SOME NOTES ON THE "TRANSFORMATION PROBLEM"[1]

THE debate initiated by Böhm-Bawerk on the alleged " great contradiction " between Volume I and Volume III of Marx's *Capital* has by no means been resolved to the satisfaction of all parties. In one form or another, and with various degrees of sophistication, a number of aspects of the question continue to be hotly disputed to-day. In particular, literature on the so-called " transformation problem " has multiplied considerably since Paul Sweezy drew the attention of English-speaking readers to it in 1946 in his *Theory of Capitalist Development*.[2]

The present article sets out to do three things. First, it examines Marx's own discussion of the transformation of " values " into " prices of production," dealing in particular with the meaning which ought properly to be ascribed to his famous statement that " total values equal total prices of production." Second, it reviews two solutions of the " transformation problem " which have recently been put forward, and suggests an alternative method of solution which (it is submitted) illustrates more effectively than the others the essential point which Marx was trying to make. Third, it says something about an important gap in Marx's argument which still remains after the " transformation problem " has been solved.

" Profit ", wrote Marx, " is . . . that disguise of surplus-value which must be removed before the real nature of surplus-value can be discovered. In the surplus-value, the relation between capital and labour is laid bare." [3] In Volume I of *Capital*, therefore, Marx presents us with an analysis of surplus value stripped of its disguise. In this first stage of his argument the surplus value produced in each branch of industry is assumed to accrue to the capitalists *in that branch* in the form of a net gain. Now, since the only possible source of this surplus value, according to Marx's account, is the surplus labour performed by the labourers actually employed on the job, it follows that the ratio of net gain to capital must be unequal in cases where

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[2] See Sweezy, *The Theory of Capitalist Development* (London, 1946), pp. 109 ff.; Ladislaus von Bortkiewicz, *Value and Price in the Marxian System* (reprinted in *International Economic Papers*, no. 2, 1952); Bortkiewicz, *On the Correction of Marx's Fundamental Theoretical Construction in the Third Volume of " Capital "* (reprinted as an appendix to Sweezy's edn. of Böhm-Bawerk's *Karl Marx and the Close of his System* and Hilferding's *Böhm-Bawerk's Criticism of Marx* (New York, 1949)); J. Winternitz, " Values and Prices : A Solution of the So-called Transformation Problem " (ECONOMIC JOURNAL, June 1948, p. 276); K. May, " Value and Price of Production : A Note on Winternitz's Solution " (ECONOMIC JOURNAL, December 1948, p. 596); Joan Robinson, in a review in ECONOMIC JOURNAL, June 1950, p. 358; Rudolf Schlesinger, *Marx : His Time and Ours* (London, 1950), pp. 139 ff.; and M. H. Dobb, *A Note on the Transformation Problem*, in *On Economic Theory and Socialism* (London, 1955), p. 273.

[3] *Capital*, Vol. III (Kerr edn.), p. 62.

the organic composition [1] of the capitals concerned is unequal.[2] In actual fact, however, the rates of profit in the different branches tend towards equality under developed capitalism, and the organic compositions of capital tend if anything towards greater inequality. It is evidently necessary, there-fore, that the Volume I assumptions should be removed at a later stage in the proceedings, and the effect of their removal upon the Volume I con-clusions duly examined.

The assumptions are removed in Parts 1 and 2 of Volume III, where the question of the relations between surplus value and profit is considered. In actual fact, Marx argues, the amount of profit which the capitalists in each branch of industry receive must be sufficient to yield them the average rate of profit on the *total* quantity of capital which they employ, so that in the majority of cases the amount of profit they receive will differ from the amount of surplus value actually generated in their own branch of industry. But this does not mean that the Volume I analysis is vitiated. On the contrary, Marx believed that without this analysis political economy would be left " without a rational basis." [3] For, according to him, the profit which the capitalists in each branch of industry receive must be conceived as accruing to them by virtue of a sort of redivision of the aggregate surplus value produced over the economy as a whole. This aggregate surplus value is, as it were, reallocated among the different branches of industry so that the capitalists in each branch share in it not in accordance with the amount of capital they have spent on wages but in accordance with the total amounts of capital they have severally employed. Without the Volume I analysis to determine the magnitude of this aggregate, Marx maintained, the average rate of profit would be, as he put it, " an average of *nothing*." [4]

In his analysis of surplus value in Volume I, Marx had assumed that the commodities in which the capitalist producers dealt were bought and sold " at their values " in the Marxian sense—*i.e.*, at equilibrium prices which were proportionate to the quantities of socially necessary simple labour required to produce them. So long as it is taken for granted that the net gain received by the capitalists in each branch of industry consists of the surplus value generated in that branch, this is a reasonable enough assump-tion. But the conversion of surplus value into average profit necessarily implies the transformation of values into what Marx called " prices of production." [5] It implies, in other words, that the majority of commodities do *not* tend to sell " at their values," but at " prices of production " which

[1] The organic composition of capital is the ratio between the part of capital spent on equip-ment, raw materials, etc., which Marx calls *constant* capital (c), and the part spent on wages, which he calls *variable* capital (v).

