Cambridge Journal of Economics 1988, **12**, 125–134

Growing subsystems, vertically hyper-integrated sectors and the labour theory of value

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1. Foreword

The concept of 'sub-system' was originally presented by Piero Sraffa in a brief non-mathematical 3/4 page appendix to his book (Sraffa, 1960, p. 89). His purpose was that of providing 'an alternative if less intuitive line of approach' to evaluate the quantities of labour that directly and indirectly enter the production of each commodity (*ibid*, p. 56). To this effect he had used, in the text of his book, the more intuitive method of 'reduction to dated quantities of labour'. But this method, while unexceptionable in the case of single products, does not work for joint production. Sraffa was proud to find that the sub-system approach works in all cases.

The concept of sub-system, however, has turned out to be of much wider relevance than Sraffa himself could have imagined. In a previous article (Pasinetti, 1973), I linked up Sraffa's direct and indirect quantities of labour with the 'direct and indirect requirements' emerging from Leontief's inverse matrix (Leontief, 1951), and I proceeded therefrom to develop the two symmetrical concepts of vertically integrated labour coefficient and vertically integrated unit of productive capacity.

The present paper goes one step further in the direction of a complete generalisation of the concepts of sub-system and vertically integrated sector. It will be shown that the analytical device of partitioning an economic system into sub-systems (and correspondingly of constructing vertically integrated sectors) can indeed be taken into dynamic analysis, with remarkable implications.

In fact, if one looks carefully at the physical quantity side, as against the price side, of the usual systems of equations representing a production economic system, one may notice a formal asymmetry. In the physical quantity system, the means of production are specific to each particular sub-system; there is a specific vector of means of production for each particular sub-system. In a sense, the physical quantities of the means of production

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0309–166X/88/010125 + 10 $03.00/0

appear as playing a sort of ancillary role with respect to the physical quantities of final demand; the former being, so to speak, 'at the service' of the latter.

By contrast, in the price system, a similar feature does not emerge. There is absolutely no difference, in the scheme earlier proposed, between the prices of the means of production and the prices of the final goods. *The vector of prices is exactly the same for all sub-systems.*

It is shown in the following pages that this asymmetry disappears when the analytical scheme is further generalised.

2. Growing or hyper-subsystems

Consider an economic system of the type sketched out in Pasinetti (1973). In each particular unit of time, m commodities are produced by means of commodities and labour. The physical quantity system at time t may be represented by:[1]

$$\mathbf{AX}(t) + g\mathbf{AX}(t) + \mathbf{C}(t) = \mathbf{BX}(t), \tag{2.1}$$

$$\mathbf{a}_n\mathbf{X}(t) = L(t), \tag{2.2}$$

$$\mathbf{AX}(t) = \mathbf{S}(t), \tag{2.3}$$

where $\mathbf{A}$ and $\mathbf{B}$ are the input and output inter-industry matrices respectively, $\mathbf{a}_n$ is the input direct-labour coefficient (row) vector, $\mathbf{X}(t)$ is the physical quantity (column) vector, $\mathbf{C}(t)$ is the consumption goods (column) vector, $\mathbf{S}(t)$ is the means of production (column) vector, $L(t)$ is the (scalar) quantity of labour and g is the (scalar) percentage rate of growth of the labour force (for simplicity no distinction is made between labour force and population). All matrices are m order matrices and all vectors have m components. The physical units are chosen so as to make the main diagonal of $\mathbf{B}$ a list of ones. The technical coefficients are such as to be economically meaningful at all levels of the population percentage rate of growth, g, from zero to its maximum, i.e. in the interval $0 \leqslant g \leqslant G$, where G is the maximum rate of growth allowed by existing technology.

