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The transformation non-problem and the non-transformation problem

> I have thrown overboard the whole doctrine of profit up till now. By mere accident . . . I leafed through Hegel's *Logic* again and found much to assist me in the *method* of analysis. (Marx, 1983a: 50)

Most criticisms of Marx's attempt to solve the transformation problem focus on his failure to separate value and price calculations. It is argued here that there is nothing wrong with Marx's approach once it is construed as an exercise in dialectics rather than some attempt to map mathematically values on to prices. Consequently, much of the debate over values and the transformation of values has been wrongly directed at matters of mathematical technique when the real issue concerns one of method.

● Marx's theory of value continues to be a subject of controversy, as recent debates on abstract labour (Gleicher, 1983, 1985; Eldred, 1984) and joint production (Roberts, 1987; Rankin, 1987) in this journal show. Through such debates, radical economists' views on value theory have seemingly crystallised into two main approaches, characterised by de Vroey (1982) as the 'technological' and 'social' paradigms. As students of a third, humanist problematic, we hope in this paper to create a dialogue with proponents of other approaches by re-examining the key unresolved question of post-Marx Marxist[1] value theory – the 'transformation problem'.

Within the 'technological' paradigm, both adherents to the labour theory of value (e.g. Meek, 1956) and those who reject value categories as redundant or worse (e.g. Steedman, 1978) confer upon technological relations a crucial role in the valuation process. Most of those working within the 'social' paradigm hold to what Elson (1979) called the 'value theory of labour'; in this theory, 'value refers to the validation of private labour through the exchange of commodities against money' (de Vroey, 1982: 40). Our own view is neither 'technicist' nor market-oriented, but a production-centred value theory of labour. In short, we take capitalist technological relations *themselves* to be social relations, class relations of dead to living labour in production. '[L]abour is expressed in value' because 'the process of production has mastery over man, instead of the opposite' (Marx, 1977: 174-75). We do not de-emphasise the quantitative aspect of Marx's value theory, however; this paper, for instance, attaches great importance to the aggregate equalities which obtain in Marx's transformation procedure.

This unorthodox interpretation of Marx's theory of value arises out of the new problematic raised since the 1950s, by mass movements and intellectuals alike, in their call for new human relations. US coal miners fighting automation asked 'what *kind* of labour should human beings do?' (Phillips & Dunayevskaya, 1984); East German workers and masses struggling against a new totalitarianism called for 'Bread and Freedom'; radical intellectuals in the 'East', 'West' and 'South' returned to Marx's humanism, rooted in the Hegelian dialectic, as a philosophic-economic totality in need of re-concretisation (see, for instance, Fromm, 1965; Dunayevskaya, 1958; Fanon, 1961).

We argue below that because they view Marx's *Capital* as a narrowly 'economic' work, post-Marx Marxists and others misread his transformation procedure in Volume III, Chapter 9, and consequently reject it as logically inconsistent. Both anti-Marxist and Marxist economists view Marx's 'failure' to separate value- and price-calculation as a logical error. Yet, whereas the former see this 'failure' as a sufficient reason to reject his analysis of the capitalist economy, the latter set out to resuscitate him. They hold that he himself was aware of the error; only his lack of mathematical sophistication and/or failure to complete Volume III prevented him from presenting the 'correct solution'.[2]

Despite the near-universal rejection of Marx's procedure, the debate has not resolved the question of the relation of values to prices of production. Rather, it has reached an impasse and degenerated into chronic indeterminacy. The constantly expanding multiplicity of 'solutions' to the 'transformation problem' has itself become part of the problem. Since each new 'solution' is necessarily opposed to all others, none of them can resolve the debate.[3]

To help break through the apparent endlessness of this debate, we undertake here a defence of Marx's own procedure and a critique of 'transformation problem' 'solutions'. Ours is, admittedly, not the first recent work to defend Marx's procedure.[4] We believe, however, that previous responses adhere too closely to the ground of Marx's critics. *All* sides in the debate focus on the questions of 'logic' and the internal (in)consistency of the labour theory of value. What is downplayed or even ignored[5] is the issue of *method*: the abstract, analytical method of formal logic *versus* concrete, dialectical self-development. The non-dialectical understanding perceives each object as isolated, uniquely itself, a whole unto itself. Thus, for instance, separate 'systems' of value- and price-calculation are demanded, 'systems' in which value equals value, price of production equals price of production. Conversely, dialectic comprehends a judgement such as 'value is price of production' because this judgement, like every other proposition taken singly, is inadequate. It must therefore continue to be developed until the original statement has undergone so much differentiation that we now fully comprehend *how* value *becomes* price of production.[6]

Though little attention has been paid to this issue, rejection of the dialectical method was an inseparable part of Boehm-Bawerk's (1949) critique of *Capital*, a critique which has set the ground for the subsequent 'transformation problem' debate. Boehm's accusation of inconsistency has been quoted widely in discussions which take the controversy to be a purely analytic one. Yet, what has been overlooked is his stipulation that 'every *logical* thinker' will agree with him (Boehm, 1949: 30, emphasis added), and his concomitant desire to distance his own view of logic from Marx's dialectical logic.

> Marx has not deduced from facts the fundamental principles of his system, either by means of sound empiricism or a solid economical-psychological analysis: *he founds it on no firmer ground than a formal dialectic. This is the great radical fault of the Marxian system at its birth: from it all the rest necessarily springs.* (101, emphasis added)

In his critique, Boehm thus recognised both the centrality of the dialectic to *Capital* and the methodological gulf which separated him from Marx. In contrast to other participants in the debate on the 'transformation problem' we, too, view the question of the 'internal consistency' of Marx's procedure to be inseparable from his method.

In Section II, we first examine Marx's transformation procedure within the context of his critique of political economy as a whole. We argue that his illustration appropriately kept values and prices unseparated, though distinct, within a dialectical totality – the *single* 'system' of capitalist production and circulation. The transformation of values into prices of production is but one of many 'transformations into opposite', successive developments of the reification of labour and the fetishism of commodities, discussed in *Capital*. Conversely, we then argue, 'transformation problem' 'solutions' are expressions of anti-dialectical methodology and a divergent view of capitalist economic relations. These general equilibrium 'solutions', however, neither compel rejection of Marx's view of the value-price relationship nor actually demonstrate any relationship themselves.

Finally, we present a multiperiod continuation of Marx's one-period transformation procedure. Where Section I defends the 'failure' to separate values and prices into two systems as methodologically apposite, Section III seeks to demonstrate that it is logically consistent as well. Retaining values and prices in a single 'system', we illustrate the 'transformation of input prices' in the context of simple reproduction.[7] The *three* aggregate equalities which result from Marx's procedure – the equalities of total value and total price, of total surplus-value and total profit,[8] *and* the equal magnitudes of the 'value' and 'price of production' rates of profit – hold in each period and even in general equilibrium. A summary and conclusions follow in Section IV.

