Neoclassical empirical evidence on employment and production laws as artefact

by

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Summary

This paper claims that empirical evidence that seems to support neoclassical production functions are spurious findings. The paper surveys previous experiments that provide strong indications to that effect. Examples involve labour demand curves and production functions. Basically, it is shown that both neoclassical labour demand curves and neoclassical production functions reproduce national accounts, and hence are devoid of any practical behavioural implications.

Introduction

Students that are taught neoclassical economics are often struck by the lack of realism of many assumptions that underlie the theory. They are usually quite relieved when they discover that there are other schools of thought in economics that entertain different, more realistic, assumptions. However the enthusiasm of students for these alternative economics paradigms is often moderated by the enormous amount of empirical evidence that seems to provide support for neoclassical theory. If neoclassical economics is wrong, they ask, why is it that so many empirical studies appear to "confirm" the main predictions of neoclassical theory?

Heterodox economists often claim that neoclassical production functions and their substitution effects make little sense in our world of fixed coefficients and income effects. Claims to that effect also arose from the Cambridge capital controversies that rocked academia in the 1960s and 1970s. Neoclassical economists, however, have responded by pointing to the large number of empirical studies that seem to "verify" neoclassical theory, in particular when fitting Cobb-Douglas production functions. The purpose of this paper is to resolve this apparent paradox, and show that the "good fits" of neoclassical number crunchers is no evidence at all. Students can embrace heterodox microeconomics and its alternative assumptions without remorse. The numerous studies of empirical "evidence" supporting neoclassical production functions or other derived constructs are worthless. This empirical evidence is nothing but spurious findings, or as the title of the paper suggests, this empirical evidence is nothing but an *artefact*.

The word *artefact* carries several definitions. The most common definition, relevant to science, says that an *artefact*, or *artifact*, is a spurious finding caused by faulty procedures. It is a finding that does not really exist but that was created inadvertently by the researcher. In particular we shall see that neoclassical economists claim to measure output elasticities with respect to capital and labour, whereas in reality they are estimating the profit and wage shares in income. The word *artifact* is also used in the fantasy literature. In the fantasy and sorcery literature, an *artifact* is a magical tool with great power, like a magic wand. This definition seems to be just as relevant to the neoclassical production function. Correlation coefficients obtained with regressions of Cobb-Douglas production functions miraculously approach unity, and all the predictions that can be drawn from a model of perfect competition applied to the Cobb-Douglas production function are usually verified, even when we know that these conditions do not hold. In other words, the

neoclassical production functions and their derived labour demand functions are not behavioural concepts that can be empirically refuted. Their magical power is enormous!

The paper is divided into three sections. In the first section I briefly recall some of the stakes of the Cambridge capital controversies. The next two sections show that the equations that could verify the validity of the neoclassical theory of production and labour demand are no different from those of national accounting. The second section deals more specifically with labour demand functions, while the third section tackles production functions.

1. The Cambridge capital controversies

1.1 Presentation of the controversies

The Cambridge capital controversies pitted a group of economists from the University of Cambridge, in England, to a group of economists from the Massachusetts Institute of Technology (MIT), in Cambridge, near Boston, in the USA. Whereas the mainstream usually views the capital controversies as some aggregation problem, it is not the point of view of the Cambridge Keynesian economists, who see them as a more fundamental problem. Joan Robinson (1975, p. vi) for instance has clearly indicated that "the real dispute in not about the *measurement* of capital but about the *meaning* of capital". Nicholas Kaldor, who only briefly engaged in the controversies, nevertheless had a similar view when arguing that the distinction between the movement along a production function and the shift in the production function is entirely arbitrary (1957, p.595).

The controversies arose as a combination of circumstances. The *coup d'envoi*, from the neoclassical side, was provided by Paul Samuelson's (1962) attempt to demonstrate that Robert Solow's (1956) growth model and (1957) empirical manipulations of the neoclassical production function were perfectly legitimate. Samuelson was also trying to respond to Joan Robinson, following her 1961 visit to MIT. One can suspect that this rare opportunity of exchange between rival research programmes was provided by the fact that both Robinson and Samuelson were dealing with linear production models, so that mainstream economists could grasp to some extent what the heterodox economists were up to. Robinson had in mind the Sraffian model that was then in the making (Sraffa 1960), while MIT economists were working on linear programming and activity analysis (Dorfman, Samuelson and Solow 1958). Samuelson claimed that the macroeconomics of aggregate production functions were "the stylized version of a certain quasi-realistic MIT model of diverse heterogeneous capital goods' processes" (1962, p. 201-2).

The controversies made use of static models, based on profit maximization, with fixed technical coefficients, but with several techniques, or even an infinity of techniques. It was finally resolved, among other things, that the main properties of aggregate production functions could not be derived from a multi-sector model with heterogeneous capital, nor for that matter even from a two-sector model with one machine but several available techniques. This put in jeopardy the neoclassical concepts of relative prices as a measure of scarcity, substitution, marginalism, the notion of the natural rate of interest, and capital as a primary factor of production.

The controversies provided examples where standard results of neoclassical theory, as presented in undergraduate textbooks or when giving policy advice, were no longer true. For instance, with aggregate production functions, it is usually argued that, economy-wide, the rate of profit is equal to the marginal productivity of capital, and that there exists an inverse relationship between the capital/labour ratio and the ratio of the profit rate to the real wage rate. Counter-examples were shown to exist (see Cohen and Harcourt 2003):

☐ Reswitching: A technique which was optimal at high profit rates (or low real wages), and then abandoned, becomes optimal again at low profit rates (or high real wages);.