Suppose, to begin with, constant technical coefficients and population growth. It is also convenient to assume immigration. Immigrant labour is assumed to be available in a practically unlimited amount. When the growth of demand is very strong, workers are allowed to come in, and when the growth of demand slackens the foreign workers leave the country, so that there always is full employment, as a result of adapting the labour force to the requirements of production (rather than the other way round).[2] It is also convenient to assume that the consumption tastes of the immigrants are different from those of the indigenous population, so that the per capita rates of growth of demand for commodities 1, 2, . . . , m, are all different from one another. This means that, if g, defined above, is the proportional rate of growth of the labour force (whether by natural growth or by immigration) and if r_i (which may be positive or negative) is defined as the per capita rate of growth of consumption demand for each commodity i, then the rate of growth of consumption demand for each commodity i will be $(g + r_i)$, and in general will be different from one consumption good to another (i.e. $r_i \neq r_j$ in general).

Let us now partition the economic system into as many such newly defined sub-systems as there are consumption goods. Since the economic system is growing, these 'newly defined sub-systems' are more comprehensive than those considered in Pasinetti (1973), because they include not only the labour and the means of production for the reproduction

[1] Bold type will be used throughout the paper to denote vectors and matrices; italic type will denote scalars.
[2] This is simply an analytical device to avoid inconsistencies in assuming differentiated growth rates for different consumption goods, in the presence of constant technical coefficients.

of each subsystem, but *also* the labour and the means of production necessary to its *expansion* at its particular rate of growth $(g+r_i)$. We call these newly defined sub-systems, the *m growing subsystems* or, more compactly, the *m hyper-subsystems* of the economic system under consideration.

We denote by $C_1(t)$, $C_2(t)$, ..., $C_m(t)$ the m consumption goods produced in period t, making up the m components of the already defined consumption goods (column) vector $\mathbf{C}(t)$. Each $C_i(t)$ is expanding through time at its particular rate of growth $(g+r_i)$, which we assume to be steady for the time being:

$$C_i(t)=C_i(0)\,e^{(g+r_i)t}, \qquad i=1,2,\ldots,m. \tag{2.4}$$

We now proceed in a way perfectly analogous to the one used in defining the earlier sub-systems in Pasinetti (1973), but with the essential difference here of including in each hyper-subsystem *all* gross investments (both replacements *and* net investments). We thereby obtain:

$$\mathbf{X}^{(i)}(t)=[\mathbf{B}-(1+g+r_i)\mathbf{A}]^{-1}\mathbf{C}^{(i)}(t), \tag{2.5}$$

$$\mathrm{L}_i(t)=\mathbf{a}_n[\mathbf{B}-(1+g+r_i)\mathbf{A}]^{-1}\mathbf{C}^{(i)}(t), \tag{2.6}$$

$$\mathbf{S}^{(i)}(t)=\mathbf{A}\,[\mathbf{B}-(1+g+r_i)\mathbf{A}]^{-1}\mathbf{C}^{(i)}(t), \qquad i=1,2,\ldots,m, \tag{2.7}$$

$$\Sigma\mathbf{C}^{(i)}(t)=\mathbf{C}(t);\ \Sigma\mathbf{X}^{(i)}(t)=\mathbf{X}(t);\ \Sigma\mathbf{S}^{(i)}(t)=\mathbf{S}(t);\ \Sigma L_i(t)=L(t), \tag{2.8}$$

where $\mathbf{C}^{(i)}(t)$ is a (column) vector, the components of which are all zeros except the ith one, which is the scalar C_i, i.e. the ith component of vector $\mathbf{C}(t)$; and $\mathbf{X}^{(i)}(t)$, $\mathbf{S}^{(i)}(t)$, $L_i(t)$ are those particular corresponding magnitudes, that, in a self-contained expanding economic system, are necessary to produce through time that physical (expanding) quantity $C_i(t)$, and at the same time to produce all the means of production necessary both to maintain in operation such an economic system and to expand it at the required rate of growth $(g+r_i)$.

In this way, each hyper-subsystem i is characterised by:

(a) production of one single consumption good i, expanding through time at its particular rate of growth $(g+r_i)$;
(b) absorption of a physical quantity of labour $L_i(t)$, expanding in time at whatever rate is required by the technical coefficients (in our case, clearly, at rate $(g+r_i)$); and
(c) maintenance of a circular production process that *both* reproduces all the means of production which are absorbed by the production process for consumption good $C_i(t)$ *and* also produces those means of production that are strictly necessary to expand such a circular process at a rate of growth $(g+r_i)$.