[2] Marx assumes here that the ratio of surplus value (s) to v is the same in all branches of industry.

[3] *Capital*, Vol. III, pp. 176-7.

[4] *Theories of Surplus Value (Selections)* (London, 1951), p. 231.

[5] " The price of production ", wrote Marx, " includes the average profit. We call it price of production. It is, as a matter of fact, the same thing which Adam Smith calls *natural price*, Ricardo *price of production*, or *cost of production*, and the physiocrats *prix nécessaire*, because it is in the long run a prerequisite of supply, of the reproduction of commodities in every individual sphere " (*Capital*, Vol. III, p. 233).

normally diverge to some extent from their values. The question immediately arises, therefore, whether Marx's Volume III analysis of exchange ratios in terms of prices of production can be regarded merely as a modification of his Volume I analysis in terms of values (as Marx himself argued), or whether it should be regarded as being in contradiction to it (as Böhm-Bawerk and his followers have insisted).

The basic point in Marx's answer to this question is as follows. The transformation of values into prices is brought about as a result of the conversion of surplus value into profit. Now the volume and rate of surplus value [1] are evidently determined by the ratio $\frac{\Sigma a}{\Sigma v}$ (where a is the total value, in the Marxist sense, of a given finished commodity); and the volume and rate of profit are determined by the ratio $\frac{\Sigma a_p}{\Sigma v_p}$ (where the subscript p indicates that a and v have been transformed from values into prices).[2] Marx argues, in effect, that $\frac{\Sigma a}{\Sigma v} = \frac{\Sigma a_p}{\Sigma v_p}$. (This, as we shall see, was what Marx had in mind when he said that " total values equal total prices." [3]) In other words, he argues that the ratio between the value of commodities in general and the value of the commodity labour power, upon which he had in Volume I conceived surplus value to depend,[4] remains unchanged when it is expressed in terms of prices rather than values, so that profit can be said to be determined in accordance with the Volume I analysis. If this is so, it can

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[1] I am using the expression " rate of surplus value " here to mean the ratio of surplus value to *total* capital. Marx normally used it to mean the ratio of surplus value to *variable* capital.

[2] It is, of course, assumed here that the national income resolves itself only into wages and profits.

[3] Cf. M. H. Dobb, *Political Economy and Capitalism*, pp. 46 and 72–3.

[4] Marx starts in Vol. I with the fundamental exploitation ratio $\frac{s}{v}$ $\left(= \frac{\text{Surplus labour}}{\text{Necessary labour}} \right)$; Adding unity to this ratio, we get $\frac{v + s}{v} = \frac{\text{Working day}}{\text{Necessary labour}}$. When the latter expression is applied to the totality of commodities, it becomes $\frac{\Sigma(v + s)}{\Sigma v}$ $\left(= \frac{\text{Total labour force}}{\text{Labour required to produce wage-goods}} \right)$: and, given conditions of equilibrium between the different branches of the economy, this ratio $\frac{\text{Total labour force}}{\text{Labour required to produce wage-goods}}$ is equal to the ratio $\frac{\text{Value of finished commodities}}{\text{Value of wage-goods}}$ $\left(= \frac{\Sigma a}{\Sigma v} \right)$. For example, in the following case Department I produces means of production and Department II consumers' goods; the ratio $\frac{s}{v}$ is the same for both Departments; and the equilibrium conditions appropriate to simple reproduction prevail between them (*i.e.*, $c_2 = v_1 + s_1$):

	c_1	v_1	s_1	a_1
I.	80	60	40	180

	c_2	v_2	s_2	a_2
II.	100	90	60	250

It will be seen that the ratios $\frac{\text{Working day}}{\text{Necessary labour}}$ $\left(= \frac{5}{3} \right)$,

$\frac{\text{Total labour force}}{\text{Labour required to produce wage-goods}}$ $\left(= \frac{250}{150} \right)$, and

$\frac{\text{Value of finished commodities}}{\text{Value of wage-goods}}$ $\left(= \frac{250}{150} \right)$ are all equal.

be plausibly argued that the very degree to which individual prices of pro-
duction diverge from values is ultimately determined according to the
Volume I analysis.   Thus the disturbance introduced into the operation
of the law of value as described in Volume I is a *calculable* disturbance, and
" in the exact sciences it is not the custom to regard a calculable disturbance
as a refutation of a certain law." [1]

Marx's discussion of this problem is developed in two stages, the first of
which has received much more attention than the second.   In the first stage
he takes " five different spheres of production," deliberately assuming that
none of the commodities concerned enters into the production of any of the
others.   Thus capitals I to V in the accompanying table [2] can be considered

