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A note on Piketty's *Capital in the Twenty-First Century*

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Thomas Piketty's *Capital* documents long-term trends in income and wealth in advanced economies. It also provides a theoretical framework for analysing the past and projecting the future. Piketty argues that the ratio of wealth to national income is on an upward trend and that this is responsible for the rising income share of wealth-owners. This note accepts Piketty's main empirical findings but questions his interpretation. The rising income share of wealth-owners is not due to the over-accumulation of capital, as he claims, but just the opposite. There has been too little real investment. The note also considers the long-term dynamics of Piketty's model and explores the effect of modifying his assumptions about savings behaviour. Finally, it considers the implications of rising asset prices, which are documented by Piketty but are not adequately taken into account in his theoretical analysis or projection of future trends.

Key words: Piketty, Distribution of Income, National Wealth, Economic Growth
JEL classifications: E21, E25, N10

1. Introduction

Thomas Piketty's new book *Capital in the Twenty-First Century* (2014) has been a stunning success. This is not surprising. It brilliantly documents long-term trends in wealth ownership and income distribution in advanced economies. In particular, it shows how the share of income accruing to wealth-owners has increased dramatically in recent decades.[1] It also provides a simple explanation of this development based on the standard neoclassical theory of factor shares. This theory establishes a link between the capital intensity of production and the share of profits in total output. The nature of this link depends on the elasticity of substitution between capital and labour. When this elasticity is greater than unity, an increase in the capital-output ratio leads to an increase in the share of profits. This, in essence, is Piketty's explanation for the increased share of wealth-owners in national income. Thus, the shift in income distribution is due to the over-accumulation of capital: there has been too much real investment.

The foregoing explanation has two related flaws. Piketty's assumption regarding the elasticity of substitution is not correct. There is considerable evidence that this

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[1] See Piketty (2014), Figure 6.5 on page 222.

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elasticity is less than unity. Moreover, his method for measuring changes in the capital-output ratio is misleading. He fails to allow for the disproportionate increase in the market value of certain assets, especially housing, in recent decades. This leads him to conclude, mistakenly, that the capital-output ratio has risen by a considerable amount. In fact, conventional measures of this ratio indicate that it has been either stationary or has fallen in most advanced economies during the period in question. This would suggest that the basic problem is not the over-accumulation of capital, but just the opposite. There has been too little real investment.

2. The determination of factor shares

This note uses a simple model to explore Piketty's analysis of income dynamics. Apart from slight notational differences, this model is similar to that described by Piketty in his technical appendix.[2] The present model also makes an explicit allowance for variations in the market valuation of real assets and for technical progress. Technical progress is of the labour-augmenting (Harrod-neutral) variety. This choice of technical progress is motivated by the desire to analyse balanced growth paths in which capital and output grow at the same rate. It is also supported by the evidence (Klump *et al.*, 2007). In the economies that Piketty considers, net income from abroad has for most of the time been a small fraction of total income. Net income from abroad is assumed to be zero in my model.

2.1 Preliminary remarks

Piketty uses the terms 'capital' and 'wealth' interchangeably to denote the total monetary value of shares, housing and other assets. 'Income' is measured in money terms. I shall reserve the term 'capital' for the totality of productive assets evaluated at constant prices. The term 'output' is used to denote the totality of net output (value-added) measured at constant prices. Piketty uses the symbol β to denote the ratio of 'wealth' to 'income' and he denotes the share of wealth-owners in total income by α. In his theoretical analysis, this share is equated to the share of profits in total output. Piketty documents how α and β have both risen by a considerable amount in recent decades. He argues that this is not mere correlation but reflects a causal link. The rise in β is responsible for the rise in α. To reach this conclusion, he first assumes that β is equal to the capital-output ratio K/Y, as conventionally understood. From his empirical finding that β has risen, he concludes that K/Y has also risen by a similar amount. According to the neoclassical theory of factor shares, an increase in K/Y will only lead to an increase in α when the elasticity of substitution between capital and labour σ is greater than unity. Piketty asserts this to be the case. Indeed, based on movements α and β, he estimates that σ is between 1.3 and 1.6.[3]

