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ARE WE APPROACHING AN ECONOMIC SINGULARITY? INFORMATION TECHNOLOGY AND THE FUTURE OF ECONOMIC GROWTH

William D. Nordhaus

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Information Technology and the Future of Economic Growth

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Abstract

What are the prospects for long-run economic growth? The present study looks at a recently launched hypothesis, which I label *Singularity*. The idea here is that rapid growth in computation and artificial intelligence will cross some boundary or Singularity after which economic growth will accelerate sharply as an ever-accelerating pace of improvements cascade through the economy. The paper develops a growth model that features Singularity and presents several tests of whether we are rapidly approaching Singularity. The key question for Singularity is the substitutability between information and conventional inputs. The tests suggest that the Singularity is not near.

I. Introduction

What are the prospects for long-run economic growth? One prominent line of economic thinking is the trend toward stagnation. Stagnationism has a long history in economics, beginning prominently with Malthus and surfacing occasionally in different guises. Prominent themes here are the following: Will economic growth slow and perhaps even reverse under the weight of resource depletion? Will overpopulation and diminishing returns lower living standards? Will unchecked CO_2 emissions lead to catastrophic changes in climate and ecosystems? Have we

[1] The author is Sterling Professor of Economics; Cowles Foundation, Yale University, New Haven, CT 06511, USA. Mailing address: P. O. Box 208268, Yale Station, New Haven, CT 06511-8268, USA. Email: william.nordhaus@yale.edu. The author declares no financial conflict. Version is Singularity_090115.docx.

depleted the store of potential great inventions? Will the aging society lead to diminished innovativeness?[2]

However, the present study looks at the opposite idea, a recently launched hypothesis which I label *Singularity*. The idea here is that rapid growth in computation and artificial intelligence will cross some boundary or Singularity, after which economic growth will accelerate sharply as an ever-increasing pace of improvements cascade through the economy. The most prominent exponents are computer scientists (see the next section for a discussion and references), but a soft version of this theory has recently been advanced by some economists as well (Brynjolfsson and McAfee, 2014)

At the outset, I want to emphasize that this is not a tract for or against Singularity. Rather, the purpose is two-fold. First, I lay out some of the history, current views, and show an analytical basis for rapidly rising economic growth. Next, I propose several diagnostic tests that might determine whether Singularity is occurring and apply these tests to recent economic behavior in the United States. In the end, I hope that the analysis and tests will allow us to keep a running scoreboard as to whether the economic universe is on a stagnationist or accelerating path … or possibly in that middle ground of steady growth.

II. Artificial Intelligence and the Singularity

For those with a background primarily in economics, the present section is likely to read like science fiction. It will explain the history and a modern view about how the rapid improvements in computation and artificial intelligence (AI) have the potential to increase its productivity and breadth to the extent that human labor and intelligence will become increasingly superfluous. The standard discussion in computer science has no explicit economic analysis and leaves open important economic issues that will be addressed in later sections.

[2] There is a vast literature on the potential sources of stagnation. In the modern era, the "Limits to Growth" school was an early computerized modeling effort that produced scenarios for overshoot and decline in living standards (see Meadows et al 1972 and Ranjens et al 1990). Gordon (2012, 2015) argues that a decline in fundamental inventions may slow growth of leading countries. Some foresee a long period of demand-side stagnation in the wake of the long recession that began in 2008 (see Summers 2014).

It will be useful to summarize the argument before giving further background. The productivity of computers and software has grown at phenomenal rates for more than a half-century, and rapid growth has continued up to the present. Developments in machine learning and artificial intelligence are taking on an increasing number of human tasks, moving from calculations to search to speech recognition, psychotherapy, and robotic activities on the road and battlefield. At the present growth of computational capabilities, some have argued, information technologies will have the skills and intelligence of the human brain itself. For discussions of the background and trends, see Moravec (1988), Kurzweil (2000, 2005), Schmid and Cohen (2013).

The foundation of the accelerationist view is the continuing rapid growth in the productivity of computing. One measure of the productivity is the cost of a standardized operation in constant prices, shown in Figure 1. The costs of a standard computation have declined at an average annual rate of 53% per year over the period 1940-2012. There may have been a slowing in the speed of chip computations over the last decade, but the growth in parallel, cloud, and high-performance clusters as well as improvements in software appear to have offset that for many applications.

Computer scientists project the trend shown in Figure 1 into the indefinite future. At some point, these projections move from computer science to computer science fiction. They involve improved conventional devices and eventually quantum computing. If high-qubit quantum computing becomes feasible, then computing will be essentially free and the constraints on artificial intelligence will largely be ones of software and engineering (see particularly Moravec 1988, Kurzweil 2005).

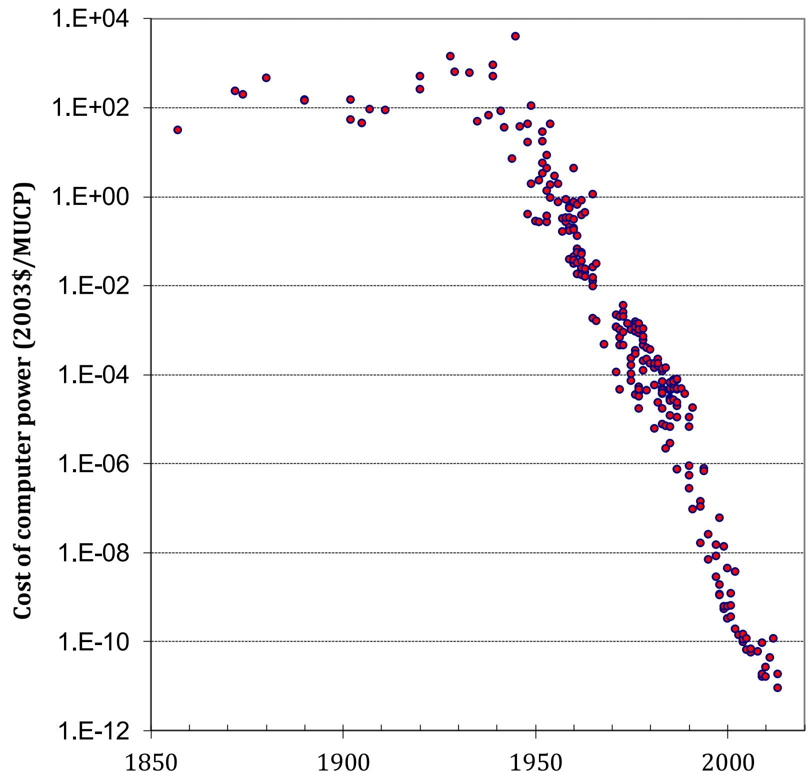

Figure 1. The progress of computing measured in cost per computation per second deflated by the price index for GDP in 2006 prices

Source: Nordhaus (2008) updated by the author using PassMark from http://www.cpubenchmark.net/, results as of April 2014.

One important milestone will be when inexpensive computers attain the computing capacity of the human brain. Current estimates are that the computational capacity of the human brain is in the range of 10^{18} computations per second (sometimes measured as "flops" or floating point operations per second). The fastest supercomputer as of 2015 was clocked at 3.4 x 10^{17} flops, and the speed of supercomputers has been growing at a rate of 82% per year over the 2007-2015 period (Top500, 2015). At this rate of increase, supercomputers will reach the upper level of 10^{18} flops by 2017. Computational speed does not easily translate into human intelligence, but it would provide the raw material for scientists to work with. Others have put the date at which human intelligence would be attained by computers from 10 to 100 years in the future.

As computer scientists look further into their crystal ball, they foresee artificial intelligence moving toward superintelligence, which denotes "intellect that is much smarter than the best human brains in practically every field, including scientific creativity, general wisdom and social skills." (Bostrum, 2006)

At the point where computers have achieved superintelligence, we have reached the "Singularity" where humans become economically superfluous in the sense that they make no difference to economic performance. Superintelligent computers are the last invention humans would make, as described by the mathematician Irving Good (1965) as follows:

> Let an ultraintelligent machine be defined as a machine that can far surpass all the intellectual activities of any man however clever. Since the design of machines is one of these intellectual activities, an ultraintelligent machine could design even better machines; there would then unquestionably be an "intelligence explosion," and the intelligence of man would be left far behind. Thus the first ultraintelligent machine is the last invention that man need ever make.

This point at which the rate and breadth of technological change will be so great is sometimes call the "Singularity" in a sense analogous to passing over the event horizon into a black hole – here the event horizon is where the forces of computer intelligence leave no room for human interventions.[3]

Before ourselves falling into the event horizon of accepting the Singularity hypothesis, we need to clarify some of the implicit economic assumptions that lie behind it. This will be the purpose of the next section.

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[3] The notion of Singularity of superintelligence is often attributed to John von Neumann. The only reference to "Singularity" comes in an appreciation by Stanislaw Ulam to von Neumann in 1958: "One conversation centered on the ever accelerating progress of technology and changes in the mode of human life, which gives the appearance of approaching some essential Singularity in the history of the race beyond which human affairs, as we know them, could not continue." Ulam (1958).

III. Historical perspectives on Singularity in economics

Societal Singularity is a recent theory, but concerns about replacement of humans by machines have been persistent for more than two centuries. The concerns tended to focus on displacement of particular skills or occupational categories. With the rise of computers, the major concern has been the replacement of unskilled labor by computers.

