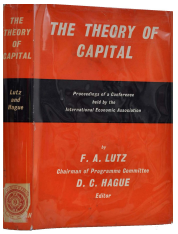

THE THEORY OF CAPITAL
Proceedings of a Conference held by
the International Economic Association
F. A. LUTZ Chairman of Programme Committee
D. C. HAGUE Editor

Chapter 10

CAPITAL ACCUMULATION AND ECONOMIC GROWTH [1, 2]

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I. INTRODUCTION

A THEORETICAL model consists of certain hypotheses concerning the causal inter-relationship between various magnitudes or forces and the sequence in which they react on each other. We all agree that the basic requirement of any model is that it should be capable of explaining the characteristic features of the economic process as we find them in reality. It is no good starting off a model with the kind of abstraction which initially excludes the influence of forces which are mainly responsible for the behaviour of the economic variables under investigation ; and upon finding that the theory leads to results contrary to what we observe in reality, attributing this contrary movement to the compensating (or more than compensating) influence of residual factors that have been assumed away in the model. In dealing with capital accumulation and economic growth, we are only too apt to begin by assuming a 'given state of knowledge' (that is to say, absence of technical progress) and the absence of 'uncertainty', and content ourselves with saying that these two factors — technical progress and uncertainty — must have been responsible for the difference between theoretical expectation and the recorded facts of experience. The interpretative value of this kind of theory must of necessity be extremely small.

Any theory must necessarily be based on abstractions ; but the type of abstraction chosen cannot be decided in a vacuum : it must be appropriate to the characteristic features of the economic process

[1] Editor's footnote : Mr. Kaldor's paper as printed here represents an extended written version of an address delivered by him orally to the conference in accordance with prior arrangement made with the I.E.A. In the subsequent discussion the members of the Round Table did not have the present text in their hands.

[2] The author is indebted to Mr. L. Pasinetti and Mr. F. H. Hahn for assistance in setting out the models in algebraic form.

as recorded by experience. Hence the theorist, in choosing a particular theoretical approach, ought to start off with a summary of the facts which he regards as relevant to his problem. Since facts, as recorded by statisticians, are always subject to numerous snags and qualifications, and for that reason are incapable of being accurately summarized, the theorist, in my view, should be free to start off with a 'stylized' view of the facts — i.e. concentrate on broad tendencies, ignoring individual detail, and proceed on the 'as if' method, i.e. construct a hypothesis that could account for these 'stylized' facts, without necessarily committing himself on the historical accuracy, or sufficiency, of the facts or tendencies thus summarized.

As regards the process of economic change and development in capitalist societies, I suggest the following 'stylized facts' as a starting-point for the construction of theoretical models :

(1) The continued growth in the aggregate volume of production and in the productivity of labour at a steady trend rate ; no recorded tendency for a *falling* rate of growth of productivity.

(2) A continued increase in the amount of capital per worker, whatever statistical measure of 'capital' is chosen in this connection.

(3) A steady rate of profit on capital, at least in the 'developed' capitalist societies ; this rate of profit being substantially higher than the 'pure' long-term rate of interest as shown by the yield of gilt-edged bonds. According to Phelps Brown and Weber [1] the rate of profit in the United Kingdom was remarkably steady around $10\frac{1}{2}$ per cent in the period 1870–1914, the annual variations being within $9\frac{1}{2}$-$11\frac{1}{2}$ per cent. A similar long-period steadiness, according to some authorities, has shown itself in the United States.

(4) Steady capital-output ratios over long periods ; at least there are no clear long-term trends, either rising or falling, if differences in the degree of utilization of capacity are allowed for. This implies, or reflects, the near-identity in the percentage rates of growth of production and of the capital stock — i.e. that for the economy as a whole, and over longer periods, income and capital tend to grow at the same rate.

(5) A high correlation between the share of profits in income and the share of investment in output ; a steady share of profits (and of wages) in societies and/or in periods in which the investment coefficient (the share of investment in output) is constant. For example, Phelps Brown and Weber found long-term steadiness in the investment coefficient, the profit share and the share of wages in the U.K., combined with a high degree of correlation in the (appreci-

[1] *Economic Journal*, 1953, pp. 263-88.

able) short period fluctuations of these magnitudes.[1] The steadiness in the *share* of wages implies, of course, a rate of increase in real wages that is proportionate to the rate of growth of (average) productivity.