(1) Capitals.	(2) Used-up c.	(3) Cost price.	(4) Surplus value.	(5) Value.	(6) Profit.	(7) Price of produc- tion.	(8) Deviation of price from value.
I. 80c + 20v	50	70	20	90	22	92	+ 2
II. 70c + 30v	51	81	30	111	22	103	− 8
III. 60c + 40v	51	91	40	131	22	113	−18
IV. 85c + 15v	40	55	15	70	22	77	+ 7
V. 95c + 5v	10	15	5	20	22	37	+17
			110	422	110	422	

as the component parts of one single capital of 500.   Each of the constituent
capitals shown in column 1 totals 100, but the cost price of each of the out-
puts is less than 100, since it is assumed that only a portion of the value of
the constant capital is transferred to the commodity in the period we are
considering.[3]   The amount so transferred is shown in column 2, and the
cost price, which is the sum of v and used-up c, is shown in column 3.   It is
assumed that the working day is everywhere equally divided between neces-
sary and surplus labour, so that surplus value (shown in column 4) is equal
to v.   The total *value* of each of the outputs being considered (shown in
column 5) represents the sum of the cost price and the surplus value.   Now
it is evident that the sale of these commodities at their values would result in
very unequal rates of profit on each of the capitals.   In actual fact, however,
Marx maintains, the total pool of surplus value, amounting to 110, is allotted
(" by means of competition " [4]) to the individual capitals in accordance
with the total size of each—in this case uniformly, so that each receives a
profit of 22 (column 6).   The " price of production " (column 7), then, at
which each output actually tends to sell, is the sum of the cost price and the
profit, and differs in each case from the value.   But since the total profit is
by definition equal to the total surplus value, it naturally follows that in the
present case the sum of the prices of production is equal to the sum of the

[1] P. Fireman, quoted by Engels in his preface to Volume III of *Capital*, p. 25.
[2] This table is an amalgamation of those on pp. 183 and 185 of *Capital*, Vol. III, with some of
the figures re-arranged.
[3] The turnover periods of v are assumed to be the same in each case.
[4] *Capital*, Vol. III, p. 186.

values, or, to put the same thing in another way, that the deviations of prices from values (column 8) cancel one another out.[1]

Marx's statement that the sum of the prices is equal to the sum of the values has come in for considerable criticism. From Böhm-Bawerk onwards, critics have questioned whether this statement can be held to be meaningful, whether it embodies a tautology and so on, and have generally concluded that Marx's " argument " is quite untenable. Some of the difficulty no doubt arises from the fact that Marx, having illustrated this equality arithmetically in the particular case just described (the case where mutual interdependence is abstracted from), immediately went on, rather rashly perhaps, to say that " in the same way the sum of all the prices of production of all commodities in society, comprising the totality of all lines of production, is equal to the sum of all their values." [2] The implication of this statement, read in its context, might seem to be that when the assumption that none of the commodities concerned enters into the production of any of the others is dropped, so that the values of input as well as those of output have to be transformed into prices of production, a transformation carried out on the basis of a redistribution of the pool of surplus value will bring out total prices equal to total values in the arithmetical sense. This is in fact not so. On any plausible set of assumptions regarding the manner in which the different branches of the economy are inter-related, it will soon be found upon experimenting with various sets of figures that if the values of input as well as those of output are to be transformed into prices of production, it is normally impossible to effect a simultaneous transformation which will make total profit equal to total surplus value and at the same time make total prices of production equal to total values. In all but very exceptional cases, we may preserve one of these equalities, but not both.[3] If Marx's attention had been drawn to this fact, he might well have reformulated some of his expressions regarding the equality of total prices and total values, while still insisting on the essential point they were designed to express—viz., that after the transformation of values into prices of production the fundamental ratio upon which profit depended [4] could still be said to be determined in accordance

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[1] It is evident that the only case in which price and value would coincide would be one in which the composition of the capital concerned coincided with the " social average."

[2] *Capital*, Vol. III, p. 188.

[3] For an example of one of these exceptional cases, see the transformation exhibited in Tables II and IIIb on pp. 111 and 120 of Sweezy's *Theory of Capitalist Development*.