Since $i=1,2,\ldots,m$, there are m such hyper-subsystems, and the sum of the physical magnitudes of these m hyper-subsystems—see (2.8)—add up to the physical magnitudes of the actual economic system.

For convenience, we may define the following vectors and matrices:

$$l^{(i)}=\mathbf{a}_n\,[\mathbf{B}-(1+g+r_i)\mathbf{A}]^{-1}, \tag{2.9}$$

$$\mathbf{M}^{(i)}=\mathbf{A}\,[\mathbf{B}-(1+g+r_i)\mathbf{A}]^{-1}, \qquad i=1,2,\ldots,m. \tag{2.10}$$

Unlike what happened in Sraffa's sub-systems, each hyper-subsystem i is now characterised by a particular vector $l^{(i)}$ and by a particular matrix $\mathbf{M}^{(i)}$, $i=1,2,\ldots,m$.

It should be noticed, however, that not all columns of matrix $\mathbf{M}^{(i)}$ are relevant, but only the ith column, which we may denote by a star:

$$\mathbf{m}_j^{(i)} = \mathbf{m}_i^{\star}. \tag{2.11}$$

This is a vector which defines a particular composite commodity that may be called *a unit of vertically hyper-integrated productive capacity* for commodity i. Similarly, not all components of vector $\boldsymbol{l}^{(i)}$ are relevant, but only the ith component, which we may equally denote by a star:

$$l_i^{(i)} = l_i^{\star}. \tag{2.12}$$

This scalar defines what may be called the *vertically hyper-integrated labour coefficient* for commodity i.

We may now ask: What then is the meaning of any other column $\mathbf{m}_j^{(i)}$ of each matrix $\mathbf{M}^{(i)}$ and of any other component $l_j^{(i)}$ of each vector $\boldsymbol{l}^{(i)}$?; $j \neq i$; $i, j = 1, 2, \ldots, m$. The answer is that their meaning takes a *hypothetical* connotation. Each vector $\mathbf{m}_j^{(i)}$ (each component $l_j^{(i)}$) in matrix $\mathbf{M}^{(i)}$ (in vector $\boldsymbol{l}^{(i)}$), where $j \neq i$, has the meaning of a unit of vertically hyper-integrated productive capacity (of a vertically hyper-integrated labour coefficient), *if* commodity j were to be produced as a consumption good (though it is not) in hyper-subsystem i.

Therefore, in each matrix $\mathbf{M}^{(i)}$, we may call the relevant column $\mathbf{m}_i^{\star}$ the *actual* unit of vertically hyper-integrated productive capacity and all the other columns $\mathbf{m}_j^{(i)}$, the *hypothetical* units of vertically hyper-integrated productive capacity in hyper-subsystem i. Similarly, in each vector $\boldsymbol{l}_i^{(i)}$, we may call the relevant component $l_i^{\star}$ the *actual* vertically hyper-integrated labour coefficient and all the other components $l_j^{(i)}$, the *hypothetical* vertically hyper-integrated labour coefficients; $j, i = 1, 2, \ldots, m$; $j \neq i$.

The generalising features of this analytical construction should now be evident. In the very particular case in which all $g + r_i = 0$, $i = 1, 2, \ldots, m$, i.e. in the particular case of a perfectly stationary economic system, all matrices $\mathbf{M}^{(i)}$ coincide with one another and with matrix $\mathbf{H}$ as defined in Pasinetti (1973). And similarly all vectors $\boldsymbol{l}^{(i)}$ coincide with one another and with vector $\mathbf{v}$ of Pasinetti (1973). In other words, when $g + r_i = 0$, the m hyper-subsystems reduce to the simpler sub-systems previously defined. The latter thereby emerge as particular cases of the former. Or, to put it the other way round, the growing or hyper-subsystems represent a truly general notion, of which Sraffa's sub-systems are particular cases.

