A Temporal Single-system Interpretation of Marx's Value Theory

ANDREW J. KLIMAN[1] & TED MCGLONE[2]

[1]*Department of Social Sciences, Pace University, 861 Bedford Road, Pleasantville, NY 10570-2799, USA*
[2]*Department of Social Sciences, St. Joseph's College, 155 W. Roe Boulevard, Patchogue, NY 11772, USA*

This paper presents an interpretation of the quantitative dimension of Marx's value theory in which prices and values are determined interdependently and within historical time. This interpretation is then shown to refute allegations that his value theory suffers from internal inconsistencies. Among the issues considered are Marx's law of the falling rate of profit, the alleged redundancy of value, value determination under joint production, and the 'transformation problem.' The 'single-system' interpretation of values and prices as interdependent eliminates the alleged inconsistencies that pertain to magnitude; the 'temporal' interpretation of value and price magnitudes as determined in historical time eliminates the alleged inconsistencies that pertain to determination.

1. The Centrality of the 'Internal Inconsistency' Issue

For more than a century, the main line of critique of Marx's value and profit rate theories, coming from both Marxist and non-Marxist economists, has been an internal one. That is, rather than arguing that these theories lack fruitfulness or empirical relevance, the critics have for the most part attempted to show that they suffer from insuperable internal inconsistencies. Indeed, this line of critique makes argumentation superfluous: if Marx's theories are untenable even in their own terms, they must necessarily be revised or rejected. As Brewer (1995, p. 140) has put the point recently,

> It might be possible to argue that *Capital* was a success in Marx's own terms. ... Even by this standard, however, *Capital* must be counted a magnificent failure. ... Much of the debate over Marx's economics has focused on [the internal coherence of his value theory and law of the tendential fall in the profit rate], and for good reason. If both fail, as they do, not much is left.

During the past decade and a half, however, researchers from around the world, often unknown to one another, have been engaged in an elemental rethinking of the issues, out of which a new interpretation of the quantitative dimension of Marx's value theory has begun to crystallize. First dubbed the 'temporal single-system' interpretation in Skillman (1995), it vindicates the internal consistency of Marx's most challenged theoretical results without relinquishing his theory's quantitative determinacy or absorbing it into the

theories of his critics.[1] The aims of the present paper are to acquaint a general audience of economists for the first time with this little known reconceptualization, and to show that the inconsistencies which have been ascribed to Marx's theory in fact stem from the standard interpretation of it, not the original.

Handed down from Bortkiewicz (1952, 1984) and codified in recent surveys such as Desai (1988) and Howard & King (1992), the standard interpretation construes the values and prices of Marx's theory as two separate and atemporally determined systems. It is *atemporal* in that the values and prices of inputs are determined simultaneously with, and are therefore necessarily equal to, the values and prices of outputs. The temporal single-system interpretation, in contrast, holds that Marx conceives of the values and prices of inputs as determined *before* the values and prices of outputs, so that the former become determinants of the latter. For instance, the value of constant capital (outlays on means of production), given at time of input, becomes a formative element in the value of the output, and the rate of profit measures the profit yielded at one time in relation to the capital advanced at an earlier time.

The standard interpretation is a *dual-system* interpretation in that price magnitudes are not determinants of values, nor are value magnitudes determinants of prices. The temporal single-system interpretation of Marx's theory, in contrast, holds that the value of capital advanced depends on the prices, not the values, of the inputs, and the aggregate price of output depends on the surplus-labor and surplus-value performed in capitalist production.[2]

Formally speaking, these two differences are all that separate the temporal single-system and standard interpretations. Simple and seemingly technical though they are, they have immense ramifications. We shall show that conceiving of values and prices as a single system eliminates the alleged inconsistencies in Marx's theory that pertain to *magnitude* (e.g. the discrepancy between the value and price rates of profit). More importantly, temporal determination of values and prices eliminates the alleged inconsistencies that pertain to *determination* (e.g. the law of the tendential fall in the profit rate).

We shall refrain from criticizing simultaneous valuation and dual-system formalisms in terms of their coherence, power or desirability as theoretical propositions or paradigms. Our focus is solely interpretative: must Marx's own value theory be judged internally inconsistent, or can it be conceptualized in

---

[1] The terms 'sequential' and 'non-dualist' have sometimes been used as alternatives to 'temporal' and 'single-system.' For an introduction to this interpretation, see the papers by Carchedi & de Haan, Freeman, Kliman, and McGlone & Kliman in Freeman & Carchedi (1996). Temporal single-system contributions by other authors include Ernst (1982), Giussani (1991–92), Maldonado-Filho (1997) and Ramos (1997). Yet because this body of work is not homogeneous, we emphasize that others are not responsible for the views contained in this paper, which is (as the title indicates) *a*, not *the*, temporal single-system interpretation. Our ideas have, however, been clarified greatly through discussions with the individuals noted above, and through intensive debates with the members of the Outline of Political Economy e-mail discussion list. We have also benefited from the comments of two anonymous referees.

[2] Several authors, including Wolff *et al.* (1984), Lee (1993), Moseley (1993) and Ramos & Rodriguez (1996), have put forth what can be called a simultaneous single-system interpretation, since they argue that prices and values in Marx's theory are interdependent, but concur with the atemporal determination of the standard interpretation.

terms that render it coherent? It is both possible and plausible that one concurs with our interpretation while rejecting the theory of Marx's emerging from the interpretation. We thus urge the reader to judge our work as an interpretation of Marx's own value theory, not as a new theory or approach in its own right, which it is not.

We shall also refrain from evaluating Marx's value theory. Not only is that a separate topic, but evaluation is, in a sense, premature. If the standard interpretation can no longer be taken for granted, it then becomes necessary first to return to the task of comprehending Marx's theory. It is also necessary first to come to grips with the charge of internal inconsistency since, as we noted above, this charge has disqualified the 'uncorrected' theory at the starting gate, thereby largely precluding an evaluation of it. If we succeed in refuting the allegations of internal inconsistency, then Marx's erstwhile critics should set the historical record straight by acknowledging that his value theory can be considered internally coherent. Only then can an evaluation of its fruitfulness and relevance begin.

In the next section, we present *Capital*'s concepts of value, price and profit in their original terms, and then show how the standard and temporal single-system formalisms result from two different readings of the same concepts. Following that, we assess these different readings in light of the textual evidence. The remainder of the paper turns to Marx's theoretical results to demonstrate that the present interpretation is able to replicate them. This demonstration fulfills two aims at once. First, by showing that these theoretical results *can* follow from Marx's concepts, it vindicates the internal consistency of his value theory. Second, since the ability of one interpretation to make sense of conclusions that other interpretations cannot accommodate clearly recommends it as a superior interpretation, the demonstration constitutes important evidence in favor of the temporal single-system interpretation.

2. Value, Price and Profit

2.1. Marx's Theory

Marx held that commodities have determinate values, expressed in money prices, *before* they enter into circulation (Marx, 1977, pp. 220, 260; 1981, p. 352). On the basis of this claim he demonstrated—even prior to addressing how either values or prices are determined—that the amount of value some commodity owners gain in exchange must be offset exactly by others' losses (Marx, 1977, pp. 260–266). A mere transfer of titles cannot cause commodities to gain or lose value in the aggregate. Although all commodities may sell at money prices greater than their 'true value[s]', Marx argued in the standard manner that this type of gain is merely 'nominal': if everyone both sells and buys everything for 10% more than it is really worth, they lose as sellers what they gain as buyers (Marx, 1977, p. 263).

'Real,' i.e. quantitative, differences between prices (the amount of value received for commodities) and commodities' true values, on the other hand, stem

from a variety of other phenomena, including rent, monopoly, interest, and the tendency for profit rates to equalize. To study the impact of these phenomena, Marx abstracted from nominal price–value differences by holding constant the relationship between the labor-time and money measures of value (see for example Marx, 1981, pp. 142, 266). We shall follow this procedure throughout the paper, taking \$1 to be equivalent to 1 labor-hour. All value and price magnitudes can thus be understood equally as amounts of money and of labor-time.[3]

In any industry i, the sums of value advanced by capitalists to acquire means of production and pay wages are what Marx calls constant capital (c_i) and variable capital (v_i), respectively. He argued that the value of constant capital only 'reappears' in the value of the product (λ_i), while living labor (l_i) generates all new value. After being hired, workers are forced to perform an amount of labor equivalent to their wages, plus an excess amount, surplus-labor, which generates a surplus-value ($s_i = l_i - v_i$).[4] Abstracting from fixed capital for simplicity, so that the entire value of constant capital is 'transferred' to the product, its value can thus be expressed as

$$\lambda = c + v + s = c + l \qquad (1)$$

where all terms are row vectors of the λ_is, etc.