Thus, Piketty's argument rests on two crucial assumptions: $\beta = K/Y$ and $\sigma > 1$. Once these assumptions are granted, the neoclassical theory of factor shares ensures that an increase in β will lead to an increase in α. In fact, neither of these assumptions is supported by the empirical evidence, which is surveyed briefly in the Appendix. This evidence implies that the large observed rise in β in recent decades is not the result of a big rise in K/Y but is primarily a valuation effect.[4]

[2] See http://piketty.pse.ens.fr/files/capital21c/en/Piketty2014TechnicalAppendix.pdf, pp. 37–39.
[3] Piketty (2014), chapter 6, p. 221. Also, see the online technical appendix p. 39.
[4] This point is made powerfully by James Galbraith (2014) in his review of Piketty.

2.2 The model

Real output is given by the following constant elasticity of substitution (CES) production function:

$$Y = \left[bK^{\frac{\sigma-1}{\sigma}} + (1-b)\left(Le^{\mu t}\right)^{\frac{\sigma-1}{\sigma}} \right]^{\frac{\sigma}{\sigma-1}} \tag{1}$$

where μ is the constant rate of labour-augmenting technical progress and σ is the constant elasticity of substitution between capital K and labour L. The parameter b is constant.

Following Piketty, assume that capital receives its marginal product. The rate of profit is thus:

$$\pi = \frac{\partial Y}{\partial K} \tag{2}$$

which yields:

$$\pi = b\left(\frac{K}{Y}\right)^{-\frac{1}{\sigma}} \tag{3}$$

The share of profits in output is given by:

$$\alpha = \frac{\pi K}{Y} = b\left(\frac{K}{Y}\right)^{\frac{\sigma-1}{\sigma}} \tag{4}$$

Growth rates of the above variables satisfy the following equation:

$$g_{\alpha} = \left(\frac{\sigma-1}{\sigma}\right) g_{K/Y} \tag{5}$$

Thus, g_{α} and $g_{K/Y}$ have the same sign if $\sigma > 1$ and opposite signs if $\sigma < 1$. This is a standard neoclassical result.

2.3 Piketty

Piketty does not measure K/Y directly but takes as a proxy the ratio of wealth to income, where wealth is the total monetary value of shares, housing and other assets; income is measured in money terms. The inclusion of housing is questionable, because housing is not combined with labour in a production process in the same way as other types of capital. There is also the question of valuation. Taking produced goods as numeraire, let W be the market value of capital (stocks and shares, housing, etc.) and define the valuation ratio as follows:

$$v = \frac{W}{K} \tag{6}$$

In the case of quoted companies this is Tobin's Q.

Piketty's wealth to income ratio is given by:

$$\beta = \frac{W}{Y} = \frac{vK}{Y} \tag{7}$$

Growth rates of the above variables satisfy the following equation:

$$g_\beta = g_{K/Y} + g_v \tag{8}$$

In his explanation for the changing distribution of income, Piketty finesses the issue of valuation by assuming, in effect, that $g_v = 0$ and hence that $g_{K/Y} = g_\beta$. Given his finding that β has increased by a great deal in recent decades, Piketty concludes that K/Y must have increased by a similar amount. However, evidence surveyed in the Appendix indicates that K/Y has been falling since around 1981–2 in the United States and has been roughly constant in most of Europe. Indeed, this is just what Piketty and Zucman (2013) find when they correct the wealth-income ratio for valuation changes (capital gains).[5] Piketty's tacit assumption that $g_v = 0$ is also at odds with his own evidence which documents the large increase in the valuation ratios of quoted companies (Tobin's Q) that has occurred since 1970.[6]