(6) Finally, there are appreciable differences in the *rate* of growth of labour productivity and of total output in different societies, the range of variation (in the fast-growing economies) being of the order of 2-5 per cent. These are associated with corresponding variations in the investment coefficient, and in the profit share, but the above propositions concerning the constancy of relative shares and of the capital-output ratio are applicable to countries with differing rates of growth.

None of these 'facts' can be plausibly 'explained' by the theoretical constructions of neo-classical theory. On the basis of the marginal productivity theory, and the capital theory of Böhm-Bawerk and followers, one would expect a continued *fall* in the rate of profit with capital accumulation, and not a steady rate of profit. (In this respect classical and neo-classical theory, arguing on different grounds, come to the same conclusion — Adam Smith, Ricardo, Marx, alike with Böhm-Bawerk and Wicksell, predicted a steady fall in the rate of profit with economic progress.) Similarly, on the basis of the neo-classical approach, one expects diminishing returns to capital accumulation which implies a steady *rise* in the capital-output ratio *pari passu* with the rise in the capital-labour ratio; and a diminishing rate of growth in the productivity of labour at any given ratio of investment to output (or savings to income). Finally, the fluctuations in the *share* of profits that are associated with fluctuations in the rate of investment cannot be accounted for at all on the basis of the marginal productivity theory — if we assume, as I believe we must, that the fluctuations in the level of investment are the causal factor, and the fluctuations in the share of profits consequential, rather than the other way round.

My purpose here is to present a model of income distribution and capital accumulation which is capable of explaining at least some of these 'stylized' facts. It differs from the prevailing approach to problems of capital accumulation in that it has more affinities with the classical approach of Ricardo and Marx, and also with the general equilibrium model of von Neumann, than with the neo-classical models of Böhm-Bawerk and Wicksell; or with the theories which start off with the Cobb-Douglas type of production function. It differs from the classical models in that it embodies the basic ideas of the Keynesian theory of income generation, and it takes the well-known 'dynamic equation' of Harrod and Domar as its starting-point.

[1] *Op. cit.* Fig. 7.

II. THE CHARACTERISTIC FEATURES OF THE CLASSICAL APPROACH

The peculiarity of classical models as against the neo-classical theories is that they treat capital and labour as if they were complementary factors rather than competitive or substitute factors. Of course Ricardo was well aware that the use of capital is not only complementary to labour but also a substitute to labour — hence the famous 'Ricardo effect'.[1] This demonstrates that with a rise in wages more machinery will tend to be employed per unit of labour, *because* the price of machinery will fall relatively to labour with any rise in the share of the produce going to labour — but he did not accord this substitution-aspect any major rôle in his distribution or growth theory. As far as his distribution theory is concerned he treated the amount of capital per unit of labour as something given for each industry (and similarly, the distribution of labour between different industries as given by the 'structural requirements' of the system). He solved the problem of distribution between wages and profits (after deduction of the share of rent which is determined quite independently of this division) by assuming that the amount going to one of these two factors, labour, is determined by its supply price, whereas the share of the other is residual — the share of profits is simply the difference between output per man (after deduction of rent) and wages per man, the latter being treated as constant, governed by the 'natural price' of labour at which alone the working population *can remain* stationary.

Since profits were assumed to be largely saved and invested, whilst wages are consumed, the share of profits in income also determines the share of investment in total production, and the rate of accumulation of capital. The rate of accumulation of capital in turn determines the rate of increase in the employment of labour (since employment was assumed to increase at the same rate as capital, there was no scope for any consequential change in the amount of capital per unit of labour) without enquiring very closely where this additional labour comes from. The model is consistent with the assumption that there is an unlimited labour reserve, say, in the form of surplus population in an under-developed country (the assumption favoured by Marx) or with assuming that the rate of increase in population is itself governed by the rate of growth in the demand for labour (the assumption favoured by Ricardo).

[1] *Principles*, ch. i, sec. v.

Von Neumann's general equilibrium model,[1] though on a very different level of sophistication, explicitly allowing for a choice of processes in the production of each commodity, and abstracting from diminishing returns due to the scarcity of natural resources to which Ricardo accorded such a major rôle, is really a variant of the classical approach of Ricardo and Marx. Von Neumann similarly assumes that labour can be expanded in unlimited quantities at a real wage determined by the cost of subsistence of the labourers, and that profits are entirely saved and re-invested. These two assumptions enable him to treat the economic problem as a completely circular process, where the outputs of productive processes are simultaneously the inputs of the productive processes of the following period; this is achieved by treating not labour, but the commodities consumed by labour, as the inputs of the productive processes, and by treating the surviving durable equipment as part of the outputs, as well as of the inputs, of the processes of unit length. Von Neumann is concerned to show that on these assumptions an equilibrium of balanced growth always exists, characterized by the equi-proportionate expansion in the production of *all* commodities with positive prices: and that this rate of expansion (under perfect competition and constant returns to scale for each process) will be the maximum attainable under the given 'technical possibilities' (the real wage forming one of the given 'technical possibilities'), and will be equal to the rate of profit (= rate of interest) earned in each of the processes actually used.[2]

The celebrated Harrod-Domar equation can be applied to the Ricardian model and the von Neumann model as well as to other models.[3] Though it can be interpreted in many ways (according to which of the factors one treats as a dependent and which as an independent variable) it is fundamentally a formula for translating the

[1] *Review of Economic Studies*, 1945–1946; originally prepared for a Princeton mathematical seminar in 1932.

[2] Von Neumann was only concerned with demonstrating the *existence* of such an equilibrium solution. Later Solow and Samuelson (*Econometrica*, 1953) have shown that on certain further assumptions this solution will be stable both 'in the large' and 'in the small' — i.e. the balanced growth equilibrium will be gradually approached from any given set of initial conditions; and it will restore itself if it is disturbed for any reason.

[3] In von Neumann's formulation, where the surviving equipment at the end of each period is treated as a part of the output, v is $1/1+g$, when Y is defined as the gross output of the period (since then K_t and Y_{t-1} are identical) whilst s is unity if Y is defined as the *net* output (since the wage bill forms part of the commodities consumed in the process of production) so that the net-output/capital ratio is equal to g, the rate of growth of the capital stock. It is possible, however, within the framework of the model, to define Y in the usual way as being the sum of profits and wages — in which case the output-capital ratio (in a state of balanced growth) is identical with the net rate of expansion of the system multiplied by the ratio of Y (thus defined) to net output (i.e. the ratio by which the sum of wages and profits exceeds profits). Given a fixed real wage, and the possibility of expanding the rate of employment at the rate dictated by the requirements of a

share of savings (and investment) in income (s) into the resulting growth rate of capital (G_K), given the capital-output ratio, $v\left(\equiv\frac{K}{Y}\right)$[1]

$$G_K=\frac{s}{v}, \tag{1}$$

which can also be written

$$s=\frac{I}{Y}=G_K v. \tag{1a}$$

It further follows that when $s=\frac{P}{Y}$, i.e. all profits are saved and all wages are consumed,

$$\frac{P}{Y}=G_K\frac{K}{Y}.$$

But since

$$\frac{P}{K}\equiv\frac{P}{Y}\cdot\frac{Y}{K}$$

$$\frac{P}{K}=G_K. \tag{2}$$

the rate of profit on capital is the same as the rate of growth of capital.

As far as Ricardo and von Neumann are concerned, this is really the end of the story, for they do not introduce any limit to the *speed* with which additional labour can be introduced into the system, so that the rate of growth of employment, and hence of income, is fully determined by the rate of growth of capital. Supposing, however, that even if the supply of labour can be increased to an indefinite extent *ultimately*, there is a maximum to the rate of increase of population and/or of employment per unit of time, determined by biological or institutional factors. Writing L for the quantity of employment, this gives us another equation

$$G_n=l, \text{ where } l=\frac{1}{L}\cdot\frac{dL}{dt}. \tag{3}$$

The Ricardo-Marx-von Neumann model clearly does not work when $G_K>G_n$ since in that case the rate of growth of production cannot be determined by G_K alone.

In a progressive economy the labour potential increases, however, not only on account of the rise in numbers, but also on account of

balanced-growth economy, the ratio of wages to profits is itself determined by the relative input-intensities of labour and non-wage commodities when (at the given wage and with the given range of available processes) the rate of expansion of the system is maximized.

[1] Time subscripts are omitted, except in the formal presentation of the models.

the rise in the productivity of labour due to technical progress. Hence, allowing for technical progress,

$$G_n = l + t, \text{ where } t = \frac{1}{Y/L} \cdot \frac{d(Y/L)}{dt}, \tag{3a}$$

which is Harrod's formula for the 'natural' rate of growth.

Harrod realized that balanced-growth equilibrium is only conceivable when his 'warranted rate of growth' equals the 'natural rate',

$$G_K = G_n,$$

in other words

$$\frac{s}{v} = l + t.$$

Since he assumed, however, that s, v, l and t are all independently given and invariant in relation to each other, such an equality, on his theory, could only be the result of a fortunate accident. Moreover, he thought that any discrepancy between $\frac{s}{v}$ and $(l+t)$ must set up cumulative forces of disequilibrium, so that a moving equilibrium of steady growth, even if momentarily attained, is necessarily unstable.

The problem takes on an entirely different aspect, however, once we recognize (as we must) that these variables are not mutually invariant, but that there are certain inter-relationships between them. Thus, as will be shown, the proportion of income saved s, is by no means independent of $(l+t)$; nor is the rate of increase in productivity, t, independent of the rate of capital accumulation, $\frac{s}{v}$.[1]

III. THE NATURE OF GROWTH EQUILIBRIA

In order to exhibit the rôle of these various factors it is best to start from a model based on a number of artificial assumptions which together produce the simplest solution to the problem of growth-equilibrium. We shall afterwards remove these assumptions one by one (with the exception of the first assumption listed below) in the reverse order in which they are presented here. The six critical assumptions of our 'basic model' are :

(1) Constant returns to scale in any particular process of production ; natural environment does not impose any limitation to

[1] In the above equation, in deference to the generally accepted use of symbols, we have denoted the rate of growth of labour by l and the rate of growth of output per man by t. In the rest of this paper, however, we shall denote the *maximum* rate of population growth by λ, and the rate of growth of productivity by G_0; reserving the letter t to denote time.

expansion (i.e. there are two factors of production, Capital and Labour (K and L), and two kinds of income, Profits and Wages (P and W)).

(2) The absence of technical progress — i.e. the function relating the output of various commodities to the input-coefficients of production remains unchanged over time.

(3) General rule of competition: the prices of commodities in relation to the prime costs of production settle at the point where the market is cleared. Capital earns the same rate of profit, and labour the same rate of wages, in all employments.

(4) All profits are saved and all wages are consumed; the division of output between equipment goods (or 'input goods') and wage goods (consumption goods) is the same as the division of income between Profits and Wages.

(5) There is strict complementarity between Capital and Labour (or commodity-inputs and labour-inputs) in the production of both equipment goods and wage goods; there is therefore a single kind of 'equipment good' for the production of each wage good, and the different kinds of wage goods are also complementary in consumption.

(6) There is an unlimited supply of labour at a constant wage in terms of wage goods.[1]

Under these assumptions the rate of growth of the capital stock, G_K, will govern the rate of growth of the economy, G_Y; and G_K in turn depends on the proportion of output saved, s, and the capital-output ratio, v. The proportion of output saved is determined by the condition that the wage rate cannot fall below a certain minimum, determined by the cost of subsistence,

$$w \equiv\equiv w_{\min} \tag{4}$$

so that the excess of output per head over the subsistence wage alone determines the share of profits. Output per head (O), the capital-output ratio (v), and hence capital per head, are given technical constants; and in addition the total amount of capital at some arbitrary point of time, $t = 0$, is taken as given.

These assumptions yield a model which can be formally stated as follows. Using our previously introduced notation[2] and denoting

[1] These six assumptions are identical (except for (5)) with those underlying Neumann's model; they are substantially the same as those implicit in Ricardo's theory (except for (1)); and Marx's theory (except of course in its 'dynamic' aspect, assumptions (2) and possibly (5)).

[2] This notation may be summarized as follows:

$$G_K = \frac{dK}{dt}\frac{1}{K} \qquad G_Y = \frac{dY}{dt}\frac{1}{Y} \qquad v = \frac{K}{Y} \qquad O = \frac{Y}{L}$$

and the symbols K, Y, L, w and s represent the stock of capital, output (or income), labour employed, wage per worker, and the proportion of income saved respectively.

output per worker by O, we obtain a system of six relationships, of which four represent assumptions, one is a definitional identity and one equation the equilibrium condition.

$$\left.\begin{aligned} O(t) &= \bar{O} \\ v(t) &= \bar{v} \\ w(t) &= w_{\min} \\ s(t) &= \frac{P(t)}{Y(t)} \end{aligned}\right\} \text{for all } t \geqslant 0 \qquad \begin{matrix}(i)\\(ii)\\(iii)\\(iv)\end{matrix}$$

$$P(t) \equiv Y(t) - w(t)L(t) \qquad (v)$$

$$s(t)Y(t) = \frac{dK(t)}{dt} \text{ for all } t \geqslant 0 \qquad (vi)$$

which are sufficient to determine the six basic variables $O(t)$, $v(t)$, $s(t)$, $P(t)$, $Y(t)$ and $w(t)$ given the initial values. From (vi) and (ii) we have

$$G_Y = \frac{s(t)}{\bar{v}} \text{ or } \bar{v}G_Y = s(t)$$

From (v) it follows

$$\frac{P(t)}{Y(t)} = \left[1 - \frac{w_{\min}}{\bar{O}}\right]$$

and hence the share of profit is independent of t. And so, by (iv), $s(t)$ is also independent of t, and hence

$$\begin{aligned} G_K &= \frac{s}{\bar{v}} \\ G_K &= G_Y \\ \frac{P}{K} &= G_K \\ \frac{P}{Y} &= G_K\bar{v} \end{aligned} \qquad (I)$$

IV. FULL EMPLOYMENT GROWTH

The first modification I shall introduce is the removal of assumption (6), that of an unlimited supply of labour. We may suppose

that there is a certain maximum rate of population growth, λ, determined by fertility rates ; so that (abstracting from technical progress) this rate determines the long-run 'natural rate of growth'. Hence

$$G_n = \lambda.$$

If we suppose, further, that initially

$$G_K > G_n,$$

i.e. the rate of capital accumulation, as determined by the conditions of our previous model, exceeds the maximum rate of growth of population, the economy can only grow at the rate G_K as long as there are reserves of unemployed labour to draw upon. But just because the economy grows at a higher rate than λ, sooner or later capital accumulation must overtake the labour supply. According to Marx this is precisely the situation which leads to a crisis. When the labour reserves are exhausted, the demand for labour will exceed (or tend to exceed) the supply of labour, since the amount of capital seeking profitable employment will be greater than the number of labourers available to employ them with. Owing to the competition between capitalists, this will cause wages to rise and profits to be wiped out, until, in consequence, capital accumulation is reduced sufficiently to restore the labour reserve and thus restore profits.

However, there is no inherent reason why this situation should involve a crisis ; nor does it follow from the assumptions that the maintenance of accumulation requires the continued existence of a labour reserve. Indeed there is no reason why this situation should not result in a neat balanced-growth equilibrium with a higher rate of wages and a lower share of profits, and with a correspondingly lower rate of capital accumulation that would no longer exceed, but be equal to, the rate of increase in the supply of labour. All that is necessary is to bear in mind that every increase in wages (in terms of commodities) lowers the share of profits in income, and every reduction in the share of profits lowers the rate of accumulation of capital and hence *the rate of increase* in the demand for labour. Hence the situation will lead to a balanced-growth equilibrium in which employment at some arbitrary point of time $t = 0$ is taken as given by the size of the working population at that point of time, and where the rate of growth of population λ is also taken as given.

This gives us an alternative model of seven relationships of which four define the assumptions, one is an identity as before and two are equilibrium conditions. Using, in addition, the notation $L^*(t)$ for the maximum amount of labour available at time t, the relationships are as follows :

$$\left.\begin{aligned} L^*(t) &= L^*_{(0)}e^{\lambda t} \\ v(t) &= \bar{v} \\ O(t) &= \bar{O} \\ s(t) &= \frac{P(t)}{Y(t)} \end{aligned}\right\} \text{for all } t \geqslant 0 \qquad \begin{matrix} \text{(i)} \\ \text{(ii)} \\ \text{(iii)} \\ \text{(iv)} \end{matrix}$$

$$P(t) \equiv Y(t) - w(t)L(t) \qquad \text{(v)}$$

$$\left.\begin{aligned} s(t)\,Y(t) &= \frac{dK}{dt} \\ L(t) &= L^*(t) \end{aligned}\right\} \text{for all } t \geqslant 0 \qquad \begin{matrix} \text{(vi)} \\ \text{(vii)} \end{matrix}$$

subject to the inequality

$$w(t) \geqslant w_{\min}$$

which are sufficient to determine the seven basic variables $O(t)$, $v(t)$, $s(t)$, $P(t)$, $Y(t)$, $w(t)$ and $L(t)$, given the initial conditions.

It follows from (i) and (vii) that

$$G_Y = \lambda.$$

From (vi), $s(t) = \lambda v(t)$ and so, by (i) and (ii), $s(t)$ is independent of t. Hence by (iv)

$$G_K = \frac{s}{\bar{v}}$$

$$G_K = G_y$$

$$\frac{P}{Y} = \lambda\bar{v}$$

$$\frac{P}{K} = \lambda$$

Also, by (v), $\qquad w(t) = (1 - \lambda\bar{v})\bar{O},$

subject to the inequality stated. (II)

The difference between this model and the previous one is that while in both, output-per-man and capital-per-man are constant (over time), in this model the rate of profit on capital and the share of profit in income (given v, which is here as a technical constant) are uniquely determined by λ, the population growth rate, which on our present assumptions will alone determine the uniform expansion rate of the economy. There is an equilibrium wage, w, which will exceed

the subsistence wage, w_{min}, by the amount necessary to reduce the share of profits to λv. But despite the similarities, this second model is the inverse of the Ricardian (or Marxian) one; for here it is not profits which form a residual after deducting subsistence-wages, but wages form the residual share after deducting profits, the amount of profits being determined independently by the requirements of the (extraneously given) balanced growth rate.[1]

Ricardo did say, in various places scattered around in the *Principles*, that as capital accumulation runs ahead of population, or the reverse, wages will rise above the 'natural price of labour' or may fall below it. But he never drew the immanent conclusion (though in several places he seemed almost on the point of saying it) that the rise or fall in wages resulting from excessive or insufficient rates of accumulation *will itself change the rate of accumulation of capital through changing the profit share*, and thereby provides a mechanism for keeping the rate of accumulation of capital in step with the rate of increase in the labour supply — i.e. that there is an 'equilibrium' level of wages which maintains the increase in the demand for labour in step with the increase in supply. (Had he said so, with some emphasis, one cannot help feeling that the subsequent development of economics, both Marxist and orthodox, might have taken a rather different turn.)

Marx's view that where excessive accumulation leads to a crisis due to the scarcity of labour there is nothing to stop wages from rising until profits are wiped out altogether, clearly assumes a *constant* supply of labour over time. If population is rising, profits cannot fall below the level which provides for a rate of accumulation that corresponds to the rate of growth in the supply of labour; and once 'full employment' has been reached (i.e. the 'reserve army' is exhausted) there is no reason why wages should not settle down to a new equilibrium level, divorced from the cost of subsistence of labour.

There is one other important assumption implicit in this, and in the other growth models, which may be conveniently introduced at this stage. In a capitalist economy continued investment and

[1] This situation is incompatible also with von Neumann's model, which, as mentioned before, implicitly assumes that the effective supply of labour can be increased at the required growth rate, whatever that rate is. But if one introduced labour explicitly as one of the 'commodities' into the von Neumann model (instead of the goods consumed by labour) and assumed that the supply of labour was growing at some autonomous rate that was lower than the maximum potential expansion rate of commodities other than labour, the same result would be reached. For then the equilibrium price system which equalized the rate of profit earned in all the 'chosen' processes would be the one which made the price of labour in terms of other commodities such as to reduce the rate of profit earned in the production of commodities (other than labour) to the expansion rate of labour.

accumulation presupposes that the rate of profit is high enough (in the words of Ricardo) to afford more than the minimum necessary compensation to the capitalists 'for their trouble, for the risk which they must necessarily encounter in employing their capital productively'.[1] Hence growth-equilibrium is subject to a further condition which can be written in the form

$$\frac{P}{K} > r + \rho, \tag{5}$$

i.e. the rate of profit as determined by the model (under our present assumption by λ alone) cannot be less than the sum of the 'pure' rate of interest on financial assets of prime security, and the additional premium required for the risks involved in productive employments of wealth.

We know, since Keynes, that there is a minimum below which the pure long-term rate of interest cannot fall, and that this is determined by the minimum necessary compensation for the illiquidity-risk entailed in holding long-term bonds as against cash (or other short-term financial assets which are close substitutes for cash). We also know (though this has received far less emphasis in the literature) that the risks (whether illiquidity risks or other risks) associated with the direct investment of capital in business ventures are quantitatively far more important than the risks entailed in holding long-term financial assets of prime security. (The rate of profit on business investments in fixed capital [in plant and equipment] in the U.S., for example, is generally taken to be 20 per cent gross, or say 10 per cent net, of taxation, when the 'pure' long-term rate of interest is around 4 per cent.)

The (expected) marginal return on investments in circulating capital (which, by universal convention, are treated as part of the 'liquid assets' of a business) is much more in line with the money rates of interest, though here also, the expected return is likely to be appreciably higher than the (pure) short-term rate of interest. It is indeed highly unlikely that in an economy *without* technical progress, and where *all* profits are saved and re-invested, the rate of profit (as determined by population growth) could be anywhere near high enough to satisfy the above condition. If it is not, there cannot be a moving equilibrium of growth, though this does not mean that the economy will lapse into perpetual stagnation. Accumulation could still take place in periodic spurts, giving rise to a higher-than-trend rate of growth for a limited period.

[1] *Principles*, Sraffa edition, p. 122.

We must now proceed with the relaxation of the various simplifying assumptions made. As we shall see, until we come to technical progress, none of these introduces a vital difference to our results.

V. NEO-CLASSICAL GROWTH

We can allow for variable proportions, instead of strict complementarity, between capital and labour, by postulating that there is a choice of processes of production involving differing quantities of capital per man (i.e. a differing ratio between 'commodities' and 'labour' as inputs). Thus output per man, O ($O \equiv Y/L$), will be a function of K/L, capital per man, the increase in the former being less than proportionate to the latter, if the production function for labour and capital together is homogeneous and linear. Hence

$$O \equiv Y/L = f_1(K/L), \text{ where } f_1' > 0, f_1'' < 0. \tag{6}$$

Assuming that each entrepreneur at any one time has a limited amount of capital at his disposal, the amount of capital per man employed will be such as to maximize the rate of profit; and this optimum amount of capital per man will be all the greater the higher are wages in terms of commodities, hence

$$K/L = f_2(w), \text{ where } f_2' > 0, f_2'' < 0. \tag{7}$$

(6) in combination with (7) also implies that the capital-output ratio in the 'chosen' process will be all the greater, the higher the rate of wages, hence

$$v \equiv \frac{K}{Y} = f_3(w), \text{ where } f_3' > 0, f_3'' < 0. \tag{8}$$

Further, it also follows that output per man will be the greater the higher the capital-output ratio

$$O = f_4(v), \text{ where } f_4' > 0, f_4'' < 0. \tag{9}$$

Hence as wages rise (with the approach to full employment and the slowing down of the rate of accumulation) v will rise as well; this in turn will increase the share of investment in output $\left(\frac{I}{Y}\right)$ at any *given* rate of growth of output, and hence the share of profits. It *may* also slow down the rise in wages in terms of commodities, but since the rise in v will increase output per man, as well as the share of profits, this does not necessarily follow. However, on the assumption of diminishing returns (which, as we shall argue later, comes to

much the same as the assumption that there is no technical progress) $f_1'' < 0$, the rise in the investment ratio and in the share of profits will not be sufficient to prevent a continued fall in the rate of growth of capital with the continued increase in v. Hence this process of adopting more labour-saving techniques by increasing capital per head will come to an end when the rate of growth of capital declines sufficiently to approach the rate of increase in the supply of labour, λ. From then onwards the system will regain a balanced-growth equilibrium with unchanging techniques and capital per head and proceeding at the uniform expansion rate λ.