The value received for a commodity, its sales price (p_i), will typically differ from its own value, due to a gain (or loss) of some amount of value, g_i, in exchange. Prices can thus be expressed as

$$p = c + v + s + g \qquad (2)$$

An industry's profit, $\pi_i = p_i - (c_i + v_i)$, will accordingly differ from the surplus-value it produces, precisely by the amount g_i; that is, $\pi - s = g$. Since g_i is also the size of price–value difference, price–value and profit–surplus value differences are necessarily equal in size. The industry's value (value produced) profit rate is $s_i/(c_i + v_i)$, and its price (value received) profit rate is $\pi_i/(c_i + v_i) = (s_i + g_i)/(c_i + v_i)$.

2.2. The Standard Interpretation

Defining $\mathbf{A}$ as a square input–output matrix and $\mathbf{b}$ as a column vector of real wage components per unit of labor performed, the standard interpretation

---

[3]   Despite some recent confusion, Marx's critics (e.g. Bortkiewicz, 1952, pp. 9, 11) have traditionally recognized that he measured both values and prices in money. Their claim that his account of the value/price transformation was self-contradictory, because inputs were left in value terms when outputs were transformed into price terms, concerns the magnitude of the capital advanced for inputs, not its unit of measurement.

[4]   Some authors (e.g. Vroey, 1982) argue to the contrary that the work that workers actually perform is not abstract and thus does not create value. Instead, value is ultimately determined in the market because it is what turns workers' concrete labor into abstract labor. We follow the interpretation of Dunayevskaya (1988, pp. 103–105, 112–115), who argued that the capitalistic dehumanization of the production process makes workers' activity abstract within that process.

translates Marx's c, v and s as $c = \lambda A$, $v = \lambda bl$, $s = l - \lambda bl$. The value of Equation (1) is thus expressed as

$$\lambda = \lambda A + \lambda bl + (l - \lambda bl) = \lambda A + l \qquad (3)$$

Clearly, both premises of the standard interpretation are embedded in Equation (3). The output values on the left-hand side are identical to the input values on the right, so that input and output values are simultaneously determined, and the values of outputs are determined independently of prices.

The standard interpretation of Marx's theory of price determination has been formalized, to our knowledge, only in the special case in which uniform profitability prevails. Yet the basis of the dual-system conception—the claim that when commodities do not exchange at values, inputs must be 'transformed' from value magnitudes into price magnitudes—has nothing to do with this special case *per se*.

Rather, the dual-system premise implies a more elemental and general critique of Marx's value theory: price–value and profit–surplus value differences cannot be identical. In Equation (3), values equal the *values* of means of production and subsistence plus surplus-value, but prices obviously equal the *prices* of means of production and subsistence plus profit. Hence, prices differ from values not only because profits differ from surplus-values, as Marx held, but also because prices and values of inputs differ. It thus follows from the dual-system premise that Marx erred when he held that gains and losses of value in exchange are the sole source of both price–value deviations and profit–surplus value deviations.

2.3. The Temporal Single-system Interpretation

The temporal single-system interpretation translates Marx's c, v and s as $c_t = p_t A$, $v_t = p_t bl$, $s_t = l - p_t bl$.[5] The vector p_t is the set of actual market prices that prevail at the moment when means of production and labor-power enter production; these inputs are therefore valued at the *current* prices of that moment. If production uniformly takes one period, then this interpretation holds that Marx's unit output values are

$$\lambda_{t+1} = p_t A + p_t bl + (l - p_t bl) = p_t A + l \qquad (4)$$

and unit output prices are

$$p_{t+1} = p_t A + l + g_t \qquad (5)$$

Equations (4) and (5) embody the claim that valuation in Marx's theory is temporal, not simultaneous, and they constitute a single system, since the values of outputs depend on the prices of inputs. Yet the other single-system inter-dependence—the dependence of output prices on value magnitudes—is not yet apparent. This is because, first, it holds only in the aggregate, and second, it is

---

[5] Although we omit time subscripts for A, b, and l, they do vary from period to period. None of the following analysis or results depends on the constancy of these coefficients, or preclude it as a special case.

not merely a tautological consequence of Marx's concepts. It instead follows from his conclusion that exchange cannot cause value to be gained or lost in the aggregate.

This conclusion can be expressed as $g_t x = 0$, where x is a column vector of gross outputs. From Equation (5) it follows that

$$p_{t+1}x = p_t Ax + lx + g_t x = p_t Ax + lx$$

so that

$$p_{t+1}x - p_t Ax = lx \tag{6}$$

Equation (6) shows that Marx's argument implies that, nominal changes aside, the sole source of value added in price terms in any period is a value magnitude—the living labor performed in the capitalistic production process.

3. Textual Evidence Concerning Marx's Concepts

The temporal single-system interpretation is certainly unconventional. Yet we think the textual evidence strongly suggests that it adequately reconstructs the quantitative dimension of Marx's own value theory. In this section, we examine one kind of textual evidence from Marx, passages in which he explicates his concepts of value, price and profit. Although space limitations do not allow us to present all of the relevant evidence, we think the selection of it reviewed below suffices to demonstrate that it is at least plausible to understand Marx's concepts of value and price as constituting a single system determined in historical time. The following sections will then turn to another type of textual evidence that we consider even more compelling: in contrast to other interpretations, the temporal single system interpretation leads to theoretical results that correspond to those of the original texts and render them internally coherent.

3.1. *One System or Two?*

The controversial feature of the single-system interpretation is that the values of constant and variable capital depend on the prices, not the values, of means of production and subsistence. Hence, we restrict our assessment to this proposition.

When first explicating the concepts of constant and variable capital, Marx (1977, p. 317) wrote that 'The means of production on the one hand, labour-power on the other, are merely different forms of existence which the value of the original capital assumed when it lost its monetary form and was transformed into the various factors of the labour process.' The value of capital is therefore not synonymous with the values of inputs purchased with it. Before being tied up in production, the capital-value firsts exists as a sum of money. The capital-value is this sum, *the sum of value advanced to acquire inputs*, which can clearly differ from the values of the inputs themselves.

To be sure, this difference was rarely discussed in *Capital* I, because from Chapter 6 onward, Marx generally assumed that commodities are bought at their values. Yet soon after the passage just quoted, Marx (1977, pp. 317–318) noted

that if the price of cotton doubles, 'it transfers to the product a [doubled] value,' thus suggesting that the value transferred from constant capital depends on the cotton's price, not its value.

In Chapter 6 of Volume III, Marx argued not only that price fluctuations affect the value profit rate, $s/(c + v)$, by altering the constant capital advanced, but also that his argument concerning 'the effect that these price fluctuations have on the profit rate … is just as valid if prices rise or fall not as a result of fluctuations in value, but rather as a result of the intervention of the credit system, competition, etc.' (Marx, 1981, pp. 201, 208). On the following page he wrote that 'the same causes that raise or lower the *price* of the product also raise or lower the *value* of the capital' (Marx, 1981, p. 209, emphases added). Similarly, a passage in Marx (1971, p. 223) argues that if the value of cotton falls but the price of cotton falls to an even greater extent 'through the law of demand and supply,' the profit rate of a firm which uses the cotton as an input increases more than 'it would have increased had the cotton which has become cheaper been sold at its value.'

Ironically, much of the evidence which indicates that Marx distinguished the value of capital from the value of the inputs it purchases is contained in the very passages that critics cite in order to argue that he admitted his error in not 'transforming' the value of capital into price terms. For instance, shortly after first discussing the divergence of production prices from values, Marx (1981, p. 265; cf. p. 309) noted that the commodity's cost price will likewise be affected:[6]

> As the price of production of a commodity can diverge from its value, so [can] the cost price of a commodity, in which the price of production of other commodities is involved … . It is necessary to bear in mind this modified significance of the cost price … if the cost price of a commodity is equated with the value of the means of production used up in producing it, it is always possible to go wrong.

This passage not only distinguishes clearly between the sum of value advanced and the value of the inputs acquired with the advance; it even cautions readers against 'go[ing] wrong,' as they have for the past 90 years, by equating the two concepts.

Another detailed passage (Marx, 1971, p. 167) observes similarly:

> the cost-price of constant capital—or of the commodities which enter into the value of the newly produced commodity … may likewise be either above or below its value. Thus … the difference between cost-price [production price] and value, insofar as it enters into the price of the new commodity independently of its own production process, is incorporated into the value of the new commodity as an antecedent element.

If, for instance, λ and p are the value and the production price of steel, then the difference, $p - \lambda$, is incorporated into the value of trucks produced with the steel, and the sum of value paid for the steel, $\lambda + (p - \lambda) = p$, reappears in the value of the trucks.

---

[6] 'Cost price' is Marx's term for the variable capital plus used-up constant capital laid out for the commodity's production. 'Price of production' is the price that yields the firm the average rate of return on capital advanced.

At least two passages suggest that the division of the workday between necessary and surplus-labor, and accordingly between variable capital and surplus-value, is affected by deviations of production prices from values: if workers consume 'commodities whose prices of production are different from their values [they] must work for a greater or lesser amount of time in order to buy back these commodities (to replace them) and must therefore perform more or less necessary labour' (Marx, 1981, p. 309; cf. p. 261). Marx (1981, p. 1001) later notes another factor that modifies the *value* of labor-power: 'If the commodity with the monopoly price is part of the workers' necessary consumption, it increases wages and thereby reduces surplus-value, as long as the workers continue to receive the value of their labour-power.' For Marx, then, 'necessary labor,' 'variable capital' and 'value of labor-power' are not determined solely by the amount of labor-time needed to reproduce (or the value of) means of subsistence. Rather, they depend on the actual amount of value laid out in wages, which in turn depends on the price of means of subsistence.

Given all of this evidence, what explains the persistence and dominance of the dual-system interpretation? Three things, we believe.

First is the tendency to read *Capital* linearly rather than dialectically. Initial statements that commodities' values are determined by the labor-time they contain, or needed to reproduce them, are read as transparent definitions requiring no enrichment of meaning, so that Marx's subsequent development of the concept of value is forced either to conform equally transparently to the definitions or to be judged self-contradictory.

Second is the tendency to read the apparent definitions in a technological determinist way, such that the amount of labor-time a commodity contains cannot be affected by price-value deviations. Yet after indicating that these deviations modify cost prices, Marx (1981, pp. 265, emphases added) immediately argued that a commodity's cost price and value are still determined by the amounts of labor they contain: 'cost price simply depends on the quantity of paid *labor it contains*, while the value depends on the total quantity of *labor it contains*.[7]

Third is the tendency, initiated by Bortkiewicz (1952, pp. 5–6), to regard value and price merely as divergent exchange ratios. This perspective makes it impossible to conceive that, although the money advanced for inputs is a 'price' sum, it is nevertheless a sum *of value*. It thus also makes it impossible to conceive that, when Marx distinguished between the values of constant capital and means of production, he was alerting readers to this distinction, not admitting having made an error.

---

[7] Was Marx unaware that he had made two contradictory statements in this one paragraph? Or, in saying that value is determined by labor-time, may he have meant all along that (a) the product's value is determined by the living labor added plus the labor represented by the money needed to *acquire* the means of production; and (b) since these means of production themselves are needed, it is the amount of labor-time needed to acquire them, rather than to reproduce them, which is part of the total labor-time needed to reproduce the product? We suggest that the preferred interpretation is that which best makes sense of the text as a whole, its concepts as well as its theoretical results.

3.2. *Temporal or Simultaneous Determination?*

The temporalist Equations (5) indicate that, rather than the two being determined simultaneously, the value of used-up constant capital is determined before, and is thus a determinant of, the value of the product. This reading is controversial because many passages in Marx's texts may seem to suggest the opposite. In these passages, he held not only that a commodity's value is determined by the cost of reproducing it rather than its actual historical cost, but also that price changes lead to the revaluation of already purchased stocks and machines. Hence, the capital-value needed to acquire them, and the value transferred from them to new products, likewise change (Marx, 1977, pp. 318–319; 1981, pp. 207–208; 1969, p. 109).

Perhaps because the alternative to historical cost in conventional accounting terminology is replacement cost, some authors (e.g. Wolff *et al.*, 1984, p. 133; Moseley, 1993, p. 168) have therefore inferred that Marx held the value of constant capital to be determined by the post-production replacement cost of means of production. Mirowski (1989, pp. 180ff), on the other hand—correctly stressing that this notion is incompatable with such concepts of Marx's as the 'preservation' of the value of constant capital through its 'reappearance' in the product—charges that his texts vacillate between historical and replacement cost valuation.

These, however, are not the only possibilities. If an item produced yesterday is warehoused until used as an input today, and the product is finished tomorrow, the cost of reproducing the item when it enters into production is neither yesterday's price nor tomorrow's price, but today's price. The present interpretation is thus consistent with Marx's rejection of historical valuation.

Yet a good deal of evidence directly suggests that Marx also rejected replacement cost valuation and the simultaneous determination implied by it. In one very important passage, he rejected an early articulation of the 'corn-ratio' theory of the profit rate. Torrens had argued that, if 100 quarters of corn are used to produce 120 quarters, the farmer's profit is 20 quarters. In his critique, Marx (1971, p. 79) objected not only to Torrens' unit of measurement, but also to his *quantitative* claim: 'the value of 100 quarters can be greater than that of 120 quarters … . Thus, on the basis of one example which has *nothing* to do with profit, with the surplus in the *value* of the product over the *value* of the capital outlay, Torrens draws conclusions about profit.' Simultaneous valuation of inputs and outputs would imply, to the contrary, that the value of the 100 quarters of input must be less than that of the 120 quarters of output, even if technology had improved in the meantime (because each quarter, whether input or output, must have the same value). It would also imply that, whatever the units of measurement used, Torrens' ratio of profit to outlays is correct.

When first introducing the concept of value transfer in *Capital*, Marx (1977, p. 314) wrote: 'In the labour process [a means of production] serves only as a use-value, a thing with useful properties, and cannot therefore transfer any value to the product unless it possessed value before its entry into the process.' Similarly, a draft manuscript states: 'that part of capital which enters into the production process … as raw material or tools, does not add more value to

the product than it possessed before production' (Marx, 1971, p. 178). These passages are again incompatible with the determination of constant capital-value at replacement cost. The latter implies that a means of production will 'transfer' more value than it possessed before production if its post-production value is greater.

Other passages in draft manuscripts similarly hold that the sum of value transferred is determined by the pre-production cost of the means of production:

> the values of the material and means of labour only re-appear in the product ... to the extent that they ... were values before they entered into the process ... . If [their] value changes before the new product of which they are the elements is finished they nevertheless relate to it as independent, given values preposited to it. (Marx, 1988, pp. 79–80)

> the value of this constant part [of capital] can fall or rise, depending on whether the commodities of which it is composed have to be reproduced at lesser or greater cost. [Yet ...] in the process of production, into which it enters as a condition of production, it is a postulated value which must reappear in the value of the product ... a definite quantity of past, materialized labour, which passes into the value of the product as a determining factor. (Marx, 1969, p. 109)

Marx thus held that the constant capital-value transferred to the product is given (preposited, postulated) by the cost of the means of production when they enter into production. As a given sum that only reappears in the product, it is a 'determining factor' of the product's value, not a simultaneously determined one.

Marx seems not to have explained why he rejected the notion that the values of capital advanced and outputs were determined simultaneously. Perhaps this is because he thought it to be self-evident that this notion is incompatible with the theory that a commodity's value is determined by the costs incurred in its production, according to which an average firm that sells its products at value recovers the full (paid and unpaid) costs incurred in producing them. Replacement cost valuation implies that, when values are falling, firms that buy and sell at value suffer losses, because the replacement cost of means of production, recovered through sale, is less than the sum of value actually advanced for them.

Another controversial aspect of the temporalist interpretation is its related claim that Marx conceived of profit as the difference between revenue and the consumed capital-value actually advanced, since others (e.g. Wolff *et al.*, 1984; Naples, 1989) hold that profit is given by the difference between revenue and the replacement cost of capital. We deal with the theoretical aspects of this important controversy in the Appendix. On a purely textual level, however, we know of no evidence that Marx defined profit as an excess over the replacement cost of capital.

On the contrary, he explicitly defined surplus-value as the difference between the sum of value that 'is finally withdrawn from circulation' and the original sum 'thrown into it at the beginning' (Marx, 1977, p. 251). Profit is likewise defined as 'an excess over and above the total capital advanced' or

invested (Marx, 1981, p. 133). Other passages indicate even more clearly that Marx used 'profit' to mean the increase in the value of capital at the end of a period over the sum of value advanced at the start: 'Profit … expresses in fact the increment of value which the total capital receives at the end of the processes of production and circulation, over and above the value it possessed before this process of production, when it entered into it' (Marx, 1991, p. 91). 'The relation between the value antecedent to production and the value which results from it—capital as antecedent value is capital in contrast to profit—constitutes the all-embracing and decisive factor in the whole process of capitalist production' (Marx, 1971, p. 131).

4. Results Pertaining to Magnitude

Although we think the textual evidence just presented is persuasive, it alone cannot be decisive. Unfortunately, no meta-interpretative standards exist that would compel proponents of one interpretation to abandon it rather than to explain away others' evidence, so discussion of isolated concepts seems inevitably to degenerate into chronically indeterminate battles of quotations. In response to this problem, we turn now to a second kind of textual evidence, theoretical results. We shall show that, whereas other interpretations often cannot replicate Marx's theoretical results (and proponents of these interpretations thus declare his value theory to be internally inconsistent), the temporal single-system interpretation can indeed do so. This evidence both vindicates the internal coherence of the original theory and suggests that the present interpretation corresponds to the original in a way that others do not. In this section, we examine those disputed theoretical results which concern magnitude—aggregate value–price equalities and the positivity of value and surplus-value under joint production—and show that the single-system dimension of our interpretation renders these results coherent. The next section shows that the temporal dimension of our interpretation vindicates those results which concern determination.

4.1. Aggregate Price/Value Equalities

Marx's result that value cannot be gained or lost in exchange ($gx = 0$) is a powerful one. Examination of his Equations (1) and (2) in light of it shows immediately that total price (px) equals total value (λx) and total profit ($\pi x = px - [cx + vx] = sx + gx$) equals total surplus-value (sx). The aggregate price and value rates of profit are also necessarily equal, since $\pi x/(cx + vx) = (sx + gx)/(cx + vx) = sx/(cx + vx)$.

   These conclusions, of course, cannot be derived from the standard interpretation. As we discussed above, the dual-system distinction between the value and the price of capital advanced implies that price–value and profit-surplus value deviations are not identical, and that at least one of these two sets of deviations is not reducible to gains and losses of value in exchange. Hence, even if $gx = 0$,

it is impossible in general to obtain at least one of the two aggregate equalities.[8] Moreover, the dual-system premise implies that the denominators of the aggregate price and value profit rates (the price and the value of capital advanced) differ, so even if the numerators (total profit and total surplus-value) are equal, the two profit rates will differ.

A ninety-year span of critique thus maintains that Marx's simple conclusions cannot be sustained, so his value theory is internally inconsistent and needs to be corrected or rejected. Yet his conclusions follow trivially from the temporal single-system translation of the terms of his argument. Computing total price and total value by means of Equations (4) and (5), it is obvious that they are equal if $g_t x = 0$. Since, in addition, the value and price of capital advanced are the same thing, it follows that both total profit and total surplus-value, and the price and value profit rates, are also equal.

These results justify what would otherwise be an arbitrary premise, namely that the price of the means of production is a sum of value, a quantum of labor-time expressed in money. The justification is of the circular, Hegelian type. It follows from the aggregate price–value equality which depends on this premise that, even though the prices of the component parts of the total product differ from their own values, these prices are nonetheless sums of value. The price sum transferred to outputs from those components that become means of production is therefore also a sum of value. What was initially taken as a premise has thus been substantiated as a result.

We have not yet said a word about the infamous 'transformation problem'. Yet because, as we have just shown, the single-system interpretation replicates— and thus demonstrates the internal coherence of—Marx's results concerning aggregate price–value equalities in the general case, it necessarily does so also in the special case in which profit rates are equalized. The logical objection to Marx's account of the transformation of values into production prices cannot be sustained.[9]

4.2. *Effect of Nominal Price Changes: a caveat*

At the risk of digressing, it is imperative to clarify that the above results *do not* mean that Marx's value theory, in the original or in its single-system interpretation, can predict the aggregate magnitudes of price and profit in money terms. These nominal sums are not constrained by total value and surplus-value as

---

[8] It is of course possible to obtain both equalities by imposing them as invariance postulates, as Naples (1989) does. This is clearly one postulate too many. The additional constraint itself makes it impossible in a two-sector economy for commodities' prices and values to differ. It can also prevent profit rates from even tending to equalize since, to satisfy both postulates, a rise in the profit rate of a less profitable sector can require an even larger rise in the profit rates of more profitable sectors. The two constraints together imply, moreover, that the aggregate values and prices of consumed means of production and subsistence are equal, a conclusion which Marx (1981, p. 309) explicitly rejected. The twin postulates thus rescue some of Marx's theoretical results only at the expense of contradicting others.

[9] Bortkiewicz's (1952, pp. 8–9) original 'proof' of Marx's inconsistency, however, was not that the aggregate equalities fail to hold, but that Marx's account leads to a spurious breakdown of the economy because his input prices differed from his output prices. We take up that objection in the Appendix.

measured in labor-time. As Marx himself noted: 'To the degree that corn is *sold above its value*, other commodities … are, to the same degree, sold *below their value*, and, to be sure, even if their own money price does *not* fall. The *sum of values* remains the same, even if the expression of that total *sum of values* were to grow in money [terms]' (Marx, 1975, pp. 187–188).

Thus if, for whatever reason, firms are able to sell their products for an aggregate money price that differs from aggregate value, as measured in money according to the pre-production money–labor time equivalence, the money–labor time relation changes by the same proportion, and therefore so do the money *values* of the products.[10] All aggregate equalities between price and value magnitudes therefore continue to hold in monetary terms. If, conversely, adjustment for the nominal change in prices is made by holding the money–labor time equivalence constant, all aggregate equalities also continue to hold (see McGlone & Kliman, 1996, pp. 43–44 for an example). Since the issue here concerns measurement alone, this argument applies to both fiat money and commodity money.

The change in the money–labor time equivalence does imply that, although the price (value received) and value (value produced) profit rates continue to be equal, whether measured in labor-time or in money, the money and labor-time measures of profitability diverge. A systematic divergence between the movements of these two measures, however, requires a continual change in the money–labor time relation. Even if this occurs, Marx's result that value cannot be gained or lost in exchange means that capitalists as a whole cannot generate extra profit simply by overcharging one another. He thus suggests that the divergence of the money profit rate from the labor-time rate is purely a nominal one (Marx, 1981, pp. 280–281).

4.3. Joint Production

By extending the standard interpretation of Marx's value theory to the joint production case, it has been shown that values and surplus-value can be negative, although prices and profits are positive (Steedman, 1977, pp. 150–162). Letting **A** and *l* now denote the total, not per unit, input requirements, and replacing the singly produced outputs *x* with a matrix of joint outputs **B**, Steedman and others define the vector of the commodities' values as

$$\lambda \mathbf{B} = \lambda \mathbf{A} + l \tag{7}$$

and, if wages are part of advanced capital, the production prices, p^*, are

$$p^*\mathbf{B} = p^*(\mathbf{A} + \mathbf{b}l)(1 + r^*) \tag{8}$$

---

[10] The interpretation of Marx's money–labor time relation popularized by the so-called New Interpretation (Duménil, 1983; Foley, 1982) yields incorrect conclusions because it overlooks changes in this relation between the beginning and end of a period. Imagine that, at time *t*, 1 labor-hour is represented by $1 and workers are paid wages of $99. If they subsequently work for 100 hours, they have been exploited, having supplied 1 hour of surplus-labor. Yet if the money–labor time relation has changed such that, at time $t + 1$, the replacement cost of the net product produced during these 100 hours is only $98, not $100, the New Interpretation holds that the wages of $99 represent $99(100 hr/$98) = 101.02 hours. Hence, the workers have exploited the capitalists.

where $r*$ is the uniform profit rate. Given the input–output data of Table 1, and equating the price of the net product to the total living labor performed, Steedman's prices and profit are positive, but the unit value of good 1 and surplus-value are negative.

He rightly argues that the absurdity of this result deprives these value magnitudes of all significance. Yet is it the cogency of Marx's own value theory that is called into question, as Steedman alleges, or merely the cogency of the standard interpretation, in particular its dual-system aspect?

The separation of values and prices into divergent systems emerged historically with Bortkiewicz's (1952, pp. 5–6) contention that, when examining quantitative price–value differences, prices and values must be regarded purely as two distinct sets of exchange ratios. The meaning of value is thus reduced to a particular kind of price, so that it becomes impossible, as we noted earlier, to conceive of advances for inputs as price sums that are likewise sums of value. The value system therefore requires its own separate measure of capital advanced, in which inputs are valued at imaginary exchange ratios proportional to the labor-times needed to produce them. To obtain this measure, one must obviously determine the value of the direct and indirect labor-time needed to produce each single commodity, as Equation (7) attempts to do.

In the case of joint production, however, single commodities simply do not have values. To attempt to determine the amount of labor-time needed to produce a commodity singly when it is only produced jointly is a meaningless and futile task. The dual-system interpretation of prices and values as mutually exclusive but symmetrical categories—each commodity has one price at which it actually exchanges and another at which it would exchange were exchanges governed by relative labor requirements—is thus revealed to be a bad analogy.

Once one rejects the notion that Marx's concept of value is a kind of shadow price, however, the non-existence of single values of joint products neither makes values meaningless nor renders them indeterminate. No less than in the case of single production, joint production in capitalism requires that sums of value be advanced and, in Marx's theory, production remains the production not only of material goods and services, but of value and surplus-value as well. By conceiving of the price sum advanced as a sum of value, the single-system interpretation enables the value of capital advanced to be determined in the case of joint production. Moreover, the sums of value and surplus-value produced by each firm and industry remain meaningful notions and determinable magnitudes.

The value produced in each industry, for instance, is the value of its total joint product, the total labor-time contained in it. Replacing λ_{t+1} with λ^J_{t+1}, a row vector of the values of each industry's joint products, and again letting **A** and **I** denote the total amounts of means of production and labor needed to produce them, System (4) becomes:

$$\lambda^J_{t+1} = p_t\mathbf{A} + p_t\mathbf{b}\mathbf{l} + (\mathbf{I} - p_t\mathbf{b}\mathbf{l}) = p_t\mathbf{A} + \mathbf{I} \qquad (9)$$

Value produced in every industry must therefore be positive if input prices are positive or living labor is performed in production. Since the aggregate prices of the various sectors can similarly be written as $p^J_{t+1} = p_t\mathbf{A} + \mathbf{I} + g_t$, the result that

Table 1. Input-output relations

Producer	Inputs				Outputs	
	Commodity I	Commodity II	Living Labor		Commodity I	Commodity II
A	10	0	4	⇒	12	3
B	0	2	4	⇒	1	4

The real wage = 1/2 unit of commodity II per unit of living labor.

