issue with the production

Figure 1:



function. With the new data they presumably would not do so.⁴ Piketty believes

⁴They do not explain how exactly this would have resolved the debate. Galbraith (2014a) discusses problems with Piketty's (2014) depiction of that debate; for accounts of how the debate actually went see Harcourt (1972), and also Cohen and Harcourt (2003), Pasinetti (2003), and Samuelson (1966.)

that his non-standard definition of capital “*is a useful, meaningful, and well-defined starting point*” (ibid.) for analyzing models of growth and distribution and the elasticity of substitution.

Empirical Differences

In how far does this change the empirical evolution of the capital/income ratio? Stefan Homburg (2014) shows for France that it is thanks to the non-standard definition that Piketty’s capital income ratio is increasing. The revaluation of land and real estate, but not the accumulated capital used in production grew at a faster rate than income. In other words, cumulated investment has increased proportionally with income in France.

Figure 1 shows that taking out the real estate and foreign capital parts of Piketty’s measure of capital, the remaining “other domestic capital” series has no upward trend for most of the eight countries for which Piketty provides data. “Other domestic capital” is still a financial valuation of capital and subject to revaluation that may have little to do with a change in value at cost of the underlying capital. But it approximates cumulated investment better by excluding residential housing that is not used for production. The non-standard definition of capital significantly increases the magnitude of and changes in the capital wealth ratio. To be able to distinguish traditional and the non-standard definitions, I shall call Piketty’s definition wealth, W , not capital, K .

The Theory of Land Rent

The theoretical support for wealth as an input into production for the purpose of elasticity substitution is questionable. It is curious that Piketty calls on the 18th-19th century or classical economists to back up his argument of subsuming all assets under the term capital. The classical economists, particularly Adam Smith, David Ricardo and their critic, Karl Marx distinguished between capital and land. According to their labor theory of value, labor creates value in the form of commodities. Some of the commodities are used as capital for further production. Capital's value (and price) arises from its cost of production, which is a function of the labor used in production. The owners of capital can claim part of the money against which the valuable commodities exchange in the form of profits. The rate of profit is the quotient of the flow of profits and the stock of capital used in production. A class of non-produced assets of which there is a limited and scarce supply, called "land", on the other hand, does not earn profits. Instead, owners of scarce land earn a rent thanks to their ability to exclude others via property rights from the free use of the land asset for production.⁵ Land type assets may be needed directly or indirectly for production, for instance in the form of agricultural land, a river for electricity generation, housing for workers, but also rights to financial or legal intermediation or patents.⁶ Through the rent, land asset owners appropriate part of the value created by labor with the aid of capital. "Substituting" land for labor means that the price of

⁵The classical theory of rent was described by Smith (1905, book 1, chapter 11) and critiqued and refined by Ricardo (1951, chapter 2). For a discussion see Lackmann (1976) and for a recent summary, Foley (2006).

⁶Foley (2013) discusses modern manifestations of the old concept of rent in financial and information services.

land associated with production rises relative to the wage share, which is expressed in Piketty's rising wealth/income ratio. But following the classical economists, this does not imply that more land is harnessed for production – by definition land is scarce and limited. Rather, it implies that the price of land increases relative to the labor share. It is a consequence of a revaluation of scarce land, not an increase in its quantity.⁷

The reason why the price of land changes, is a change in the rate of return on produced assets. Land, like any other asset, can be sold on the market. Its market price (that goes into Piketty's wealth) is determined by what rate of return it can earn. It earns the rate of return that produced assets, or capital, in the same risk class earn. Since

$$\text{rate of return} = \frac{\text{rent}}{\text{price of land}}$$

and with rate of return on assets determined by the return on capital, and the rent of land determined by its scarcity, the land's price will be bid to the price that results from the ratio of the two.

$$\text{price of land} = \frac{\text{rent}}{\text{rate of return}}$$

If scarcity remains constant and the rate of return on produced capital falls – as

⁷All rent discussed here is average rent. Particular land assets will generate above and below average amounts of rent, as Ricardo highlights in his theory of differential rent on different qualities of land.

it usually does when produced capital is substituted for labor as Piketty points out –, the price of land or its “capitalization” must increase to lower its rate of return on the constant amount of rent. As a consequence, both the value of capital stock used in production and the value of land increase.