3. The natural price sub-systems

We may now turn to the system of prices. Here the general attitude (which was also followed by Sraffa) has been that of accepting a uniform exogenously given rate of profits. In this case we should be back to the usual price system:

$$\mathbf{pB} = \mathbf{a}_n w + \mathbf{pA} + \mathbf{pA}\pi, \tag{3.1}$$

where $\mathbf{p}$ is the (m-component, row) price vector, and w and π are scalars for the wage rate and the rate of profits respectively. Equation system (3.1) contains two degrees of freedom, which may be closed by choosing a particular *numéraire* of prices, for example by fixing:

$$w = 1, \tag{3.2}$$

which means expressing all prices in terms of 'labour commanded', and by considering the (uniform) rate of profits π as determined from outside. Thereby we should indeed be back to the usual price system, but at the same time we should be back to the asymmetry pointed out at the beginning.

The relevant, and significant, alternative is to depart from the traditional assumption of a uniform rate or profits. Following the approach of Pasinetti (1981), I shall therefore introduce now the notion of a natural rate of profits. A *natural* rate of profits, denoted by a star, $\pi^\star$, is defined as a rate of profits which is equal to the rate of growth of demand for the corresponding consumption good. This means that in general we shall have, in the whole economic system, m different *natural rates of profits:*

$$\pi_i^\star = g + r_i, \qquad i = 1, 2, \ldots, m. \tag{3.3}$$

More specifically, this means that there will be a particular natural rate of profits for each hyper-subsystem.

It follows that there will also be a specific set of *natural prices* for each particular hyper-subsystem. In total, there will be m sets of natural prices, each one referring to the corresponding hyper-subsystem.

A crucial distinction emerges at this point which did not arise before. The distinction is between the natural prices of the consumption goods and the natural prices of the means of production.

In each hyper-subsystem i, we can see that its specific set of natural prices refers, to begin with, to the means of production (which may well include the particular good i, which is also used as consumption good). Calling $\mathbf{p}^{(i)}$ this specific set of natural prices, we have the particular price vector:

$$\mathbf{p}^{(i)} = \mathbf{a}_n \, [\mathbf{B} - (1 + \pi_i^\star) \, \mathbf{A}]^{-1} w, \tag{3.4}$$

or, from definitions (2.9) (and 3.3)

$$\mathbf{p}^{(i)} = \boldsymbol{l}^{(i)} w. \tag{3.4$'$}$$

But $i = 1, 2, \ldots, m$, i.e. there are m hyper-subsystems. It follows that there exist m sets of natural prices for the same means of production, each set of natural prices being specific to a particular hyper-subsystem.

The situation is remarkably different for the m consumption goods. Each consumption good *only* appears in one hyper-subsystem and therefore only has *one* natural price. The consumption good natural price is the ith price in each price system (3.4). By calling it $p_i^\star$, it clearly emerges as:

$$p_i^\star(t) = \mathbf{a}_n \, [\mathbf{B} - (1 + \pi_i^\star)\mathbf{A}]^{-1} \mathbf{e}_i w = l_i^\star(t) w, \qquad i = 1, 2, \ldots, m. \tag{3.5}$$

where $\mathbf{e}_i$ is the unit (column) vector and $l_i^\star(t)$ is the corresponding actual vertically hyper-integrated labour coefficient, defined by (2.12).

In all hyper-subsystems taken together, i.e. in the whole economic system, each good therefore has only one natural price as a consumption good, although it may have up to m different natural prices as a capital good.

Notice, from (3.5) and (2.6) that, since $\pi_i^\star = g + r_i$, when we put $w = 1$, the price $p_i^\star$ of commodity i; $i = 1, 2, \ldots, m$, (i.e. its price as a consumption good), is simply equal to $L_i(t)/C_i(t)$, which means that $p_i^\star \, C_i(t)$, the value of $C_i(t)$, is simply equal to $L_i(t)$. The value of the physical quantity of each consumption good that is produced turns out to be equal to the physical quantity of labour which has directly, indirectly and—as I have called it elsewhere (see Pasinetti, 1977 and 1981)—hyper-indirectly been required by its production. Because of linearity, the consumption goods produced by the m hyper-subsystems exhaust all the produced consumption goods and the quantities of labour allocated to the m hyper-subsystems exhaust all the available labour.