Steedman's Value System

Producer	c	v	s	Output Value $(c + v + s)$
A	-40	8	-4	-36
B	8	8	-4	12
Total	-32	16	-8	-24

$\lambda_I = -4$, $\lambda_{II} = 4$

Steedman's Price System

Producer	c'	v'	Avg. Profit (π)	Output Price $(c' + v' + \pi)$	Rate of Profit $\pi/(c' + v')$
A	10	2	3	15	25.0%
B	2	2	1	5	25.0%
Total	12	4	4	20	25.0%

$p_I = 1$, $p_{II} = 1$

Single value-price system

Producer	c	v	s	Avg. Profit (π)	Output Value $(c + v + s)$	Output Price $(c + v + \pi)$	Rates of Profit	
							Value $s/(c + v)$	Price $\pi/(c + v)$
A	10	2	2	3	14	15	16.7%	25.0%
B	2	2	2	1	6	5	50.0%	25.0%
Total	12	4	4	4	20	20	25.0%	25.0%

$p_{I \text{ (input)}} = 1$, $p_{II \text{ (input)}} = 1$

value cannot be gained or lost in exchange also implies not only that profit cannot be positive unless surplus-value is positive, but that their aggregates are again equal—total profit is $(\boldsymbol{p}_{t+1}^{J} - \boldsymbol{p}_t\mathbf{A} - \boldsymbol{p}_t\boldsymbol{bl})\mathbf{1}$, equal to $(\boldsymbol{l} - \boldsymbol{p}_t\boldsymbol{bl})\mathbf{1}$, total surplus-value, where $\mathbf{1}$ is a column vector of ones. Aggregate price and value, and the aggregate price and value profit rates, are again also equal.

The bottom of Table 1 illustrates these conclusions for the same physical data, assuming that the input prices are the same as Steedman's. Due to this assumption, input and output prices are equal, and all price and profit magnitudes are the same as his. Yet since, in our interpretation, the production of value precedes its distribution, these conclusions hold even if prices are not stationary or the profit rate is not uniform.

5. Results Pertaining to Determination

The last section showed that, with respect to issues of magnitude, the alleged internal inconsistency of Marx's value theory stems from a dual-system interpretation of its concepts, so that a single-system interpretation—whether simultaneous or temporal—vindicates the theory's internal coherence. With respect to issues of determination, on the other hand, its alleged redundancy and internal inconsistency stem from simultaneous valuation formalisms, so that a temporal interpretation of valuation is needed to vindicate the theory's internal coherence.

5.1. What Makes Value Production Redundant?

Samuelson (1971), followed by Steedman (1977), noted that the magnitudes of the standard interpretation's value system (3) (which they attributed to Marx) is an irrelevant detour for quantitative price theory. The relative prices and profit rate of the standard interpretation are actually determined directly by physical quantities—technological and real wage coefficients. Although we concur with this assessment, it is crucial to emphasize that what makes value magnitudes redundant in the standard interpretation is not the discrepancy between the price and value systems—not, in other words, its dual-system aspect. Redundancy of value stems instead from its simultaneous determination of input and output prices (and values).[11]

The larger the profit rate, the higher output prices are relative to input prices, and vice-versa. Infinitely many profit rates are therefore associated with the same set of physical quantities. Yet by equating input and output prices, simultaneous valuation prevents output prices from rising above or falling below input prices. It thereby also fixes the profit rate at one particular level. In other words, a unique profit rate is associated with each set of physical quantities—the latter is the sole proximate determinant of the former—only because simultaneous valuation rules out the myriad other possible profit rates.

---

[11] Thus, the value magnitudes of the simultaneous single-system interpretation are likewise redundant, even though that interpretation is able to replicate Marx's results concerning magnitude discussed above.

Once input and output prices are not constrained to be equal, physical quantities no longer suffice to determine the levels of the profit rate and output prices. It is then possible to argue that the profit rate and output prices also depend on a value magnitude, the value added by living labor in the production process, *lx* of System (6). The more living labor is performed, the greater is the increase in the total price (= total value) of the output over the total price of the inputs, and the greater is the profit rate.

That it is simultaneous valuation, not price–value deviations, that makes value and labor-time redundant is further underscored by the fact that they are redundant even within the value system itself, i.e. even when prices equal values. This is clearly seen in the case of a one-commodity economy; the profit rate of the standard value system is $r = \lambda/(\lambda k) - 1 = 1/k - 1$, where k is the commodity input (including real wage) requirement per unit of output. The profit rate depends upon k alone; changes in the amount of living labor needed to produce the commodity thus have no effect on the profit rate unless they happen to alter k.

This result applies equally to a multisector economy: when commodity inputs (including real wages) per unit of output are held constant, proportionate changes in living labor requirements will have no effect on the profit rate of the standard value system. Each of the unit values of System (3) will change by the same proportion. The numerator and denominator of the profit rate will therefore change by the same proportion as well, leaving the ratio unchanged. Hence, only changes in *relative* values affect simultaneously determined value rates of profit—the absolute amount of labor-time needed to produce commodities becomes irrelevant. Even disproportionate changes in living labor requirements will thus have no effect on the value rate of profit of an economy in which all commodities' economy-wide input–output ratios are equal, since its profit rate is invariant to changes in relative prices (values). In all other cases, a fall in the amount of labor needed to produce a commodity is as likely to cause a rise in the value system's profit rate—which contradicts Marx's theory—as it is to cause a fall; the effect depends only on whether the commodity's input–output ratio is greater or less than average.

What is it about simultaneous valuation that makes value production irrelevant? Simply this: when one constrains commodities' output values (prices) to equal their input values (prices), no matter how much the labor-time needed to produce them changes between time of input and time of output, one thereby constrains labor-time to be irrelevant to the determination of their values (prices).

Simultaneous valuation thus gives rise to what may be called a use-value productivity theory of the profit rate. According to this theory, rising labor productivity will have no effect on profitability unless it alters per-unit commodity input (including real wage) requirements. Even more significantly, whereas Marx (1981, p. 347) held that 'The profit rate does not fall because labour becomes less productive but rather because it becomes more productive', simultaneous valuation leads to the conclusion that increases in productivity which do lower per-unit commodity input requirements will cause the profit rate to rise.

5.2. *The Self-expansion of Value over Time*

Prior to Marx, theorists had often attempted to account for commodities' 'normal' prices by reducing them to their costs of production, defined to include average profit. He criticized such explanations for their failure to account for the magnitude of average profit. The level of the profit rate, he held, must be determined independently of its equalization, because otherwise 'the *average profit*, and therefore also the [production] prices, would be purely imaginary and untenable. The [average profit] could then equally well be 1,000 per cent or 10 per cent' (Marx, 1968, p. 190). Accordingly, the principal quantitative questions addressed by Marx's value and profit rate theories are: what enables capitalists to get back a sum of value that exceeds the expenses they incurred?, and what determines the size of this difference 'between the value antecedent to production and the value which results from it' (Marx, 1971, p. 131)?

These questions actually cannot even be posed, much less answered, when valuation is postulated to be simultaneous. Instead, a different question is addressed: what determines the size of the difference between sales revenue and inputs' replacement cost at a single moment in time? Once one returns to Marx's intertemporal questions, however, it is clear that the use-value productivity theory of the profit rate can provide no answers unless input and output prices do in fact happen to be equal, since, as was noted above, it is only in that special case that the profit rate is reducible to quantities of use-values.

Marx's own answer, of course, was that value 'self-expands' through the extraction of surplus-labor in capitalist production. The magnitude of surplus-value is fully determined upon the completion of the production process, before products go to market, because it depends only on the difference between the value of the wages advanced before production and the amount of labor performed during production. It follows from this, together with the aggregate price-value equalities, that the level of the general profit rate (profit divided by the capital advanced before production), is likewise determined before output prices of individual commodities are determined. Output price variations merely lead to different divisions of the 'profit ... produced *before* this division takes place' (Marx, 1981, p. 505, emphasis added).

As we have shown, the temporal single-system interpretation reproduces this sequence of determination. It vindicates the internal coherence of Marx's disputed results concerning determination precisely because it does so. This is important to emphasize because it has sometimes been thought (e.g. Skillman, 1995, p. 10) that this interpretation instead eliminates the inconsistency in Marx's value theory by supplying extra unknowns, in effect by modeling a perpetual disequilibrium in which 'anything goes'. Yet if we examine the 'simultaneist' and temporalist production price equations

$$p^* = p^*(\mathbf{A} + \mathbf{b}l)(1 + r^*) \tag{10}$$

$$p^*_{t+1} = p_t(\mathbf{A} + \mathbf{b}l)(1 + r^*_{[t,t+1]}) \tag{11}$$

each system contains n equations and $n + 1$ unknowns, the n prices plus the uniform profit rate. Although there are only n prices in System (10) and $2n$ prices in System (11), the n input prices of the latter are data, not unknowns,

because they are the output prices of the preceding period (or initial conditions). The reason these two systems yield different results is rather that, in System (10), the profit rate is determined simultaneously with the $n-1$ relative output prices, while, in System (11), determination cannot be simultaneous. Either the level of the output prices determine the profit rate or, as Marx held, the profit rate, itself already determined as a result of the production process, determines the level of output prices.

What, however, are these different results? When System (11) is closed by the temporal single-system interpretation of Marx's general profit rate

$$r^*_{[t,t+1]} = s_t x/(c_t x + v_t x) = (lx - p_t blx)/[p_t(A + bl)x] \qquad (12)$$

four major differences result:

(a) As we have already stressed, the extraction of surplus-labor ($lx - p_t blx$) in production is no longer redundant; on the contrary, it re-emerges as a crucial determinant of the level of the aggregate profit rate and of the self-expansion of value.
(b) The magnitude of the rate of profit is invariant to alternative distributions of profit, as Marx (1981, Chapter 9) held, precisely because the profit rate is determined before and independently of output prices, the movements in which lead to different distributions of profit. In contrast, when input and output prices are determined simultaneously, movements in output prices cause the replacement cost of capital and therefore the average rate of profit to change.
(c) The level of the general rate of profit depends on the productivity of luxury industries, as Marx (1971, pp. 349–351) maintained in opposition to the Ricardians, because the general rate is a weighted average of the value rates of profit of all industries. In contrast, when input and output prices are determined simultaneously, technical and real wage coefficients in non-luxury industries become the sole determinants of those industries' profit rates and therefore of the general rate, if profit rates are equal.
(d) Cost-reducing, labor-saving technical change can cause a fall in the general rate of profit. Due to the importance of this result, we will discuss it in more detail presently.

In light of these results, it is surprising that some dual-system theorists have alleged that, because input prices and therefore value magnitudes are influenced by exchange phenomena in the temporal single-system interpretation, it is incompatible with the determination of value by labor-time (see, e.g. Naples, 1993, and the reply by Kliman, 1993). In the dual-system interpretation, determination of value by labor-time has come to mean that a cause-and-effect relation exists between two mutually exclusive sets of variables, such that surface price phenomena are a veil that obscures the underlying determination of economic relations by technology. As we have noted, however, not only did Marx not suggest that commodities' values are invariant to different distributions of already-existing sums of value, but he explicitly cautioned that to draw such an inference is to 'go wrong.' We therefore think that when he

himself argued that the level of prices and profits must be explained on the basis of the determination of value by labor-time, he meant something both more plausible and of much greater real-world significance: the amount of surplus-labor performed in capitalist production determines the aggregate difference 'between the value antecedent to production and the value which results from it.'

5.3. *The Falling Rate of Profit*

It is widely thought that Okishio's (1961) theorem has proven the falsity of Marx's law of the tendential fall in the profit rate. The theorem purportedly demonstrates that, no matter how much labor is saved or how much the technical composition of capital (means of production per worker) is increased by new techniques, they themselves cannot cause the uniform profit rate to fall if they are cost reducing at current prices. Although a fall in the profit rate may indeed follow the introduction of such new techniques if real wages also rise, it is then this rise, not the technical changes *themselves*, which are the cause of the decline in profitability.

This claim can be illustrated by means of the simple example of Table 2, which is not intended as a realistic model of capital accumulation. We assume away fixed capital and abstract from the influence of changes in the real wage by holding it constant. In both sectors, output and non-labor inputs increase at one rate, while living labor increases at a slower rate. Since the new techniques introduced in each period require no more non-labor inputs, and less living labor, per unit of output, they both save on labor and raise the technical composition of capital. Since, in addition, the real wage is constant, they also reduce unit costs of production as evaluated at current prices.

Assuming, finally, that the profit rate is continually equalized, the Okishian profit rate is determined by the simultaneous Equations (10). As Table 2 shows, the simultaneist profit rate, initially 0.212, rises monotonically over time and eventually approaches 0.250.

This result depends crucially on the simultaneous determination of input and output prices. First, as we noted above, it is simultaneous valuation that turns the profit rate into a function of technical and real wage coefficients alone. When, in addition, the real wage rate is held constant, the profit rate becomes purely an index of use-value productivity. The reason Okishio's profit rate rises continually is therefore that each new technique is more productive than the last—commodity inputs (including real wages) per unit of output continually decline.

Second, when used to assess profitability, simultaneous valuation is by no means a neutral technique. Instead, as we also noted above, it constitutes an implicit claim that every commodity is worth exactly as much as an output as it was worth as an input, no matter how much the amount of labor needed to produce it has changed between the time of input and the time of output. Hence, when one equates input and output prices, one imposes an exogenous constraint on output prices. By preventing them from falling relative to input prices, the

Table 2. Tendencies of sumultaneist and temporalist rates of profit

| | Physical data | | | |
| | Inputs | | | |
	Good 1	Living Labor	Real Wage*	Output
Sector 1				
initial	44	11	11/32	55
growth rate	(10%)	(3.4%)	(0%)	(10%)
Sector 2				
initial	6	24	11/32	30
growth rate	(10%)	(3.4%)	(0%)	(10%)

*Units of good 2 per unit of living labor.

| | Time Paths | | | |
| | Rate of Profit (%) | | | |
Period	Simultaneist	Temporalist	P_{1t+1}/P_{1t}	P_{2t+1}/P_{2t}
0	21.2	21.2	1.000	1.000
1	21.6	20.0	0.988	0.970
2	21.9	19.4	0.981	0.959
3	22.1	19.0	0.975	0.953
5	22.6	18.4	0.968	0.948
10	23.3	17.9	0.957	0.944
20	24.2	17.6	0.948	0.941
30	24.6	17.6	0.944	0.941
50	24.9	17.5	0.941	0.940
70	25.0	17.5	0.940	0.940

constraint also prevents the decline in profitability that accompanies falling output prices.[12]

    Neither Okishio's original paper nor subsequent extensions of his theorem explicitly invoke this constraint as a premise. They claim to prove a general result, not one that holds true only if input and output prices happen to be equal. As we shall demonstrate presently, however, it need not hold otherwise and, since a contrary result will be derived without violating any explicitly stated premise of the theorem, the demonstration constitutes a refutation of it in the strict logico-mathematical sense.

---

[12] The stipulation that the price of one commodity (or some aggregate) is constant throughout time constitutes a very similar claim and acts to constrain the aggregate price of output and the profit rate in a very similar manner.

Once the *a priori* constraint that prices are stationary is lifted, the tendency of the profit rate obviously must be assessed by means of Equation (11) instead of (10). An additional equation is needed in order to close system (11); we employ our interpretation of Marx's general rate of profit (12) for that purpose. This choice is certainly crucial to our results, but no premise of the Okishio theorem precludes it. Indeed, were it precluded, then the theorem could no longer be considered a critique of the *internal* logic of Marx's law of the falling tendency of the profit rate, but merely a conflicting conclusion which results from a conflicting theory.

System (11) also requires initial input prices. We assume that the prices of goods 1 and 2 equal 2.2 and 0.8, respectively. This ensures that, given the same physical data, the temporalist profit rate starts off at the same level as the simultaneist rate (which facilitates comparison of their paths). As Table 2 indicates, however, whereas the simultaneist profit rate rises continually to 0.250, the temporalist rate continually falls and eventually approaches 0.175. The logical coherence of Marx's law is thus confirmed.

In accordance with his theory, moreover, the fall in the profit rate is traceable directly to rising productivity. Note that

$$1 + 0.175 = \frac{1 + 0.250}{(1 + 0.1)/(1 + 0.034)}$$

The left-hand side is one plus the limit of the temporalist rate, and the numerator of the right-hand side is one plus the limit of the simultaneist rate. The denominator of the right-hand side is an index of productivity growth (one plus the growth rate of labor productivity), since the growth rate of output is 0.1 and the growth rate of living labor extraction is 0.034. The faster the rate of productivity growth, then, the greater is the gap between the two profit rates in the long-run.

This relationship can also be explained in terms of input and output prices: the gap between the two rates corresponds to the fall in output prices relative to input prices. The reciprocal of the right-hand side denominator is $(1 + 0.034)/(1 + 0.1) = 0.940$, so the relationship between the limits of the two profit rates can also be written as $1 + 0.175 = (1 + 0.250)(0.940)$. As Table 2 shows, 0.940 is the value to which the output-to-input price ratio of both goods converges. Given that value is determined by labor-time and that profit and surplus-value are equal in the aggregate, the continual increases in productivity cause the prices continually to fall (abstracting, of course, from nominal changes in money values and prices relative to their labor-time counterparts). Output prices therefore continually fall below input prices and, accordingly, the temporalist profit rate is lower than the simultaneist rate. The greater the fall in output prices relative to input prices, the greater is the gap between the two rates in the long run.