Although the mainstream of economics has subsequently criticized the labor theory of value, the concept of rent on scarce resources is widely accepted.⁸ The narrower definition of capital used by economists that have estimated the elasticity before Piketty recognized the important distinction between capital that earns a profit and land that earns a rent, and focused on accumulated capital only. The next subsection illustrates why the land-capital distinction matters.

Elasticity Measurement Consequences

Including land into a measure of capital has an important bearing on elasticity estimates. Revaluations of the massive amount of land assets, in high income economies (think of the US residential housing market) as a response to a small change in the rate of return to capital may lead to significant changes in the wealth/income ratio, β , but not in the amount of accumulated capital relevant for production. In particular, the housing assets that Piketty includes in his wealth measure of capital contain large share of land type assets; a change in the rate of return from 11% to 10% induces a ten percent rise in the land price, a change from 6% to 5% even a twenty percent increase. That multiplied by the share of land in wealth, which is

⁸Modern treatments of rent on land or scarce resources motivated by utility maximization considerations of scarce resource owners can be found in textbooks on resource economics, e.g. Fisher(1981); Lackman (1977) traces its pedigree to the classicals.

considerable as Figure 1 adumbrates, is the fluctuation in β stemming from a revaluation. The elasticity is an increasing function of the change in β ; measures of it that include land assets will reflect those additional fluctuations.

For the Piketty and Zucman data it seems that capital has increased enormously while the rate of return has only fallen slightly. In reality much of it may be a revaluation of the land capitalization. As Figure 1 shows, the actual underlying capital used for production may have changed little and it is to be expected that the elasticity measure discussed in Piketty and Zucman’s paper is inflated. In the rest of this note I will compare empirical estimates of the elasticity of substitution using wealth and the other domestic capital measure as a proxy for cumulated investment.

Estimating the Elasticity of Substitution

I will carry out a simple estimate of correlations in the data that are purely empirical, but can be interpreted as the elasticity of substitution assuming a two-factor production function underlying the data. The estimate will be of time series for the last four to five decades, which is the time frame for which Piketty has annual data and also the time during which he makes the argument for the higher elasticity. I have not found any existing estimates of the elasticity for the Piketty and Zucman dataset. I will estimate it alternatively using Piketty’s non-standard capital measure that I call “wealth” and the subset measure that is denoted as “other domestic capital” in the dataset. I fit a line to the logarithms of the pairs of rate of return and the wealth/income ratio. The equation underlying the fit follows from the deriva-

tive of output with respect to capital in a constant elasticity of substitution (CES) production function. Alternatively it can be derived from the derivative of cost with respect to the profit rate in the dual CES cost function. The derivation is in the mathematical appendix. In particular I estimate

$$\log \frac{W}{X} = -\sigma_W \log(r) + c_W + \epsilon \quad (1)$$

$$\log \frac{K}{X} = -\sigma_K \log(r) + c_K + \epsilon \quad (2)$$

where W/X is the ratio of “wealth” to income, σ_W is the elasticity of substitution between wealth and labor in production, and r is the rate of return. c_W comprises constant factors that determine the wealth/income ratio and ϵ are technology and distributional shocks. The second equation displays analogous quantities for the “other domestic capital” series, K .

For the purpose of this short note the simple estimation suffices. An econometric study trying to make the best use of the information the data provides should not only test the data for violation of the assumptions of the linear model but also compare the likelihood of the data for the assumed constant returns to scale with that under the more general assumption of non-constant returns to scale, for which Kmenta (1976) gives an estimable approximation. Robert Chirinko (2008) has a discussion of further elasticity estimation issues. The present simple equation, however, does represent the constant return function that Piketty refers to in the online technical appendix to his book.

Data

W is Piketty’s definition of capital, which I am calling “wealth”. K is the narrower definition of capital that Piketty calls “other domestic capital” and which I will use as a proxy for cumulated investment for lack of better data. With X the “net national income” series used by Piketty, this yields the two time series W/X , K/X . I will use Piketty’s “capital share excluding government interest earned” series that is also used by Piketty (2014, technical appendix p. 36), and label it α following his notation. Then, like in Piketty, the rate of return to wealth time series is

$$r = \alpha/\beta = \alpha W/X \tag{3}$$

The precise sources of the data are in the Data Appendix.

Results

The log-log plots in Figures 2 and 3 summarize the simple estimate of the elasticity of substitution, σ for Piketty’s data for eight countries. The rate of return to wealth is on the x-axes, the wealth/income and capital/income ratios on the y-axes. Black dots have Piketty’s “wealth” measure and blue dots Piketty’s “other domestic capital” measure as the numerator. The slopes of the linear fits are the estimate of σ multiplied by minus one. The x-axis scale varies from plot to plot, the y-axis scale is the same throughout for comparison, except for Japan, which has some data points with $W/X > 7$.