This is a complete generalisation of the pure labour theory of value. Each physical quantity of any consumption good is unambiguously related to a physical quantity of labour; and the two have, in between them, a physically defined self-replacing, and expanding, circular process.

In terms of the compact notation defined by (2.9), each $p_i^{\star}$ is simply proportional—and in the case in which w is used as *numéraire* it is exactly equal—to the corresponding actual vertically hyper-integrated labour coefficient $l_i^{\star}$, which precisely represents the quantity of labour which is necessary directly, indirectly and hyper-indirectly to its production.

But the interesting result that follows immediately from this compact notation is that the natural prices $\mathbf{p}^{(i)}$ of the corresponding means of production have exactly the same meaning, although assuming a *hypothetical* connotation. As emerges from (3.4), (3.4′), these prices are proportional (or actually equal, in the case in which w is used as *numéraire*), to the quantity of labour that *would* be necessary—directly, indirectly and hyper-indirectly—to produce them as consumption goods, *if* they were produced as consumption goods in the corresponding hyper-subsystem (though in fact they are produced as capital goods, and not as consumption goods). In other words, they are proportional to the corresponding *hypothetical* vertically hyper-integrated labour coefficients, as defined in the previous section.

At this point, the whole set of natural prices of the means of production appear as performing in each hyper-subsystem a sort of ancillary role with respect to the corresponding price of the consumption good. Formal symmetry has been re-established perfectly between all aspects concerning physical quantities and all aspects concerning prices.

4. A complete generalisation of the pure labour theory of value from a different viewpoint

The elaborations of the previous section, which provide a complete generalisation of the pure labour theory of value, may be further illustrated by taking an alternative route.

Let us start from the general price system (3.1) in which there is a uniform rate of profits, π. We can always re-write it as:

$$\mathbf{pB} = \mathbf{a}_n w + \mathbf{pA} + (g + r_i)\,\mathbf{pA} + (\pi - g - r_i)\,\mathbf{pA}, \tag{4.1}$$

where $(g + r_i)$ is the rate of growth of demand for any one of consumption goods, which we have called i. Of course, since $i = 1, 2, \ldots, m$, there are *m different* ways, expressed by (4.1), of re-writing *the same* general price system (3.1), each one with reference to the particular consumption good i; $i = 1, 2, \ldots, m$; whose rate of growth $(g + r_i)$ is explicitly introduced. Let us proceed with one of them—the one which we have called i. By re-arranging, and by using previous notation, we obtain:

$$\mathbf{p}[\mathbf{B} - (1 + g + r_i)\mathbf{A}] = \mathbf{a}_n w + (\pi - g - r_i)\,\mathbf{pA}, \tag{4.2}$$

$$\mathbf{p} = \mathbf{a}_n[\mathbf{B} - (1 + g + r_i)\mathbf{A}]^{-1} w + (\pi - g - r_i)\,\mathbf{pA}[\mathbf{B} - (1 + g + r_i)\mathbf{A}]^{-1}. \tag{4.3}$$

This is a remarkable expression. On the right hand side, the first addendum contains the vector of the vertically hyper-integrated labour coefficients for hyper-subsystem i, and similarly the second addendum contains the matrix of the hyper-integrated units of productive capacity for the same hyper-subsystem i. Using definitions (2.9) and (2.10), expression (4.3) may be re-written as:

$$\mathbf{p} = \boldsymbol{l}^{(i)} w + (\pi - g - r_i)\mathbf{pM}^{(i)}. \tag{4.3'}$$

Clearly, in the very particular case in which

$$\pi = g + r_i, \tag{4.4}$$

the second addendum, on the right hand side of (4.3′), disappears and the expression comes to coincide with $\mathbf{p}^{(i)}$ in (3.4′)—the natural prices for hyper-subsystem i.

It is important to stress that, since we can write expression (4.1) in m different ways, with reference to each specific r_i—i.e. to the rate of growth of per capita demand for consumption good i; and in general $r_i \neq r_j$, for $i, j = 1, 2, \ldots, m$—then it follows that if it were to happen that (4.4) were satisfied for consumption good i it would *not* in general be satisfied for any of the other $(m-1)$ consumption goods.

In general, therefore, if π is uniform, the second addendum on the right-hand side of (4.3′) does not vanish, and we will be able to re-write (4.3′) as

$$\mathbf{p} = \boldsymbol{l}^{(i)} [\mathbf{I} - (\pi - g - r_i) \mathbf{M}^{(i)}]^{-1} w; \qquad i = 1, 2, \ldots, m \tag{4.5}$$

(which of course reduces to

$$\mathbf{p} = \boldsymbol{l}^{(i)} w, \qquad i = 1, 2, \ldots, m, \tag{4.6}$$

coinciding with the natural prices for hyper-subsystem i, only in the very particular case in which $\pi = g + r_i$).

We can see now, in another way (i.e. from the point of view of the solution of the price system) that there are m different ways expressed by (4.5)—each one with reference to each of the consumption goods and therefore to each of the particular hyper-subsystems—of writing the *same* general price system (3.1) in which there is a uniform rate of profits.

It also emerges very clearly at this point that expression (4.5), when $w = 1$, can also be regarded as providing a complete generalisation of Marx's 'transformation problem'.[1] It 'transforms' the physical quantities of labour $\boldsymbol{l}^{(i)}$ into the prices $\mathbf{p}$ of an economic system in which there is a uniform rate of profits π.

There are m expressions (4.5), for any given uniform rate of profits π, which means that there is a particular 'transformation', carried out by the corresponding hyper-subsystem, for each particular consumption good produced in the economic system. This transformation concerns *both* the price of the consumption good *and* the prices of the corresponding means of production.

We should now be able to look from yet another point of view at the meaning of the complete generalisation of the labour theory of value emerging from the foregoing elaborations.

In a hypothetical economic system in which we were able *actually to separate* the various hyper-subsystems, it would be possible at least in principle to envisage m different natural price sub-systems, which would embody a perfectly general pure labour theory of value. Such a labour theory of value would be general in the sense that it would realise, for an advanced society, the fundamental characteristic of the pure labour theory of value originally proposed by Adam Smith with reference to a primitive society: namely a set of values that realise a universal equality of 'labour commanded' and 'labour embodied'. The analytical step that allows the achievement of this result is of course a re-definition of the concept of 'labour embodied', which must be intended as the quantity of labour required

[1] This complete generalisation concerns the version of Marx's transformation problem as presented in Pasinetti, 1973 (pp. 9, 16, 17n).

directly, indirectly *and* hyper-indirectly to obtain the corresponding commodity as a consumption good.

In any economic system in which the analytical process of deriving the various hyper-subsystems is to be taken only as a *conceptual* (not a real) process, any rule of price determination inevitably entails a 'transformation' process, no matter whether the economic system itself is organisationally centralised or decentralised. In such a context, the general labour theory of value that has emerged from the foregoing elaboration has to be taken as providing a logical frame of reference—a conceptual construction which defines a series, actually a family of series, of ideal *natural* prices, which possess an extraordinarily high number of remarkable, analytical, and normative, properties.

5. Choice of efficient techniques

There is a further normative property of natural prices that emerges with reference to the problem of choice of techniques.

