THE NONSWITCHING THEOREM IS FALSE

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We wish to make clear for the record that the nonswitching theorem associated with us is definitely false. We are grateful to Dr. Pasinetti for first giving examples that raise legitimate doubts about the theorem's truth, and for discussion of flaws in the purported proof of the theorem and of the bearing for general economics of the reswitching phenomenon. And we are grateful to Professor Morishima, Professor Garegnani and Mr. Sheshinski for independent counterexamples that settle this matter definitively.

Some comments on the theorem are offered to minimize misunderstanding.

1. The February 1965 paper contained two parts: a nonsubstitution theorem and an alleged nonswitching theorem. The falsity of the latter in no way affects the validity of the former.[1]

2. This paper took for granted the possibility of a decomposable system's reswitching back at a low interest rate to the technique that had been competitively viable at a much higher interest rate. Ruth Cohen, Joan Robinson, and Piero Sraffa had established this without question.

What the 1965 paper attempted to show — we now agree, wrongly — was that this could not happen in an "indecomposable"

1. Morishima is correct in saying that the nonsubstitution theorem is valid without regard to the "indecomposability" assumptions of the February 1965 paper. Only because it appeared in the same paper with the nonswitching theorem, which was thought to be related to indecomposability, was the exposition so worded. For early recognition of the nonsubstitution theorem, see M. Morishima, *Equilibrium, Stability and Growth* (Oxford: Clarendon Press, 1964), p. 97, Theorem 1; "A Dynamic Analysis of Structural Change in a Leontief Model," *Economica*, XXV (May 1958), 120; "Prices, Interest, and Profits in a Dynamic Leontief System," *Econometrica*, Vol. 26 (July 1958), pp. 358–80; "Some Properties of a Dynamic Leontief System with a Spectrum of Techniques," *Econometrica*, Vol. 27 (Oct. 1959), pp. 626–37. Some of the Morishima conditions on positive direct labor requirements can also be lightened. See also J. T. Schwartz, *Lectures on the Mathematical Method in Analytical Economics* (New York: Gordon and Breach, 1961). P. A. Samuelson, "A Modern Treatment of the Ricardian Economy: II," this *Journal*, LXXIII (May 1959), 228 shows that durable capital goods can give rise to a nonsubstitution theorem, as does P. A. Samuelson, "Equalization by Trade of the Interest Rate Along with the Real Wage," in Essays in Honor of Gottfried Haberler, *Trade, Growth, and the Balance of Payments* (Chicago: Rand McNally, 1965). We understand that Morishima and Murata, and Burmeister and Sheshinski, have work in process relating to extension to durable-goods models. Once one recognizes that a system with positive interest r behaves in all of its pecuniary relations just like a zero-interest system but with every input coefficient raised from a_{ij} to $a_{ij}(1+r)$, the 1949 formulation of the earlier nonsubstitution theorem provides a convenient reference.

or "irreducible" technology (which means a situation in which *every single* output requires, directly or indirectly as input for its production, something *positive* of *every single* other output).

Morishima's example, involving two goods and labor, meets this requirement, and so do the examples in the Sheshinski-Bruno-Burmeister and Garegnani papers. The Pasinetti-Sraffa example, in which labor over eight periods makes a product or alternatively makes it over twenty-five periods, would have to be modified before it would meet the indecomposability requirement of the (false) non-reswitching theorem. Thus, call the output of labor after one period in the eight-period process *thread* and the output of labor after three periods in the twenty-five-period process *yarn*. It is clear that thread requires no yarn, directly or indirectly, in its production.

In his original 1965 Rome paper, Pasinetti went most of the way toward modifying his example to get rid of its decomposability. And now that we realize how false the nonswitching theorem is, it is clear to us that making the trifling change in coefficients sufficient to rule out decomposability can be seen, *from continuity considerations alone*, to leave us with the same reswitching possibility known to prevail in the decomposable case.

3. Once a theorem is recognized to be false, there is not much point in hashing over errors in its proof. Still, to avoid confusion in the literature, we wish to emphasize that the fundamental lemma, used in the proof of the false theorem, is itself definitely false.[2]

4. Conditions that are sufficient to make a nonswitching theorem valid — like those of Weitsman and Bruno-Burmeister-Sheshinski — are of interest to economists for their own sake. But, of course, they do not absolve from error the false nonswitching theorem.

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2. Professor Solow has pointed out that an arbitrary matrix can be written as the difference between two positive matrixes. Hence, we can make $a-b$ have a row of negative elements and another row of positive elements, thereby making $(a-b)x$ be, for all positive x, a vector of more than one sign — negating the lemma in a stronger way than the Pasinetti counterexample, which leaves $(a-b)x$ semidefinite. Still we agree with Pasinetti and other writers in this symposium that even if a version of the lemma could be shown to be true along the factor-price frontier, this could be of no avail in providing a proof of what is essentially a false theorem. See Bruno-Burmeister-Sheshinski for a discussion of the fact that, when an x exists such that $(a-b)x \geqq 0$ or $(b-a)x \geqq 0$ for any admissible (a,b), sufficient conditions for nonswitching are realized.