Figure 2:

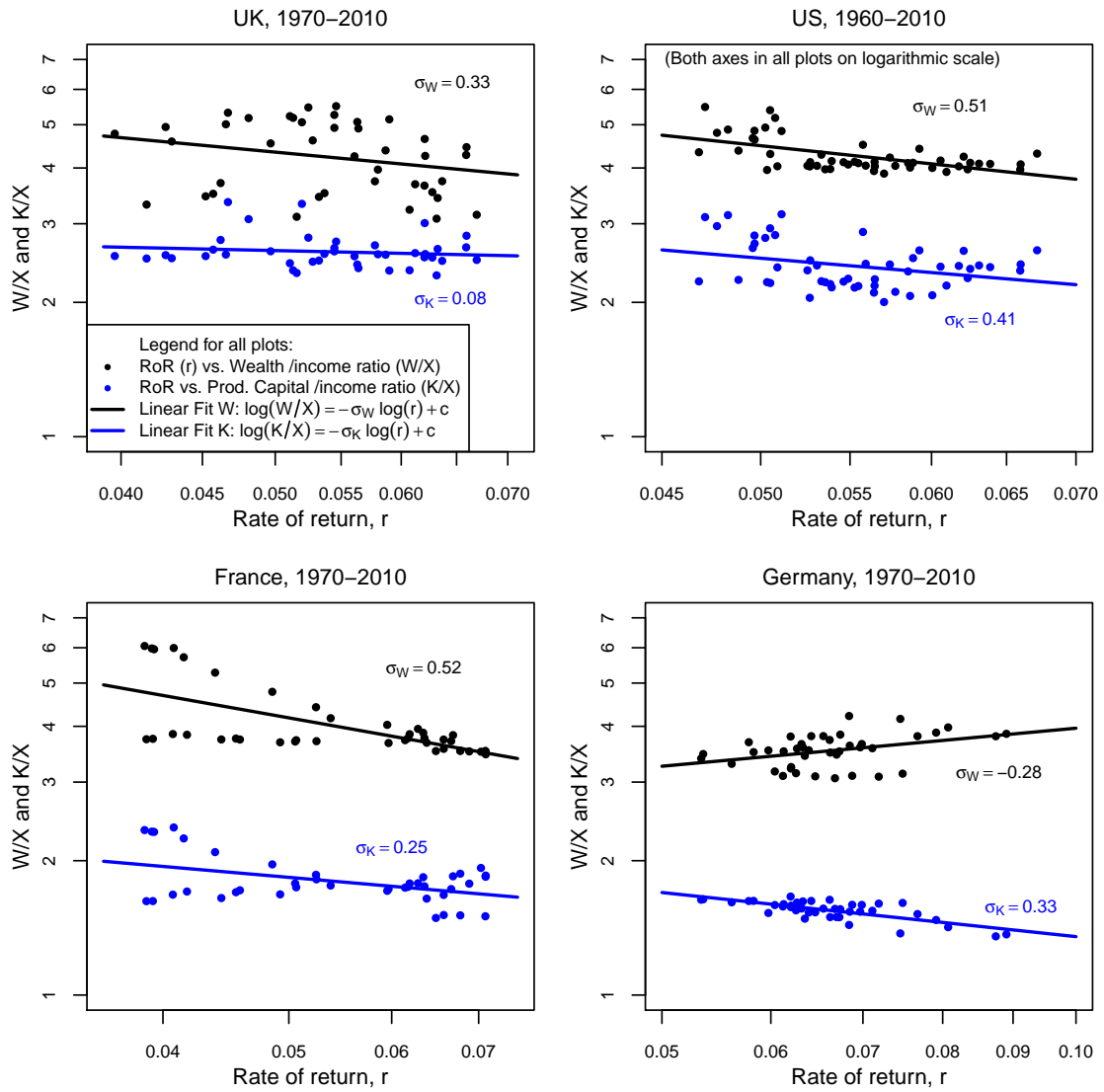