[4] There is a slight technical difficulty here. When Marx said that " total values equal total prices " it is fairly clear that what he had in mind was the equality of the ratios $\frac{\Sigma a}{\Sigma v}$ and $\frac{\Sigma a_p}{\Sigma v_p}$, each calculated over the economy as a whole. (Cf. Dobb, *loc. cit.*). Given conditions of equilibrium between the different Departments, these ratios will be equal to the basic exploitation ratio $\frac{\Sigma(v + s)}{\Sigma v}$. In the case we have just considered, however, where the information which we are given covers only a part of the economy, it is obvious that the numerical value of the ratio $\frac{\Sigma a}{\Sigma v}$ derived from this information alone (assuming that we are able to derive it at all) is likely to differ from the numerical value of $\frac{\Sigma a}{\Sigma v}$ which we could derive from complete information regarding the economy as a whole. (For example, if we assume that the table gives us complete information regarding

with the Volume I analysis. In the special case where none of the commodities concerned enters into the production of any of the others, he might have said, the ratio remains the same for the simple reason that the relevant quantities remain the same—the denominator remains the same by hypothesis, and the numerator remains the same because *in this case* the sum of the prices necessarily equals the sum of the values. In the more difficult case where the various branches of production are mutually interdependent, he might have said, the sum of the prices does not necessarily " come out " equal to the sum of the values, but the fundamental ratio can still be said to be determined in accordance with the Volume I analysis. And it would have been possible for him to illustrate this, as I shall show below, by an arithmetical example rather similar in character to that described above.

However, it would be wrong to suggest that Marx simply ignored this more difficult case. On the contrary, his examination of it, although by no means detailed, was sufficiently well organised to be said to constitute that second stage in his argument of which I have spoken above. He begins by dropping the assumption that none of the commodities concerned enters into the production of any of the others. In actual fact, he writes, " the elements of productive capital are, as a rule, bought on the market," so that " the price of production of one line of production passes, with the profit contained in it, over into the cost-price of another line of production." At first sight it might seem as if this would mean that the profit accruing to each capitalist might be counted several times in a calculation such as that which has just been described, but Marx has little difficulty in disposing of this superficial objection. The dropping of the assumption, however, does indeed make one " essential difference," which Marx describes as follows:

> " Aside from the fact that the price of a certain product, for instance the product of capital B, differs from its value, because the surplus-value realized in B may be greater or smaller than the profit of others contained in the product of B, the same fact applies also to those commodities which form the constant part of its capital, and which indirectly, as necessities of life for the labourers, form its variable part. So far as the constant part is concerned, it is itself equal to the cost-price plus surplus-

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the output of finished goods, but of no other branches of production, Σa will be the same but Σv will be under-estimated.) A similar sort of difficulty arises in the second stage of the argument (to be considered shortly), where we have full information concerning the economy, but where it is not desirable to postulate equilibrium conditions. In both these cases our calculation of $\frac{\Sigma a}{\Sigma v}$ from the information which we are given is likely to differ from that of the basic exploitation ratio $\frac{\Sigma(v + s)}{\Sigma v}$. In such cases, then, if we want to illustrate by an arithmetical example what Marx had in mind when he said that " total values equal total prices," the best we can do is to start with a ratio whose numerator is the sum of the total values of all the commodities (whether finished or otherwise) about which we are given information, and whose denominator is the sum of all the v's which we are given ; and then to show that the numerical value of this ratio remains the same when those values which the particular problem requires to be transformed into prices are so transformed. The numerical value of this ratio will not normally be identical with that of $\frac{\Sigma a}{\Sigma v}$ calculated for the economy as a whole, but it will express the same underlying idea. In what follows the symbol a will be used for the total value of any commodity, whether finished or not.

value, which now means cost-price plus profit, and this profit may again be greater or smaller than the surplus-value in whose place it stands. And so far as the variable capital is concerned, it is true that the average daily wage is equal to the values produced by the labourers in the time which they must work in order to produce their necessities of life. But this time is in its turn modified by the deviation of the prices of production of the necessities of life from their values. However, this always amounts in the end to saying that one commodity receives too little of the surplus-value while another receives too much, so that the deviations from the value shown by the prices of production mutually compensate one another. In short, under capitalist production, the general law of value enforces itself merely as the prevailing tendency, in a very complicated and approximate manner, as a never ascertainable average of ceaseless fluctuations." [1]

Marx returned to the same point a few pages later, emphasising that the transformation process involves a modification of the Volume I assumption that " the cost-price of a commodity is equal to the value of the commodities consumed in its production." The price of production of a given commodity, he writes—

" is its cost-price for the buyer, and this price may pass into other commodities and become an element of their prices. Since the price of production may vary from the value of a commodity, it follows that the cost-price of a commodity containing this price of production may also stand above or below that portion of its total value which is formed by the value of the means of production consumed by it. It is necessary to remember this modified significance of the cost-price, and to bear in mind that there is always the possibility of an error, if we assume that the cost-price of the commodities of any particular sphere is equal to the value of the means of production consumed by it. Our present analysis does not necessitate a closer examination of this point." [2]

And in a later passage, repeating the same point once more, Marx argues that " this possibility does not alter the correctness of the rules laid down for commodities of average composition." [3]

This is where the so-called " transformation problem " comes into the picture. Marx's " method of transforming values into prices," it is said, meaning by this his original calculation outlined in the table above, contains an " error," since it does not take account of the fact that the values of elements of input as well as those of elements of output have to be transformed into prices.[4] It is then claimed that Marx can be rescued from this " error " simply by showing the *formal possibility* of a consistent derivation of prices from values in the case of mutual interdependence. When values are

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[1] Quotations from *Capital*, Vol. III, pp. 188–90. There is a similar passage at the end of Marx's comments on Bailey in the *Theories of Surplus Value* (not included in the English edn.) which shows that the point had occurred to Marx several years before the publication of the first volume of *Capital*.