The following is a plausible story, at least for the United States, where the capital-output ratio has fallen a great deal and thus $g_{K/Y} < 0$. Evidence reported in the Appendix indicates that $\sigma < 1$. Suppose this is the case. Suppose also that $g_v > -g_{K/Y}$. Then, in line with Piketty's empirical findings, $g_\beta = g_{K/Y} + g_v > 0$. Given that $\sigma < 1$ and $g_{K/Y} < 0$, the neoclassical theory of factor shares implies that $g_\alpha > 0$, which is also in line with Piketty's empirical findings. In this example, the income share of wealth-owners is increasing because of a low rate of real capital formation and a falling capital-output ratio. However, the wealth-income ratio is increasing because of a rapid growth in asset prices. This story, it must be said, assumes that the neoclassical theory is correct. In particular, it assumes that capital receives its marginal product. If this assumption is incorrect, a different or more complex explanation for the rising income share of wealth-owners is required. Such an explanation might include the declining economic and political power of organised labour in most advanced economies. However, this would not preclude low real investment as a contributing factor behind the observed shift in income distribution.

3. Balanced growth

Following Piketty, assume that a constant fraction s of income is saved. The growth rate of the real capital stock is given by:

$$g_K = \frac{sY}{K} \tag{9}$$

[5] Piketty and Zucman (2013), appendix figure A133, available at http://piketty.pse.ens.fr/en/capitalisback.

[6] See Piketty (2014), chapter 6, Figure 5.7 and his discussion on pp. 187–91.

Suppose that employment grows at the exogenous rate ℓ. With the assumed savings propensity, the economy will converge to a balanced growth path on which capital and output grow at the same rate g, where:

$$g = \ell + \mu \tag{10}$$

In his book, Piketty refers to g as the 'structural growth rate of the economy', although elsewhere he uses the conventional term 'natural growth rate' (Piketty and Zucman, 2013, p. 6).

Thus, on the balanced growth path:

$$g_K = g_L = g \tag{11}$$

The capital-output rate is:

$$\frac{K}{Y} = \frac{s}{g} \tag{12}$$

The rate of profit is:

$$\pi = \left(\frac{s}{g}\right)^{\frac{1}{\sigma}} \tag{13}$$

The share of profits is:

$$\frac{\pi K}{Y} = b\left(\frac{s}{g}\right)^{\frac{\sigma-1}{\sigma}} \tag{14}$$

A change in s/g does not affect the long-run (balanced) growth rate, but it does affect the capital-output ratio and the distribution of income. The share of profits is an increasing function of s/g if $\sigma > 1$ and a decreasing function if $\sigma < 1$. This is an important result. If $\sigma > 1$, as Piketty assumes, a reduction in the natural growth rate g (or increase in s), for reasons of technology or demography, will as he claims lead to a new balanced growth path on which the share of profits is higher than on the original path. Conversely, if $\sigma < 1$, the share of profits on the new balanced growth will be lower than before. Note that whatever the elasticity of substitution, a higher value of s/g leads to a higher capital-output ratio and a lower profit rate.

3.1 An alternative savings function

Piketty argues that there is too much capital accumulation because the owners of capital have plenty of income to save. This would suggest the use of an alternative savings function. Suppose there is no saving out of wages and that a constant fraction s_π of profits is saved.[7] Total saving, which equals total investment, is therefore $s_\pi \pi K$ and the growth rate of capital stock is given by:

$$g_K = \frac{s_\pi \pi K}{K} = s_\pi \pi \tag{15}$$

[7] This point was made by David Soskice in a personal communication to the author.

This equation will yield convergence to a balanced growth path on which, as before, capital and output grow at the rate $g = \ell + \mu$. On this path the profit rate is:

$$\pi = \frac{g}{s_\pi} \tag{16}$$

Equations (3) and (16) imply that the capital-output ratio is:

$$\frac{K}{Y} = \left(\frac{b}{\pi}\right)^\sigma = \left(\frac{s_\pi b}{g}\right)^\sigma \tag{17}$$

The share of profits is:

$$\frac{\pi K}{Y} = b^\sigma \left(\frac{s_\pi}{g}\right)^{\sigma-1} \tag{18}$$

The equation implies that the profit share is an increasing function of s_π / g if $\sigma > 1$ and a decreasing function if $\sigma < 1$. In the previous example, using a different savings function, the profit share was an increasing function of s / g for $\sigma > 1$ and a decreasing function for $\sigma < 1$ (see equation (14)). Changing the savings function does not alter the long-run growth rate or the qualitative results regarding the capital-output ratio, the rate of profit or the profit share.