Macroeconomic concerns about rapid productivity growth and "automation," as it was called in the early days, focused first on the potential for satiation of human wants and a crisis either of unemployment or superabundant leisure. This was the theme of J.M. Keynes's essay, "The Economic Prospects for Our Grandchildren" (1930). He wrote:

> Suppose that a hundred years hence we are eight times better off than today. Assuming no important wars and no important increase in population, the economic problem may be solved. This means that the economic problem is not—if we look into the future—the permanent problem of the human race….Must we not expect a general "nervous breakdown"? Thus for the first time since his creation man will be faced with his real, his permanent problem—how to use his freedom from pressing economic cares, how to occupy the leisure, which science and compound interest will have won for him, to live wisely and agreeably and well.

Although we are close to the hundred-year mark, perhaps humans are suffering from a nervous breakdown, but there is no sign that humans have found themselves satiated with goods or overendowed with leisure.

One of the most impressive attempts to deal with the macroeconomic implications of computerization was Herbert Simon in "The Shape of Automation" (1965). Simon was unique in the intellectual history of the accelerationist debate in being a pioneering computer scientist as well as a leading economist. Writing a half-century ago, he was a self-described "technological radical." He wrote, "I believe that, in our time, computers will be able to do anything a man can do." (p. xii-xiii) At the same time, he was not what I will call an accelerationist, holding that "computers and automation will contribute to a continuing, but not greatly accelerated, rise in productivity." (p. xiii) As we show below, it seems likely that if, as Simon believed, computers can duplicate humans, then productivity would greatly accelerate.

Simon's analysis was very simple, relying on what is known as the "factor price frontier." This is the concept that, under highly stylized conditions, factor rewards can be summarized by the equation:

$$w a_L + (1+r)\, a_K = 1$$

In Simon's analysis (similar to the second model used below), output is produced by labor and capital, there are constant returns to scale, there is one good that can be used for either consumption or new capital. In this equation, $w =$ wage rate, $a_L =$ labor input coefficient, $a_K =$ capital input coefficient, and r = real interest rate. The price of goods is normalized to one.

Simon correctly argues that technological change affects unit inputs by lowering the labor and/or capital input coefficients so that at existing factor prices, the cost of production with the new technology is less than 1. Using the notation of the factor-price equation (where subscripts 0 are original factor prices, and asterisks denote the new technology) with an innovation, $w_0\, a_L^* + (1+r_0)\, a_L^* < 1$. Under competition, factor prices will rise until the cost will be equal to the price at 1, so in equilibrium, an innovative technology will raise either wage rates or interest rates or both.

Simon does not deploy a formal model for his critical next step. He argues that labor is inelastically supplied while capital is elastically supplied (so r is close to constant). This leads him to conclude that future changes in technology from automation will lead to nearly constant interest rates. He further argues (mistakenly) for a near-constant share of capital in national income, which then implies that "almost all the increased productivity will go to labor." (p. 15)

Simon's pathbreaking analysis pointed to an important result about factor prices – that it impossible in the neoclassical framework to have both a falling rate of profit and immiseration of the working classes (a formal analysis is in Samuelson 1957). But his analysis was unable to deal with the potential of rapidly growing capital productivity in the case where the *share* of capital in national output is rising rather than stable.

There is remarkably little writing on Singularity in the modern macroeconomic literature. While trend productivity growth has clearly risen from the period before the Industrial Revolution, the workhorse models today assume steady productivity and real income growth into the future.

The potential for accelerating economic growth has arisen occasionally as a curiosity in the economic literature. Explosive growth was explored in studies on endogenous technological change. (Similar but less explosive results are found in the "AK model," but those are not examined here.) The key feature of the endogenous technology models is that knowledge is a produced input. One formulation would be that knowledge growth is proportional to the inputs into the production process. Here A_t is technological knowledge, Y_t is output, a fraction λ of output is devoted to inventive inputs, dA_t / dt is knowledge growth, and its growth is a function of inventive inputs, as in $dA_t / dt = \phi(\lambda Y_t)^\beta$. To simplify this greatly, assume that output is produced with labor, and that labor grows at a constant growth rate n. If $\beta \geq 1$, which corresponds to increasing returns to inventive inputs, then the growth rate of output tends to infinity (see particularly Romer 1986, 1990).

The prospect of unbounded growth rates has not been taken seriously in the empirical growth literature for both technical and empirical reasons. The empirical reasons are that productivity growth has not accelerated in recent years. The technical reason is that it has unattractive assumptions about the knowledge-generation function, particularly the lack of diminishing returns to inventive inputs. For useful discussions of the shortcomings of the model, see Jones (1995, 1995a).

A final potential source of rising productivity growth comes from the benign version of Baumol's cost disease. Baumol and his co-authors emphasized the potential for low-productivity-growth industries to have rising costs, and potentially to slow aggregate economic growth (see Baumol and Bowen 1965, Baumol 1967, Baumol, Blackman, and Wolff 1985). However, depending upon the substitution parameters, the impact could be to raise rather than lower aggregate productivity growth. This might be called Baumol's cost euphoria and will be examined below.

IV. Singularity from the demand and the supply side

To begin with, I emphasize that rapid growth in the productivity of computers or information technology such as shown in Figure 1 has no necessary implication for aggregate economic growth. The reason is that the economy does not run on bits alone, either on the demand side or the supply side. Consumers may love their iPhones, but they cannot eat the electronic output. Similarly, at least with today's technologies, production requires scarce inputs ("stuff") in the form of labor, energy, and natural resources as well as information for most goods and services.

The question for the long run is the substitution properties between information and other stuff such as conventional, non-informational inputs or outputs. Here is the general result:

> Major insight: If information and conventional stuff are elastic substitutes either in consumption *or* in production, then growth will rise, perhaps extremely rapidly. However, if information and conventional stuff are inelastic in production *and* consumption, then rapid improvements in information technology will eventually be irrelevant to the economy.

Put more precisely, and as will be developed below, Singularity can arise from either the demand or the supply side. Both are the results of substitution toward high-growth inputs or outputs and away from stagnant inputs or outputs. On the demand side, Singularity would occur if preferences are such that consumer spending move increasingly toward high-productivity-growth industries as relative prices change. This is Baumol's cost euphoria. On the supply side, Singularity would occur if production has sufficient substitutability that the input bundle moves toward rapidly improving information capital as growth proceeds. Both, as we will see, will lead to rapid growth over time. I begin with the demand side and then move to the supply side.

V. The Baumol effect and demand-side Singularity

I begin by describing the forces from the demand side that might lead to rapid growth. These are the mirror image of Baumol's cost disease, and will be called Baumol's cost euphoria. The idea at the simplest level is that sectors with relatively rapid productivity growth have relatively rapid price declines and will therefore generally experience a rise in relative consumption levels. The key question for the growth in aggregate consumption is whether those sectors with relatively rapid productivity growth have rising or falling shares in nominal expenditures.

Baumol and his co-authors appeared to hold that the trend pointed toward stagnationism because of rising expenditure shares of stagnant sectors. For example, Baumol, Blackman, and Wolff (1985), p. 815-816 concluded as follows: [4]

> The [real] output shares of the progressive and stagnant sectors have in fact remained fairly constant in the postwar period, so that with rising relative prices, the share of total expenditures on the (stagnant) services and their share of the labor force have risen dramatically….

Unfortunately, their analysis was made with old-style (Laspeyres) output indexes, so the calculations using real output shares were biased.

We can use a two-sector example to understand Baumol-type Singularity. Assume that the economy has two sectors– call them information and handicrafts – produced by a single composite input. The rates of productivity growth are very high and very low, respectively. According to the Baumol mechanism, the relatively prices will be changing rapidly in favor of information.

If demand substitution is inelastic (technically, if the elasticity of substitution in demand between to two goods is less than one and the income elasticities are unity), then handicrafts eventually dominate expenditures, and the rate of growth of consumption will approach the rate of growth of productivity in the handicrafts sector. By contrast, if substitution is elastic (the elasticity of substitution in demand between to two goods is greater than one with unit income elasticities), then information dominates consumption, and the growth in consumption tends to the growth rate in the information sector. So here the critical parameter is the elasticity of substitution in the demand between the two kinds of goods.

Analysis of the Baumol effect

A more rigorous statement is as follows for the two-sector example. Assume that there are two consumption goods (C_1 and C_2) that are information and handicrafts, respectively. Outputs are competitively produced with a single exogenously growing composite input, L. Productivity growth is assumed constant in each industry (at rates h_1 and h_2). Preferences are homothetic with a constant elasticity of substitution between the two goods, σ. Given these assumptions, prices in the two sectors are falling at rate h_1 and h_2 relative to wages. Total consumption

[4] Op. cit., p. 815-816.

as measured by an ideal index (such as the Tornqvist index) will be growing at rate $\beta_1 h_1 + \beta_2 h_2$, where β_1 and β_2 are the relative expenditure shares of the two goods. With some work we can show that the ratio of the shares of the two industries is changing at the logarithmic rate of $(h_1 - h_2)(\sigma - 1)$.

So for example, if σ = 1.25, h_1 = 10% per year and h_2 =0% per year, then the share of information will be rising at approximately 10 $\times$ 0.25 = 2.5% per year (percent, not percentage points). Or to take a specific example of computers (formally, Information processing equipment), the relative price decrease over the last decade has been about 10% per year relative to other consumption. The share of computers in 2000 was approximately 2.0%. If the elasticity of substitution between computers and other goods was 1.25, then the share would grow to 2.6% after a decade. This is almost exactly the actual pattern over this period.

We can also easily calculate the Baumol effect for the two sector example. The growth in consumption (in the superlatively measured Tornqvist index) equals the weighted growth of consumption, $\beta_1(t)h_1(t) + \beta_2(t)h_2(t)$. Under the assumptions in the last paragraph, the growth in the index of consumption over the decade would increase from 1.20% to 1.26% per year, or an increase of 0.006% per year per year. This is equal to the change in shares times the difference in the growth rates (change in shares = 0.06 %-points per year $\times$ growth rate difference of 10% per year). Note that with elastic substitution the growth rate in this model tends toward the growth in the high-productivity-growth industry. The share of computers tends to one, so the weighted growth rate tends toward 10% per year in the simple example.