☐ Capital reversal or real Wicksell effects: A lower profit rate is associated with a technique that is less mechanized (the capital/labour ratio is low), even without reswitching;

☐ Discontinuity or rejection of the discrete postulate: An infinitely small change in the profit rate can generate an enormous change in the capital/labour ratio.

Figures 1 illustrate the implications of these results for the theory of labour demand. Neoclassical authors thought that an infinite number of fixed-coefficient techniques would yield a labour demand curve that has the standard downward-sloping shape shown in Figure 1A. However, Pierangelo Garegnani, who was a student of Sraffa, has shown that it is quite possible to build examples of a continuum of techniques that do not generate the downward-sloping curves that are needed by neoclassical theorists to assert their faith in the stability of the market mechanisms. Garegnani (1970) provides a numerical example that gives rise to the labour demand curve shown in Figure 1B, and Garegnani (1990) suggests the possible existence of a labour demand curve that would have the shape shown in Figure 1C. Because the neoclassical theories of value and output are, nearly by definition, one and the same thing, it should be clear that these results have destructive

consequences not only for neoclassical price theory but also for neoclassical macroeconomic theory, which relies on substitution and relative price effects.

Insert Figure 1 here
Figure 1: Conventional and unconventional shapes of the labour demand curve arising out of the Cambridge capital controversies

1.2 The neoclassical response to the resolution of the controversies

What has been the response of neoclassical authors to the Cambridge-Sraffian arguments? This response can be summarized under five headings:

- Neoclassical authors minimize the capital paradoxes, making an analogy with Giffen goods in microeconomics, which do not question the entire neoclassical edifice;
- They look for the mathematical conditions that would be required to keep production functions as 'well behaved', or they claim that this is a simple aggregation problem that can be resolved;
- They claim that Walrasian general equilibrium theory is impervious to the critique;
- They claim that they have the faith, or they plead ignorance;
- Empiricism (It works, therefore it exists).

Today, the last two responses are the most common. General equilibrium theory is now facing problems of its own, those tied to the Sonnenschein-Mantel-Debreu theorem (Kirman 1989, Guerrien 1989). The theorem asserts that it is impossible to insure uniqueness and global stability in general equilibrium models of supply and demand. In other words, there may exist a Walrasian equilibrium, but unless economically-irrelevant restrictions are added, nothing guarantees that a multi-agent competitive economy with flexible prices will ever converge to it. Macroeconomic models of the representative agent, based on Walrasian microeconomic foundations, are thus entirely bogus. As a result of this dead end, general equilibrium theory has quickly dropped out of the picture and is not even taught in most graduate programs. But since the Cambridge capital controversies are not discussed either, many economists can plead ignorance on both fronts. Those who are aware of the capital controversies and of the Sonnenschein-Debreu-Mantel impossibility theorem usually take a pragmatic view, claiming that the neoclassical model "works" or that nothing has yet been proposed to replace it.

Empiricism was a line of defence of neoclassical economics from the very beginning. For those neoclassical economists who rely on empiricism, the validity of neoclassical theory is an *empirical* issue, not a *theoretical* issue, in contrast to the arguments made by critics of the neoclassical production function in the course of the Cambridge controversies. This was because neoclassical theory, in its aggregate form, was recognized to be false, or to be a very special case, valid only under unlikely conditions. As a result, once they had lost the theoretical argument, as was recognized in the 1966 symposium of the *Quarterly Journal of Economics*, neoclassical authors quickly moved on to the empirical front. Sato (1974, p. 383) put it in a straightforward way: "The neoclassical postulate is itself in principle empirically testable in the form of a production function estimation of the CES and other varieties. This can make us go beyond purely theoretical speculations on this matter".

Some neoclassical authors did not hesitate to argue that empirical tests had already provided support to the parables of neoclassical theory. Bronfenbrenner (1971, p. 474) for instance, a staunch defender of marginal productivity theory, argued that the Cobb-Douglas production function works, not for magical reasons, but because its many applications had demonstrated that it could explain empirical facts fairly well. Ferguson, who had earlier stated that "placing reliance upon neoclassical economic theory is a matter of faith" (Ferguson, 1969, p. xvii), adding that he had faith, made the following claims, which probably represent the viewpoint of most of his mainstream colleagues, since most of them continue to use aggregate production functions.

"But to empirically-minded economists such as Douglas and Solow, the [neoclassical] parable has meant something more. In particular, it offers a set of hypotheses that can be subjected to statistical examination and evaluation. Assume the existence of an aggregate production function, such as Cobb-Douglas or CES, that meets the requirements of the Clark parable. In such circumstances, do the conventionally defined aggregates furnished by the OBE and other government statistical agencies tend to confirm or reject the inferences of the neoclassical parable? Without documentation, which is readily available, I will assert that the answer is 'Confirm'." (Ferguson, 1972, p. 174)

Not all neoclassical economists were enthusiastic about this empirical defence. Frank Hahn, a neoclassical economist from Cambridge, U.K., was, at least initially, quite critical of the empiricist defence, claiming that the simplicity of the aggregate neoclassical theory "is obtained at the cost of logical coherence" and that "the view that nonetheless it 'may work in practice' sounds a little bogus and in any case the onus of proof is on those who maintain this" (Hahn, 1972, p. 8).

However in the end the empiricist view has prevailed, with modern neoclassical authors justifying their use of aggregate production functions on the basis of past successful

regressions of neoclassical production functions. As Nobel Prize recipient Prescott (1998, p. 532) points out, "the neoclassical production function is the cornerstone of the [neoclassical] theory and is used in virtually all applied aggregate analyses". Without it, very little or no applied aggregate economic analysis can be pursued by neoclassical economists. And very little policy advice could be offered, because, for instance, as again pointed out by Prescott (1998, p. 532), "the aggregate production function is used in public finance exercises to evaluate the consequence of alternative tax policies". This is why it is so important for mainstream economists, even well-known ones such as Hamermesh (1986, p. 454, 467), to claim that "the estimated elasticities that seem to confirm the central prediction of the theory of labor demand are not entirely an artefact produced by aggregating data …. The Cobb-Douglas function is not a very severe departure from reality in describing production relations".

2. Labour demand theory and empiricism

Hamermesh seems to believe that neoclassical labour demand theory is well verified. In the present section, I present two examples of how neoclassical theories of labour demand seem to be supported by empirical studies, but really are not. Both examples are related to the influential work of Layard, Nickell and Jackman (1991) – their famous price-setting/wage-setting (PS-WS) model. These British neoclassical authors, whom we will denote by LNJ from now on, essentially conclude that the "equilibrium" rate of unemployment in Europe (the *NAIRU*) has risen because real wages have grown too fast relative to labour productivity. We shall see that, most likely, the empirical work that sustains such a conclusion is a spurious result, and hence that no economic policy should be based on it.

2.1 Confusing identities for behavioural relations

Let us first consider the work of three French authors, Cotis, Méary and Sobszak (1998), which we will call CMS from now on. Note that their work is not that of neoclassical economists who are far away from the edge of science. Jean-Philippe Cotis has been the chief economist at the OECD, and has just been named head of the main French statistical agency, the INSEE. The OECD provides influential policy advice on the basis of its economic research. CMS wish to provide a small econometric model of the PS-WS model, using cointegration analysis. Their model is given by the following two equations:

WS : $w - p = a_1 U + a_4 wedge + \gamma t$ (1)

PS : $w - p = b_1 U + b_2(q - n) + b_5 t$ (2)

where *w, p, q, n*, are all logarithmic values, with w = wage rate; p = prices; q = output; n = active population; U = unemployment rate. The *wedge* variable in equation (1) is the overall tax rate on wage income and wage cost.

CMS recall, based on the work of LNJ, that when the profit-maximizing first-order conditions of a well-behaved neoclassical production function (with diminishing marginal product of labour, perfect competition, factor pricing at the value of the marginal product, etc.) are fulfilled, the PS equation will take a particular form, with $b_1 = b_2 = 1$. Thus with these additional constraints, the following PS equation must hold:

PS : $w - p = U + (q - n) + b_5 t$ (3)

CMS marvel at the fact that their regressions yield support to their assumed parameters. But it turns out that the PS equation, as given by equation (3), can also precisely be derived from the national accounting identities, without resort to any behavioural equation or neoclassical assumption!

Start with the national accounts:

$$PQ = WL + RPM$$ (4)

where P = prices; Q = output; W = wage rate; L = labour; R = profit rate; M = the stock of machines. Take the logarithmic derivative of equation (4), denoting the growth rate of a variable X by X' , where $X' = (dX/dt)/X$. Equation (4) can then be rewritten as:

$$P' + Q' = \alpha (W' + L') + (1-\alpha)(R' + P' + M')$$ (5)

where α is the share of labour in income. By rearranging it, equation (5) can be written in terms of the growth rate of real wages. One obtains:

$$W' - P' = (Q' - L') + \{(1-\alpha)/\alpha\}(Q' - M' - R')$$ (6)

At this stage, we may make use of two approximations which are used by LNJ to obtain their equation (3). These two approximations are the following. First, LNJ define the rate of unemployment U as the ratio:

$$U = (N - L)/L \qquad\qquad\qquad (7)$$

where N is active population, instead of using the standard definition of the rate of unemployment, which would be: $U = (N - L)/N$.

Secondly, LNJ make use of a mathematical approximation, noting that $x = \log(1+x)$, when x tends towards zero. Now calling

$$U = (N - L)/L = x$$

it follows that:

$$1+x = 1+(N - L)/L = N/L$$

Hence

$$U = \log (N/L) = \log N - \log L$$

and

$$dU/dt = N' - L'$$

or else

$$L' = N' - dU/dt \qquad\qquad\qquad (8)$$

Thus, combining equations (6) and (8), the identities of national accounting become :

$$W' - P' = dU/dt + (Q' - N') + \{(1\text{-}\alpha)/\alpha\}(Q' - M' - R') \qquad\qquad (9)$$

Integrating equation (9), and omitting the constant, the national account equation becomes:

$$w - p = U + (q - n) + \{(1 - \alpha)/\alpha\}ht \qquad\qquad (10)$$

where once again small letters represent the logarithmic values of the equivalent capital letters, and where $h = Q' - M' - R'$.

Equation (10), derived from the national accounts, is no different from equation (3), which was derived from a neoclassical model of labour demand with all the well-behaved conditions. Thus it comes as no surprise that CMS conclude that their model is "not rejected by the data". And it not surprising that the regressions of LNJ allow them to verify that in equation (2), indeed, $b_1 = b_2 = 1$. Any time one runs a regression that incorporates the variables found in equation (2), including a time trend, as would be the case of a PS equation, the regression cannot but yield a positive result between the log of real wages and the rate of unemployment. This result comes out directly from the national accounts.

Thus, one may conclude that the empirical results drawn from the PS-WS model do not (necessarily) depend on behavioural relations based on profit maximization with well-behaved production function, neutral technical progress and diminishing returns. Instead, quite the opposite is the likely outcome. The correlations and signs hat have been obtained rest on the national income identities, and as such, they have no causal or explanatory power. These estimates of the neoclassical labour demand are only artefacts. They are meaningless.

In other words, economists that use *PS-WS* models are only providing estimates of what the determinants of the equilibrium rate of unemployment would be (a kind of *NAIRU* or natural rate of unemployment) *if* the neoclassical theory of labour demand, based on aggregate production functions and decreasing returns, were valid. These estimates cannot provide *any* support for neoclassical theories of equilibrium unemployment. Thus, paraphrasing Nicholas Kaldor (1972, p. 1239), we see that the estimates based on PS equations or similar equations can only help to "illustrate" or "decorate" neoclassical theory and its assumptions of profit-maximization, decreasing returns, and equilibrium unemployment. In no way can these estimates confirm or corroborate neoclassical theory. They cannot, in any way, be used as a basis for economic policy advice.

2.2 A "reductio ad absurdum" proof

Anadyke-Danes and Godley (1989) have provided a *reductio ad absurdum* proof that questions the relevance of the kind of regression analysis that has been pursued by those economists, LNJ in particular, who are convinced that overly high real wages are the main cause of the consistently high European unemployment rates. Godley and his associate intend to demonstrate that even when, *by construction*, an hypothetical economy has no relationship whatsoever between employment and real wages, standard econometric

analysis (based on OLS estimates) will give the impression that it verifies a negative relationship between employment and real wages.

Before we move on to their econometric results, we can show, once more, how easy it is to get the labour demand PS equation of the LNJ model. Start this time with the simplest of the cost-plus pricing equation – the mark-up equation:

$$PQ = (1+\theta)WL \qquad\qquad (11)$$

$$P = (1+\theta)W(L/Q) \qquad\qquad (12)$$

where θ is the costing margin. In logs, with the log of θ being called θ_1, we have:

$$p = \theta_1 + w - q + l \qquad\qquad (13)$$

or rearranging in terms of labour employment, and dropping the constant, we get:

$$l = - (w - p) + q \qquad\qquad (14)$$

Equation (14) reminds us that, for a given output level, we automatically get a negative relationship between employment and real wages when prices are set through a cost-plus procedure. But this negative relationship only reflects the fact, that, with a given costing margin, the real wage will be lower if labour productivity (measured by $q - l$) is lowered. It has nothing to do with a demand for labour function. It is simply an arithmetic relation that arises from the cost-plus pricing formula. Rewriting equation (14) yet once more, and dropping the constant, we see that :

$$(w - p) = q - l \qquad\qquad (15)$$

And with the Layard approximation, $U = n - l$, equation (15) once more can be transformed into the miraculous PS equation, $(w - p) = U + (q - n)$! Thus having started from the simplest cost-plus pricing equation, with no marginalism content whatever, we can recover the PS equation that links high real wages to high unemployment rates – a result which neoclassical economists attribute to the profit-maximizing behaviour of firms making hiring decisions.

Anadyke-Danes and Godley (1989) go one step further. Here is their *reductio ad absurdum* proof. They start by assuming, *by construction*, that nominal wages, output and

employment all grow independently of each other, with prices set on the basis of a mark-up on current and lagged labour unit costs (75% of sales are assumed to be based on current output and 25% of sales arise from held inventories, produced in the previous period, and hence, in the pricing equation below, $\varphi = .75$). Wage rates, output and employment each grow at some specific trend rate (7%, 5% and 1% respectively), with random fluctuations around it. We have:

$$w = (1.07 + \text{random}) + w_{-1}$$
$$q = (1.05 + \text{random}) + q_{-1}$$
$$l = (1.01 + \text{random}) + l_{-1}$$
$$p = \theta_2 + \varphi(w - q + l) + (1-\varphi)(w_{-1} - q_{-1} + l_{-1})$$

Anadyke-Danes and Godley then run a regression on the data generated by this hypothetical economy. They get the following result, with the absolute *t*-statistics in parentheses:

$$l = 1.3 - 0.94\,(w - p) - 0.12l_{-1} + .73q + .01t$$
$$(7.4)(1.0)(1.0)(4.2)$$

According to the regression equation, employment seems to entertain a statistically significant negative relationship with real wages, as well as a positive time trend, as LNJ and their neoclassical colleagues would like it to be. In addition, note that employment does not seem to depend on actual output q, in contrast to what post-Keynesians would argue, and that it does not depend on past employment l_{-1}, since these two variables do not have statistically significant coefficients in the regression.

But we know that, *by construction*, employment l is completely independent of real wages, and that the current level of employment *only* depends on past employment. As Anyadike-Danes and Godley (1989, p. 178) put it, "real wage terms turn out to be large, negative and strongly significant although we know, as Creator, that real wages have no direct causal role whatever in the determination of employment". Thus empirical studies can manage to give support to the neoclassical theory of labour demand even in those cases where we know that, by *construction*, neoclassical theory is completely irrelevant (i.e., when real wages and employment are independent of each other, while prices are set on a cost-plus basis and not on marginal pricing principles).

3. Production functions and empiricism

While the above results are certainly a cause for concern for those neoclassical economists who claim that neoclassical theory holds because it seems to "work", other exercises have shown even greater gaps in the empiricist defence of neoclassical theory. In the previous section we saw that it is impossible to falsify the neoclassical theory of labour employment, since the econometric version of neoclassical labour demand is no different from an equation derived from the national accounts, and also because econometric regressions will yield results consistent with the neoclassical view even when we know by construction that this theory does not hold. In the current section, we show that the same can be said about neoclassical production functions: they cannot be falsified, as long as technical progress is adequately represented. But we also show an additional damning result: the coefficients of neoclassical production functions, for instance those of the Cobb-Douglas type, as obtained through econometric analysis, do not even measure the elasticities of factors of production which they claim to measure.

3.1 Reversing causality

One can draw a long list of authors who have argued, in one way or another, and with more or less conviction, that neoclassical production functions (such as the Cobb-Douglas function, the Constant Elasticity of Substitution (CES) function, or the translog production function) often provide good empirical results because they simply reproduce the underlying identities of the national accounts. The argument applies both to cross-industry estimates and to time series. The list goes back to Phelps-Brown (1957). It incorporates previous recipients of the so-called Nobel Prize in Economics, Simon (1979) and Samuelson (1979). And of course, as one would suspect, some heterodox economists have driven the point on numerous occasions: Shaikh (1974, 1980, 2005a), Sylos Labini (1995), McCombie (1987, 1998, 2000-1, 2001), McCombie and Dixon (1991), Felipe and McCombie (2001, 2002, 2005a, 2005b, 2006). I have myself made the point in two of my books (Lavoie 1987, 1992), besides my critique of CMS and LNJ (Lavoie 2000).[1]

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[1] When I wrote Lavoie (1987) I came across the paper written by Herbert Simon (1979), where he argued that the good fit of Cobb-Douglas and CES production functions is a statistical artefact. I noticed that he did not cite Shaikh (1974), despite the fact that Shaikh's arguments were highly similar to those of Simon. I wrote to Simon to point this out, and he replied that, being less connected with economics, he had to rely on friends and colleagues to keep track of the literature. The strangest thing however, is that Simon, in the first footnote of his paper, thanks Robert Solow for his comments. Solow had to be aware of the highly relevant Shaikh (1974) paper, since Solow's work was Shaikh's main target and because Solow had published a reply to it, but for some reason he preferred not to mention it to Simon. In 2007, Shaikh told me that when

Before getting into the link between neoclassical production functions and national accounting identities, let us recall another related line of research, which was pursued by Franklin M. Fisher, tied to the problem of aggregation. Fisher (1971) built an hypothetical economy, made up of several profit-maximizing firms with Cobb-Douglas production functions, but each with different output elasticities, such that conditions of aggregation clearly did not hold. Despite this, running regressions on the aggregate data, Fisher found that the aggregate Cobb-Douglas function did seem to « work » properly, provided the wage share was constant enough within the set of data.

Fisher concluded that one must reverse the usual argument. Rather than saying that the wage share in national income is constant because technology is of the Cobb-Douglas type, one should say instead that the apparent success of the Cobb-Douglas production function is due to the approximate constancy of the wage share. Or as Felipe and Fisher (2003, p. 237) revisit this issue more recently, the fact that the Cobb-Douglas production function works even when conditions for successful aggregation are violated suggests that "the (standard) view that constancy of the labor share is due to the presence of an aggregate Cobb-Douglas production function is wrong. The argument runs the other way around, i.e., the aggregate Cobb-Douglas works well *because* labor's share is roughly constant". Thus any economic or institutional force that would tend to keep labour and profit shares relatively constant through time or across industries or countries would provide favourable evidence for an aggregate Cobb-Douglas production function.

Similarly, as pointed out by Fisher (1971, p. 325), the positive relationship between output per labour and the real wage – a key feature of CES production functions – "occurs not because such functions really represent the true state of technology but rather because their implications as to the stylized facts of wage behaviour agree with what happens to be going on anyway. The development of the CES, for example, began with the observation that wages are an increasing function of output per man and that the function involved can be approximated by one linear in the logarithms. The present results suggest...that the explanation of that wage-output per-man relationship may not be in the existence of an aggregate CES but rather that the apparent existence of an aggregate CES may be explained by that relationship". Indeed, Felipe and McCombie (2001) argue that the popularity of the CES production function arose out of a statistical artefact.

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attending a recent meeting in honour of Ando Modigliani, at the New School University, Solow refused to shake hands.

3.2 Another "reductio ad absurdum" *proof*

Although the Fisher experiments are enlightening, they somewhat leave up in the air why Cobb-Douglas production functions seem to work so well. Before we consider these fundamental reasons, let us consider some especially compelling arguments that question the empirical evidence favouring the use of neoclassical aggregate production functions. These arguments are based on yet another *reductio ad absurdum* proof. This proof is proposed by John McCombie, who has devoted quite a bit of attention to these issues.

McCombie (2001) takes two firms *i* each producing in line with a Cobb-Douglas function:

$$Q_{it} = A_0 L^{\alpha}_{it} M^{\beta}_{it}$$

with $\alpha = 0.25$.

Thus α is the output elasticity of labour and is equal to 0.25 for both firms. Similarly, for both firms, the output elasticity of capital, β, is equal to 0.75 since the sum of the two elasticities is assumed to be unity (there are constant returns to scale). Inputs and outputs of the two firms are perfectly identical. Hence there is no aggregation problem; in other words the Fisher (1971) problem is avoided.