This vindication of the logical coherence of Marx's law and refutation of the Okishio theorem cannot be dismissed on the ground that the temporalist profit rate, as a value rate of profit, is the profit rate of an imaginary economy in which commodities exchange in proportion to the labor-times needed to produce them. Although it is certainly the general value rate of profit, the

temporalist rate is likewise a price rate—each sector receives as profit a sum that differs from the surplus-value it extracts, and input costs are based on the inputs' actual prices, not their values.

　　To be sure, the temporalist rate is not a *nominal*, money, rate of profit, since it adjusts money figures for changes in the money–labor time relationship. Yet if that makes it an imaginary rate, then the simultaneist profit rate must be judged equally imaginary since, by revaluing inputs at replacement cost in parallel fashion, it adjusts money figures for changes in the money–use value relationship. Underlying these two methods of adjustment are two different theories of what determines the real magnitude of profit, the amount of surplus-labor performed or the amount of physical surplus produced. Because attempts to exclude the former theory on a priori 'logical' grounds have proved to be untenable, theoretical and empirical investigations are needed to determine which theory, if either, is right.

6. Conclusions

This paper has presented an interpretation of Marx's value theory that refutes a century of allegations that it is internally inconsistent. In this temporal single-system interpretation, value and price magnitudes are determined interdependently and in historical time, in contrast to the standard interpretation of them as two separate and simultaneously determined systems. Although the present interpretation is certainly unorthodox, we have presented textual evidence which shows it to be, at minimum, a plausible interpretation of the concepts Marx employed. More importantly and conclusively, this interpretation conforms to Marx's own theory in another respect: it replicates his theoretical conclusions.

　　Specifically, we have shown that, under the temporal single-system interpretation: (a) all of Marx's aggregate value–price equalities hold; (b) values cannot be negative; (c) profit cannot be positive unless surplus-value is positive; (d) value production is no longer irrelevant to price and profit determination; (e) the profit rate is invariant to the distribution of profit; (f) productivity in luxury industries affects the general rate of profit; and (g) labor-saving technical change itself can cause the profit rate to fall. As we have noted, a single-system interpretation is necessary to replicate the first three conclusions, which concern magnitude, while a temporal interpretation is necessary to replicate the rest, which concern determination.

　　By replicating Marx's theoretical conclusions, we have demonstrated that it is not Marx's value theory *per se* which is self-contradictory, as his Marxist and non-Marxist critics have persistently alleged, but the standard interpretation of that theory. Precisely because the temporal single-system interpretation is able to make sense out of crucial aspects of his value theory that the standard interpretation (and others) have always found to be incoherent, we submit that it marks a significant advance as an interpretation of Marx's own texts.

　　Clearly, under our interpretation, Marx's theory does not address some questions that his critics make central. For instance, it does not determine static equilibrium prices or purport to show that relative values underlie relative prices. Our interpretation instead opens the way for the re-integration of Marx's theories

of value and surplus-value with his theory of capital accumulation and crisis. Even proponents of atemporal interpretations (e.g. Duménil & Lévy, 1997, p. 16) recognize that they grant value theory no explanatory role with respect to dynamic questions. They are thus unable to explain the difference 'between the value antecedent to production and the value which results from it,' while the present interpretation suggests, to the contrary, that the principal quantitative function of Marx's theories of value and price determination was precisely to explain this difference as resulting from the exploitation of workers in capitalist production.

Appendix: Non-stationary Production Prices

The secondary literature has systematically misconstrued the nature of Bortkiewicz's alleged proof—which to this day remains the sole one—that Marx's account of the transformation of values into production prices is internally contradictory. It is generally said that Bortkiewicz demonstrated the conceptual incoherence of valuing outputs in price terms while inputs are valued in value terms.

Yet his actual objection was economic, not 'logical'. Specifically, Bortkiewicz (1952, pp. 6–7) claimed to have proved that, because input and output prices differ in Marx's theory of production price determination, the formation of production prices itself would disturb social reproduction. In another version of the same argument, Bortkiewicz (1984, pp. 212–213) purported to demonstrate that each industry's sales and purchases will fail to coincide unless prices are stationary. Were these demonstrations valid, they would be decisive, since a uniform profit rate and production prices do require the equality between supplies and demands of all produced commodities. Although only by accident do commodities exchange at their production prices, even one example of self-contradiction is sufficient to require that a general theory be rejected or corrected.

Table 3 illustrates Marx's 'uncorrected' value–price transformation in the context of simple reproduction. The three departments produce means of production, means of subsistence, and luxury goods, respectively.[13] Period 1's inputs are bought at their values, so Bortkiewicz contends

Table 3. Marx's uncorrected value–price transformation

Period	Dept.	Revenue (*m*)	*c*	*v*	*s*	Output Value (*c* + *v* + *s*)	Avg. Profit (*π*)	Output Price (*c* + *v* + *π*)	'Value' *s*/(*c* + *v*)	'Price' *π*/(*c* + *v*)
									Rates of Profit	
1	I		140	36	24	200	44	220	13.6%	25.0%
	II		40	48	32	120	22	110	36.4%	25.0%
	III		20	36	24	80	14	70	42.9%	25.0%
	Total		200	120	80	400	80	400	25.0%	25.0%
2	I	33	154	33	27	214	51	238	14.4%	27.3%
	II	22	44	44	36	124	24	112	40.9%	27.3%
	III	15	22	33	27	82	15	70	49.1%	27.3%
	Total	70	220	110	90	420	90	420	27.3%	27.3%

---

[13] The value–price transformation has nothing to do with simple reproduction. Bortkiewicz linked them in order to illustrate Marx's alleged error. The three-department reproduction scheme, which privileges the distributional struggle between workers' and capitalists' consumption, is also a post-Marx construction. Marx developed a two-department scheme that accentuates the antagonism between production for production and production for consumption.

that simple reproduction requires that outputs sell at values. If Department I's output were priced at 220, for instance, some would go unsold, since purchases of constant capital only total 200. He also claims that if exchanges took place at production prices, Department I would sell commodities priced at 60 (IIc + IIIc) but buy commodities priced at 80 (Iv + Iπ). Department II's sales of 72 (Iv + IIIv) and purchases of 62 (IIc + IIπ), and Department III's sales of 66 (Iπ + IIπ) and purchases of 56 (IIc + IIIv) would also fail to match.

This reasoning is characteristic of underconsumptionism: the social product must be 'bought back' with prior income. Actually, however, reproduction requires the *advance* of capital for the next period, which likewise enables the current period's outputs to be sold. Output prices thus need to equal, not the input prices of the same period, but those of the next period.

Once this is understood, Bortkiewicz's argument can be refuted immediately, as was first shown by Kliman & McGlone (1988). Table 3 is based on the assumption that all physical quantities are the same in periods 1 and 2, but the value figures change between periods because the price of means of production has risen from 200 to 220 and the price of means of subsistence has fallen from 120 to 110. After advancing sums of value (c and v) sufficient to obtain the same inputs at the changed prices of period 2, capitalists have residual proceeds (which Marx calls revenue, m) left over from the sale of period 1's outputs, which they spend on luxury goods. It is clear that the whole social product is bought and sold at the new, changed prices, so production can resume on the same scale and in the same proportions. Moreover, Department I's sales (IIc + IIIc) and purchases (Iv + Im) both total 66, as do Department II's sales (Iv + IIIv) and purchases (IIc + IIm). Department III's sales (Im + IIm) and purchases (IIIc + IIIv) both total 55.

Bortkiewicz thus erred in assuming that equality of input and output prices is necessary for reproduction and market clearing to take place. One period's output prices *are* the next period's input prices, so if the physical amounts each industry supplies to the others is matched by an equal demand on their parts, then the monetary balance follows automatically, however prices may have changed over the production period. The key condition for balance is a uniform profit rate; as Table 3 illustrates, this likewise does not require that input and output prices be equal[14]

Yet we have persistently encountered a widespread objection (e.g. Naples,1993; Laibman, 1998) to this refutation of Bortkiewicz's argument: the profit rate equalized in Table 3, the rate of return on capital advanced, is not the appropriate rate. The most common version of the objection is that firms seek to maximize, and therefore competition tends to equalize, the replacement cost profit rate (RCPR), which values inputs as well as outputs at output prices. Bortkiewicz was therefore right all along in insisting on the equality of input and output prices.

We find no textual support for the view that Marx held that competition tends to equalize the RCPR instead of the rate of return on capital advanced. As we showed above, his concepts of profit and rate of profit compare sums of value produced (or received) at one time with other sums of value advanced at earlier times.