If we move to a multi-sector example, the analysis is analogous but more complicated. The analysis is laid out in Nordhaus (2008) and will be summarized here. Assume the growth rate of the ideal index of consumption is given by an almost ideal demand system in which consumption growth in each sector is a function of the growth in relative prices of the good and an income effect. If we assume that the income elasticities are uncorrelated with the changes in relative prices, then the average change in shares for each good will be determined by the average change in the relative price of that good times the price-elasticity of demand minus 1 times the relative price movement. So this is the analog of the two-sector example where the price-elasticity replaces the elasticity of substitution. The aggregate effect is then the weighted average of this term plus errors due to exogenous growth rates plus income effects.

VI. Empirical tests of demand-side Singularity

We can test for the Baumol or demand-side Singularity by looking at the relationship between the shares of different goods in total consumption and the trends in relative prices.

In a prior study of trends of major industries for the U.S., I determined that there was a tendency for industries with falling relative productivity and rising relative prices to have rising nominal shares and shares of employment. This was consistent with the trend identified by Baumol and his colleagues cited above of the cost disease. I concluded, "There is a negative association of productivity growth with the growth in nominal output. In other words, stagnant industries tend to take a rising share of nominal output; however, the relationship is only marginally statistically significant."

An alternative approach for this study focuses on consumption as that seems a more natural place to examine substitution patterns. We can test the impact of the composition by examining whether those sectors that have the most rapid decline in prices tend to have rising or declining shares in expenditures.

The BEA has developed long-term data on consumption expenditures and prices starting in 1929. These data include 89 distinct sectors ranging in size from owner-occupied housing to food provided on the farm. In our analysis, we take a simple regression of the log of expenditure change on the log of price change for different periods. The results are shown in Table 1, which looks at both sub-periods and the total period over the 1929-2012 record.

While there is no consistent and significant sign, the general pattern is for positive coefficients, indicating inelasticity of substitution. If we examine the entire period from 1929 to 2012 or pooled sub-periods of the total period, there is a clear indication of inelasticity. There are no sub-periods with significant coefficients that indicate elasticity, although the last period shows elasticity with marginal significance. These results are consistent with the analysis in Nordhaus (2008), which focuses on production patterns.

An alternative would be to look at major information-technology sectors, shown in Table 2. These are elusive to define, but for this purpose I included telecommunications, video services, information equipment, internet services, telephone, and photographic services. This new economy group shows a different pattern from the totality of industries. The prices of the new economy services in total have been declining steadily, and the shares have risen during all subperiods. However, a statistical analysis of the 6 new-economy sectors along the lines of Table 1 does not show a consistent pattern of elastic demand.

Period	Coefficient	t-statistic	Observations	P-value
1929-1948	0.25	1.10	48	0.012
1948-1969	0.90	2.59	54	0.012
1969-1990	0.06	0.37	83	0.714
1990-2013	-0.17	-1.58	90	0.118
1929-1969	0.15	0.50	48	0.617
1969-2012	-0.02	-0.26	83	0.796
1929-2012	0.44	2.04	48	0.047
Pooled, all subperiods	0.19	2.10	246	0.037

Table 1. Coefficient of log price in expenditure equation
This table reports a regression of following:

$$\Delta \ln[\text{expend}_i(t)] = \alpha_0 + \alpha_1 \Delta \ln[\text{price}_i(t)/\text{price}_i(t)] + \varepsilon_i(t).$$

Note that a positive price indicates that a rising relative price increases the expenditures share and is therefore an indication of inelasticity of demand.

	Change in prices	Change in share
Telecommunications	-2.9%	-1.3%
Video equipment	-11.1%	-1.8%
Information equipment	-21.1%	4.7%
Internet	-5.4%	24.7%
Telecommunications	-6.3%	3.3%
Photographic equipment	-3.2%	-6.1%

Table 2. Share and price change for new economy sectors
This table shows the average change in relative prices and in the shares of six information-technology sectors.

The size of the Baumol stagnation effect is small for the estimates that are provided here. We show in Figure 2 a calculation of the Baumol effect for selected well-measured industries.[5] This is the sum of the changes in shares times the logarithmic price change. A positive number indicates a cost disease. For these industries, the Baumol effect subtracts 0.098% per year from aggregate consumption growth if gasoline is included, and subtracts 0.015% per year without gasoline. In both cases the effect is small, but in neither case is the effect to increase economic growth. The dominant effect of gasoline arises because it not only has a large share but is extremely price-inelastic in the short run.

[5] These are selected both because the price indexes and real output measures are reliable and because they show a relatively large composition effect with large differences in output growth. The industries are Food, Imputed rental of owner-occupied nonfarm housing, Electricity, Pharmaceutical products, New motor vehicles, Motor vehicle fuels, Telecommunication services, Internet access, Video and audio equipment, Information processing equipment, Magazines, newspapers, books, and stationery, and Tobacco. They comprise about one-third of GDP in 2012.

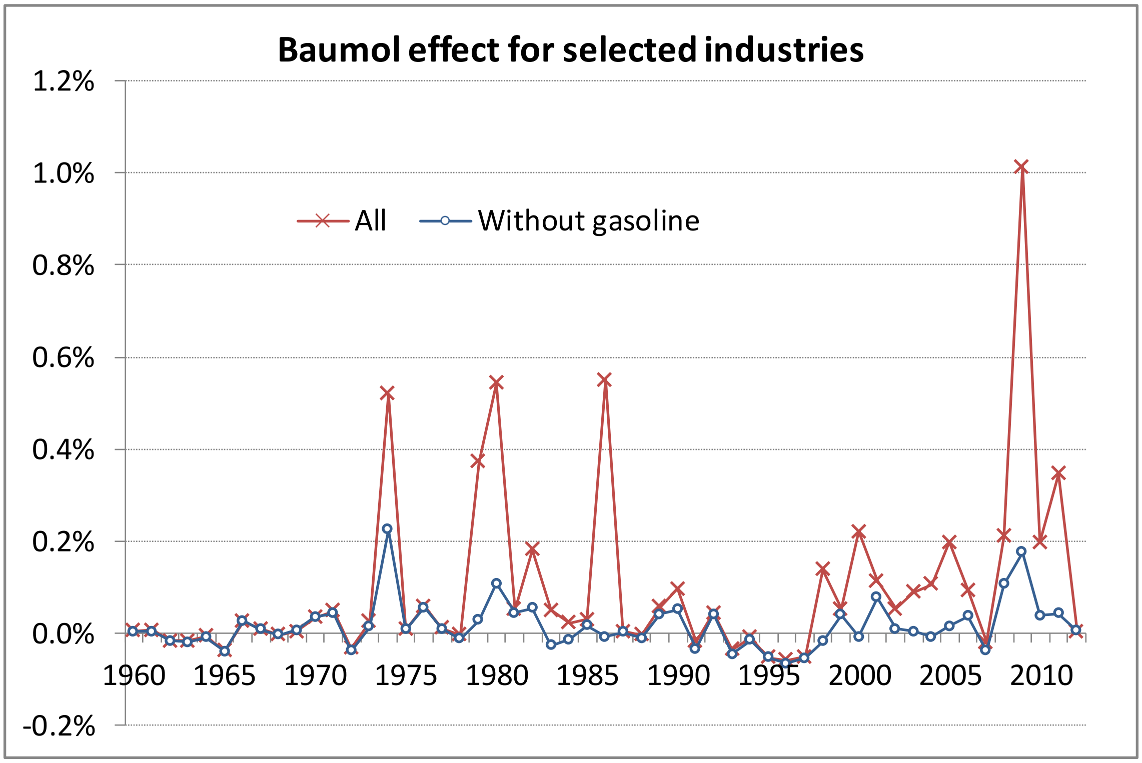

Figure 2. Baumol effect for selected well-measured industries
This shows the net effect of changing shares on growth in consumption (measured as a Tornqvist index). A positive number indicates in this graph indicates reduced overall growth. This signifies that on average industries with rising relative prices have rising shares of expenditures. Therefore, a positive number is a stagnationist force.

These results indicate that the Baumol effect of changing shares in consumption is a force for stagnation rather than acceleration. In plain English, the sectors that are experiencing the most rapid price declines are also experiencing slight declines in expenditure shares. This tendency means that growth in aggregate consumption would slow over time if the underlying technological trends were stable within individual industries. However, the impact of changing shares on the aggregate growth in consumption has historically been extremely small – in the order of minus 0.1% per year. The reason is that the shares of high- and low-productivity-growth industries have not changed appreciably over the last two decades. So this first test indicates no sign of demand-side Singularity.

VII. Supply-side Singularity

A second accelerationist mechanism involves substitution in production. We can again start with a two-sector model, similar to that of Simon above, to motivate the analysis. In this model, there are two factors of production and a single composite output that can be used for either consumption or investment. One input is either fixed or slowly growing, and it is usefully thought of as labor. The other is produced capital, which is produced by a rapidly improving technology. The natural produced input to consider is information technology, and will be identified in practice as "information capital."

In the simple two-input model, analogous to the Baumol effect, the key parameter is the elasticity of substitution in production. If the elasticity of substitution is greater than one, then information capital takes an increasing share of inputs, and the growth of productivity rises. If the elasticity of substitution is less than one, then information capital's share in production declines over time, and the growth in aggregate productivity tends toward the growth of the relatively fixed factor, labor. In the unit-elastic Cobb-Douglas case, productivity growth tends to a constant rate.