4. Capital appreciation

The consumption of wealth-owners may be influenced by the gains they make from capital appreciation. For example, a landlord may increase his consumption out of rental income because of the capital gain he is making from rising house prices. In aggregate, capital gains are assumed to be unrealised. This ensures that the consumption of wealth-owners cannot exceed their current income from profits.[8] When such gains are included, the real rate of return on wealth is given by:

$$r = \frac{\pi K + \frac{dv}{dt} K}{vK} \tag{19}$$

Hence:

$$r = \frac{\pi}{v} + g_v \tag{20}$$

where g_v is the growth rate of v.

Suppose that workers do not save and that wealth-owners consume a constant fraction $1 - s_r$ of their real income including capital appreciation. Consumption by

[8] Wealth-owners could in aggregate realise some of their capital gains by selling some of their capital assets to workers. This possibility is ruled out by my assumption that workers do not save (see later discussion).

wealth-owners is then equal to $(1-s_r)rW = (1-s_r)rvK$. Subtracting this amount from the real profit flow πK yields the following expression for the real savings to be invested in additional capital stock:

$$\begin{aligned} S &= \pi K - (1-s_r)rvK \\ &= \pi K - (1-s_r)\left(\frac{\pi}{v} + g_v\right)vK \\ &= \left(s_r\pi - (1-s_r)g_v v\right)K \end{aligned} \tag{21}$$

Dividing by K yields:

$$g_K = s_r\pi - (1-s_r)g_v v \tag{22}$$

The assumption that in aggregate capital gains are not realised ensures that $S \geq 0$ and hence $g_K \geq 0$. If g_v is positive, the final term indicates that real investment is reduced because wealth-owners are consuming more as a result of capital appreciation. Their real wealth is increasing because of rising asset prices, and they have less need to save out of their current income (profits).

Suppose that employment grows at the exogenous rate ℓ. The growth rate of output is then:

$$g_Y = \alpha g_K + (1-\alpha)g \tag{23}$$

where $g = \ell + \mu$ is the natural growth rate. The capital-output ratio grows at the following rate:

$$g_{K/Y} = (1-\alpha)(g_K - g) \tag{24}$$

The left-hand side will be negative if $g_K < g$. Equation (22) implies that this will occur when:

$$s_r\pi - g < (1-s_r)g_v v \tag{25}$$

Piketty's wealth-income ratio is $\beta = W/Y = vK/Y$ which has growth rate:

$$\begin{aligned} g_\beta &= g_{K/Y} + g_v \\ &= (1-\alpha)(s_r\pi - (1-s_r)g_v v - g) + g_v \\ &= (1-\alpha)(s_r\pi - g) + \left[1 - (1-\alpha)(1-s_r)v\right]g_v \end{aligned} \tag{26}$$

The above growth rate is positive if:

$$s_r\pi - g > (1-s_r)vg_v - \frac{g_v}{1-\alpha} \tag{27}$$

The condition for the two inequalities $g_{K/Y} < 0$ and $g_\beta > 0$ to hold simultaneously is

$$(1-s_r)vg_v - \frac{g_v}{1-\alpha} < s_r\pi - g < (1-s_r)vg_v \tag{28}$$

As always:

$$g_\alpha = \left(\frac{\sigma-1}{\sigma}\right) g_{K/Y} \tag{29}$$

If σ < 1 and the inequalities (28) are satisfied, the capital-output ratio will fall in the course of time and the share of wealth-owners in total income will rise. The capital-output falls because there is so little real investment. Because of capital appreciation, wealth-owners are able to enjoy a high level of consumption and at the same time see their wealth growing faster than total income. This is a fair description of what has happened in a number of countries.

5. Conclusions

Piketty argues that the higher income share of wealth-owners is due to an increase in the capital-output ratio resulting from a high rate of capital accumulation. The evidence suggests just the contrary. The capital-output ratio as conventionally measured has either fallen or been constant in recent decades. The apparent increase in the capital-output ratio identified by Piketty is a valuation effect reflecting a disproportionate increase in the market value of certain assets. A more plausible explanation for the increased income share of wealth-owners is an unduly low rate of investment in real capital. These alternative explanations may have distinct policy implications which it is beyond the scope of this note to explore.