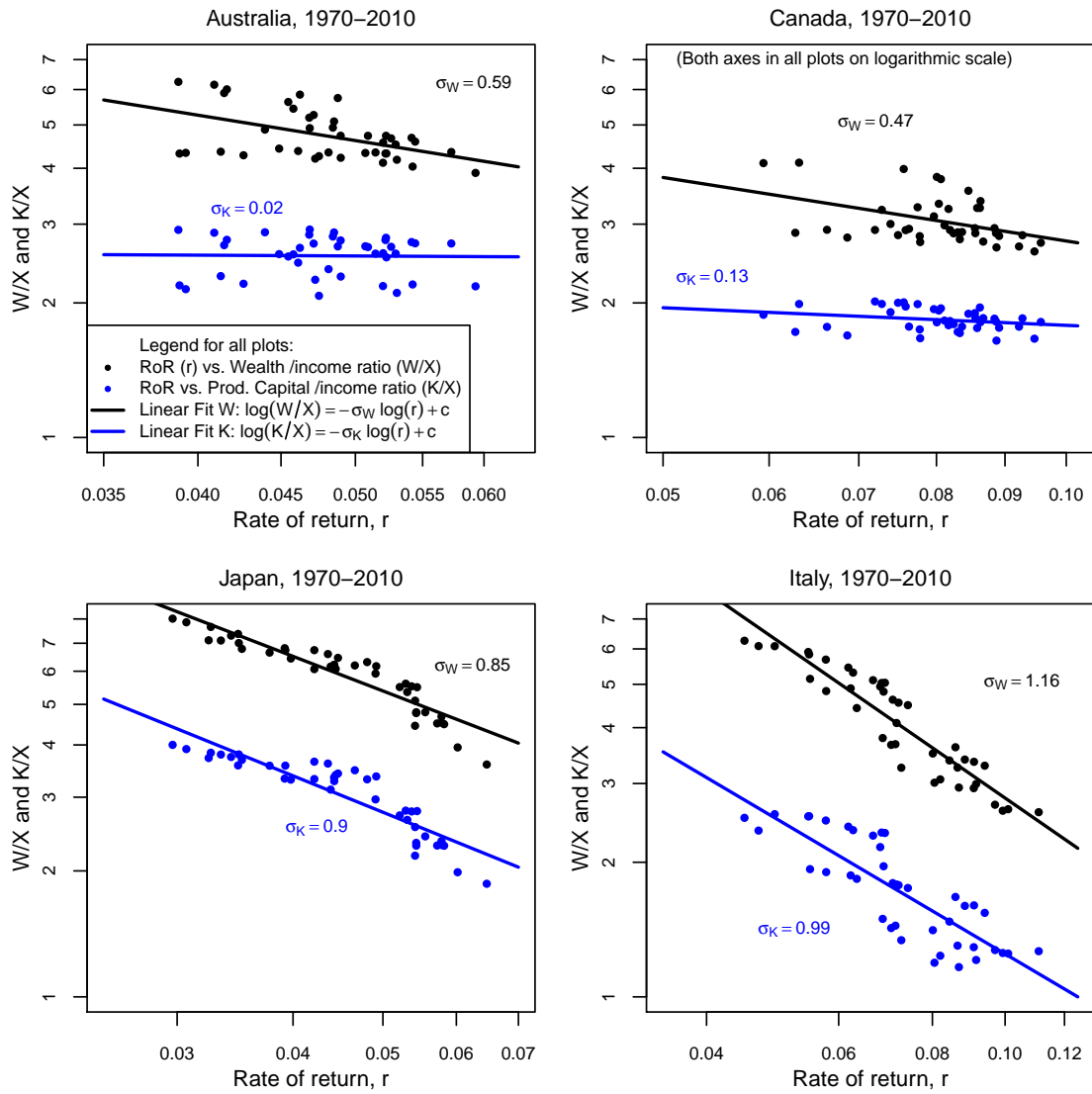