There are clearly other cases as well, such as multiple goods and multiple inputs, which are discussed below. However, the analysis is extremely simple in the one-good/two-input case. And the empirical tests are relatively clean. So it seems best to start here and see what we find.

To develop the model further, we use a standard closed-economy neoclassical growth model with a constant savings rate and with a particular modification. Assume that labor is growing at a constant rate *n* and that all technological change is capital-augmenting at a constant and rapid rate. In effect, we consider only information capital as an endogenous variable and sweep all other capital into labor.

The model is straightforward. Output and capital growth are given by

$$(1)\quad Y_t = F(A_t K_t, L_t)$$

$$(2)\quad \partial K_t / \partial t = sY_t - \delta K_t$$

So the growth of output is:

(3) $g(Y_t) = \alpha_t[g(A_t) + g(K_t)] + (1-\alpha_t)g(L_t)$

Here, α_t is the elasticity of output with respect to capital, which would equal capital's share of national income in a perfectly competitive economy. Combining the equations, we get:

(4) $g(Y_t) = \alpha_t[g(A_t) + sY_t / K_t - \delta] + (1-\alpha_t)n$

For a Cobb-Douglas economy, $\alpha_t = \alpha = \text{ constant}$, which implies that

(5) $g(Y_t) = n + [\alpha / (1-\alpha)]g(A_t)$

This is a straightforward balanced growth path.

For our purposes, the more interesting cases are where the elasticity of substitution between capital and labor (σ) is bounded away from one. In the case of inelastic production $(\sigma < \underline{\sigma} < 1)$, the competitive share of capital tends to zero, and the growth rate tends to the stagnationist case of zero growth in per capita output:

(6) $g(Y_t) \to (1-\alpha)n \to n$

The accelerationist case is where the elasticity of substitution is bounded above one $(\sigma > \bar{\sigma} > 1)$. The algebra in the general case is complicated, so simplify by assuming that the rate of growth in information productivity is a constant *h.* Because production shows elastic substitution, the elasticity of output with respect to capital (or the competitive share of capital) tends to one. As $\alpha \to 1$, production becomes linear in capital, or $Y_t \to cA_tK_t$, so $Y_t / K_t \to ce^{ht}$. This leads to the Singularity result:

(7) $g(Y_t) \to h + sY_t / K_t - \delta \to h + s(Y_T / K_T)e^{h(t-T)} - \delta \to \infty.$

The surprise here is that the growth of output is unbounded. In effect, the economy is just information produced by information capital, which is produced by information, which in turn is producing information ever faster every year. We don't need to push this result to the absurd limit. Rather the three key points are (1) the value share of information capital in the input bundle is tending toward unity, (2) as a result the contribution of information capital is rising, and finally (3) because information capital is a produced input, the growth rate of output is accelerating.

A numerical example

This result is so surprising that we can perform numerical analyses to make sure it is not a mistake or a possibility for distant millennia. To get a flavor for the dynamics, perform a simple simulation. Assume that labor is constant, that all technological change is capital-augmenting at 10% per year, and that the elasticity of substitution between labor and information capital is 1.25. Figure 3 shows a typical simulation of the share of capital and the growth rates of output and wages. Growth goes off the charts after about 70 years.

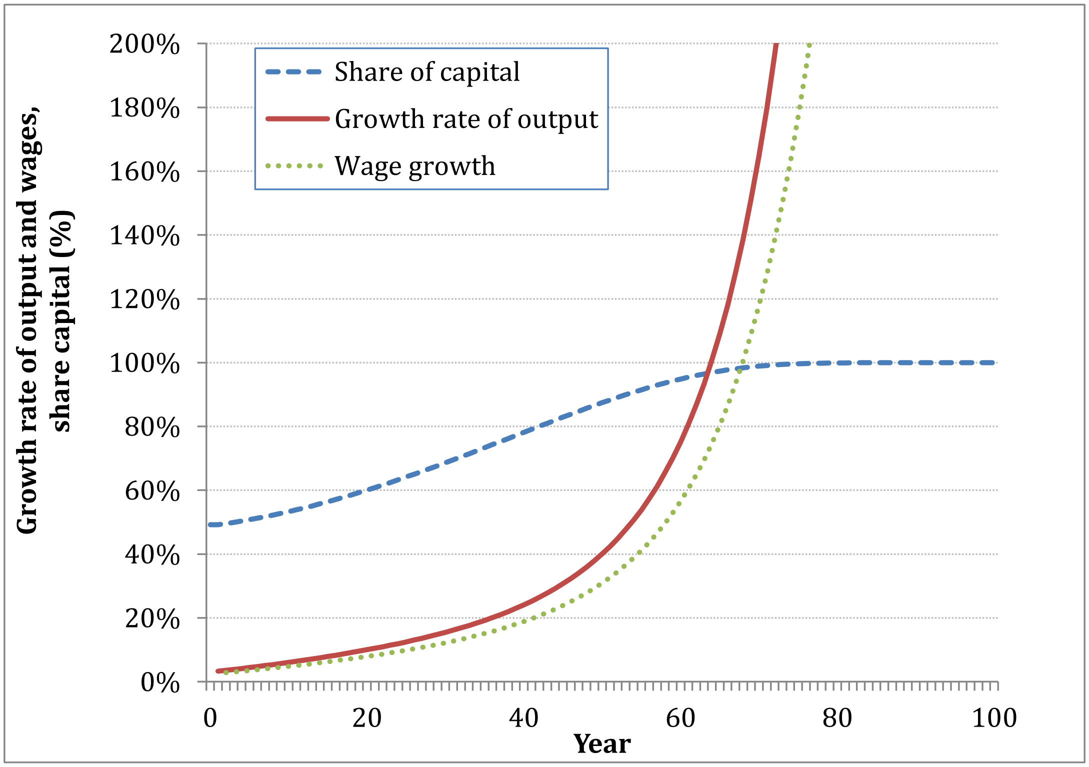

Figure 3. Simulation of a growth model with rapid technological change in capital and elastic substitution between labor and capital.

A second surprising result concerns the impact of rapidly growing growth on wages. Wages grow increasingly rapidly in this specification: wage growth reaches 200% per year in year 80. Capital eventually gets virtually all the cake, but the crumbs left for labor – which are really small parts of the increasingly large mountains of cake –are still growing at a phenomenal rate. The exact timing depends upon the parameters, but with elastic production and rapid capital productivity, the pattern always looks like Figure 3.

VIII. Tests for Supply-Side Singularity

Are we heading for the Singularity? If so, how far off is our rendezvous? Optimists believe that superintelligence could be achieved by the middle of the 21st century based on the progress in computing power. We can apply the economic models developed above to examine observable economic variables that can distinguish supply-side accelerationism from stagnation or steady growth.

The Simon-style growth model has several predictions that are consistent with a Singularity in economic growth. Among the most salient are the following six diagnostic signals.

1. The most important implication of the accelerationist growth model is that output growth is rising. This will show up as either rising labor productivity (LP) growth or rising total factor productivity (TFP) growth. While this is clearly a central prediction, it does not provide a strong differential diagnosis because the rising productivity growth could come from other sources.
2. A second important and cleaner differential diagnosis concerns the share of information capital in inputs. The clear prediction of the accelerationist view is that the nominal capital share in the value of inputs is rising, and should eventually rise to one.
3. A third prediction is that the relative prices of investment and capital goods are falling relative to output. Indeed, the price decline of total capital should trend toward the price decline of information capital as information capital gradually invades the entire economy.
4. A further prediction is that the real capital-output ratio should be rising at a very rapid rate.
5. The share of information capital in total capital in the accelerationist economy will be growing toward one.
6. The rise in wages will depend upon the elasticity of substitution between capital and labor. In cases of plausible elasticities, wage growth becomes extremely rapid.

We will discuss below measurement issues that may cloud the data in a fashion that hides the accelerating growth of productivity. While the discussion suggests that this is to date empirically relatively small for information-based output, this criticism would apply primarily to test 1 and 6. Tests 2 and 5 are based on nominal measures and are largely unaffected by errors in price and output measurement. Tests 3 and 4 both require that the price of investment (or capital) relative to output is accurately measured. This is similar to the reliability of tests 2 and 5 but is probably less open to bias because some of the output-price bias will be offset by investment-price bias since these are the same goods. In any case, at least two of the tests (2 and 5), and perhaps the most useful ones, are largely immune to measurement problems.

Test 1: Accelerating productivity growth?

A first question is whether productivity growth is accelerating. Figure 4 shows an estimate of multi-factor productivity for the U.S. private business sector. The 1949-2014 period shows an average MFP growth rate of 1.3% per year. MFP growth has been on a slight declining trend over the period. The period since 1990 saw a rise in productivity until the mid-2000s, but then growth has declined. The average MFP growth over the 1990-2012 period is slightly below (-0.03 ± 0.03 % per year) the earlier period.

The summary on MFP growth is that there is no sign of any acceleration of multifactor productivity as of the most recent data for the U.S. Even with the potential biases discussed in the next section, it would be difficult to discern any noticeable upturn in TFP growth.

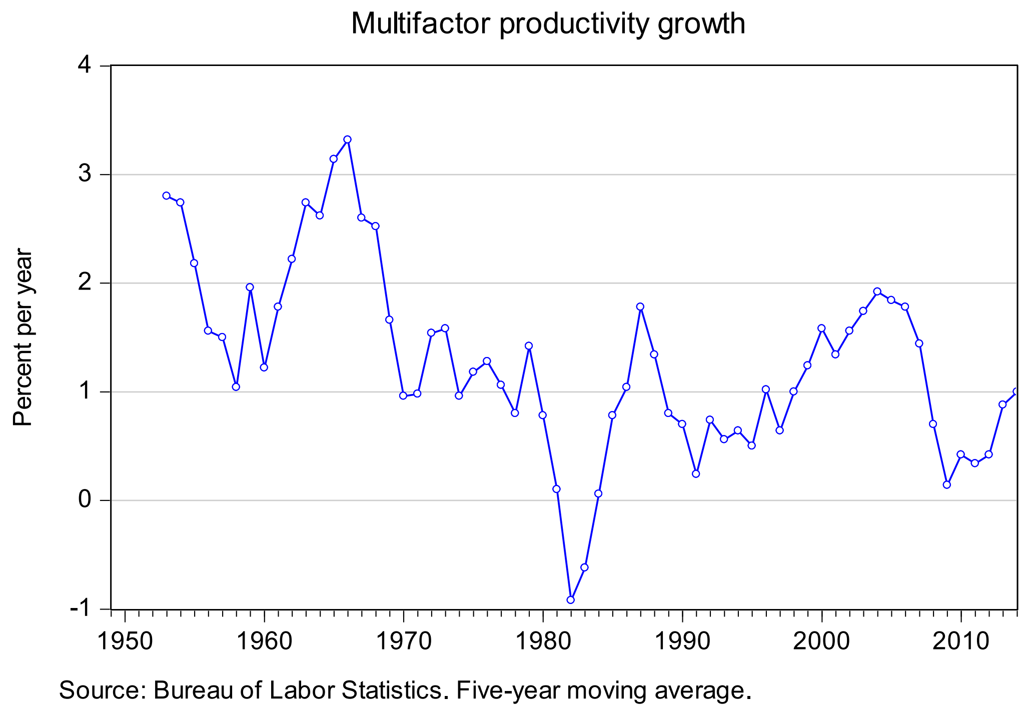

Figure 4. Multifactor productivity growth, 1949-2014
Multifactor productivity measures total output growth minus total input growth using a Tornqvist index.
Source: Bureau of Labor Statistics at www.bls.gov.

Test 2: Rising share of capital?

A central diagnostic forecast of Singularity is a rising share of capital in national income. (Note that the "share" in the growth model is the elasticity of output with respect to capital. That parameter is not observable, so we use the income share, which would equal the elasticity under competitive conditions.)

Figure 5 shows the trend in the income share of capital (strictly speaking, all income other than labor compensation) over the period 1948-2013. One sectoral concept is the entire economy, while the other is the non-farm business sector. The latter is better measured and provides a cleaner definition of capital income than the former, which includes a large component in owner-occupied housing as well as government capital. Note that capital income in the data include many elements other than the net return to capital, such as depreciation, royalties on minerals, interest income, income of proprietors, and some labor income. Some analysts suspect that a substantial part of the increase in capital's share is either mismeasurement or is due to housing, so the estimates here are probably an upper bound on the share change (Elsby, Hobijn, and Şahin 2103 and Rognlie 2015).

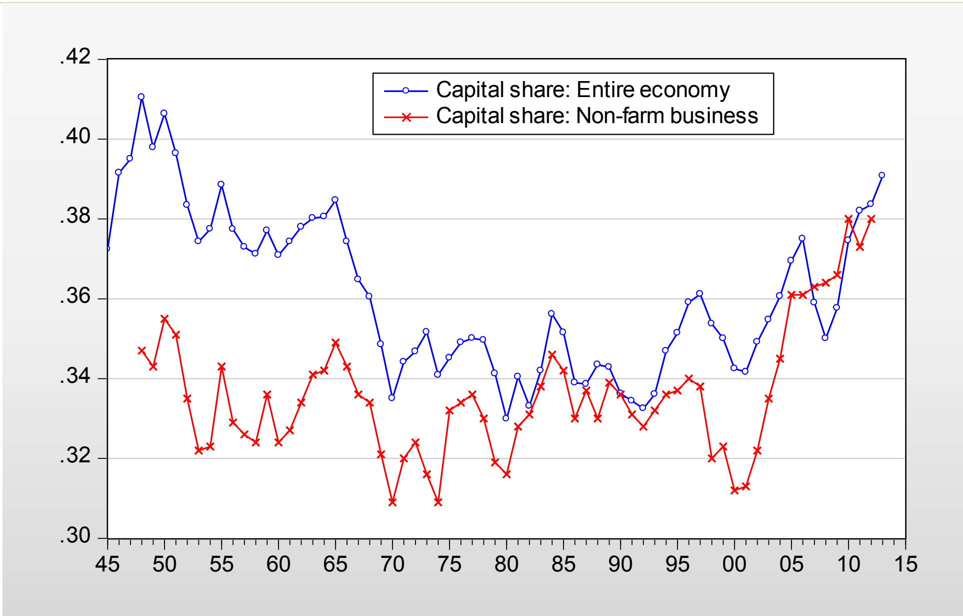

Figure 5. Trend in share of capital for US in overall economy and non-farm business sector
Source: U.S. Bureau of Economic Analysis and U.S. Bureau of Labor Statistics

Table 3 shows regressions of the shares in the two sectors with and without breaks in trend in 1990 and 2000. Both show a small upward trend of about 0.2 percentage points per year since 1990 and close to 0.5 percentage points per year since 2000. This trend is supportive of the accelerationist hypothesis at the raw data level. However, since we do not have a good understanding of the reasons for the rise in capital's share, further research would be necessary to determine whether there is a link between this rise and a rapid rise in capital productivity, and particularly in information capital.

Projecting future trends such as those of capital's share in Figure 5 is clearly a primitive exercise. However, projections are useful to give some perspective on when the Singularity might become more apparent. Our simulation model shown in

Figure 2 indicates that the acceleration in output becomes quite apparent (with the growth rate crossing the 20% per year threshold) when capital's share crosses the 80% level. At the rate of increase from a forecast using the regression model underlying the last set of estimates in Table 3 (+0.47% per year), the 80% rate will not be reached until 2100 (plus or minus 20 years depending on the sample period of the regression). So while the test is positive, Singularity is apparently many decades in the future using this diagnostic test.

	Trend	Trend since 1990	Trend since 2000
Non-farm business			
Coefficient	-0.04%		
t-statistic	-3.71		
Coefficient	-0.15%	0.21%	
t-statistic	12.33	12.33	
Coefficient	-0.10%		0.42%
t-statistic	-9.66		8.59
Overall economy			
Coefficient	0.03%		
t-statistic	2.84		
Coefficient	-0.03%	0.19%	
t-statistic	-2.40	5.77	
Coefficient	-0.02%		0.47%
t-statistic	-2.20		8.98

Table 3. Regression coefficients for equation with share of capital as dependent variable and time and breaks in the trends in 1990 and 2000. Number under coefficient is t-statistic.

Source: Data from sources in Figure 5.

Test 3: Accelerating decline in capital goods prices?

The prices of investment and capital goods have been falling relative to output (GDP), consumption, and labor for most of the last half-century. Since the accelerationist hypothesis holds that the price of capital goods will be falling ever more rapidly, we examine the price of various definitions of capital goods. To calculate the price index, I take the ratio of the current-cost capital stock to the quantity index of capital developed by the BEA.[6]

Figure 6 shows the rate of decline for seven important series. This shows the decline in the price of the capital stock relative to labor's wages, this being the important variable for the production function in the growth model. While capital prices have continued to decline, there has been no significant change in the last decade; indeed all six sectors show either the same or slower relative price declines in the last period.

From the point of view of the accelerationist hypothesis, the key variable to look at is the decline in the price of all capital goods relative to wages. For this purpose, we examine all "private fixed assets," which include business structures and equipment, residential structures, and intellectual property. The relative price decline here (shown as the first set of bars in Figure 6) has been 1.4, 1.5, and 0.5 percent per year for the periods 1960-1990, 1990-2000, and 2000-2012. Clearly, these are not only small declines but not accelerationist at all.

[6] Current-cost estimates of capital stocks are derived by converting the constant-dollar estimates of stocks to the prices of the current period. Chain-type quantity indexes are computed using the Fisher quantity index formula. The ratio should be the Fisher ideal price index under the self-reflexive property of Fisher indexes. See BEA (2003).

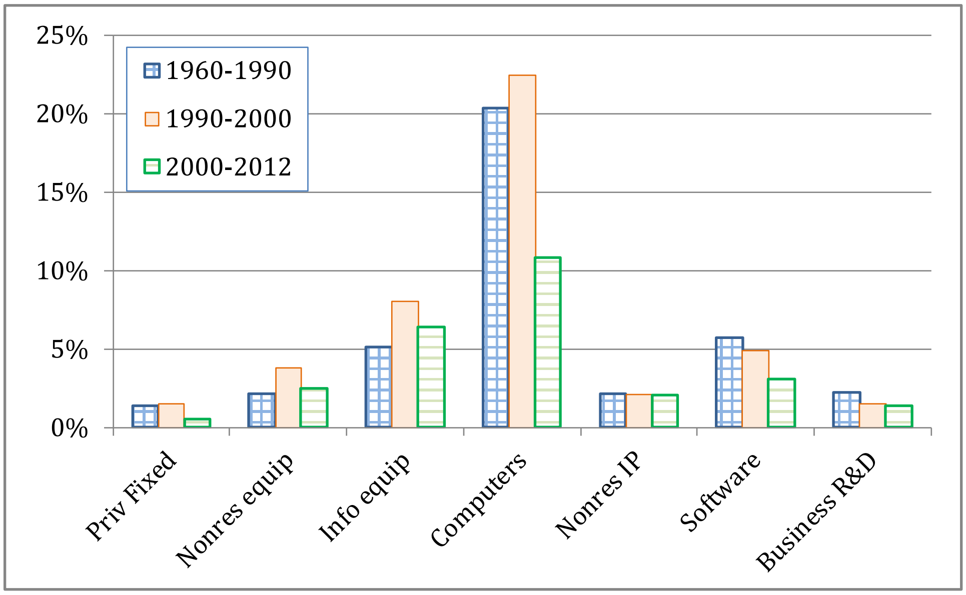

Figure 6. Rate of decline of capital prices relative to labor's wage rate. Source: Bureau of Economic Analysis.

Test 4: Rapidly rising capital-output ratio?

Another important diagnostic concerns the real capital-output ratio. As is seen in the growth model sketched above, the rapidly rising quality of capital implies that the capital stock (in efficiency units) will rise increasingly rapidly relative to output. The rise will come because informational capital grows rapidly, and also because informational capital takes a larger share of the capital stock.[7]

Table 4 shows the trends in the real capital-output ratio since 1960. The capital stocks shown are different components of private capital corrected for quality by the Bureau of Economic Analysis and other government agencies. The output measure is gross business product. The first line shows that the overall

[7] The relationship between tests 3 and 4 is the following. Let p_K = the price of capital goods, V = current-cost value of capital, K = quantity of capital, p_Y = the price of output, Y = quantity of output, and Q = nominal value of output. Quantities in both cases are measured as nominal values divided by price indexes. The real capital output ratio in test 4 is $K/Y = (p_K K/p_Y Y)/(p_K/p_Y)$, while the price relatives in test 3 are p_K/p_Y. So the difference is the share of capital in national output.

capital-output ratio has been falling slowly over this period, although it has been close to constant in the last subperiod. Looking at the information capital components, these have been rising relative to output, but only at modest rates. In any case, the overall contribution of informational capital has been too small to lead to a rising capital-output ratio.

Sector	1960-1990	1990-2000	2000-2012
Private fixed assets	-0.5%	-1.2%	-0.2%
Equipment	0.7%	0.4%	0.6%
Nonresidential equipment	0.6%	0.4%	0.6%
Information processing equipment	6.4%	5.5%	3.9%
Computers and peripheral equipment	na	21.1%	6.3%
Intellectual property products	4.3%	4.4%	4.1%
Nonresidential intellectual property products	2.0%	2.1%	1.8%
Software	18.2%	10.2%	3.2%
Research and development	2.5%	0.4%	1.4%

Table 4. Growth rates of the real capital output ratio, different sectors

Source: Bureau of Economic Analysis.

Test 5: Share of information capital rising?

A further test is that informational capital should be a rising share of the capital stock. Indeed, as the economy approaches the Singularity, the share of informational capital should approach 100%.

Table 5 shows the shares of informational capital in total private assets. (These are the current-cost net stock of private fixed assets.) It is clear that informational capital is becoming a more important part of the capital stock. The growth is particularly strong in intellectual property products. Surprisingly, computers and information processing equipment have seen a declining share over the latest period. So this test would appear to conform to the Singularity view, although there is still a long way to go before these sectors dominate investment.

To determine whether an inflection point is in the near future, we project the share of informational capital into the future at the growth rate for the 1960-2012 period. Our numerical example suggests that the growth rate begins to accelerate when the capital share exceeds 80% of income. Our extrapolation of Table 5 indicates that this would not occur within the next century, so the Singularity appears at best distant by this test.

Sector	1960	1990	2012
Equipment	17.6%	19.2%	15.7%
Nonresidential equipment	17.3%	19.0%	15.5%
Information processing equipment	1.8%	4.5%	3.7%
Computers and peripheral equipment	0.0%	0.7%	0.5%
Intellectual property products	2.8%	4.6%	6.4%
Nonresidential intellectual property products	2.8%	4.6%	6.4%
Software	0.0%	0.7%	1.6%
Research and development	1.5%	2.8%	3.6%

Table 5. Share of information capital in total capital

Source: Bureau of Economic Analysis

Test 6: Rising wage growth?

A final test is that wages should be growing more rapidly. The extent of acceleration of wages will depend upon the elasticity of substitution, but as long as the elasticity is not too high, wages will grow at a rate close to that of output per hour worked.

Table 6 shows trend for two alternative measures of real compensation, for the entire economy and for the private business sector. (Note that this divides by the output price index rather than the consumer price index, so the results differ from those usually cited. Additionally, this measure is compensation including fringes rather than just wages or wages and salaries.) Real wages accelerated slightly in the first decade of the new economy (1990-2000), but growth then slowed over the subsequent period. So the real wage test clearly fails to show any signs of Singularity.

Increase in real wages (annual average % per year)

Period	Total economy	Private business
1960-1990	1.8%	2.3%
1990-2000	2.0%	2.6%
2000-2013	1.1%	1.5%

Table 6. Increase in real product wages, different periods
Source: Bureau of Economic Analysis and Bureau of Labor Statistics

IX. Summary of Tests for Singularity

Table 7 shows a summary of tests of Singularity. Five of the seven tests are negative for Singularity while two are positive. We can also calculate for the two positive tests how far we are from the point of Singularity. I define Singularity as a time when the economic growth rate crosses 20% per year. Using simple extrapolation for the two positive tests, the time at which the economy might plausibly cross the Singularity is 100 years or more.

Source	Result of test	Time until singularity
Demand side		
Baumol effect on shares of high-productivity industries	Negative	x
Supple side		
Test 1: Accelerating productivity growth	Negative	x
Test 2: Rising share of capital	Positive	100 years ± 20 years
Test 3: Increasing decline of capital goods prices	Negative	x
Test 4: Rapidly rising capital-output ratio	Negative	x
Test 5: Share of information capital rising	Positive	> 100 years
Test 6: Rising wage growth	Negative	x

Table 7. Results of the Singularity tests and time to Singularity

Source: Earlier figures and tables.

X. Interpretations and Elaborations

The tests and theory above raise several issues of interpretation. I consider the question of minimal resource inputs, heterogeneous labor, measurement problems, the social structure of a Singular economy, and concerns about evil agents.

a. Violations of basic physical laws

An objection to all of this analysis that immediately comes to mind is whether accelerationism violates basic laws of nature. All processes need minimal energy, and energy is limited if superabundant. Other potential limiting resources are fresh and clean water, oxygen, and exotic minerals to build machines. Some would invoke the second law of thermodynamics, which holds that increasing order must be offset by increasing disorder elsewhere.

The issues here are too deep to be adequately treated in the present study. While some resources are indeed needed for all production processes, the inputs can in theory be reduced sharply, and potentially as rapidly as production increases. This is vividly illustrated for computation. An early computer was the ENIAC (shown at the upper left in Figure 1). It required about 150 kW to operate, or approximately 55 watts per floating point operation (flop). A desktop computer today requires about 75 watts to produce 10^{13} flops. While this is only an approximation, this calculation indicates that the energy requirement for computation has declined by a factor of 10,000,000,000,000. In recent years, the energy use has declined at approximately the rate of improvement of computers.

So the bottom line on resources is that *at least in theory* improvements in material use and miniaturization can overcome the physical limitations on accelerating growth. As Richard Feynman said, "There is plenty of room at the bottom."

b. Heterogeneous labor in the growth model

The Simon-type growth model of information and productivity analyzed above has the shortcoming that it assumes heterogeneous capital and labor. Heterogeneous output is considered in the Baumol example. We consider in this section the interesting implications of adding heterogeneous labor to the analysis.

Economists have generally found that skilled workers are more adaptable to rapid changes in information technology than middle-skilled, manual, or unskilled workers. The process is summarized nicely by Autor (2014):

> "Routine tasks" [are ones] that follow an exhaustive set of rules and hence are readily amenable to computerization. Routine tasks characteristic of many middle-skilled cognitive and manual activities, such as bookkeeping, clerical work and repetitive production tasks. Because the core tasks of these occupations follow precise, well-understood procedures, they are increasingly codified in computer software and performed by machines. This force has led to a substantial decline in employment in clerical, administrative support and, to a lesser degree, production and operative employment… [135]

We can extend the Simon model to include heterogeneous labor by considering some polar cases. Assume as one example that unskilled labor is a perfect substitute for informational capital, while the other input is skilled labor. As above, skilled labor has high but imperfect substitutability with capital. We then directly apply the analysis above. The marginal product and wage of unskilled labor fall proportionally with capital prices. More realistically, if there is a reservation wage for unskilled labor, say because of income support, the unemployment rate of unskilled labor rises to unity, and the employment of unskilled labor approaches zero.

As a historical analog, consider the fate of human adding machines of the nineteenth century. As explained in Nordhaus (2007), there was a revolution in the employment of human calculators around 1900. In his book on calculation, Orton writes, "To be able to add two, three or four columns of figures at once, is deemed by many to be a Herculean task, and only to be accomplished by the gifted few, or in other words, by mathematical prodigies." (Orton 1866, p. v) "Lightning calculators," the prodigies who could add up columns of numbers rapidly, were at a premium. Indeed, John D. Rockefeller was a champion lightning calculator before he turned to being a champion monopolist. The advent of calculators changed all that. Aside from quiz shows, there is zero demand today for lightning calculators. Such would be the fate of unskilled labor in this simple two-labor model as we approached the Singularity.

What of skilled labor? In the simple two-labor-input model described here, skilled labor would have the same future as labor in the one-labor Simon model. Its share in national income would tend to zero as capital took over the economy. But skilled labor would be fully employed, and its wages would begin to rise rapidly as shown in Figure 1. We would see social and economic polarization with a vengeance.

Perhaps the pattern of impacts would be reversed, as is suggested by Autor (2014). Perhaps the work of skilled labor would be substituted by information technology while unskilled labor would be the only group not susceptible to substitution by information technology. Perhaps, patients would be diagnosed and treated by computers rather than doctors. Classes would be taught online by computerized instructors and virtual teaching assistants rather than Ph. Ds. Central banks would finally, in Milton Friedman's vision, be run by a computerized rule rather than imperfect discretion. Workers just hook up the monitors, plug in the machines, and make sure that the Fed has the latest operating system. Since the skill ladder is a two-way street, skilled workers would abandon their professional degrees as the skilled jobs disappear and all humanity becomes unskilled apprentices to computers. We are then back to the Simon model, but in this case with the one factor being unskilled labor. Surprisingly, there is much greater labor-market equality than in the first example.

c. Measurement issues

One concern about the empirical tests of accelerationism is that the major increases in productivity are hidden by poor measurement. Hal Varian, the chief economist at Google, argues that there is an explosion of productivity underway because of the devices, apps, and other digital innovations coming out of Silicon Valley. "There is a lack of appreciation for what's happening in Silicon Valley because we don't have a good way to measure it." (WSJ 2015).

The issues involved in measuring the contribution of new and improved goods and services have been carefully studied and raise several thorny issues. The most important shortcomings arise from improper measurement of the prices for goods that are either new or show rapid improvement. (Recall that "real output" growth is nominal output growth less the rate of change of the price of the good. So if price increases are overstated, as is the case with improper quality adjustment, then real output increases will be understated.) Additional questions arise when goods are free.

We can illustrate the issue for the case of cell phones. At the beginning, these involved a new good, so it would not be possible to have an accurate comparison of how much the price of "cell phone service" was falling. If cell phones are introduced late in the product cycle, the increases in consumer welfare from the falling prices will be missed. A second issue involves quality change. It is difficult to measure the improvement in quality (which implies a fall in price of a standardized good) because of the rapid improvements in cell phone design along with the many bundled applications. A third issue arises because many of the services provided on a smartphone (such as having a flashlight, map, weather forecast, and the like) have zero prices. Under the conventions of national output accounting, the value of those services is also zero because goods are valued at their market prices. The presence of these factors lies behind the contention of Varian and others that actual (as opposed to measured) productivity is actually growing rapidly.

Business v consumer uses

What are we to make of these contentions? We can quickly dispose of one part of the issue, which involves the use of IT by companies. To the extent that IT is increasing the productivity of companies as an incorrectly measured intermediate good, then that would show up as productivity for the industry. If for example free Internet services vastly increased the ability of airlines to utilize their fleet more efficiently, then measured productivity growth of airlines would rise. So the IT going as intermediate products or capital services to business would not lead to underestimated aggregate productivity growth.

What are the proportions of consumer versus business in information technology? We can look at detailed input-output tables to get an idea of the magnitudes. Taking the major eleven IT sectors,[8] we can divide gross output into that part going to consumers and that going to businesses. The former are included as personal consumption expenditures, while the latter are investment or intermediate purchases. Looking at the input-output structure for 2002, there were $1,217 billion in domestically purchased IT goods and services (about 11% of GDP). Of these, 77% were purchased by businesses, 23% were by consumers. The major consumer service was telecommunications, where consumers purchased about half

[8] The sectors were Computer and peripheral equipment, Audio, video, and communications equipment, Semiconductors and electronic components, Electronic instruments, Software publishers, Cable networks and program distribution, Internet publishing and broadcasting, Telecommunications, Data processing services, Other information services, and Computer systems design and related services. Data are from www.bea.gov.

of total output. Given these numbers, it seems likely that most of the productivity impacts of IT will be captured in either business output or business productivity.

Considering the IT purchases by consumers, these comprise about 2½ percent of GDP. If productivity growth for these products were underestimated by 10 % per year (surely an upper bound on the number), aggregate productivity would be underestimated by 0.025 % per year. This hardly makes a dent on the productivity slowdown over the last decade.

Measurement of consumer surplus

A second issue is the provision of free services to consumers (free services to businesses are covered by the last section and can be excluded). Perhaps the consumer surplus from provision of these services is enormous. This is an ancient problem in national income accounting. If the price is zero, then the marginal value to consumers is zero, and that is the customary valuation. But perhaps there is great value to the inframarginal units, and these were not available in earlier periods.

Two issues arise here. First, we should ask how the value of the unmeasured value of IT compares with the new products and services of earlier periods. Gordon (2012, 2015) persuasively argues that the unmeasured value of inventions of the 19th and 20th century dwarfs the value of IT. We might point to examples like indoor plumbing, anesthetics, electricity, radio, motor vehicles, lighting, photography, antibiotics, and even the lowly zipper as examples of goods with vast unmeasured consumer surplus.

Second, the issue of including consumer surplus raises insuperable obstacles of measurement. If we follow this road, we run into the "zero problem" that arises when we attempt to measure total utility or happiness rather than value using marginal values. Here is an explanation of the issue using the example of the consumer surplus of water.[9] Suppose that we want to measure the total value of consumption of water services in the national accounts. We then need to integrate the marginal surpluses between some "zero" level and current consumption. But what do we mean by zero? Is it literally zero water consumption in which case consumer surplus is equal to the value of life itself and is infinite? Or is it the level of

[9] This is drawn from William Nordhaus, "Principles of National Accounting for Nonmarket Accounts," in Dale W. Jorgenson, J. Steven Landefeld, and William D. Nordhaus, Eds., *A New Architecture for the U.S. National Accounts,* Chicago, University of Chicago Press, 2006, pp. 143-160.

consumption in pre-industrial times? If so, should pre-industrial times relate to the 1700s, when water in the U.S. was plentiful? Or to the time when humans first crossed the Bering land bridge, when ice was plentiful but water was scarce? If we attempt to measure total surpluses for necessities in too many areas with low "zeroes," we will undoubtedly find ourselves with multiple infinities of the value of critical goods and services. Once we travel even a few thoughts down this road, we rapidly come to the conclusion that, for purposes of measuring output and income, we had best rely on the standard approach of using marginal valuations in all sectors.

Time use as a complement for free goods and services

A final way of looking at the role of IT as unmeasured output is to examine the time spent on information activities (this approach was pioneered by Goolsbee and Klenow 2005). We might employ this approach to estimate consumer surplus (despite our reservations), or more appropriately in our context it might be used to estimate the errors introduced by mismeasurement of prices and outputs of these services.

There are comprehensive data on time use by Americans collected since 2003. While it includes email use, it does not include total Internet use. Figure 7 shows the history of household use for televisions (reduced by a factor of 20), telephone, and email. This does not include a comprehensive survey on Internet use, unfortunately. Two points are striking. One is that popular culture has a vastly exaggerated notion of how much time on average Americans spend on email. The figure here is 0.03 hours per day, orders of magnitude less than the 3 hours a day watching television. A second striking feature is that the time spent on email is actually declining over the last decade, while TV time has been slightly rising.

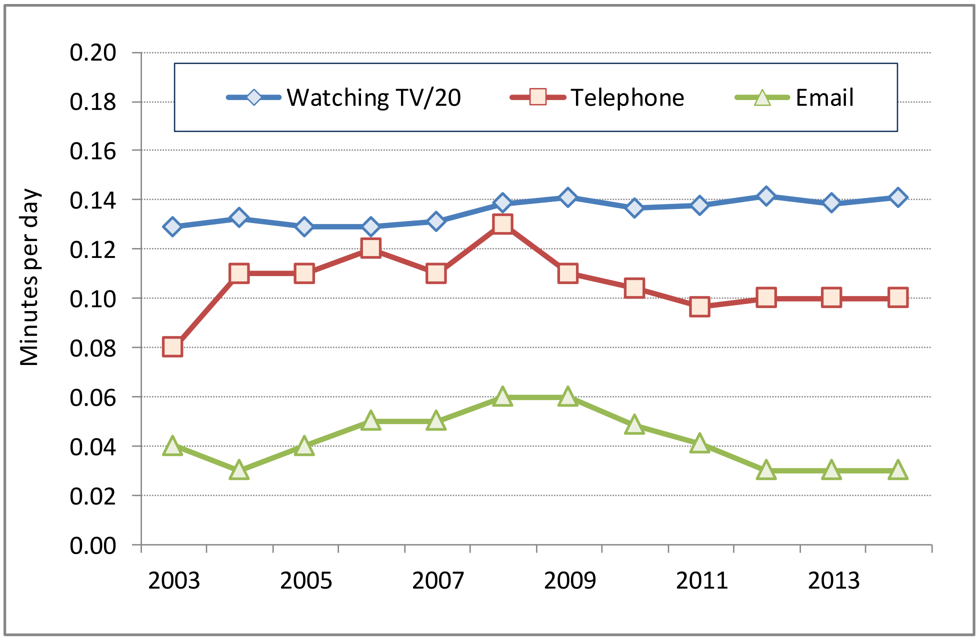

Figure 7. Time use of electronic media by US household

Source: American Time Use Survey (atus.bea.gov)

Based on other less comprehensive data, these numbers are likely to be seriously biased underestimates of Internet use. Data from Nielson (2015) adjusted for over-reporting suggests that people spend slightly under two hours a day on the Internet, roughly equally divided between computers and smartphones. Here is an illustrative calculation. Assume the average person spends 1.5 hours a day on the Internet. Further suppose that time is valued at the marginal post-tax hourly compensation of $18 per hour, and that it replaced other time valued at half of that value (say watching TV or housework). Further assume that one-third of the Internet time is for personal rather than business or instrumental purposes. Then for the 245 million persons in the adult population, the total unmeasured value would be slightly above $135 billion in 2015, or about 0.7% of GDP. If this unmeasured value started at zero in 1995, then productivity growth over this period would be underestimated by 0.04% per year. While these numbers are just suggestive, they indicate that compared to other goods and services, the unmeasured value of "apps and gadgets" is unlikely to make a substantial dent on overall growth of national output and productivity.[10]

d. <u>The euthanasia of the laboring classes</u>

As growth accelerates with superintelligent capital, the rate of return on capital and real interest rates fall to zero. This was an outcome envisioned by J.M. Keynes in a chapter from *The General Theory* (Keynes 1935).

> [There would be an] increase the stock of capital up to a point where its [marginal product] had fallen to a very low figure…. Now, [this] would mean the euthanasia of the rentier, and, consequently, the euthanasia of the cumulative oppressive power of the capitalist to exploit the scarcity-value of capital. Interest today rewards no genuine sacrifice, any more than does the rent of land.
>
> I see, therefore, the rentier aspect of capitalism as a transitional phase which will disappear when it has done its work. And with the disappearance of its rentier aspect much else in it besides will suffer a sea-change. It will be, moreover, a great advantage of the order of events which I am advocating, that the euthanasia of the rentier, of the functionless investor, will be nothing sudden, merely a gradual but prolonged continuance of what we have seen recently … and will need no revolution.

Keynes's analysis predated the pioneering work on production functions that clarified the key role of the elasticity of substitution on factor shares, and as a result he saw only one of several possible outcomes. Keynes's scenario described a growth path in which the elasticity of substitution between labor and capital is less than one; accumulation in the inelastic case therefore drives not only the rate of return to zero but also the share of capital to zero.

However, the accelerationist case leads to the opposite outcome, where the share of capital goes to unity. In this outcome, we thus would see the euthanasia of the laboring classes in the sense that all of income eventually goes to the owners of

[10] Goolsbee and Klenow (2005) have substantially higher numbers because they estimate a non-linear demand curve for Internet services, which has a much higher intercept and higher inframarginal values. They also do not correct for business usage. If their estimate is corrected for non-consumption usage, the estimate here is about one-half of their linear estimate. The Nielson estimate is reduced by about one-third to reflect the higher estimate of TV usage in the Nielson compared to the ATUS estimate of time spend watching TV.

capital. Workers would be well-paid but would control a vanishing part of national output through the fruits of their labor. However, as long as corporations own most of the capital, and people or human institutions (including governments through taxation) own corporations, capital income will indirectly flow through to humans. Since national income equals national output, average income will be growing increasingly rapidly.

How this will play out in terms of individual equality or inequality goes beyond economics to politics, tax and benefit systems, and the nature of dynastic savings. It is clear that the Piketty condition for growing inequality (that $r > g$) definitely will not hold, but beyond that little is clear. Will the incomes be captured by the Schumpeterian classes – the innovators who design machines and write software for them? Or by the wealthy who subvert institutions to increase their wealth? By those who are the last humans who are complements rather than substitutes for information, perhaps as gardeners or butlers? Perhaps by those who control the intelligent machines before they take over?

Fortunately, the euthanasia of the laboring classes is far off and will flash sufficient warning signals so that, if it does occur, humans will have time to contemplate the social structures of such an era.

e. Autonomous agents in warfare

The sector which has invested most heavily and is most advanced in substitution of information technology for conventional inputs (outside of IT itself) is in warfare. There are very powerful incentives to develop autonomous and robotic activities because of the winner-take-all nature of military technologies and because the dangers of war make nations averse to risking lives.

The key word in the last paragraph is autonomous. The US Department of Defense defined these as "weapon systems that, once activated, can select and engage targets without further intervention by a human operator." This definition suggests the ability of such systems to assess the situational context on a battlefield and to decide on the required attack according to pre-programmed rules and battlefield information.

Some of the key developments in IT warfare are the following. Drone aircraft such as the Predator have the capability to identify targets and fire missiles. Daksh is a battery-operated remote-controlled robot on wheels that can recover and defuse bombs. Guardium is a small Israeli tank-like surveillance vehicle that operates completely autonomously to guard the Gaza border. PackBots are a series of small robots used to identify bombs, collect air samples in hazardous sites, and sniff for explosives. SWORDS is a small American tank-like vehicle that is remote controlled at this time. The Samsung SGR-A1 is a South Korean military robot sentry, armed with sensors and a machine gun, that can operate autonomously and is designed to replace human counterparts in the demilitarized zone at the South and North Korea border. More advanced versions of these are under development. It is possible to envision that a rogue nation will develop genetically engineered super-humans to fight alongside robots.

While the automation of warfare is only in its infancy, we can examine the impact to date. The share of compensation in total output for US defense spending has risen slightly over the last two decades, so on that test the accelerationist hypothesis is not supported. Battle deaths in recent wars (in Iraq and Afghanistan) are down sharply from earlier wars (Vietnam and Korea), and this is undoubtedly in part due to better information and smart weapons. The success of cyberweapons is (as far as we can tell from public sources) minimal, perhaps setting back Iran's nuclear program by a year or so. So the bottom line on the role of IT in military technologies is that it has not moved substantially toward replacing human labor.

f. The complication of evil agents

The discussion up to now has ignored one major specter haunting information technology – the presence of increasingly powerful and dangerous hacking, cybercrime, and evil agents in cyberspace. The parallel here is to the game-theoretic dynamics of the development of more powerful weaponry in warfare. Even though the innovators (of bows and arrows, machine guns, tanks, and nuclear weapons) have an initial advantage over their adversaries, the advantage is temporary. Even the most closely held technological secret slowly diffuses around the world.

We must therefore assume that those who develop the engines of superintelligence will eventually find they are sharing them with evil agents – with their military, commercial, and political adversaries. And as we have learned through the Snowden leaks, our own governments are likely to be in the vanguard of use and potential abuse of advancing computational powers. The issues concerning the ethics and law of armed conflict with autonomous agents (discussed in the last section) have been extensively debated (see for example Singer 2009).

A further complication is this: The development of superintelligence raises a new concern not contemplated before in the development of political and military spying and weapons. We must be concerned that to the list of adversaries will be added the superintelligent machines themselves. If we are to take seriously Good's description above of superintelligence, we must consider that superintelligent machines will develop their own ethical systems, laws, sanctions, and governance.

If we consider how much moral, legal, and economic systems have evolved over the last millennium at the slow pace of human thinking, then we would have to believe that superintelligent reasoning would evolve many times more rapidly once it began to tackle the thorny issues that human struggle with. Just as theologians worry whether a powerful God is just by our primitive human standards, we should also worry about whether superintelligent machines will be just – or more accurately, whether their sense of justice will resemble our own. If they become irritated with humans, what will they do? Will the superintelligent treat us as flies to wonton boys?

So the point here is that the approaching Singularity is not one of unambiguous economic and social improvement. This was appreciated by nuclear weapons developer John von Neumann (1955):

> Useful and harmful techniques lie everywhere so close together that it is never possible to separate the lions from the lambs. This is known to all who have so laboriously tried to separate secret, classified science or technology (military) from the open kind; success is never more nor intended to be more than transient, lasting perhaps half a decade. Similarly, a separation into useful and harmful subjects in any technological sphere would probably diffuse into nothing in a decade.

XI. Concluding Comments on Singularity

So the conclusion as of today is that "the Singularity is not near." This conclusion is based on several tests that place the theory of the Singularity within the context of economic growth theory. Much of the computer science literature on the Singularity examines the growth in specific sectors or processes (such as flops or storage), but the economic perspective insists that the growth must be weighted by the economic valuation of the good or service.

The major insight of economics is to emphasize the heterogeneity of both inputs and outputs of the economic system. It is surely true that technological change in production of raw computation has been phenomenal over the last century. We can process information at a speed that is millions of billions times faster and cheaper than was possible for the fastest lightning calculators of the nineteenth century.

Suppose that trend continues indefinitely, including the ability to devise ever more ingenious software and artificial intelligence (AI). For increasing capabilities of computers to lead to the Singularity would require that AI could encompass all human activities, not just add numbers, solve equations, play chess, and interpret speech; but also lay hands on patients, read bedtime stories to children, and change flat tires.

Whereas computerized AI might do many routine tasks, the non-routine tasks are less easily programmed, and they evolve over time in response to the economic environment, including the environment of artificial intelligence itself. Particularly if we view the world with potential superintelligence as a competition between humans and machines, then we definitely would need a team of humans to consider how to protect humans from machines. We routinely spend 5% of output on defense, and this might rise to a much larger number when faced with a more powerful enemies like superintelligent machines. So one occupation at least would survive into the Era of Singularity.

Whether other sectors and tasks would be immune to the rise of superintelligence is an open question. The empirical question is the degree of substitutability between information and human labor. Given the complexity of both humans and jobs, it is unlikely that the question can be decided a priori. The analysis above indicates that information and computers will come to dominate the economy only if the information inputs or outputs take a rising share of consumption or inputs. This requires that the expenditure shares or input cost shares of information rise over time, which in turn requires that the volume of expenditures or inputs rise more rapidly than the relative prices fall. We can call these the “substitution tests” to be concise.

There are six tests on the supply side and one test on the demand side. The conclusions from the empirical tests proposed here is that the substitution tests fail for five of seven tests and succeed for two of the five tests. However, the growth trajectories of the variables which pass the test (the share of capital in total income and the share of informational capital in total capital) are relatively slow. Projecting the trends of the last decade or more, it would be in the order of a century before these variables would reach the level associated with the growth Singularity.

The conclusion is therefore that the growth Singularity is not near. However, this conclusion is tentative and is based on economic trends to date. Those who are concerned about the coming Singularity can use these tests on an ongoing basis to test whether the trends are changing in a favorable or unfavorable direction.

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