One 'system' or two?: the transformation of values into prices of production *vs* the 'transformation problem'

> . . . the material, the *opposed* determinations in *one* relation, is already posited and at hand for thought. But formal thinking makes identity its law, and allows the contradictory content before it to sink into the sphere of ordinary conception . . . in which the contradictories are held *asunder* . . . and so come before consciousness without reciprocal contact. (Hegel, 1969: 835)
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> 'Come before consciousness without mutual contact' (the object) – that is the essence of anti-dialectics . . . thought must *apprehend* the whole 'representation' in its movement . . . (Lenin, 1961: 228)[9]

A single charge has dominated the criticism of Marx's transformation procedure: 'it fails to keep separate rigorously enough the two principles of value- and price-calculation' (Bortkiewicz, 1952: 8). To correct this 'failure', understood to be an error in logic, 'solutions' to the 'transformation problem' hold values and prices 'asunder', 'without reciprocal contact', in *two* opposed systems of calculation. We believe, however, that the demanded separation of value- and price-calculation is not a simple matter of 'logic'. It reveals, on the contrary, the critics' methodological opposition to Marx's procedure as well as their divergent view of what a transformation procedure must illustrate. We turn now to these issues, first by examining Marx's transformation procedure in the context of his critique of political economy as a whole, and then by contrasting it to the host of 'solutions'.

The meaning and method of Marx's transformation procedure

It is well-known that classical political economy adhered to two opposing principles which it was unable to reconcile and that, in Marx's view, this failure led to its disintegration. On the one hand, it discovered that labour is the substance of value and that the magnitude of a commodity's value is determined by the labour-time needed for its production. On the other hand, it adhered to the *prima facie* contradictory view that profit rates tend toward equality and that a commodity's price therefore tends to be equal to the costs of its production plus an average profit. Even Ricardo failed to account for the determination of the *level* of the profit rate and held the disproportionality of prices and values to be an 'exception' to the law of value.

It is also well-known that Marx insisted that, rather than attempting to 'rescue' the law of value by means of a 'violent abstraction' of this sort, the existence of prices of production and a general rate of profit 'have to be explained through a number of intermediate stages' (Marx, 1977: 421; 1968: 174). However, this stipulation is often interpreted as a call for successive relaxation of assumptions, for an even stricter adherence to Ricardo's method – the analytic method rooted in formal logic. In this view, the law of value is a 'first approximation' based on assumptions, such as equal compositions of capital, which do not hold in the real world and which must be dropped as the model becomes more realistic.

We view Marx's stipulation, instead, as an indication of his dialectical method. In *Capital*, Volume I, he sharply distinguishes this method from that of the 'abstract materialists' whose 'materialism . . . excludes the historical process':

> Even a history of religion that is written in abstraction from [its] material basis is uncritical. It is, in reality, much easier to discover by analysis the earthly kernel of the misty creations of religion than to do the opposite, i.e. to develop from the actual, given relations of life the forms in which these have been apotheosized. The latter method is the only materialist, and therefore the only scientific one. (Marx, 1977: 493-94n)[10]

What Feuerbach had done in the analysis of religion,[10] Ricardo and the classicists had done in the analysis of economic life. They 'discover[ed] by analysis' the earthly kernel – labour – of the mystery of commodity-value. The manifold phenomena of price relations were reduced abstractly, without mediation, to this undifferentiated substance, labour. Yet, the starting-point in reality (prices) persisted *in contradistinction to* the starting-point in theory (labour). The gulf between the 'real world' and the theoretical world, between appearance and essence, was not overcome.[11]

Marx's approach was 'to do the opposite, i.e. to develop from the actual, given relations of life the forms in which these have been apotheosized'. The difference is not only that Marx maintained a consistent starting-point whereas the classicals vacillated between two inconsistent principles. Rather, instead of being a method of *reconciliation*, Marx's method is one of *development through contradiction*. His starting-point thus contains *within itself* a duality – the dual character of labour revealed within its product, the commodity. The duality between the concrete potentiality of the living workers and the abstract, value-producing character of their actual activity, i.e. alienated labour, is ever-present in capitalist production. It is as isolated, independent individuals that the workers 'enter into relations with the capitalist . . . Their co-operation only begins with the labour process, but by then they have ceased to belong to themselves' (Marx, 1977: 451). Their activity is not their own, but is subjected to the domination of dead labour. The social relations between persons at work have been *transformed* into thing-like relations (Marx, 1977: 166).

Through a succession of 'intermediate stages', Marx traced the development of the fetishised forms in which this reification of labour manifests itself. The first of these forms is the commodity-product, the materialisation of the labour which is an 'objective' factor of production rather than the workers' self-expression. Each subsequent 'stage' is still another transformation, an inversion in which the worker's subjectivity takes on yet another form of a false 'objectivity', a 'social relation between things' (Marx, 1977: 166).[12] However, capitalism manifests itself not only in industrial relations, but in the market and in the categories of even 'scientific' political economy. Thus, in these realms which Marx examines in Volume III, still more transformations are revealed. As he writes in Chapter 2:

> the way that surplus-value is transformed into the form of profit, by way of the rate of profit, is only a further extension of that inversion of subject and object which already occurs in the course of the production process itself. We saw in that case how all the subjective productive forces of labour present themselves as productive forces of capital. On the one hand, value, i.e. the past labour that dominates living labour, is personified into the capitalist; on the other hand, the worker conversely appears as mere objectified labour-power, as a commodity. This inverted relationship necessarily gives rise, even in the simple relation of production itself, to a correspondingly inverted conception of the situation, a transposed consciousness, which is further developed by the transformations and modifications of the circulation process proper. (Marx, 1981b: 136)

Thus, in Chapter 9 of Volume III, Marx argued that the consciousness of capitalists and bourgeois economists, though 'transposed', is grounded in reality's appearance.[13] Even in the form of price of production (in which considerations of disequilibrium of supply and demand, interest, rent, etc. are excluded), price and profit for an individual capital differ quantitatively as well as qualitatively from value and surplus-value. Because price appears to be *determined* by (not only *equal* to) the costs of production plus profit, and profit appears as a pure mark-up over costs, the law of value/surplus-value seems false. Nevertheless, the alien reality of capitalist production relations remains the essential determinant of these new forms *and makes its presence felt*. By making the *total* social capital the object of analysis, viewing capital as if it 'belong[ed] to one and the same person',[14] Marx was able once again to see the capital-labour relationship through the appearance of 'many capitals'. Total value and surplus-value are proportional to total price and profit, respectively; the general rate of profit is the ratio of total surplus-value to total capital advanced.

Throughout Volume III, rather than analysing market phenomena as *self-subsistent*, in their seeming independence from the sphere of production, these phenomena are developed as transformed forms of production relations. Thus, in Marx's illustration of the 'transformation *of* commodity values *into* prices of production',[15] value and price are conceived of as contradictory terms in *one* relation. Value takes on a transformed form of appearance, a form of appearance which differs from itself.

The dialectical meaning of the term 'transformation' thus differs from its use as a synonym for a mathematical mapping. Many, if not most, of Marx's critics view his transformation procedure precisely as a failed attempt to map a self-contained set of values onto another, self-contained set of prices of production (or general equilibrium prices). Curiously, however, what goes unrecognised is that this transformation is but one of many transformations of the same sort discussed throughout the three volumes of *Capital*, none of which are mappings. Were this fact better understood, perhaps this particular transformation would not have been singled-out for criticism.

Moreover, the failure to recognise that many transformations have preceded the transformation of values into prices of production is one factor that leads critics to charge Marx with logical inconsistency. Lacking this recognition, their misconceptions regarding the latter transformation's starting-point are significant. Firstly, some critics of Marx's procedure still interpret Volume III's reference to 'value' as a reference solely to labour and labour-time, and thus claim that the dimensionality of values and prices of production are inconsistent (see, for example, Abraham-Frois & Berrebi, 1979: 26-27). Actually, after tracing the development of the value-form into the price-form in Volume I, Chapter I, Marx regularly (albeit confusingly) referred to sums of money as 'values'.[16] Moreover, in a letter to Engels explaining the transformation of 'value' into price of production, Marx explicitly equates 'cost-price' with the '*price* of constant part of capital + *wages*' and notes that this transformation 'presupposes' that various value magnitudes appear as sums of money (Marx, 1983a: 109; emphases added).

The development of the form of value is not our present concern. However, inasmuch as the relationship between the value- and price-forms will play a crucial role in Section III of this paper, let us briefly indicate what is involved. The value congealed in a commodity is *always* expressed as a money price, a sum of money, because it is *always* related to the value of the *universal* measure of value, money. Conversely, of course, a sum of money always represents a sum of value. As the *universal* measure of value, money is ever-present, even in the absence of an exchange, since it 'serves only in an imaginary or ideal capacity' (Marx, 1977: 190). Hence, the initial input 'values' in Marx's illustration of the transformation of 'value' into price of production are actually sums of money which, through the ideal presence of money, implicitly *represent* sums of value. Therefore, both before and after the transformation of magnitudes, the value- and price-forms are related through a unique, necessary numeraire; inputs and outputs have the same, dual, dimensionality.

Secondly and relatedly, in Volume III 'commodities are not exchanged simply as *commodities*, but as the *products of capitals*', as results of capital's process of production (Marx, 1981b: 275). Capital-values, not the value of means of production and labour-power, constitute the starting-point of Marx's illustration. In circulation, capital is a sum of money which purchases means of production and labour-power. As Yaffe (1975: 45-46) has noted, the value of the *capital* is the value represented by that sum of money, not the combined value of the *means of production and labour-power*. Clearly, the capital advanced to production does not cease to be a sum of value merely because it differs from the values of its material elements.

At the beginning of Volume III, in discussing the transformation of value into cost-price plus profit, Marx *did* assume that cost-price equalled the combined values of the labour-power and means of production used up in producing the commodity. This assumption was made in order to grasp the qualitative transformation in its 'purity', independently of any quantitative disproportionality. On the other hand, when he discussed the quantitative transformation of Chapter 9, Marx dropped this assumption, noting that '*if* the cost price of a commodity is equated with the value of the means of production used up in producing it, it is always possible to go wrong' (1981b: 265, emphasis added). Because they interpret his procedure as having wrongly equated the two, his critics universally view this stipulation as an admission of error which, to be rectified, requires that values and prices be held apart in two 'systems'.

The passage, however, continues: '. . . The cost-price of the commodity is a given precondition, independent of his, the capitalist's, production . . .' Marx thereby indicated that he took the cost-price as a *datum*, a *given* magnitude of value represented by a *given price*, *without* assuming that this magnitude equals the value of the means of production (and labour-power) used up. Hence, neither his transformation procedure nor its resulting aggregate equalities depend on this assumption, as is often supposed. As we shall see in Section III, this procedure accounts for prices of production and the aggregate equalities obtain even when inputs are purchased at their prices of production.

That the initial magnitudes of value and price are *data*, established in the immediate past, implies that Marx's illustration was not a 'system' which abstracted from time. Rather, it depicted *one particular period* of capitalist production and circulation *within* the process of history. History as a process contains and releases two aspects of time. Time is always complete at this point, a continuously moving 'here and now', in human development. And it is simultaneously differentiated: past and future are sharply discontinuous within the 'here and now'. The present is therefore different from both the past and the potential for a totally different future.

> [W]hen the limited bourgeois form is stripped away, what is wealth other than the [situation in which the human being] does not reproduce himself in one specificity, but produces his totality? Strives not to remain something he has become, but is in the absolute movement of becoming? (Marx, 1973: 488)

The non-transformation problem

Marx's transformation procedure retains values and prices in one relation; 'solutions' to the 'transformation problem' separate them into two opposed equational systems. We have defended the former procedure as the one appropriate to Marx's purpose – illustration of the dialectical transformation of value into its opposite, price of production. Are not, however, the various 'solutions' merely different means of achieving the same end which Marx sought? To answer this question, we now examine their method, what they set out to demonstrate, and what they actually demonstrate.

There is no doubt that these 'solutions' correctly calculate general equilibrium relative prices and the general equilibrium profit rate. Were that their only purpose, they would be unobjectionable. Yet, insofar as they attempt also to disclose the relationship of equilibrium prices to values, it must be asked whether they do, in fact, achieve this additional goal.

The very form of the 'solutions', i.e. the separation of values and prices into two 'systems', is not without its implications. We turn first to the value 'system'. Values appear here as a set of price relations ('value prices') opposed to equilibrium price relations. Rather than conceiving of price as a form of value, value becomes another form of price. The question to be answered thus becomes: in what way are these two pricing systems related? While there is nothing 'wrong' with this question, its source is Boehm-Bawerk's (1949) misconception of Marx's value theory rather than that theory itself. Boehm, as is well-known, misinterpreted Volumes I and III as advancing two contradictory theories of exchange ratios. In response to this critique, 'solutions' to the 'transformation problem' attempt to affirm that these two systems of exchange are indeed related. But, unlike Ricardo, Marx *did not* advance a labour theory of exchange ratios. The 'value price' system therefore has no basis in Marx's theory and the question of its relation to equilibrium prices is, from this standpoint, moot.

In contrast to all schools of bourgeois economics, Marx refused to look at the market as a world unto itself, possessing its own reality, distinct from and unaffected by the underworld of production relations. Instead, value relations, the thing-like relations which distinguish the capitalist mode of production, are so essential to it that they pervade all realms, assert themselves through all appearances. Circulation is but one moment of capital's process of production and reproduction, the means through which value relations continually re-create themselves. The question Marx (1983a: 148) asked, therefore, was precisely *how* value relations assert themselves; his transformation procedure was part of the answer to this question.

When value is conceived of as a form of price and isolated into a separate 'system', this question cannot be answered. The market and the factory never come into contact; the unity of production and circulation is broken, *a priori*; the analysis becomes focussed on different market forms alone. Moreover, since 'value *prices*' are abstracted from real prices, there has arisen a tendency to view value *relations* as abstractions from price relations, rather than as the reality of the factory. For instance, constant capital may indeed be *mathematically* equivalent to past wage outlays plus compound interest. But the *economic* significance of the former in production – the specific manner in which, as a use-value, it confronts the workers and its impact on their fate as a class – evaporates when reduced to the latter.[17] Marx's analysis of capital's process of production becomes an overly elaborate statement that workers do not receive the full product of their labour, for which Morishima's (1973) 'Fundamental Marxian Theorem' serves as an elegant substitute.

We now turn to the price system. 'Solutions' which begin with two sets of simultaneous equations formulate the price system as a set of n equations and n+1 unknowns – the prices of the n commodities (which appear on both sides of each equation) and the equilibrium profit rate. Though the magnitudes of these variables are unknown, the variables themselves are taken as given, postulated. In other words, the existence of general equilibrium prices and the general equilibrium profit rate is assumed. This procedure is legitimate if one wishes only to calculate the magnitudes of these variables. If, however, the objective is to show these variables to be forms of value – to show, in other words, how they come into existence through the operation of the law of value – then, *in this context*, their existence cannot be postulated. To do so is tantamount to assuming what must be demonstrated. The meaning of these variables can only be revealed through the conceptualisation of the real process of their determination. Mathematical 'determination' – i.e. calculation of magnitudes – not only differs from the real process of determination (Shaikh, 1982); by its nature, it cannot investigate the *meaning* of the variables which it assumes.

The manner in which simultaneous 'solutions' obtain prices and the equilibrium rate of profit provides additional evidence of this conflation of mathematical and conceptual questions. All relative prices are 'determined' within the price system, without reference to the value 'system' which lies beside it. And, since prices appear on both sides of the equations, the system is circular: prices determine prices.[18] Again, no fault can be found with this procedure on grounds of mathematics or logic. Yet what does it offer as economic *theory*, as inquiry into the meaning of prices and their relation to values?

Secondly, the conception of the rate of profit as an unknown, to be solved within the price system, differs markedly from Marx's conception. That the latter's transformation procedure leaves the rate of profit unaltered has received little notice, as if this result were a mere 'by-product' of that procedure. However, its significance for Volume III of *Capital* is crucial. At pains to dispel the illusions which competition creates, Marx sought to demonstrate that, given a certain advance of capital, the *level* of the profit rate depends only on the degree to which capital succeeds in pumping-out surplus-labour. It is therefore determinable upon the completion of the production process, before commodities go to market. Competition merely effects the equalisation of profit rates *at this previously determined level*.

The mathematical results of simultaneous 'solutions' seem to discredit these contentions. The rate of profit appears to be 'determined' by the price system itself – that is, either by competitively determined prices or by planning which utilises shadow prices. Since this rate differs from the profit rate obtained through the value 'system', its appearance as a magnitude relatively independent of production relations is reinforced. However, *inasmuch as the value 'system' is an irrelevancy, so too is the discrepancy between its profit rate and the equilibrium profit rate.* As we seek to demonstrate in the next section, when the value of the capital advanced is not confused with the value of its material elements, the (theoretical) existence of the general equilibrium profit rate no longer implies its determination outside of production relations.

Calculation, then, cannot replace the 'power of abstraction' (Marx, 1977: 90) which is needed to comprehend the value-price relationship. The claim that Marx could not 'solve' the 'transformation problem' because he lacked the mathematical tools overlooks the difference between the two. Though Marx was not familiar with simultaneous equations, his *Mathematical Manuscripts* (Marx, 1983b) reveal not only his mathematical sophistication, but his critique of mathematicians' method. Newton, for example, discovered the differential operation. Though he used it to obtain correct *results*, Marx criticised his lack of rigour in working out the *process* of its derivation.[19] Furthermore, even the equations of the relative and equivalent forms of value, which Marx employs at the beginning of *Capital*, are not abstract identities. They are, rather, asymmetrical and imply a specific direction of movement.[20] Our present critique of 'transformation problem' 'solutions' must be judged on its own, independently of any 'appeal to authority'. But this evidence of Marx's mathematical practice is relevant to the history of thought: it indicates that he would not have considered the use of simultaneous equations as either the necessary corrective to, or an adequate substitute for, his own transformation procedure.

Even if they are of the iterative form instead of the simultaneous form,[21] 'solutions' to the 'transformation problem' must employ one or another 'normalisation condition' or 'invariance postulate'. Because the value and price 'systems' are in themselves unrelated and the dimensionalities of values and relative prices are inconsistent,[22] only the adoption of a normalisation condition can create some relation between the two. It is generally recognised that, since 'there does not seem to be an objective basis for choosing any particular invariance postulate in preference to all the others . . . the transformation problem may be said to fall short of complete determinacy' (Seton, 1957: 153, emphasis omitted). This indeterminacy indeed turns the 'transformation problem' into an endless exercise. The number of possible normalisation conditions (and therefore 'solutions') is limitless and each is, objectively, as good as any other.[23] Even in principle, then, the 'transformation problem' cannot resolve the question of the relation of values to prices.

Perhaps even more significant is the fact that *none of the 'solutions' actually demonstrates* any *relation of values to prices.* Whereas Marx's procedure obtains aggregate equalities on the basis of the *given data*, the value-price relationships which result from 'transformation problem' 'solutions' come from the theorists' heads alone. Because normalisation conditions are asserted *a priori* and imposed externally on the *otherwise unrelated* value and price 'systems',[24] the resulting relationships are only assumed ones. In short, first the theorists negate the internal relation of values to prices, then they substitute whatever arbitrary relation they choose. Marx characterised this 'tendency to form arbitrary unmediated connections between things that belong together in an organic union' as '[c]rudeness and conceptual nullity'.[25]

In the 'transformation problem', the theorist is the 'external mediator' who comes from outside of the problem bearing a normalisation condition that dictates how values will be reconciled with prices. The external mediator in actual life, however, must be some social force, 'independent' of both capitalists and workers, that can dictate a reconciliation of production with the market – in other words, the 'classless technical intelligentsia' responsible for planning the economy and establishing social equilibrium. Indeed, use of input-output models and equilibrium shadow-pricing form the foundation of the state planning of our day. Are not 'solutions' to the 'transformation problem' therefore the *ideological* representations of a harmonious, state-planned economy?

Yet, *in production itself*, there has been no reconciliation. To those who remain inside the factory, the plan is not classless but represents a 'social formation in which the process of production has mastery over man, instead of the opposite . . . production by freely associated men, [which] stands under *their* conscious and planned control' (Marx, 1977: 175, 173, emphasis added). There is only the domination of labour by capital *or* the *internal* transformation of this reality, by those who live under it, into its opposite.

The transformation of input prices: an illustration

> [Ricardo's *Principles*] gives rise to weariness and boredom. As the work proceeds, there is no further development. Where it does not consist of monotonous formal application of the same principles to various extraneous matters, or of polemical vindication of these principles, there is only repetition or amplification . . . (Marx, 1968: 169)

The foregoing discussion has indicated that Marx's concern was to show the transformation of values into prices of production to be only a 'further extension' of the transformation of workers' subjectivity into an antagonistic economic 'objectivity'. Comprehending this process of 'transformation into opposite' requires that values and prices be retained in a single relationship, not separated into different 'systems' of calculation. The charge of logical inconsistency, which derives from Marx's 'failure' to keep values and prices separate, is therefore misplaced.

While we reject this central criticism of Marx's transformation procedure, so often referred to as his failure to 'transform input prices', in another – quite real – sense the issue of 'input price transformation' remains. One capital's output *does* become the other's input and, in this interchange, the commodity's price generally *does* diverge from its value. Our defence of Marx's procedure can, admittedly, only retain its force if we can account for this process *without* separating value and prices into separate 'systems'. We therefore show presently that a simple continuation of Marx's own procedure, as interpreted above, can illustrate both the transformation of input prices and the transformation of values into prices of production.

In a general sense, then, our illustration does constitute a response to the criticism that Marx 'failed' to account for the transformation of input prices. Like many 'solutions', moreover, it will depict the transformation of input prices as occurring within the context of simple reproduction (without fixed capital or joint production).[26] We, however, will adopt *Marx's* conception of simple reproduction, not the conception formalised in the general equilibrium, input-output pricing models utilised by 'transformation problem' 'solutions'. Some marked differences exist. Space does not permit full discussion of this issue, which will be taken up in a future paper. We must nonetheless briefly digress in order to outline the following conceptual distinctions between 'transformation problem' 'solutions' and our illustration:

(1) **'Buying back vs advance of capital.** Were Marx to have assumed technologically inter-related departments in his illustra-

tion of the transformation of values into prices of production, the aggregate *output* price of any component of social production (e.g. wage goods) generally would not have equalled the aggregate *input* price of that same component (e.g. the total wage bill). This fact, originally noted by Bortkiewicz (1952), is taken as 'proof' of logical inconsistency and all such inequalities are indeed absent from 'transformation problem' 'solutions'. Actually, these sort of inequalities entail disruption of social reproduction only if one accepts the implicit underconsumptionist premise that the components of social output must be 'bought back'. For Marx, conversely, social reproduction is a question of the *advance* of capital, of 'investment'. Money advanced for means of production and subsistence enables the previous period's outputs both to be sold and to serve as inputs (directly or indirectly) in the *upcoming* period. *There is thus no reason why the input price of either means of production or means of subsistence in any period must equal its output price in that same period.*

(2) **General equilibrium prices vs prices of production.** 'Solutions' to the 'transformation problem' look for a set of *unique*, equilibrium (relative) prices as the means of achieving the necessary interdependence of the various industries. These prices alone are called 'prices of production'. In contrast, we do not regard the prices of production to which Marx refers as general equilibrium prices.[27] They are indeed the prices that (a) obtain when supplies and demands are equal, (b) permit each capital to receive the same profit rate, and (c) form the 'centre' around which market prices oscillate. As will be seen, however, many *different* sets of prices can at *different* times fulfill conditions (a) and (b), even when technology and real wages remain unchanged. (Abstracting from the process of competition, we will show no market price oscillations.)

(3) **Reproduction of prices vs reproduction of use-values.** As unique, general equilibrium magnitudes, the relative prices obtained in 'transformation problem' 'solutions' are continually reproduced in a timeless fashion. Implicitly, these 'solutions' assume either that material reproduction cannot occur under other prices or that this sort of 'price stability' constitutes an additional condition which must be fulfilled if the system is to be truly in balance. When Marx discussed reproduction, on the other hand, he was concerned with a prior question: in what specific quantities and proportions must the system produce two distinct use-values, means of production and articles of consumption, if it is to materially reproduce itself at a certain level? The technical relations of production were thereby considered as a class relation – the relation of dead to living labour in production – irreducible to exchange relations among capitalists. Marx

(1981a: 469-70) explicitly abstracted away from changes in values and deviations of prices from values, regarding them as irrelevant to the question at hand, and *therefore* held values fixed. In principle, the simple reproduction of material relations can take place at any set of prices. Moreover, if all profit rates are equal at the prevailing prices – whether or not they are general equilibrium prices – it is reasonable to suppose that no further incentives for capital flows exist and that supplies and demands should therefore actually equilibrate at these prices.

Other than assuming simple reproduction, in the above sense, we modify Marx's own illustration in one substantive respect alone: we continue his one-period illustration into successive periods. This is necessary because, whereas the transformation of values into prices of production can be depicted in a single period, the transformation of outputs into inputs and thus the 'transformation' of output prices into input prices takes place between *one* period of production and the *next*. It is clear that Marx (1977: 711, 716) regarded simple reproduction as a continuously renewed process taking place in real time. And, though he did not illustrate the transformation of input prices mathematically, his verbal discussions indicate that he viewed it similarly:

> [E]ven if a commodity's cost price may diverge from the value of the means of production consumed in it, this error *in the past* is a matter of indifference to the capitalist. The cost price . . . is a given pre-condition, independent of his, the capitalist's, production, while the result of his production is a commodity that contains surplus-value . . . (1981b: 265, emphasis added)

> [E]very commodity which enters into another commodity as constant capital, itself emerges as the result, the product, of another production process. And so the commodity appears *alternately* as a pre-condition . . . and as the result of a process . . . In agriculture (cattle-breeding), the same commodity appears *at one point of time* as a product and *at another* as a condition of production. (1971: 167-68, emphases added)

These passages conceive of input price transformation as part of a real process: commodity prices are not *simultaneously* the pre-conditions and results of the same period of production, but *alternately* the results of one period and the pre-conditions for the next. Hence, cost-price is always a given pre-condition in each period – *inputs cannot be* retroactively *re-priced*. Each period therefore has *new* pre-conditions which enable it to emerge as a *new* period, distinct from those in the past.

To illustrate this process, we must therefore begin (and indeed begin again in each new period) with given input prices. These prices are money prices, sums of money. As we noted in Section II A, the appearance of values as sums of money and hence the existence of money (which possesses a definite value) are necessarily presupposed. We assume that the value of money equals 1; every number in our illustration therefore signifies both a price and an amount of value.

Table 1 presents a specific two-department illustration.[28] We assume that, in every period, each department obtains half of the means of production that have been produced by Department I, and that the consumption goods produced by Department II are allocated as follows: one sixth and one third go to the workers in Departments I and II, respectively; half are consumed by the capitalists who represent the two departments. The definitions of the symbols used in Table 1 (opposite) are:

m capitalists' personal revenue
M money-capital before production
C commodity-capital before production
MP price of means of production
L price of labour-power
P productive-capital: process of production
LL (price expression of) hours of living labour added; generation of new value (not shown in table)
s (price expression of) surplus-value
C′ commodity-capital after production
M′ price of production; money-capital after production
π profit
e rate of exploitation; rate of surplus-value
r general rate of profit

Beginning with money (M), the collective capitalists of Departments I and II each purchase two commodities (C), means of production (MP) and labour-power (L), at *given* prices which represent *given* sums of value. (*Solely* in order to facilitate comparison with 'transformation problem' 'solutions', we begin without any 'error[s] in the past; i.e. initial values are equal to the values of means of production and labour-power.)

In production (P), the means ofproduction become constant capital and labour-power becomes labour, the labourers' activity as variable capital. Upon entrance into the sphere of production, no change in material or value occurs. But production results in new outputs of greater value (C+s), due to the extraction of surplus-value (s) – labour for which no equivalent has been paid. These outputs are priced, not at their value, but at their price of production (C′–M′), which equals their cost-price (C) plus an

Table 1

Period	*Department*	m	M-C	MP	L	. . . P	s . . .	C + s	C′-M′	π <	*General rates of profit and surplus-value*
	I	—	150.00	100.00	50.00	150.00	50.00	200.00	214.29	64.29	r = .4286
1	II	—	200.00	100.00	100.00	200.00	100.00	300.00	285.71	85.71	e = 1.0000
	Total	—	350.00	200.00	150.00	350.00	150.00	500.00	500.00	150.00	
	I	59.52	154.76	107.14	47.62	154.76	52.38	207.14	222.86	68.10	r = .4400
2	II	83.33	202.38	107.14	95.24	202.38	104.76	307.14	291.43	89.05	e = 1.1000
	Total	142.86	357.14	214.29	142.86	357.14	157.14	514.29	514.29	157.14	
	I	62.86	160.00	111.43	48.57	160.00	51.43	211.43	226.98	66.98	r = .4186
3	II	82.86	208.57	111.43	97.14	208.57	102.86	311.43	295.88	87.31	e = 1.0588
	Total	145.71	368.57	222.86	145.71	368.57	154.29	522.86	522.86	154.29	
	I	65.14	165.14	115.14	50.00	165.14	50.00	215.14	230.28	65.14	r = .3945
13	II	84.86	215.14	115.14	100.00	215.14	100.00	315.14	300.00	84.86	e = 1.0000
	Total	150.00	380.27	230.28	150.00	380.27	150.00	530.28	530.28	150.00	
	I	65.14	165.14	115.14	50.00	165.14	50.00	215.14	230.28	65.14	r = .3945
14	II	84.86	215.14	115.14	100.00	215.14	100.00	315.14	300.00	84.86	e = 1.0000
	Total	150.00	380.28	230.28	150.00	380.28	150.00	530.28	530.28	150.00	

average profit (π). Average profit differs from the surplus-value each department extracts, such that each receives the general rate of profit – the ratio of (1) the price expression of the surplus-labour extracted by the total social capital in production (total s) and (2) the total cost-price of the social capital (total M−C). The general rate is determined *in production*, before circulation commences, so that its magnitude would be the same were outputs sold at their values. This process of pricing and profit determination, identical to Marx's own, will hold for all periods. We note that the sum of values (total C+s) equals the sum of prices (total C′−M′) and the sum of surplus-value (total s) equals the sum of profits (total π).[29]

The first circuit of money capital is now completed. For simple reproduction to occur, each department must replace the precise quantities of the specific use-values which have been used up in this period. In our illustration, the collective capitalist of each department must lay out 107.14 to obtain 1/2 = 107.14/214.29 of the means of production which have been produced in Department I. Further, Department I's collective capitalist must lay out 47.62 in wages, enabling its workers once more to purchase 1/6 = 47.62/285.71 of the consumer goods produced by Department II. Similarly, Department II's collective capitalist must advance wages of 95.24, enabling its workers to purchase 1/3 = 95.24/285.71 of the consumer goods. The productive consumption of these means of subsistence reproduces the labour-powers of these workers. Each department's total price (C′−M′) in period 1, minus the sum of its advances to production (M−C) in period 2, equals the revenue (m) which the collective capitalists consume unproductively on output purchased from Department II. We note that the sum of the revenue, though less than the sum of profit, nonetheless enables them to purchase 1/2 = 142.86/285.71 of the output of Department II. The entire social product has thus been bought and sold at its price of production. Furthermore, each department's sales equal its purchases: 107.14 worth of means of production are purchased by Department II and 47.62 + 59.52 = 107.14 worth of consumer goods are purchased indirectly by Department I.

Since the elements of cost-price have been obtained by the two departments, their use-values can be consumed productively – the activity of production can recommence. And since we are considering simple reproduction, and thus the reproduction of the same labour-powers, the use-values of these labour-powers which accrue to the respective departments are the same in this period as in the last. Put differently, workers in Department I again perform 100 hours of labour (the sum of necessary and surplus-labour) at the same intensity, and workers in Department II perform 200 hours. The portion of this labour for which no equivalent has been paid is different in this period than the last, because means of subsistence are cheaper and capital can obtain the use-value of labour-power at a lower price.

Since the total price and the total value of the first period are the same, 500, the wages and capitalists' consumption expenditures, totalling 285.71, which exit the circuit of capital between periods of production, are sums of value as well as price. The remainder, 500 − 285.71 = 214.29, is thus *both the value and price* of that part of *capital* which recirculates, the constant *capital* – even though this diverges from the value of the *means of production*. When we add to this 214.29 the 100 + 200 = 300 in newly generated value, we obtain 514.29, the total value of the second period. This total differs from the total value produced during the first period, but only because an additional value of 14.29 has been incorporated into the social capital in the second period.[30]

The sum of surplus-value in the second period, 157.14, does not equal the newly-added labour hours minus the *value of labour-power*. As the sum of the newly-added labour hours for which no equivalent has been paid it is, nevertheless, a sum of surplus-*value*.[31] And, since the value and price of the capital advanced in the second period are equal, it follows that the general rate of profit for the second period – the ratio of surplus-value to capital advanced – is a 'price of production rate of profit', but also a 'value rate of profit'. In this period as in the first, the addition of the resulting average profits to the cost-price yields a total price which equals total value, and the subtraction of the total cost-price from this total price of production yields a total profit which equals total surplus-value.

The process we have just outlined continues in the same manner for all periods. In each period, new prices of production are formed. They differ from the prices of production of the preceding period because more value is incorporated into capital. This divergence continues as long as the sum of profits exceeds the sum of revenues and the rate of surplus-value is higher than its rate in the preceding period.

Eventually, these differences become extremely small and, in this illustration, a general equilibrium is approached. At the terminal point of this convergence, the value rate of profit would equal the 'equilibrium rate of profit' and prices of production would become general equilibrium prices. *However, we caution against interpretating our continuation of Marx's transformation procedure over time as an iterative (or any other sort of) 'solution' to the 'transformation problem'*. It is noteworthy, first of all, that such 'solutions' illustrate only movements in prices, not movements in values. In other respects, however, our continuation is undeniably identical to an iterative 'solution' on a purely *formal, mathematical* level. Yet, it begins from different *conceptual* premises and its results are, accordingly, *conceptually* different as well.

As we have emphasised, period 14's prices and profit rate *should not* be compared, as a separate 'system', to the values and profit rate of period 1. *Each period is discrete.* The difference between the total price of the 14th period and the total value of the first period is due, *not* to a deviation of total price from total value, but to the incorporation of additional value into the social capital in the interim, so that the total price and value *of the 14th period* are equal. The incorporation of additional value into capital also accounts for the different profit rates in these two periods. Similarly, the difference between the total profit of any period and the total surplus-value of the first period is due to the difference in the surplus-values extracted in these periods, *not* to any deviation of total profit from total surplus-value. Finally, we consider *each* period's prices of production to be 'correct' for that period, since both departments obtain the general rate of profit and demand equal supplies.

Because our procedure accounts for the determination of prices of production *in the absence of general equilibrium*, convergence to equilibrium is not necessary. The feature of our illustration which produced convergence, the constancy of total living labour hours over time, was therefore *not* imposed as an equilibrium-producing normalisation condition. Rather, it follows from the assumption of simple reproduction in every period.

Illustration of convergence was important for a different reason. We earlier objected to the view that the mathematical equalities of general equilibrium contradict Marx's transformation procedure or, indeed, the law of value. Our illustration has now indicated a diametrically opposed view of general equilibrium relations. Beginning with cost-price as a *known* magnitude of value, and proceeding 'through a number of intermediate stages' on the basis of the law of value and Marx's procedure, it arrived at general equilibrium prices and the general equilibrium profit rate *as forms and determinations of value*. Even in general equilibrium, the sums of price and profit equal the sums of value and surplus-value, respectively, and the general profit rate is the ratio of surplus-value to the value of capital advanced.

Summary and conclusion

We have defended Marx's transformation procedure neither on 'doctrinal' ground, nor solely on logical ground, but as a procedure which was both internally consistent and appropriate to his purpose – illustration of the dialectical transformation of value into its opposite, price of production. Criticisms lodged by post-Marx Marxists, neo-Ricardians, neoclassicists, etc. reveal their methodological and theoretical differences with Marx, not the 'logical inconsistency' of his procedure.

The value-form of labour's product, we argued, is the immediate result of the transformation of workers' subjectivity into an antagonistic, falsely 'objective' 'thing'. This inversion takes place in production; subsequent transformations in *Capital* are only its 'further extension', including price of production as its form of appearance in the market. Consequently, the transformation of values into prices of production cannot be understood apart from the transformations developed earlier in *Capital*. Moreover, comprehension of price of production as a form of appearance of value – a form in which value relations *do* manifest themselves in the market – requires that values and prices be retained 'in one relation', not kept apart in two 'systems'.

'Solutions' to the 'transformation problem' neither develop Marx's critique of political economy nor even demonstrate that an alternative conception of the value-price relationship is warranted. While they do correctly calculate general equilibrium relative prices and the general equilibrium profit rate, the theorists arbitrarily link the price and value 'systems' by imposing an assumed value-price relationship onto these otherwise unrelated opposites.

Finally, we accounted for the 'transformation of input prices' without separating values and prices into two 'systems'. A simple multiperiod continuation of Marx's own transformation procedure, developed on the basis of his conception of capitalist reproduction, easily resolved this allegedly 'logical' problem. In particular, our procedure illustrated that (1) the aggregate equalities of value and price, surplus-value and profit, and 'value' and 'price' rates of profit hold in each period as well as in general equilibrium; and (2) inter-period changes in total price are due only to changes in the magnitude of value incorporated into the total social capital. These results depend crucially on Marx's distinction between the value of capital advanced and the value of the capital's material elements.

The point of this exercise was not to defend 'the' labour theory of value, but to comprehend price phenomena as necessary manifestations of alienated labour. It is only in the *aggregate*, however, only when the *total social capital* is considered as 'belong[ing] to one and the same person' – that the real relations of capital to labour, centralisation and concentration of capital at one pole and the revolt of labour at the other, pierce through the illusions created by competition.

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Notes

1. The term 'post-Marx Marxists', as a pejorative designation, comes from Dunayevskaya (1981).
2. This view appears charitable. Yet, it implies incompetence or even disingenuousness on Marx's part, since he relied on the specific conclusions of his transformation procedure to develop further his view of price of production and profit as forms of appearance of value and surplus-value. Thus, Marx's critics imply that he deduced certain features of capitalist society solely on the basis of a mathematical 'model' he constructed. Even worse, he proceeded in these deductions despite his awareness that the 'model' was flawed. Bortkiewicz (1952: 13) for one was well aware of these implications of his critique of Marx: '[H]e holds the nature of the object to which his theoretical construction refers, responsible for the inner contradictions afflicting this construction.'
3. Failure to resolve the relationship of prices to values led to the disintegration of the Ricardian school. Is it not possible that the persistence of the 'transformation problem' is a sign that post-Marx Marxist economics is likewise disintegrating? That the 'transformation problem' appears to post-Marx Marxists as a technical issue to be solved by the specialist's 'box of tools' is only one expression of the predominance in our day of what Lenin called the 'administrative mentality'. However, just as the many crises of our age have not yielded to technical 'solutions', neither has the 'transformation problem'.
4. We know of no thorough review of 'transformation problem' literature, but Hunt (1979) discusses the history of this approach to the value-price relationship, with reference to early 'solutions'. Recent works defending Marx's procedure include Mattick (1972); Yaffe (1975); Mandel (1981); Mandel & Freeman (eds) (1984); Fine (1986); and Carchedi (1986).
5. See, for instance, Mandel & Freeman (eds) (1984). Mandel's introduction in particular ignores the issue of method while emphasising those of 'science', 'logic', and the labour theory of value: it even goes so far as to imply that rejection of Marx's transformation procedure is purely a neo-Ricardian (and neoclassical) vice. In his haste to unify 'orthodox Marxists' against neo-Ricardianism, Mandel not only neglects to ask *why* almost all 'orthodox' adherents to the labour theory of value likewise reject Marx's procedure, he even ignores the fact itself.

One recent article (Carchedi, 1986) which we discovered just before this paper was accepted for publication, does stress that methodological differences between Marx and his critics are crucial to the 'transformation problem' debate. We also recommend Carchedi's discussion of the starting-point of Marx's transformation procedure as complementary to our own. However, we cannot accept Carchedi's central claim that prices of production are 'social values', the realised forms of potential, 'individual values'. Because it fails to recognise that the commodity as a value is itself the *social* 'realisation' of alienated labour, this novel formulation denies the actuality of value as distinct from price. *Capital* Volume 1 is not fully appreciated as Marx's theoretical penetration into the reality of capitalist production relations, but is regarded as a more or less abstract reflection of concrete market phenomena.

6. We hasten to add that Marx's philosophy of human liberation is neither separate from nor reducible to his methodology. Unfortunately, the particular content of this paper does not permit discussion of either the entirety of Marx's work or the dialectic of freedom in *Capital*.

7. 'Transformation of input prices' refers to the re-evaluation of the constant and (usually) variable capital advanced to production. This is held to be necessary because the prices of production of the material elements of the advanced capital, means of production and subsistence, are not proportional to their values. Proponents of the 'new solution' to the 'transformation problem' (Dumenil, 1983; Foley, 1982; Lipietz, 1982) do not re-value the variable capital.

8. As we will discuss below, the transformation is, strictly speaking, a transformation of the simple price-expression of value into price of production. These equalities thus refer to the fact that the aggregate price of output and aggregate profit are not affected by the formation of a general rate of profit and prices of production. We will follow the shorthand convention of referring to the simple money-forms as 'value' and 'surplus-value', except where confusion might result.

9. With the simultaneous outbreak of World War I and the collapse of the Second International, Lenin, through his study of Hegel's philosophy, transformed his philosophical views. 'Unity of opposites' and 'transformation into opposite' became especially significant to his new comprehension of the movement of reality and thought. His subsequent analyses of the transformation of competition into monopoly and of part of the proletariat into a 'labour aristocracy', his call to transform the imperialist war into a civil war, and his support of national liberation struggles against imperialism all testify to this fact.

10. Cf. Marx's fourth thesis on Feuerbach. Feuerbach 'resolv[ed] the religious world into its secular basis. But that the secular basis detaches itself from itself and establishes itself as an independent realm . . . can only be explained by the cleavages and self-contradictions within this secular basis [which] must then itself be destroyed in theory and in practice.' McGlone (1985) provides a more thorough discussion of Marx's break with Feuerbach and abstract materialism.

11. '[I]f the commodity has a double character . . . then labour contained in the commodity must also be of double character, while mere analysis of labour as such, as with Smith, Ricardo, etc., must everywhere come up against the inexplicable. This is indeed the whole secret of the critical conception.' (Marx, 1983a: 125)

12. We are here reviewing the development of the fetish. The development in *Capital* always remains two-sided, precisely because Marx grasped labour in its duality – as objectified labour (the substance of value) and as the living labourer (the human being as subject of history). Capital's oppressive reduction of the labourer to 'one-dimensionality' is itself what drives her to oppose it in her struggle to become a 'totally developed individual' (Marx, 1977: 618).

13. Banaji (1979) correctly stresses the objectivity of appearance in Hegel's and Marx's work.

14. Marx (1981b: 259). Cf. Marx (1981a: 509): 'it is necessary . . . to avoid looking at things as if a society based on the capitalist mode of

production lost its specific historical and economic character when considered . . . as a totality. This is not the case at all. What we have to deal with is the collective capitalist.'

15. This phrase (without emphases) is part of the title of *Capital* Vol. III, Chapter 9.

16. For just one example, see Marx (1977: 417).

17. For examples of this reduction, see Bortkiewicz (1952) and Lipietz (1982).

18. Bródy (1974) acknowledges that the definition of prices is circular. However, he considers circular definitions to be both fruitful and indispensable.

19. Regarding this differential operation, Brokmayer (1986: 46) notes that 'Marx characterises the process of its derivation as negation of the negation which was hidden in the mystifying methods of mathematics because they could not conceive how something could come out of nothing.'

20. Arthur (1979) stresses the asymmetry (as well as the non-reflexivity and intransitivity) of these equations. Their implicit undirectional movement was pointed out to us by Ron Brokmeyer, in a personal communication.

21. The best-known iterative 'solution' is that of Shaikh (1977). Morishima (1973: 77n) and Brody (1974: 90) also propose iterative 'solutions'. It is to Shaikh's credit that he does not begin by severing values and prices into two distinct 'systems'. In addition to adopting a normalisation condition, however, he takes only the final, general equilibrium prices to be ' "correct" ' prices of production, thereby interpreting Marx's procedure as only the first approximation to an adequate 'solution'.

22. The dimensionality of unit values is (labour hours/commodity unit), and the dimensionality of their price expressions is (monetary unit/commodity unit). The dimensionality of relative prices is (unit of numeraire commodity/commodity unit).

23. For instance, one could let total value equal the total price of means of production and subsistence, thereby 'showing' that profit is a pure 'mark-up' on top of commodities' real value. Though this would violate the entire spirit of Marx's theory, it constitutes a 'solution' to the 'transformation problem' which is no less legitimate than any other.

24. That a numeraire is needed to obtain absolute prices does not justify the arbitrary imposition of a normalisation condition. While the price-form itself entails that a commodity find expression in *some* amount of money, the declaration that a *specific* aggregate of commodities must equal a *specific* sum of money is only the theorist's whim imposed upon the actual data.

25. Marx, quoted in Lukács (1971: 9). Cf. Nicolaus's translation in Marx (1973: 88). Ironically, though Samuelson (1971) himself does not recognise the 'organic union' of values and prices, his suggestion that an eraser be used to effect the transformation indicates that he considers the use of normalisation conditions to be an arbitrary way of relating values to prices.

26. This does not imply acceptance of Bortkiewicz's (1952) or subsequent 'proofs' of error on Marx's part. For the 'general case' Marx examined, in which no particular input-output relations are specified, no critic has proposed an alternative to his transformation procedure. All

such 'proofs' depend crucially on the substitution of technologically *inter*dependent 'departments' or 'sectors' for Marx's *in*dependent 'branches'. We therefore consider these proofs illegitimate: though they alter the *problem* Marx's procedure is made to address, they do not permit the *procedure* to be modified accordingly.

27. To interpret Marx's concept of prices of production as general equilibrium prices has become common, if not standard. Ironically, however, those who advance this interpretation generally charge that his transformation procedure is internally inconsistent – because the prices obtained are not general equilibrium prices! 'Analytical Marxists' and Eastern European planners may *wish* that Marx had formulated a general equilibrium pricing system. Unless it can be *demonstrated* that he attempted to depict such a system, however, the charge of internal inconsistency is not justified.

28. That Marx's procedure and its aggregate equalities hold for any number of technologically interrelated sectors is shown in an Appendix available upon request from the authors (Department of Economics, University of Utah, Salt Lake City, UT 84112, USA; and Department of Economics and Finance, St John's University, Staten Island, NY 10301, USA, respectively).

29. Each department's price of production equals its cost-price x (1 + general profit rate). No normalisation condition is needed as an additional 'closing equation' because the *known data* in each equation serve to calculate a price of production. Input prices and therefore cost-prices are given initially. As the price expression of total surplus-labour divided by cost-price, the general profit rate is calculated on the basis of the known total expenditure of living labour and the initially given input prices and value of money (see the Appendix for the exact mathematical expressions). We also note that, because cost-prices are sums of money and the profit rate is a pure number, the equations directly express prices of production as sums of money, not relative prices.

30. Since, for all t,

$$(1)\quad \Sigma M'^t_i = \Sigma C^t_i + \Sigma s^t_i,$$

$$(2)\quad \Sigma C^t_i = \Sigma M^t_i,$$

$$(3)\quad \Sigma M^{t+1}_{i,} = \Sigma M'^t_i - \Sigma m^{t+1}_i, \text{ and}$$

$$(4)\quad \Sigma s^t_i = \Sigma \pi^t_i = \Sigma M'^t_i - \Sigma M^t_{i},$$

it follows that

$$\Sigma M'^{t+1}_i - \Sigma M'^t_t = (\Sigma C^{t+1}_i - \Sigma C^t_i) + (\Sigma s^{t+1}_i - \Sigma s^t_i)$$

$$= (\Sigma M^{t+1}_i - \Sigma M^t_i) + (\Sigma s^{t+1}_i - \Sigma s^t_i)$$

$$= (\Sigma M'^t_i - \Sigma m^{t=}_i) - \Sigma M^t_i) + (\Sigma s^{t+1}_i - \Sigma s^t_i)$$

$$= (\Sigma s^t_i - \Sigma m^{t+1}_i) + (\Sigma s^{t+1}_i - \Sigma s^t_i),$$

(from (1), (2), (3), and (4), respectively. Thus, the increase in total price between any two periods is due to (a) the portion of surplus-value which does not exit the circuit of capital as revenue, and (b) the difference in surplus value between periods. The latter is the result of the difference in money wages which exit the circuit of capital. More briefly, the increase in total price is due only to the incorporation of additional value into the total social capital.

31. The workers must work for a greater or lesser amount of time in order to buy back these commodities (to replace them) and must therefore *perform more or less necessary labour* than would be needed if the prices of production of their necessary means of subsistence did coincide with their values.' (Marx, 1981b: 309, emphasis added)

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