Figure 3:



Discussion of Results

The black linear fits show that except in Italy, the σ estimate for Piketty's wealth measure is below one, most of the times barely one half. Germany appears anomalous, which perhaps shows the limits of trying to estimate capital value using wealth. Using the other domestic capital series to proxy capital, which fluctuates less as it excludes housing and land, the elasticity drops further everywhere except in Japan. Germany now shows a positive but small elasticity of substitution.

Naturally the above estimators for σ are only a first approximation. However, a more sophisticated analysis would hardly catapult a slope from, say, 0.33 to above 1. Using Piketty's theory of a two-factor production function, there is little evidence of an elasticity of substitution above one especially in Piketty's most discussed countries: France, UK, US.

Matt Rognlie (2014) offers an explanation for these surprisingly low elasticities, by highlighting that Piketty's discussion is in net of depreciation terms and that for net quantities the elasticity of substitution is lower than for gross quantities, since the depreciation is paid for out of profits and not labor income. The "Gross vs. Net Appendix" at the very end of the document illustrates Rognlie's logical argument at the example of Britain, which has a higher estimate of the elasticity when using gross output and capital share. Yet, precisely because the discussion is about net quantities, the graphs in Figures 2 and 3 are the ones of interest that contradict Piketty's theory.

Conclusion

Thomas Piketty’s empirical and analytical brilliance in his sweeping history of income and wealth inequality at the micro-level is remarkable. However, his elasticity of substitution theory of a rising capital share relying on a “wealth” measure of capital at the *macro*-level is both theoretically and empirically questionable. Even if one uses wealth instead of capital for the capital/income ratio and thus inflates elasticity estimates, Piketty’s data do not support an elasticity of substitution above one. Ridding the capital measure from housing assets, the evidence becomes even more contradicting. Other macroeconomic arguments will be needed to grapple with the rising capital share and the inequality it implies. It is to Piketty’s great credit that he has fueled the debate about this pressing issue.

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Data Appendix

The data series are taken from <http://piketty.pse.ens.fr/en/capitalisback>

| | |
|---|-----------------------------------|
| Wealth/Net Income (W/X) | AppendixTables.xls sheet Table A1 |
| Capital/Net Income (K/X) | AppendixTables.xls sheet Table A1 |
| Net Capital Share excl. gov't interest α | [country].xls sheet Table 11a |

For the gross computations for the UK I used gross national income as well as the sums of components of gross wealth and profit shares that can all be found in UK.xls in the sheets DataUK1 and DataUK3.

Mathematical Appendix: Deriving the Equation for Estimation

The equation to be estimated can be derived both from a CES production function or its dual CES cost function. I will carry out the production function derivation, which remains close to Piketty (2014) and Rowthorn (2014.)

Assuming with Piketty that output, X , comes from a CES production function with inputs K and L , and the elasticity of substitution, σ , the CES functional form is

$$X = \left[\underbrace{d(\rho K)^{\frac{\sigma-1}{\sigma}} + (1-d)(\xi L)^{\frac{\sigma-1}{\sigma}}}_{\Omega} \right]^{\frac{\sigma}{\sigma-1}} \quad (\text{A.1})$$

where $d \in (0, 1)$ is a distribution parameter that determines the relative importance of each factor in production and ρ and ξ are productivities. Assuming that factors are remunerated equal to their marginal products, the remuneration of capital or rate of return, r , is

$$\frac{\partial X}{\partial K} = \left(\left([\Omega]^{\frac{\sigma}{\sigma-1}} - 1 \right)^\sigma \right)^{\frac{1}{\sigma}} d\rho(\rho K)^{\frac{\sigma-1}{\sigma}-1} \quad (\text{A.2})$$

$$= X^{\frac{1}{\sigma}} d\rho(\rho K)^{\frac{-1}{\sigma}} \quad (\text{A.3})$$

$$r = d\rho^{\frac{\sigma-1}{\sigma}} \left(\frac{K}{X} \right)^{\frac{-1}{\sigma}} \quad (\text{A.4})$$

Writing the marginal product in terms of K/X this gives

$$\frac{K}{X} = \left(\frac{r}{d\rho^{\frac{\sigma-1}{\sigma}}} \right)^{-\sigma} \quad (\text{A.5})$$

Taking logarithms gives

$$\log \frac{K}{X} = -\sigma \log(r) - \sigma \log(d) - (1 - \sigma) \log(\rho) \quad (\text{A.6})$$

This expression can be estimated as

$$\log \frac{K}{X} = -\sigma \log(r) + c + \varepsilon \quad (\text{A.7})$$

where c the constant part of the remaining terms and ε are possible technology shocks to capital productivity, ρ and to the distribution parameter d .

Gross vs. Net Appendix

Figure 4 shows two plots with pairs of rate of return and wealth/nat. income (black) and capital/nat. income (blue) ratio observations for the UK for 1970-2010 and linear fits estimated as in the text body. The left plot shows net national income and the net rate of return. The right plot show gross national income and gross rate of return. As above, the slopes of the regression lines are the elasticity of substitution estimates times minus one. The plots show that gross elasticities are higher than their net counterparts.

Figure 4:

