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The Profit Rate Under Continuous Technological Change

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INTRODUCTION

The global crisis of automated capitalist production is now a generation old. Yet since the beginning of the 1950s, workers have battled automation continuously, raising the new question: "what *kind* of labor is human?" (Phillips and Dunayevskaya 1984). They have thus imbued the thought of both the young and the mature Marx with new significance and, indeed, urgency. For it was the mature Marx of *Capital,* Volume III who analysed the falling rate of profit thusly:

> The *true barrier* to capitalist production is *capital itself*. . . . production is production only for *capital,* and not the reverse, i.e. the means of production are not simply means for a steadily expanding pattern of life for the *society* of the producers (Marx 1981:358).

In a quite opposite manner, the Okishio (1961) theorem has turned radical theorists' attention away from the mode of production, towards the mode of distribution and the form of competition. Exerting a decisive influence over recent theories of the falling rate of profit and the contemporary world economic crisis, the theorem ostensibly shows that the falling profit rate cannot "be due to technical innovation itself, independent of changes in the real wage" (Roemer 1981:113). Marx's contention that the rate of profit must fall because of incessant mechanization, even if workers labored 24 hours a day at zero wages (Marx 1981:523), is simply wrong.

Responses to the Okishio theorem have shown that the profit rate can in fact fall (see Roemer 1981, Chap. 5 for a partial review). Since it is something other than mechanization *itself* that causes the profit rate to fall in almost all of these models, however, they fail to defend *Marx's* theory of the falling profit rate against the Okishio theorem. Yet even a cursory reading of Marx's exposition of the law of the falling profit rate reveals that it refers to *continuous* mechanization (Marx 1981:317ff), while Okishio's static equilibrium model necessarily treats technical change as a *one-time-only* "disturbance" of the system. The theorem, therefore, neither refutes the law nor even bears any clear relationship to it.

Ernst (1982) has developed a model of continuous technological change in which the rate of profit can fall, even though capitalists maximize profit rates and the real wage rate remains constant. By continually reducing unit values, continuous mechanization *itself* poses a barrier to the rate of "self-expansion" of

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value — i.e., the profit rate. The full implications of Ernst's argument have not been recognized, perhaps in part because his model employed some very specific assumptions that may have made it appear that they, rather than the continuous character of mechanization, produced the falling profit rate. The present paper develops a simpler and more general continuous mechanization model and, in contrast to Ernst's contribution, explicitly utilizes an intertemporal pricing equation. By posing the difference between continuous and one-time-only mechanization more sharply, I hope to focus attention on the issues Ernst raised and, more fundamentally, on the alien mode of labor as the true source of capitalist crisis.

A MODEL OF CONTINUOUS TECHNOLOGICAL CHANGE

I wish to show that, unless mechanization is only an episodic "disturbance," the unit price (in a one-output model) will not adjust to a static equilibrium level. A systematic discrepancy between historical and replacement costs of capital therefore arises, making the latter inappropriate for measuring the tendency of the actual rate of return on the original outlay of money capital. Under continuous mechanization, the actual profit rate thus diverges systematically from the static equilibrium counterpart. Even when the latter rises, the actual rate may fall and, if extraction of living labor does not grow, it will fall to zero.

Roemer (1981, Chap. 5) has generalized Okishio's theorem to the case in which nondepreciating fixed capital serves as a means of production in addition to circulating capital. A single-capital/one-output version of his model is developed below, though modified for continuous mechanization.

Define, for the t-th period:

F_t is the amount of nondepreciating fixed capital used in production
A_t is the amount of circulating constant capital used in production
N_t is the amount of living labor used in production
Q_t is the amount of output yielded by (F_t, A_t, N_t)

Let b indicate the growth factor of F, A, and Q, and let d indicate the growth factor of N, where b and d are positive constants and $b > d$. This sort of continuous mechanization is living labor-saving *without being constant capital-using*. The output-capital ratio remains constant throughout time, while the output-labor ratio and the technical composition of capital both increase continuously. The following are solutions to difference equations of the form $F_t - bF_{t-1} = 0$:

$$F_t = F_0 b^t \quad (1)$$
$$A_t = A_0 b^t \quad (2)$$
$$N_t = N_0 d^t \quad (3)$$
$$Q_t = Q_0 b^t, \quad (4)$$

where F_0 and similar terms indicate initial, pre-mechanization magnitudes.

To determine the path that the unit price, p_t, takes over time, Marx's concept of price formation is adopted. He holds that the total value of output is the sum of the value of the constant capital, plus the value added (labor-time extracted from workers). Because, by assumption, the fixed capital is nondepreciating, none of its value is transferred to the value of output. Even if input and output values differ and/or the value of the *constant capital* diverges from the value of the *means of production* (i.e., the total labor time required to reproduce the means of production),[1] the value of the constant capital — the labor time represented in its purchase price — is nonetheless transferred to the value of output. Define the unit *input* value in period t as V_t; the unit input value of period $t+1$ is, then, V_{t+1}. Because one period's inputs consist of the previous period's outputs, V_{t+1} must also be the unit *output* value of period t. We can therefore write:

$$V_{t+1}Q_t = V_tA_t + N_t. \tag{5}$$

Substituting the solution values from eqs. (2)–(4) and dividing by Q_t, we obtain a difference equation for unit values

$$V_{t+1} = V_ta + n(d/b)^t \tag{5'}$$

(where $a = A_o/Q_o$ and $n = N_o/Q_o$), for which the solution is

$$V_t = (V_o - n/[(d/b) - a])a^t + (n/[(d/b) - a])(d/b)^t;\ a < 1 \tag{5''}$$

(V_o is the unit value corresponding to the initial static equilibrium unit price, p_o.) For the economy to yield more output than the amount of material input used in production, it is necessary that $a < 1$.

The unit price is simply the unit value divided by the value of money (labor-time represented by the monetary unit). Letting μ stand for the value of money (a constant, since purely nominal price changes are ignored), the unit price in period t is

$$p_t = (p_o-\pi)a^t + \pi(d/b)^t;\ a \neq d/b, \tag{6}$$

where $\pi = (1/\mu)n/([d/b]) - a)$. Eq. (6) indicates that the level of the unit price over time depends principally on the growth of living labor relative to the growth of output, expressed by the ratio d/b. Since $d < b$, the amount of living labor required to produce a unit of output falls over time, and this increase in productivity due to mechanization leads the unit price to fall continuously as mechanization proceeds. Yet, although unit prices fall asymptotically to zero, it is incorrect to infer that a static equilibrium price — an identity of input and output prices — is approached. We evaluate the limit of the ratio p_{t+1}/p_t as $t \quad \infty$, noting that if and only if this limit equals one do input and output prices converge. From eq. (6), we obtain

$$\lim_{t \quad \infty} (p_{t+1}/p_t) = \begin{cases} d/b & \text{if } ab < d \\ a & \text{if } ab > d. \end{cases} \tag{7}$$

Eq. (7) shows that the unit price does converge to an equilibrium level — but to a *moving* equilibrium level, not a static equilibrium. Given continuous mechanization, d/b, as well as a, is less than one. The term in eq. (6) containing the smaller of d/b and a (in the "normal" case, a is smaller) vanishes as t becomes infinite and each period's output price approaches a constant fraction (less than one) of the period's input price, either d/b or a.

Under a regime of continuous mechanization, then, historical and replacement costs of capital do not converge, but increasingly diverge. The replacement cost becomes lower and lower, while the historical cost, of course, remains unchanged. According to the Okishio theorem, this cheapening of the constant capital can only raise the profit rate. Everything else being equal, however, the rate of return on the *original* outlay of financial capital must be less than the rate of return on the *devalued* capital.

It is true that capital does eventually become revalued according to the cost of reproducing it, and that capital devaluation therefore tends to raise the profit rate. The "resolution" of the discrepancy between original production costs and current reproduction costs, however, takes place through the many mechanisms of crisis, through the *forcible* adjustment of old values to the new. The underlying continuous tendency of the profit rate to fall therefore necessarily manifests itself discontinuously, in periodic crises. (The discussion below is confined to the underlying tendencies of the profit rate and the unit price, independently of periodic disruptions. It is assumed, in other words, that all purchases and sales are made at the commodity's true value.)

In the initial static equilibrium we assume exists prior to mechanization, the profit rate is

$$r_o = (Q_o\text{-}A_o\text{-}wN_o)/(A_o + wN_o + F_o) = (1\text{-}a\text{-}wn)/(a + wn + f), \quad (8)$$

where w is the constant real wage rate and $f = F_o/Q_o$. Eq. (8) will be useful as a benchmark with which to compare the tendencies of the "material rate of profit" and the actual rate of profit over time. The material rate of profit, r^m, results from the calculation of the profit rate on the basis of replacement costs, as in the Okishio theorem, when only a single output is produced. Fixed capital, inputs, and output all have the same unit price. In profit rate calculations, the unit price cancels out, leaving a profit rate that expresses a ratio of physical quantities alone. For the sort of continuous mechanization under consideration, the material rate of profit can be written as

$$r^m{}_t = (1\text{-}a\text{-}wn[d/b]^t)/(a + wn[d/b]^t + f), \quad (9)$$

so that, as t approaches ∞

$$\lim_{t \to \infty} r^m{}_t = (1\text{-}a)/(a + f). \quad (9')$$

The limit of the material rate of profit under continuous mechanization is clearly greater than r_o; the material rate of profit rises continuously throughout time as the wage cost per unit of output falls.

But let us examine the actual profit rate, calculated on the basis of historical costs. The actual profit rate in period t is

$$r_t = (p_{t+1}Q_t - p_tA_t - p_twN_t)/(p_tA_t + P_twN_t + \overline{p_t(\Delta F\ )_t}), \quad (10)$$

with the magnitude of the unit price obtained through eq. (6). A bit of algebraic manipulation lets us express the actual profit rate as

$$r_t = \frac{(1/\mu)n-(p_o-\pi)a^twn-\pi(d/b)^twn}{a(p_o-\pi)(ab/d)^t + \pi a + (p_o-\pi)a^twn + \pi(d/b)^twn + \dfrac{\overline{P_t(\Delta F)}_t}{Q_od^t}}. \quad (10')$$

$\overline{P_t(\Delta F)_t}$ gives the total historical cost of fixed capital in period t. $\overline{P_t}$ is a row vector $= [p_o\, p_1\, p_2 \ldots p_t]$, the elements of which are simply the varying prices at which additions to the fixed capital stock are acquired over time. $\overline{(\Delta F)_t}$ is a column vector $= [F_o\,(F_1\text{-}F_0)\,(F_2\text{-}F_1) \ldots (F_t\text{-}F_{t-1})]'$. It specifies the amount that is added to the capital stock in each period. Manipulation of the dot product of these two vectors gives

$$\overline{P_t(\Delta F)_t} = F_o[p_o + (p_o-\pi)(b-1)(1/b)\Sigma^t{}_1(ab)^i + \pi(b-1)(1/b)\Sigma^t{}_1(d)^i] \quad (11)$$

(where periods are 0, 1, . . . , i, . . . , t), so that as $t \longrightarrow \infty$,

$$\lim_{t\to\infty} \overline{(P_t(\Delta F)}_t/Q_od^t) = F_o[p_o/Q_od^t + (p_o-\pi)(b-1)a/(1-ab)Q_od^t + (\pi(b-1)d/bQ_o)\Sigma^t{}_1(1/d)^i]. \quad (11')$$

If $d < 1$ — that is, if mechanization reduces the extraction of living labor absolutely — the actual profit rate not only fails to rise, in the manner of the material profit rate, it actually falls to zero. All terms in (11') become infinitely large over time; thus the entire denominator of the profit rate goes to infinity, while the numerator goes to $(1/\mu)n$. Even if $d = 1$, so that extraction of living labor remains constant, the profit rate still falls to zero, since the final term in square brackets in eq. (11') becomes

$$(F_o\pi(b-1)/bQ_o)\Sigma^t{}_1(1)^i = f\pi(b-1)(1/b)\times t.$$

As t goes to infinity, this term and therefore the denominator of the profit rate also go to infinity.

Hence, if extraction of living labor fails to increase, the profit rate must fall to zero, irrespective of any and all increases in productivity or cuts in money wages, and in striking contrast to the continuous rise in the material profit rate. To understand the economic-philosophic reason why, it is instructive to return to eq. (5), which we rewrite now in the following form:

$$V_{t+1}Q_ob^t\text{-}V_tA_ob^t = N_od^t. \quad (5''')$$

Eq. (5''') shows clearly that the total value of the net product in period t is simply the living labor added in that period. If the extraction of living labor fails to increase, then profit must stagnate, no matter how low money wages are, no matter how productive the technology is. It should now be clear that eqs. (5)–(5''') do not use labor-time as a convenient numeraire or "accounting framework," but rather express the fundamental proposition of Marx's value/surplus-value theory. Control and use of other people's labor is the organizing principle of the capitalist system. It is the only fuel on which the capitalist engine runs. Expulsion of living labor through mechanization spells the doom of the system.

If the expulsion of living labor is relative rather than absolute, i.e., if $d > 1$, then the first two terms in eq. (11') go to zero over time. The final term goes to the finite limit $[f\pi d(b-1)/b(d-1)]$. After a bit more manipulation,[2] the limit of the profit rate can be expressed as

$$\lim_{t \to \infty} r_t = \frac{1-a(b/d)}{a(b/d) + f[(b-1)/(d-1)]}. \qquad (10'')$$

The actual and material profit rates thus tend to two different limits under continuous mechanization. The limit of the material rate is *always* higher than the limit of the actual rate. Computer simulations reveal that the material profit rate always rises continuously and asymptotically to its limit. The actual profit rate takes one of two paths. It may fall continuously and asymptotically to its limit. Or, when wages are initially high *and* mechanization is rapid (d/b is low), an initial upward rise will occur, always followed by an asymptotic fall of the profit rate to its limit. In both cases, the actual rate in any period is always lower than the material rate.

As eq. (10") indicates, moreover, the tendency of the actual profit rate is sensitive to the pace of mechanization — of which the term [(b-1)/(d-1)] provides an index. The greater the pace of mechanization, the greater is the tendency of the profit rate to fall. Finally, because the input price of constant circulating capital never converges to the output price of the same period, the actual profit rate may fall (i.e., its limit may be less than r_o) even in the absence of fixed capital.

This exercise has not demonstrated that the rate of profit *must* fall, though it has shown that if the pace of mechanization is rapid enough or if extraction of living labor fails to increase, the profit rate *will* fall. It bears repeating, however, that the law of the falling profit rate has faced two very strong tests here; not only the constancy of the real wage rate, but also the constancy of the output-capital ratio.

Would a profit-maximizing capitalist or state planner actually adopt the sort of continuous mechanization modelled above? As long as the same set of prices is used to value output, inputs, and fixed capital, as is assumed in the Okishio theorem, computations would always indicate that each period's new technique should be adopted. Because the unit price will cancel out of the formula for the expected profit rate, the expected profit rate will always equal next period's material profit rate. As we have seen, the latter is higher than both the current material profit rate and the current actual rate. If, however, the fall in the unit price is anticipated, the new techniques may still be adopted because the innovating capitalist's profit rate may rise, due to the difference between the individual and social values of the commodity, while his/her competitors' profit rates and the general profit rate fall. It is important to recognize that the Okishio theorem seemed to refute this argument *only* because it seemed to show that any new technique that raised the individual profit rate would also raise the general rate (given the constancy of the real wage rate, etc.).

Finally, in contrast to the Okishio theorem, the payment of wages in real capitalism bears no monotonic relation to the amount of labor sweated out of the workers. The power of workers — united, disciplined, and organized by the mechanism of production — always threatens to raise the wage rate *per unit of actual labor activity* to uncontrollable and unacceptable levels, through strikes, slowdowns, increased supervisory costs, and so forth, not to mention the potential of workers to rise up and take control of production. In such an environment of "uncertainty," very good *microeconomic* reasons suggest to the captialist that profitability depends on reducing this uncertainty. Mechanization — the reduction of the worker to an appendage of the machine — is the key way in which the capitalist tries to gain control of the factory, to further the implementation of his/her (microeconomic) plan, and thus to raise expected profitability.

NOTES

1. For a fuller discussion of the difference between the value of capital and the value of means of production, see Kliman and McGlone (1988).
2. Note that $(ab/d)^t$ in eq. (10') goes to zero as t goes to infinity since, if the output in any period is to exceed the constant circulating capital required for the next period, ab must be less than one.

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