The objection is also quite flawed on economic grounds. The criteria that businesses and investors use to make investment decisions, net present value (NPV) and internal rate of return (IRR) criteria, likewise compare sums of value of different periods. Table 3's equalized profit rates *are* IRRs. Given the input costs, moreover, Table 3's relative output prices are the only ones that equalize the NPV of a dollar invested in each department (assuming any uniform cost of capital) and the only ones that equalize the IRRs.[15]

The current RCPR is, in contrast, of no use to profit-maximizing investors. It is certainly true that current costs, not past costs, guide current investment decisions, but so do (expected) *future* returns. The current RCPR, however, compares current costs with current returns. Equally

---

[14] This also implies that, contrary to what readers of Sraffa such as Newman (1977, pp. 347–352) and Varri (1990, p. 180) have concluded, the exchange ratios that permit simple reproduction and a uniform rate of return on capital advanced are not uniquely determined. The case of 'production without a surplus' is the *only* one in which 'exchange-values … spring directly from the methods of production' (Sraffa, 1960, p.3).

[15] The absolute magnitudes of the NPVs and IRRs, however, are not uniquely determined by input costs and the uniform profitability assumption. This underscores the need for a *theory* of the determination of the level of the profit rate, i.e. of the aggregate increase in returns at one time over the value invested at an earlier time.

irrelevant is the future RCPR, which compares the costs and returns of some future moment with one another. If one can invest today for one year either in firm A, where the future RCPR is higher, but the IRR is lower, or in firm B, the latter is always more profitable. If price changes happen to cease after the end of this year, then (but only then) the future RCPR becomes an accurate criterion for *subsequent* investments. Even in this case, however, it is obviously more profitable to invest in B this year and redirect future investment to A than to invest in A from the start. To the extent that competition tends to equalize rates of return, then, it tends to equalize the rates of return about which profit seekers care, the rate of return on the capital actually advanced, not on the replacement cost of inputs.

References

Bortkiewicz, L. von (1952) Value and price in the Marxian system, *International Economic Papers*, 2, pp. 5–60.

Bortkiewicz, L. von (1984) On the correction of Marx's fundamental theoretical construction in the third volume of *Capital*, in: E. von Böhm-Bawerk, *Karl Marx and the Close of his System* (Philadelphia, Orion Editions).

Brewer, A. (1995) A minor Post-Ricardian? Marx as an economist, *History of Political Economy*, 27, pp. 111–145.

Carchedi, G. & W. de Haan (1996) The transformation procedure: a non-equilibrium approach, in: A. Freeman & G. Carchedi (Eds) *Marx and Non-equilibrium Economics* (Cheltenham, UK, Edward Elgar).

Desai, M. (1988) The transformation problem, *Journal of Economic Surveys*, 2, pp. 295–333.

Duménil, G. (1983) Beyond the transformation riddle: a labor theory of value, *Science and Society*, 47, pp. 427–450.

Duménil, G. & D. Lévy (1997) The conservation of value: A rejoinder to Alan Freeman. Submitted to the mini-conference of the International Working Group on Value Theory at the Eastern Economic Association Convention, Washington, DC, April.

Dunayevskaya, R. (1988) *Marxism and Freedom: from 1776 until today* (New York, Columbia University Press).

Ernst, J. R (1982) Simultaneous valuation extirpated: a contribution to the critique of the neo-Ricardian concept of value, *Review of Radical Political Economics*, 14, pp. 85–94.

Foley, D.K. (1982) The value of money, the value of labor power, and the Marxian Transformation Problem, *Review of Radical Political Economics*, 14, pp. 37–47.

Freeman, A. (1996a) The psychopathology of Walrasian Marxism, in: A. Freeman & G. Carchedi (Eds) *Marx and Non-equilibrium Economics* (Cheltenham, UK, Edward Elgar).

Freeman, A. (1996b) Price, value and profit—a continuous, general treatment, in: A. Freeman & G. Carchedi (Eds) *Marx and Non-equilibrium Economics* (Cheltenham, UK, Edward Elgar).

Freeman, A. & G. Carchedi (Eds) (1996) *Marx and Non-equilibrium Economics* (Cheltenham, UK, Edward Elgar).

Giussani, P. (1991–92) The determination of prices of production, *International Journal of Political Economy*, 21, pp. 67–87.

Howard, M. C. & J. E. King (1992) *A History of Marxian Economics: Volume II, 1929–1990* (Princeton, NJ, Princeton University Press).

Kliman, A.J. (1993) 'Value in Process': A reply to Naples, *Capital and Class*, 51, pp. 139–151.

Kliman, A.J. (1996) A value-theoretic critique of the Okishio theorem, in: A. Freeman & G. Carchedi (Eds) *Marx and Non-equilibrium Economics* (Cheltenham, UK, Edward Elgar).

Kliman, A.J. & T. McGlone (1988) The transformation non-problem and the non-transformation problem, *Capital and Class*, 35, pp. 56–83.

Laibman, D. (1998) Rhetoric and substance in value theory: an appraisal of the new orthodox Marxism, in J. Wells, A. Kliman & A. Freeman (Eds) *The New Value Controversy and the Foundations of Economics* (Cheltenham, UK, Edward Elgar); forthcoming.

Lee, C.-O. (1993) Marx's labour theory of value revisited, *Cambridge Journal of Economics*, 17, pp. 463–478.

Maldonado-Filho, E. (1997) Release and tying up of productive capital and the 'transformation problem.' Presented at the mini-conference of the International Working Group on Value Theory at the Eastern Economics Association Convention, Washington, DC, April.

Marx, K. (1968) *Theories of Surplus-Value*, Part II (Moscow, Progress Publishers).

Marx, K. (1969) *Theories of Surplus-Value*, Part I (Moscow, Progress Publishers).

Marx, K. (1971) *Theories of Surplus-Value*, Part III (Moscow, Progress Publishers).

Marx, K. (1975) *Karl Marx: texts on method*, T. Carver (Ed.) (Oxford, Basil Blackwell).

Marx, K. (1977) *Capital: A critique of political economy*, Vol. I (New York, Vintage Books).

Marx, K. (1981) *Capital: A critique of political economy*, Vol. III (New York, Vintage Books).

Marx, K. (1988) *Karl Marx, Frederick Engels: Collected Works*, Vol. 30 (New York, International Publishers).

Marx, K. (1991) *Karl Marx, Frederick Engels: Collected Works*, Vol. 33 (New York, International Publishers).

McGlone, T. & Kliman, A.J. (1996) One system or two?: The transformation of values into prices of production vs the transformation problem, in: A. Freeman & G. Carchedi (Eds) *Marx and Non-equilibrium Economics* (Cheltenham, UK, Edward Elgar).

Mirowski, P. (1989) *More Heat Than Light: economics as social physics, physics as nature's economics* (Cambridge, Cambridge University Press).

Moseley, F. (1993) Marx's logical method and the 'transformation problem,' in: F. Moseley (Ed.) *Marx's Method in Capital: a reexamination* (Atlantic Highlands, NJ, Humanities Press).

Naples, M.I. (1989) A radical economic revision of the transformation problem, *Review of Radical Political Economics*, 21, pp. 137–158.

Naples, M.I. (1993) Unperceived inflation in Shaikh, and Kliman and McGlone: equilibrium, disequilibrium, or nonequilibrium?, *Capital and Class*, 51, pp. 119–137.

Newman, P. (1977) Production of commodities by means of commodities: a review, in J. Schwartz (Ed.) *The Subtle Anatomy of Capitalism* (Santa Monica, CA, Goodyear).

Okishio, N. (1961) Technical changes and the rate of profit, *Kobe University Economic Review*, 7, pp. 86–99.

Ramos, A. (1997) Labor, money, labor-saving innovation, and the falling rate of profit. Presented at the mini-conference of the International Working Group on Value Theory at the Eastern Economics Association Convention, Washington, DC, April.

Ramos, A. & Rodriguez, A. (1996) The transformation of values into prices of production: a different reading of Marx's texts, in: A. Freeman & G. Carchedi (Eds) *Marx and Non-equilibrium Economics* (Cheltenham, UK, Edward Elgar).

Samuelson, P.A. (1971) Understanding the Marxian notion of exploitation: a summary of the so-called transformation problem between Marxian values and competitive prices, *Journal of Economic Literature*, 9, pp. 399–431.

Skillman, G. (1995) Some comments on the 'temporal single-system' approach to the (non-)transformation (non-)problem in Marx. Presented at URPE Summer Conference, Bantam, CT, USA, August.

Sraffa, P. (1960) *Production of Commodities by Means of Commodities* (Cambridge, Cambridge University Press).

Steedman, I. (1977) *Marx After Sraffa* (London, New Left Books).

Varri, P. (l990) Fixed capital, in: J. Eatwell, M. Milgate & P. Newman (Eds), *The New Palgrave: capital theory* (New York & London, W. W Norton).

Vroey, M. de (1982) On the obsolescence of the Marxian theory of value: a critical review, *Capital and Class*, 17, pp. 34–59.

Wolff, R., Callari, A. & Roberts B. (1984) A Marxian alternative to the traditional transformation problem, *Review of Radical Political Economics*, 16, pp. 115–135.