[2] *Capital*, Vol. III, pp. 194–5.

[3] *Ibid.*, Vol. III, pp. 241–3.

[4] As will be clear from what has been said above, it was not intended to take account of this fact, since mutual interdependence was specifically abstracted from.

transformed into prices, the ratio of price to value must be the same when a given commodity is considered as input as when it is considered as output; and after the transformation the rate of profit must come out equal in the case of each capital concerned. These ratios of price to value, and the rate of profit, are regarded as the main unknowns. The "transformation problem" then reduces itself to this: can the relations between the various branches of production, and the various conditions which are to be fulfilled as a result of the transformation, be expressed in the form of an equational system which is "determinate" in the mathematical sense—*i.e.*, in which the number of equations is equal to the number of unknowns? The assumption lying behind these researches is that if the relations and conditions can in fact be so expressed, Marx's "method of transforming values into prices" is itself transformed from an invalid to a valid one.

The best-known solution, that of Bortkiewicz, commences with the particular set of value relationships postulated by Marx as existing between the three main Departments of the economy (I = means of production; II = workers' consumption goods; III = capitalists' consumption goods) under conditions of simple reproduction. Employing the usual notation, these value relationships can be expressed in the form of three equations:

$$\text{I.} \quad c_1 + v_1 + s_1 = c_1 + c_2 + c_3$$
$$\text{II.} \quad c_2 + v_2 + s_2 = v_1 + v_2 + v_3$$
$$\text{III.} \quad c_3 + v_3 + s_3 = s_1 + s_2 + s_3$$

If we take the ratio of price to value to be x in the case of means of production, y in the case of workers' consumption goods and z in the case of capitalists' consumption goods; if we further call the average rate of profit r; and if we state as a condition of the problem that the relations appropriate to simple reproduction should continue to obtain after the transformation of values into prices as before it, then the following equalities must hold:

$$\text{I.} \quad c_1x + v_1y + r(c_1x + v_1y) = (c_1 + c_2 + c_3)x$$
$$\text{II.} \quad c_2x + v_2y + r(c_2x + v_2y) = (v_1 + v_2 + v_3)y$$
$$\text{III.} \quad c_3x + v_3y + r(c_3x + v_3y) = (s_1 + s_2 + s_3)z$$

Here there are four unknowns (x, y, z and r), and only three equations. Bortkiewicz reduces the unknowns to three by the ingenious expedient of assuming: (*a*) that the value scheme was expressed in terms of money, and (*b*) that gold is the money commodity, and is produced in Department III, in which case z may reasonably be taken as $= 1$. The equational system thereupon becomes determinate, and solutions for x, y and r can be fairly readily derived. Upon applying these solutions to various sets of figures, it is seen that total profit comes out equal to total surplus value, but that total prices normally diverge from total values. Neither the equality nor the inequality, however, has anything more than formal significance. As Bortkiewicz says, in relation to a particular set of figures,

> "That the total price exceeds the total value arises from the fact that Department III, from which the good serving as value and price

measure is taken, has a relatively low organic composition of capital. But the fact that total profit is numerically identical with total surplus value is a consequence of the fact that the good used as value and price measure belongs to Department III." [1]

It is only in the special case where the organic composition of the capital employed in Department III is equal to the social average that the sum of the prices will come out equal to the sum of the values.

Winternitz adopts the same general attitude towards the problem as Bortkiewicz, but clears the Bortkiewicz solution of certain redundancies and unnecessary artificialities. He commences with the usual value scheme in the three Departments:

$$\text{I.} \quad c_1 + v_1 + s_1 = a_1$$
$$\text{II.} \quad c_2 + v_2 + s_2 = a_2$$
$$\text{III.} \quad c_3 + v_3 + s_3 = a_3$$

But instead of assuming the equilibrium conditions appropriate to Marx's reproduction schemes, he assumes merely that when a_1 varies by x (the price–value ratio for means of production), then c_1, c_2 and c_3 also vary by x; and that when a_2 varies by y (the price–value ratio for workers' consumption goods), then v_1, v_2 and v_3 also vary by y. Thus he arrives at the following simple equational system:

$$\text{I.} \quad c_1 x + v_1 y + S_1 = a_1 x$$
$$\text{II.} \quad c_2 x + v_2 y + S_2 = a_2 y$$
$$\text{III.} \quad c_3 x + v_3 y + S_3 = a_3 z$$

By putting $\dfrac{a_1 x}{c_1 x + v_1 y} = \dfrac{a_2 y}{c_2 x + v_2 y}$ (each of these expressions being equal to $1 + r$), solutions for $x : y$ and for r are easily obtained. A further set of relationships between x, y and z must then be postulated in order to determine the price level for the system as a whole. From a purely logical point of view, it obviously does not matter what relationships are postulated, but Winternitz puts

$$a_1 x + a_2 y + a_3 z = a_1 + a_2 + a_3$$

(*i.e.*, sum of prices = sum of values) because in his opinion this is " the obvious proposition in the spirit of the Marxian system." [2] Solutions for x, y and z are then yielded immediately without any special difficulty. When applied to various sets of figures, these solutions naturally bring out the sum of prices equal to the sum of values, but total profit normally diverges from total surplus value.

Winternitz's solution, although in essence very similar to Bortkiewicz's, is evidently simpler and therefore more acceptable from a purely mathematical point of view. Indeed, it is the special merit of Winternitz to have exposed the triviality of the whole problem as so posed—a triviality which tended to be hidden by Bortkiewicz's over-elaborate and confusing method.

[1] Bortkiewicz, in Sweezy's edn. of Böhm-Bawerk's *Karl Marx and the Close of his System*, etc., p. 205.

[2] ECONOMIC JOURNAL, June 1948, p. 279.

The Winternitz solution is an effective reply to those who said that it was not formally possible to transform values into prices when elements of input as well as output were involved.   But it seems to me that something more is required before a transformation of the Bortkiewicz–Winternitz type can properly be used to *illustrate* the second stage of Marx's Volume III argument.[1]   The essential point for Marx, as we have seen, was that after aggregate surplus value had been converted into profit, and values consequently transformed into prices, the ratio $\frac{\Sigma a}{\Sigma v}$ should be equal to the ratio $\frac{\Sigma a_p}{\Sigma v_p}$.   Is it possible to effect a transformation which brings these ratios out equal, and if so under what conditions?

This problem can be dealt with as follows.   Select three sets of quantities for c, v and s in Departments I, II and III, such that the rate of surplus value in the Marxist sense $\left(\frac{s}{v}\right)$ is equal in each case, and that the organic composition of capital in Department II is equal to the social average [2]— for example:

$$\begin{array}{ccccccc} & c_1 & + & v_1 & + & s_1 & = a_1 \\ \text{I.} & 3 & + & 4 & + & 4 & = 11 \end{array}$$

$$\begin{array}{ccccccc} & c_2 & + & v_2 & + & s_2 & = a_2 \\ \text{II.} & 18 & + & 15 & + & 15 & = 48 \end{array}$$

$$\begin{array}{ccccccc} & c_3 & + & v_3 & + & s_3 & = a_3 \\ \text{III.} & 9 & + & 6 & + & 6 & = 21 \end{array}$$

Proceed now to transform these expressions into the following:

$$\begin{array}{ll} \text{I.} & c_1x + v_1y + S_1 = a_1x \\ \text{II.} & c_2x + v_2y + S_2 = a_2y \\ \text{III.} & c_3x + v_3y + S_3 = a_3z \end{array}$$

on the basis of the following equalities: [3]

$$\frac{S_1}{c_1x + v_1y} = \frac{S_2}{c_2x + v_2y} = \frac{S_3}{c_3x + v_3y}$$

and

$$S_1 + S_2 + S_3 = s_1 + s_2 + s_3$$

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[1] The Bortkiewicz–Winternitz solutions can certainly be used to *prove* the proposition that a consistent transformation of values into prices is formally possible, but they cannot be used to do any more than *illustrate* Marx's own argument.   To suggest that any argument in *Capital* stands or falls by Marx's arithmetical illustrations is to betray a serious misunderstanding of his method, and it would be equally wrong-headed, I think, to set out to " rescue " Marx from his errors with the aid of mathematical formulæ.   As May has said (*op. cit.*, p. 598): " Marx . . . used calculations primarily as illustrations to accompany verbal arguments which combined process and cross-section analysis in a way which could hardly be fitted to the mathematical techniques available even to-day."

[2] *I.e.*, that $\dfrac{c_2}{c_2 + v_2} = \dfrac{\Sigma c}{\Sigma c + \Sigma v}$.

[3] These two equalities express, of course, the equality of profit rates, and the equality of the sum of profits with the sum of surplus values.

The result of this calculation in the given case is as follows:

$$\begin{array}{ccccc} & c_1 x & v_1 y & S_1 & a_1 x \\ \text{I.} & 2\cdot592 + & 3\cdot710 + & 3\cdot202 = & 9\cdot504 \end{array}$$

$$\begin{array}{ccccc} & c_2 x & v_2 y & S_2 & a_2 y \\ \text{II.} & 15\cdot552 + & 13\cdot911 + & 15\cdot052 = & 44\cdot515 \end{array}$$

$$\begin{array}{ccccc} & c_3 x & v_3 y & S_3 & a_3 z \\ \text{III.} & 7\cdot776 + & 5\cdot564 + & 6\cdot784 = & 20\cdot124 \end{array}$$

This calculation, like Marx's original one in the case where mutual interdependence was abstracted from, shows the result when a fixed aggregate of surplus value is re-allocated in the form of profit at the average rate among the various capitals concerned. The sum of prices diverges from the sum of values, but the real point to which Marx wished to draw attention when he emphasised the equality between total prices and total values in the original case—*i.e.*, that after the transformation of values into prices the fundamental ratio upon which profit depended [1] could still be said to be determined in accordance with the Volume I analysis—is illustrated in this case too. It is no longer true that the numerator and the denominator of the ratio remain unchanged as a result of the transformation, but under the assumed conditions *both will always change in the same proportion*, so that $\dfrac{a_1 x + a_2 y + a_3 z}{v_1 y + v_2 y + v_3 y}$ remains equal to $\dfrac{a_1 + a_2 + a_3}{v_1 + v_2 + v_3}$. The achievement of this result is dependent (in the great majority of cases) upon the equality initially postulated between $\dfrac{c_2}{c_2 + v_2}$ and $\dfrac{\Sigma c}{\Sigma c + \Sigma v}$—*i.e.*, upon the assumption that the organic composition of capital in the wage-goods industries is equal to the social average.[2, 3]

Such an illustration, however, would fill only part of the gap in Marx's analysis. To fill the rest of it, one must turn to economic history rather than to mathematics. The " derivation of prices from values," according to Marx's general economic method,[4] must be regarded as a historical as well

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[1] See footnote 4 above, p. 98.

[2] I am indebted to Dr. Wyllie for a mathematical proof both of this general result and of its dependence in normal cases upon the condition $\dfrac{c_2}{c_2 + v_2} = \dfrac{\Sigma c}{\Sigma c + \Sigma v}$. While the result will always be reached when this condition is satisfied, there may be a few special cases in which it could be reached without the condition being satisfied.

[3] In Marx's arithmetical illustration to the first stage of his argument, the conditions laid down do not require that the values of the elements of input should be transformed into prices. It is possible to re-interpret his figures, however, so that they illustrate a situation in which the values of v (but not of c), as well as those of a, have to be transformed into prices, and in which the organic composition of capital in the wage-goods industries is equal to the social average, so that v is the same whether expressed in price or in value terms (*i.e.*, that $y = 1$). In the present case, where c, v and a have all to be transformed into prices, the fact that the organic composition of capital in the wage-goods industries is equal to the social average no longer necessarily means that $y = 1$.

[4] The best short description of Marx's general economic method is that given by Engels in a review of Marx's *Critique of Political Economy* which appears as an appendix in the English edn. of Engels's *Ludwig Feuerbach*, pp. 98–101.

as a logical process.   In " deriving prices from values " we are really repro-
ducing in our minds, in logical and simplified form, a process which has
actually happened in history.   Marx began with the assumption that goods
sold " at their values " under capitalism (so that profit rates in the various
branches of production were often very different), not only because this
appeared to be the proper starting-point from the logical point of view but
also because he believed that it had " originally " [1] been so.   He proceeded
on this basis to transform values into prices, not only because this course
appeared to be logically necessary but also because he believed that history
itself had effected such a transformation.   The exchange of commodities
at their values, or approximately at their values, Marx wrote—

> " requires . . . a much lower stage than their exchange at their prices
> of production, which requires a relatively high development of capitalist
> production. . . .
>   " Aside from the fact that prices and their movements are dominated
> by the law of value, it is quite appropriate, under these circumstances,
> to regard the value of commodities not only theoretically, but also
> historically, as existing prior to the prices of production.   This applies
> to conditions, in which the labourer owns his means of production, and
> this is the condition of the land-owning farmer and of the craftsman in
> the old world as the new.   This agrees also with the view formerly
> expressed by me that the development of product into commodities
> arises through the exchange between different communes, not through
> that between the members of the same commune.   It applies not only
> to this primitive condition, but also to subsequent conditions based on
> slavery or serfdom, and to the guild organisation of handicrafts, so long
> as the means of production installed in one line of production cannot be
> transferred to another line except under difficulties, so that the various
> lines of production maintain, to a certain degree, the same mutual rela-
> tions as foreign countries or communistic groups." [2]