In the previous sections, all analysis has been carried out with reference to an economic system with a *given* technique, represented by the technical triple $[\mathbf{a}_n, \mathbf{A}, \mathbf{B}]$. In general, however, many alternative techniques of production, which we may call $\alpha, \beta, \ldots, \omega$, may be available. More specifically, alternative technical triples:

$$[\mathbf{a}_n, \mathbf{A}, \mathbf{B}]^{(\alpha)}, [\mathbf{a}_n, \mathbf{A}, \mathbf{B}]^{(\beta)}, \ldots [\mathbf{a}_n, \mathbf{A}, \mathbf{B}]^{(\omega)}, \tag{5.1}$$

may be open to choice. A criterion of choice becomes necessary. Now, if the criterion of choice to be adopted is the usual one of minimisation of costs, then it follows that the natural system of prices, by virtue of expressing physical quantities of labour (directly, indirectly and hyper-indirectly required), has the normative property of leading to the choice of that technique which entails the minimisation of labour inputs. This means that natural prices possess the normative property of being *efficient prices*.

But in a growing economic system, as has been shown, the quantities of labour that are required directly, indirectly, and hyper-indirectly for the corresponding commodities are *different* (for the same commodities) in the various hyper-subsystems, just in the same ways as the corresponding natural prices are different in the various hyper-subsystems.

It follows that the efficient technique—i.e. that technique the choice of which entails the minimisation of labour inputs—may turn out to be different from one hyper-subsystem to another. If it is technique α that minimises labour inputs for hyper-subsystem 1, it may well be that another technique, let us say technique β, is the technique that minimises labour inputs for hyper-subsystem 2, and so on.

A further consequence will now be noticed. The actual choice of different techniques in different hyper-subsystems will *a fortiori* entail different natural price systems. In other words, the natural prices, in general, will turn out to be different from one hyper-subsystem to another, not only because of the differences in the natural rates of profit, which would be the case even if the efficient technique were to remain the same for all hyper-subsystems, as has been considered up to this section, but also, and more fundamentally, because of differences in the efficient techniques themselves. For, as a consequence of cost minimisation, the efficient technique may well turn out to be different from one hyper-subsystem to another.

Notice, again, that the differences in the natural prices concern the prices of the means of production, not the prices of consumption goods. Each consumption good continues to have one single natural price—the one that emerges as reflecting the physical quantity of labour directly, indirectly and hyper-indirectly required in the corresponding hyper-subsystem. Of course the same physical good, to the extent that it is also used as capital good in other hyper-subsystems, may also have other natural prices *as a capital good.* But its natural price as a consumption good is unique.

This property further enhances *the ancillary role,* already pointed out, of the natural prices of capital goods, with respect to the natural prices of consumption goods.

But of course, at this point, it becomes necessary to go back to consider the meaning to be attributed to the logical construction of the hyper-subsystems. The partitioning of the whole economic system into m hyper-subsystems has been presented, since the beginning, as a *logical* operation *not* as a *factual* one.

In practice, that is the way in which hyper-subsystems will have to be considered, viz. as *logical* constructions. Thus the foregoing analysis allows us to state that, if it were actually possible to separate the various hyper-subsystems from each other, then a whole series of families of natural prices could actually come into existence, with the normative properties that have been brought into evidence by the foregoing analysis. It goes without saying that, when actual separation is not possible, the choice of technique induced by current prices (which may deviate from natural prices) will entail deviations from efficient positions. An interesting point to stress is that this will be so in any economic system, whether market-oriented (i.e. decentralised) or centrally-planned.

More specifically, there are two distinct obstacles to the achievement of efficient positions through the choice of efficient techniques, as has emerged from the foregoing analysis. The first obstacle is represented by the difficulty of applying differentiated rates of profit in the various hyper-subsystems. The second obstacle is represented by all those factors that are opposed to actually carrying out a particular choice of technique in each growing hyper-subsystem. While the first obstacle is typical and specific to decentralised (e.g. market-oriented) economies, the second obstacle is common to both decentralised and centralised economic systems.

6. Vertically hyper-integrated sectors

There remains one further aspect to be explored, and that is the relation of the hyper-subsystems to the concept of vertically integrated sectors defined in Pasinetti (1973).