McCombie (2001) constructs an hypothetical economy, where *L* and *M* grow through time, with no technical progress, but with some random fluctuations. Running an econometric regression directly on this constructed *physical* data set (the *Q*, *L*, and *M* variables) yields an α coefficient close to 0.25, as expected. Running the equation in log values, McCombie obtains:

$$q = -0.02 + .277l + .722m$$
$$ (22.5) \quad (55.5)$$

In this case, the estimate is based on *physical* data, and there is no problem: the regression estimates of the output elasticities correspond to those that exist by construction. Things are entirely different however, when monetary values are used.

McCombie (2001) reconstructs the same hypothetical economy, with the same two firms, each again with identical output elasticities, but this time he tries to estimate an aggregate production function using deflated monetary values, as must always be done in macroeconomics and most often in applied microeconomics. To do so, he assumes, by construction, using a mark-up equation similar to equation (12), that firms impose a mark-

up equal to 1.33 (the costing margin $\theta = 0.33$). This implies that the wage share is 75% and that the profit share is 25%. With this new regression, based on deflated monetary values, which we will denote with the subscript d to make this clear, the regression yields an estimate of the α coefficient – the apparent labour output elasticity – that turns out to be 0.75, as shown in the regression equation that follows:

$$q_d = +1.8 \; + .752l \; + .248m_d$$
$$\qquad\qquad (1198) \quad (403)$$

.

Thus, we started with production functions and physical data according to which we know that, by construction, the labour output elasticity is 0.25. Yet, the estimated aggregate production function (in deflated monetary terms) tells us that this elasticity is 0.75 – which is the wage share in income. In other words, estimates of aggregate production functions – both at the industry or macro levels, since they are necessarily based on deflated values and not on direct physical data – measure factor shares, not the output elasticities of factors of production, in contrast to what neoclassical authors would like us to believe.

These empirical estimates of aggregate production functions are completely useless to provide any information about the kind of technology in use, or about the values of output elasticities and elasticities of substitution. McCombie (2001) provides additional proof of this. He starts with the base year data of the two firms mentioned above, but assuming now, by construction, that the inputs and outputs of these firms grow in a completely random way. Not surprisingly, when a regression is run on the physical variables of each firm, correlation coefficients are near zero and estimates of output elasticities are statistically insignificant, as they should be, since there is no relationship between inputs and output, by construction.

By contrast, when the same physical data set is combined to monetary value data obtained by assuming the same mark-up in each firm, with again a 75% profit share and assuming a constant profit rate, the regression on the aggregated deflated values yields very promising results. The correlation coefficient is nearly unity, and the regression coefficients yield statistically significant values that reflect once more the labour and profit shares:

$$q_d = constant \; + .751l \; + .248m_d$$
$$\qquad\qquad (514) \quad (354)$$

.

Thus, as McCombie (2001, p. 598) concludes, "no matter what form the underlying micro or engineering production functions take, so long as the average mark-up is roughly

constant over time (so that factor shares are constant), a reasonable fit to the Cobb-Douglas relationship will always be found. However, this says nothing about the underlying technology of the economy". So even if technology is from Mars, and that Martians manage to produce output independently of inputs, provided Martian firms follow some form of cost-plus pricing, the regressions over deflated data will tell us that Martians use Cobb-Douglas production technology, with diminishing returns, constant returns to scale, and factor pricing following marginal principles.

Why is this so? It turns out, as was the case with the PS equation and as we shall see in the next section, that regressions over the deflated variables of production functions, when they are correctly estimated, only reproduce the relationships of the national accounts. If the wage share is approximately constant, and if there is no technical progress or if technical progress is adequately estimated, one will always discover that a Cobb-Douglas production function provides a good fit. If the wage share is not constant, for instance when the wage share trends upwards along with the capital to labour ratio, the CES function will yield better fits. But the CES production function, along with the translog production function, are subject to the very same criticisms that apply to the Cobb-Douglas function (Dixon and McCombie 1991; Felipe and McCombie 2001).

If technical progress is misrepresented (for instance through a linear function in time, rather than by a non-linear one), the output elasticity estimates will not equal the profit and wage shares, and the elasticities may even turn out to be negative. This explains why Cobb-Douglas functions sometimes seem to misrepresent production relations, giving the illusion that neoclassical production functions can be falsified by empirical research.

3.3 Confusing again identities with behavioural relations

Neo-classical authors often marvel at the apparent key result that their estimates of the output elasticities of capital and labour turn out to be nearly equal to the shares of profit and wages in national income. Since neoclassical theory predicts that this will be so in a competitive economy with diminishing returns and constant returns to scale, where production factors are paid at the value of their marginal product, neoclassical economists usually conclude that, even thus they know that the real word is made up of oligopolies and labour unions, in the end it behaves *as if* it were subjected to competitive forces. This assertion is rather hard to swallow, but all kinds of reasons will be advanced to justify such a result, such as the theory of contestable markets, whereby the threat of entry by

newcomers will be sufficient to insure that incumbent members of an industry behave in a competitive way. The (apparent) amazingly successful estimates of neoclassical production functions thus reinforce the belief of many neoclassical economists that the idealized supply and demand analysis is good enough to describe the real world, since economic agents ultimately behave *as if* pure competition prevailed.

The reality is that estimates of the production function based on deflated values simply reproduce the identities of the national accounts and that the *pseudo* estimates of the output elasticity of capital (labour) are really approximations of the profit (wage) share. This can be seen in the following way, by rewriting the Cobb-Douglas production function and the national accounts in logs or in growth terms. Start with the Cobb-Douglas function, but this time one that includes technical progress:

$$Q_t = A_0 e^{\mu t} L_t{}^{\alpha} M_t{}^{\beta}$$

Assume constant returns to scale, so that: $\alpha + \beta = 1$. Now consider output per head and capital per head, $y = Q/L$ and $k = M/L$. Taking logs, the Cobb-Douglas function yields:

$$\log y = \mu t + \beta \log k \qquad (16)$$

Or in growth terms, taking the log difference, $\Delta \log$, we have:

$$y' = \mu + \beta k' \qquad (17)$$

where once again a prime sign signals the growth rate of the variable.

We may now compare the two equations (16) and (17) with those obtained from the national accounts. Start with the national account identity, given by equation (4), and divide through by the prices and the number of workers. One gets:

$$Q/L = W/P + R(M/L)$$

or

$$y = W/P + Rk \qquad\qquad (18)$$

Taking the log derivative of equation (18), and denoting the profit share in national income by the Greek letter π, one gets:

$$y' = \tau + \pi k' \qquad\qquad (19)$$

with $\quad \tau = \alpha(W/P)' + \pi R' \qquad\qquad$ (19A)

Or else, in logs:

$$\log y = \tau t + \pi \log k \qquad\qquad (20)$$

Equations (19) and (20), derived from the national identities, are highly similar to equations (17) and (16), which came from the Cobb-Douglas production function. Thus it is not surprising that these equations will perform well, as long as technical progress μ in equations (16) or (17) is adequately represented. Indeed, Shaikh (1974) has shown that even a production relation that would trace the word HUMBUG (*tonteras*), with capital per head on the horizontal axis and output per head on the vertical axis, can be successfully represented by a Cobb-Douglas production function, using the method advocated by Solow (1957). Thus, as should now be clear following the exercises of Fisher (1971) and McCombie (2001), any technological relation will yield the appearance of a Cobb-Douglas production function as long as its income shares are relatively constant.

3.4 Yet another "reductio ad absurdum" *demonstration*

Still, there are cases where Cobb-Douglas functions with yield nonsense, and hence are not "verified", as pointed out by various authors such as Lucas, Romer and Solow. Such a situation does not normally occur when there is no technical progress. The problem is

that technical progress is sometimes represented by a linear trend, whereas in reality the growth rate of labour productivity is highly variable. This can be shown by Figure 2 below, which represents the growth rate of technical progress in the USA and in a Goodwin-cycle model constructed by Shaikh (2005a). Clearly, technical progress cannot be represented by some linear function; one must introduce a non-linear trend, given by a Fournier series or some trigonometric function, because the rate of technical progress is fluctuating in a wild way.

Insert Figure 2 Here
Figure 2: Rates of growth of technical progress
Source: Shaikh (2005b, Figure 7).

In the article that started the growth-accounting craze, Solow (1957) managed to overcome this problem by constructing a variable measuring technical progress. Solow's favourite equation is the log version of the Cobb-Douglas production function, given by equation (18) above, which we repeat here for convenience: $\log y = \mu t + \beta \log k$. Then, for each period, he introduces a value for the technical progress growth rate, μ, which he defines in a way which is analogous to equation (19A), thus deriving the measure of his μ straight from the national accounts (more precisely, he derived it from the quantity dual of equation (19A)). In other words Solow tested the national accounts identity, while claiming to have corroborated the neoclassical theory of income distribution and neoclassical production functions, as well as claiming to have found a simple way to distinguish between shifts of aggregate production functions and movements along the production function. No wonder he got a good fit!

Indeed, nowadays, neoclassical economists that still "test" the Cobb-Douglas production function adjust the data by making corrections to the capital stock, deflating the capital index by taking into account the rate of capacity utilization, which is tightly correlated to the rate of technical progress, thus obtaining a good "fit" with their regressions. Otherwise regression results would be catastrophic, as can be seen from

Table 1, and as was experienced by a student of mine who once gave a try at such exercises.

The table represents two sets of regressions, with two different means of estimating technical progress. All shown estimations are done in growth rates, as per equation (17), as these yield better Durbin-Watson statistics. In the left-hand side columns, technical progress is given by a constant time trend; in the right-hand side columns, technical progress is represented by a non-linear variable. Clearly, the left-side columns with the constant trend in technical progress show a dismal fit. In both of the left-side regressions, the adjusted R^2 are near zero and the measured presumed output elasticities of capital – the β coefficients – have no relationship whatsoever with the actual profit share π of the US economy or that of the fictitious Goodwin economy.

The Goodwin data has been compiled by constructing a hypothetical economy, the variables of which have been generated by a Goodwin-cycle model, with Leontief input-output technology (fixed technical coefficients), Harrod-neutral technical progress, and mark-up pricing. Hence none of the usual neoclassical constructs exist (diminishing returns, marginal productivity, marginal cost pricing). Still, as the regressions on the right-hand side demonstrate, once technical progress is introduced in an adequate way, any data can appear to be fittingly represented by a Cobb-Douglas function. This is the case of the US data, with a nearly perfect adjusted R^2 and an estimated output elasticity of capital that nearly perfectly equates the actual profit share, as neoclassical theory would have it; but more surprisingly it is also the case of the Goodwin data, which *by construction*, violates all the standard neoclassical assumptions.