But Marx did not pursue the historical aspects of the problem of the trans-
formation of values into prices very much further than this, and his critics
have taken full advantage of the fact that a number of problems still remain
unsolved.[3]   Böhm-Bawerk, for example, argued that if the derivation of
prices from values had in fact proceeded in the manner which Marx's analysis
suggests, " there must be traces of the actual fact that *before* the equalization
of the rates of profit the branches of production with the relatively greater
amounts of constant capital have won and do win the smallest rates of profit,

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[1] *Capital*, Vol. III, p. 186.
[2] *Ibid.*, Vol. III, pp. 208-9.   Cf. p. 212 : " Competition first brings about, in a certain individual
sphere, the establishment of an equal market-value and market-price by averaging the various
individual values of the commodities.   The competition of the capitals in the different spheres then
results in the price of production which equalises the rates of profit between the different spheres.
This last process requires a higher development of capitalist production than the previous process."   (My italics.)
Cf. also pp. 207-8.
[3] Engels, referring to the passage just quoted, said that " if Marx had had an opportunity to
work over the third volume once more, he would doubtless have extended this passage considerably.
As it stands it gives only the sketchy outline of what is to be said on the point in question " (*Engels
on " Capital,"* p. 102).

while those branches with the smaller amounts of constant capital win the largest rates of profit "; and he went on to assert (following Sombart) that there are in fact " no traces of this to be found anywhere, either in the historical past or in the present." [1]   Engels attempted to deal with this and other related problems in his important " Supplement " to Volume III of *Capital*,[2] giving a suggestive account of the manner in which, in the formative years of the development of capitalism, the prices of commodities were adjusted above or below their values in order to bring the surplus value into equality with the average rate of profit.   This essay is certainly the most ambitious attempt to bridge the gap in Marx's argument which we possess.   But even this is really little more than a preliminary sketch, and many details still remain to be filled in.

It is, of course, quite open to the Marxist, if he wishes, to by-pass this question by characterising the view expressed by Marx in the passage just quoted as a sort of " Robinsonade."   Marx, it might be argued, was really doing little more than take over the traditional Classical idea that exchange ratios were proportional to embodied labour ratios only in that " early and rude state of society " of which Adam Smith spoke.   Such a characterisation would not affect the utility of the labour theory of value as a tool for the analysis of *capitalist* society—given a satisfactory solution to the logical problem of the transformation of values into prices; but on the other hand it does not seem likely that Marx himself would have been prepared to accept it.   Marx and Engels always insisted very strongly that the logical method of treatment which they adopted in their work on political economy was " nothing else than the historical method, only divested of its historical form and disturbing fortuities."   The chain of thought, said Engels,

> " must begin with the same thing that this history begins with and its further course will be nothing but the mirror-image of the historical course in abstract and theoretically consistent form, a corrected mirror-image but corrected according to laws furnished by the real course of history itself, in that each factor can be considered at its ripest point of development, in its classic form." [3]

Given this approach, it seems probable that Marx would have continued to take the view that his *logical* transformation of values into prices was the " corrected-image " of some actual *historical* transformation.

Engels, in the " Supplement " referred to above, tried to solve the problem by suggesting that up to the time when the capitalist form of production came upon the scene, commodity prices in actual fact normally tended to " gravitate towards the values fixed by the Marxian law and oscillate around these values." [4]   This suggestion does not on the whole seem very plausible, for fairly obvious reasons connected with the prevalence

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[1] *Karl Marx and the Close of his System* (Sweezy's edn.), p. 49.   See also Hilferding's reply on pp. 169–72.

[2] Reprinted in *Engels on " Capital*," pp. 94 ff.

[3] Engels, appendix to *Ludwig Feuerbach*, p. 99.

[4] *Engels on " Capital*," p. 106.

of various forms of monopoly, the low degree of factor mobility, etc., in most pre-capitalist societies. Fortunately, however, it is not necessary to follow Engels all the way in this matter: it is quite sufficient to show that history has in fact effected a transformation of one type of *supply price* into another.[1] Broadly speaking, there are two main types of supply price to be found in the history of commodity exchange—first, that of the producer who thinks of his net receipts as a reward for his labour, and, second, that of the producer who thinks of his net receipts as a profit on his capital. What Marx did, in effect, was to assume that the first type of supply price was characteristic of all pre-capitalist forms of society (abstracting here from those specific features differentiating pre-capitalist societies from one another which in other contexts he was especially concerned to emphasise), and to concentrate on the task of showing how the coming of capitalism, with its conversion of labour power into a commodity, accomplished the actual transformation of the first type of supply price into the second.[2] This, I think, is the historical transformation of which the logical transformation considered above must be regarded as the counterpart.

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[1] The labour theory of value, like all cost theories, approaches the value problem via the supply price, and can afford a determinate explanation of actual prices only in so far as these are equal to or tend towards supply prices.

[2] See on this point *Capital*, Vol. I (Allen & Unwin edn.), pp. 148–9.