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Appendix

Valuation

For any group of assets define the valuation ratio v as follows:

$$v = \frac{\text{total market value of assets}}{\text{total replacement cost of assets}}$$

In the case of quoted companies, this ratio is usually known as Tobin's Q, although in fact the term valuation ratio was earlier coined by Marris (1964).

Suppose that 'real' capital K is measured in such a way that for the whole economy or for the private sector as a whole, its unit replacement cost is on average equal to the unit price of real output Y. Then:

$$\beta = \frac{W}{Y} = \frac{vK}{Y}$$

By assuming that $\beta = K/Y$, Piketty is in effect assuming that $v = 1$. Casual observation suggests that this is not true for house prices, which have risen much faster than building costs in many countries due to rising land values. Using data from Canada, France, the USA and the IL, Bonnet *et al.* (2014) show that the increase in identified by Piketty was mainly due to the rapid growth in house prices. Preliminary estimates by Bill Martin confirm this finding for the UK.[9] Note that when Piketty and Zucman simulate the effect of excluding capital gains, they find that the adjusted private wealth-income ratio for the United States falls almost continuously over the period 1982–2010 and remained virtually flat for a weighted group of European counties.[10] Estimates by Klump *et al.* (2014) of the conventional capital-output ratio for these countries reveal a similar picture. Estimates by Thwaites (2014) of the real capital to gross value-added ratio for an average of 11 industrialised countries indicate that this ratio fell up to the mid-1970s and since then has been more or less flat.

The combination of a rising β and a falling or stationary K/Y implies that the valuation ratio v must have been increasing.

[9] Piketty estimates that the UK ratio of private wealth to income rose by 69% between 1970 and 2010 (online technical appendix table S3.1). When adjusted for changes in the relative price of housing, the increase was 24% (personal communication from Bill Martin to the author).

[10] Piketty and Zucman (2013), appendix figure A133, available at http://piketty.pse.ens.fr/en/capitalisback.

Elasticity of substitution

Piketty (2014, p. 221) claims that on the basis of historical data one can infer that σ lies between 1.3 and 1.6. However, this inference is unreliable because it is based on the assumption that $K/Y = \beta$. Conventional measures of K/Y typically yield values of σ that are much lower than 1.

Rowthorn (1996) and Rowthorn (1999) report the results of 33 econometric studies that estimate the value of σ or from which estimates of this parameter can be derived. Most of these studies contain a variety of estimates referring to different industries, regions or countries, or to alternative equation specifications. Their findings are summarised by means of employment-weighted averages or medians. Out of a total of 33 studies, in only 7 cases does the summary value exceed 0.8, and the overall median of the summary values (median of the medians) is equal to 0.58. A more recent survey by Klump *et al.* (2007) reports similar findings for aggregate elasticities. These authors' own estimates for the private non-housing sector imply elasticities in the range 0.60–0.67 in both the USA and the Eurozone. A survey by Chirinko (2008) concludes, 'While some estimates of σ are above one, the weight of the evidence suggests that σ lies in the range between 0.40 and 0.60'. Allowing for biased technical change, Antràs (2004, p. 26) concludes that for the USA σ 'is likely to be considerably below one, and may even be lower than 0.5'. Allowing for mark-up pricing, Raurich *et al.* (2011) estimate an elasticity of 0.63 for the USA. Using firm-level data, Barnes *et al.* (2008) find a long-run elasticity of 0.4 for the UK. Using data for New Zealand, Tipper (2012) obtains mostly low elasticities for individual industries and estimates the aggregate elasticity to be in the region of 0.8.The latter estimate is not statistically significantly different from unity. Using data for the USA, Balistreri *et al.* (2002) find a wide variety of long-run elasticities of substitution at the industry level, with a median of around 1. Allowing for changes in the relative price of investment goods, Karabarbounis and Neiman (2014) estimate the elasticity of substitution in a large sample of countries to be in the region of 1.26.