One way to understand what is going on is to look at Figure 3 which represents a Leontief production function with fixed coefficients, with a dominant technology at each point of time. With technical progress arising at a constant capital to output ratio (given by $1/\rho$), i.e., technical progress is of the Harrod neutral sort, the wage-profit frontier rotates to the north-east, as shown on the left-hand side of the figure.[2] On the right-hand side of the figure, one can observe what is the true relationship between output per head and capital per head: it is a simple straight line, $y = \rho k$. Neoclassical analysis, however, will assume that there exists a standard production function, with diminishing returns and

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[2] The wage-profit frontier is assumed to be linear for simplicity; in general it will not be.

the standard curvature, so that it needs to distinguish between a shift of the production function and a move along the production function, from k_0 to k_2. Even when technology is of the Leontief type, as depicted in Figure 2, neoclassical economists running standard regressions on deflated variables will manage to "prove" the existence of a well-behaved pseudo neoclassical production function.

Insert Figure 3 here
Figure 3: True Leontief technology and fitted neoclassical production functions
Source: Shaikh (1990, p. 193).

Conclusion

The studies of Shaikh, McCombie, Felipe and others show that the econometric estimates of neoclassical production functions based on deflated monetary values, as is the case at the macro and industry levels when direct physical data is not used, yield pure artefacts, that is, purely imaginary results. This affects all of neoclassical applied aggregate work that relies in some way on well-behaved production functions and profit-maximizing conditions: labour demand functions and NAIRU measures; investment theory; measures of multifactor productivity or total factor productivity growth; estimates of endogenous growth; theories of economic development; theories of income distribution; measures of output elasticities with respect to labour and capital; measures of potential output; theories of real business cycles; estimates of the impact of changes in the minimum wage, social programs, or in tax rates. Even when setting aside problems of aggregation, these estimates are either completely off target (if the world is made up of neoclassical production functions) or imaginary (if economies are run on fixed technical coefficients, as I believe they are essentially).

Instrumentalism is the philosophy of science that claims that assumptions need not be realistic, as long as they help making predictions. It combines the ability to start from idealized imaginary models and the need to resort to empiricism. Instrumentalism is endorsed by Milton Friedman (1953) and most neoclassical economists (often without realizing it). The VAR methodology used in time-series econometrics is a modern example of instrumentalism. In the case of well-behaved production functions and their implied

labour demand functions, neoclassical economists are pushing instrumentalism to the hilt. What counts is their ability to make predictions, based on estimates of elasticities, even if these predictions are meaningless because these estimates do not measure output elasticities, measuring instead factor shares! Neoclassical economists are claiming to measure something, but are really measuring something entirely different. Their theories, such as the necessary negative relationship between real wages and employment, seem to be supported by the data, whereas the negative relationship arises straight from the identities of the national accounts, with no behavioural implication as to the effect of higher real wages on employment.

I have discussed some of these issues with a few of my neoclassical colleagues – those that I thought would be most open to dialogue. Amazingly, their response has been to fake that they did not understand the implications of the Shaikh or McCombie papers that I emailed them. The most genuine answers have been that without these elasticity estimates they could not say anything anymore. But they would rather continue making policy proposals based on false information than make no proposition at all. In other words, they would rather be precisely wrong than approximately right.

To conclude on the theme with which I started, heterodox economists and their students need not fear the mountains of empirical evidence that seems to give support to neoclassical theory. Most, probably all, of this evidence is an artefact. The tons of regressions conducted on just-identified neoclassical production functions can only provide estimates of the model's parameters, but they can in no way provide support for the theory. Neoclassical production theory, and its offshoots, cannot be falsified by econometric research, and hence, if we are to believe the philosopher of science, Karl Popper, they are not truly scientific. Even worse than that, the experiments recalled here have shown that estimates based on constant price values do not measure what neoclassical economists claim to be measuring. Policy advice based on these estimates is bogus.

Obviously, alternative microeconomic foundations – heterodox ones – are needed to understand both microeconomic and macroeconomic issues.

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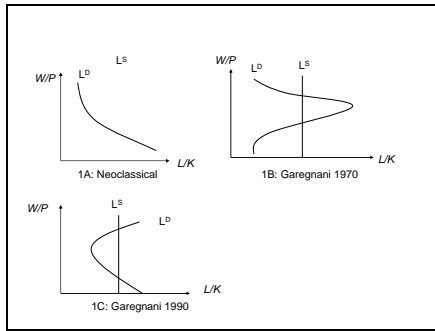
Table 1

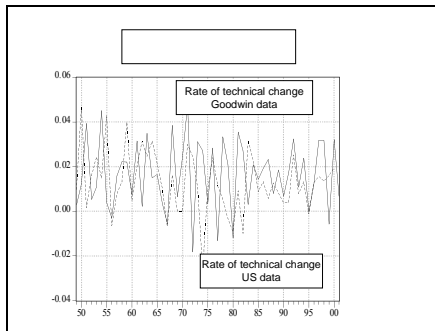Regressions on US data and Goodwin data, with linear and non-linear technical trends

Variable	With a linear time trend		With a non-linear time trend	
y'_t	Goodwin data	US data	Goodwin data	US data
Constant	0.019* (.005)	0.022* (.004)	-0.068* (.0002)	-0.053* (.0001)
Time	0.00004 (.0001)	-0.0002 (.0001)		
μ_t			5.05* (.012)	4.99* (.009)
k'_t	-0.024 (.106)	0.043 (0.098)	0.158 (0.002)	0.193 (0.001)
Adj R²	-0.038	0.027	0.9998	0.9998
D.W.	2.93	2.04	1.91	1.51
Implied Profit Share β [Actual Profit Share π]	-0.024 [0.160]	0.043 [0.190]	0.158 [0.160]	0.193 [0.190]

Source: Shaikh (2005), Tables 1 and 2.

Slide 1

1A: Neoclassical   1B: Garegnani 1970   1C: Garegnani 1990

Slide 2

Slide 3