The generalisation of previous analysis to vertically hyper-integrated sectors becomes obvious and immediate at this stage. If we return, for clarity of exposition, to the case of a single technique $[\mathbf{a}_n, \mathbf{A}, \mathbf{B}]$ in the whole economic system, and thus common to all hyper-subsystems, we can distinguish very clearly the two points of view, described in the previous literature, from which any production economic system may be looked at.

The point of view of the circularity of the production process is evinced by the construction of the hyper-subsystems (which now acquire completeness by inclusion of the relations concerning the expansion of the means of production, besides those concerning their replacement). The point of view of final demand is evinced in an even sharper way. Even in a growing economic system, *consumption* appears at one extreme of the production process and labour appears at the other extreme, and the two are immediately and directly put into relation with each other. The complex circular (expanding) production process, which is in between, is taken for granted, as it is closed onto itself and merely fulfils an intermediate and ancillary function.

From the point of view of final demand, all intermediate circular processes fulfil an ancillary role. The economic system appears simply as being made up of as many vertically hyper-integrated sectors as there are consumption goods. Each vertically hyper-integrated sector i, when operating at unit activity, may be represented by the elementary vector:

$$[1, 1, l_i, (g+r+i)], \qquad i=1,2,\ldots,m, \tag{6.1}$$

where the first component refers to one unit of consumption good i, the second component to one unit of vertically hyper-integrated productive capacity for consumption good i, the third to the physical quantity of (vertically hyper-integrated) labour for consumption good i and the fourth to the growth rate of demand for consumption good i.

The corresponding magnitudes in the price system continue to be represented by the simple vector:

$$[p_i, p_{k_i}, w, \pi], \qquad i=1,2,\ldots,m, \tag{6.2}$$

where p_i is the price of one physical unit of consumption good i, and p_{k_i} is the price of one unit of vertically hyper-integrated productive capacity for consumption good i (while w and π are, as usual, the wage rate and the rate of profits). All asymmetries have now disappeared: vectors (6.1) and (6.2) are perfectly symmetrical.

One should notice the extreme simplicity with which the unit of vertically hyper-integrated productive capacity in (6.1), and the corresponding price p_{k_i} in (6.2), have been introduced. They have been introduced without any specification of the composition of the unit concerned. In fact, at any point of time, t, given the technique $\mathbf{A}(t)$, $\mathbf{B}(t)$, $\mathbf{a}_n(t)$, the relation of the unit of vertically hyper-integrated productive capacity to the physical commodities of which it is composed, and correspondingly the relation of the natural price $\mathbf{p}_{k_i}$ to the natural price $\mathbf{p}_i$ of the various commodities measured in physical ordinary units, are rather complicated. The former is obtained directly from (2.7) while the latter is obtained by multiplying (3.4) by (2.10), i.e.:

$$\mathbf{p}_{k_i}(t) = \mathbf{p}^{(i)}(t)\,\mathbf{M}^{(i)}(t) = \boldsymbol{l}^{(i)}(t)\,\mathbf{M}^{(i)}(t)\,w(t),$$

or, by expansion,

$$\mathbf{p}_{k_i}(t) = \mathbf{a}_n(t)\,[\mathbf{B}(t)-(1+\pi^\star)\,\mathbf{A}(t)]^{-1}\,\mathbf{A}(t)\,[\mathbf{B}(t)-(1+\pi^\star)\,\mathbf{A}(t)]^{-1}\,w(t). \tag{6.3}$$

Now, the crucial point to notice is that this relation will continually be upset, as time goes on, because of technical change.

Yet, essential though these complicated relations are for any analysis concerning the interrelations of the circular (expanding) process at any given point of time, they become irrelevant for all those dynamic analyses that concern the movements through time of the final consumption goods and of the corresponding physical quantities of labour, as well as for all those relations that follow therefrom—think for example of the analyses concerning the evolution of sectoral employment and income distribution.

As far as the analyses of these problems are concerned, complete emancipation is obtained from the constraints of fixed technical coefficients. The analysis is open to the absorption of technical progress in whatever form it may take place.

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