Thus the introduction of a choice of processes permitting the substitution of capital for labour will mean that there will be an intermediate stage between the equilibrium of Model I (where G_Y was determined by G_K) and of Model II (where G_Y was determined by G_n, and G_K by G_Y), characterized by the condition

$$G_K > G_Y > G_n,$$

i.e. where the actual rate of growth is greater than the natural rate, as determined by population growth, and lower than the rate of capital accumulation. In other words, the rate of growth of capital will be higher than that of output, and the latter will be declining. The difference thus introduced is best shown in a diagram (Fig. 12)

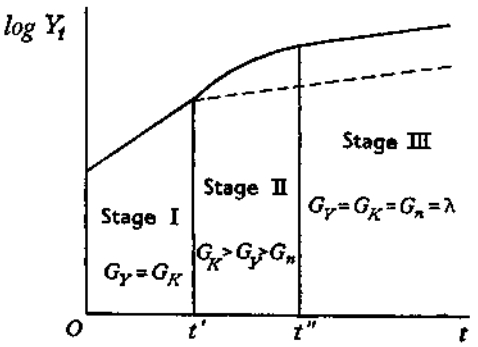

FIG. 12

where output (Y_t) is shown vertically (on a logarithmic scale) and time horizontally. Assuming that from $t = 0$ onwards the economy is in a growth equilibrium with unlimited supplies of labour with $G_Y = G_K$, G_K being determined by the ratio of savings to income when wages are at the minimum subsistence level; and assuming further that the labour reserves are exhausted at the point of time t',

then, in the absence of a choice of 'techniques' of a more or less labour-saving character, wages will immediately rise to the point where the share of profits is cut down to the level where the rate of accumulation is brought down to $G_n = \lambda$ and the system attains a new balanced growth equilibrium at this lower rate. If we assume, however, that there are technical possibilities for increasing output per head by using more capital per unit of labour, the transition will be gradual. Wages will rise more gradually, and accumulation will be maintained (temporarily) at a higher rate, serving both the requirements of the growing working population and the increasing amount of capital per unit of labour. But since during this stage the rate of growth of production will be declining, and will be constantly smaller than the rate of capital accumulation, balanced-growth equilibrium will be regained at a certain point (shown by t'' in the diagram). This will occur when wages have risen to the point at which accumulation is brought down to the rate corresponding to the rate of growth of population, and from then onwards the economy will attain the same constant growth rate, determined by λ.[1]

Given the range of alternative processes represented by our f functions, it follows that there is a unique relationship between output per worker and the capital-output ratio (as stated in equation (9) above) and also between the *desired* capital-output ratio and the rate of profit on capital. Hence for balanced growth equilibria (where the actual capital-output ratio corresponds to the desired ratio) we have the further relationship

$$v = \phi\left(\frac{P}{K}\right), \text{ where } \phi' < 0,\ \phi'' > 0. \tag{8a}$$

Writing these relationships in this form, this model will be characterized by seven relationships, of which three are equilibrium conditions.

$$\left.\begin{aligned} L^*(t) &= L^*(o)e^{\lambda t} && \text{(i)}\\ O(t) &= f(v(t)),\ f' > 0,\ f'' < 0 && \text{(ii)}\\ s(t) &= \frac{P(t)}{Y(t)} && \text{(iii)} \end{aligned}\right\} \text{for all } t \geqslant 0$$

$$P(t) \equiv Y(t) - w(t)L(t) \tag{iv}$$

[1] The first of our three stages may be termed the 'classical' stage, the second the 'neo-classical' stage (since it will be characterized by rising capital per man, a rising capital-output ratio, and a declining rate of growth and profit) and the third stage, for reasons set out below, the 'Keynesian' stage.

$$\left.\begin{aligned} s(t)\,Y(t) &= \frac{dK(t)}{dt} && \text{(v)}\\ L(t) &= L^*(t) && \text{(vi)}\\ v(t) &= \phi\left(\frac{P(t)}{K(t)}\right) && \text{(vii)} \end{aligned}\right\} \text{for all } t \geqslant 0$$

where $\phi' < 0,\ \phi'' > 0$

subject to the inequalities

$$w(t) \geqslant w_{\min}$$

$$\frac{P(t)}{K(t)} \geqslant r + \rho$$

By the same argument as employed in Model II above it follows that

$$G_Y = \lambda$$

Hence by (v), $\frac{s(t)}{v(t)}$ is independent of t. By (iii) we have $\frac{P(t)}{Y(t)} = \lambda v(t)$ and so

$$\frac{P}{K} = \lambda$$

and using (vii) we obtain $$\frac{P}{Y} = \lambda\phi(\lambda) \qquad \text{(III)}$$

As a comparison with the corresponding equations for Model II shows, the introduction of a 'production function' which makes the capital-output ratio dependent on the rate of profit will not affect the equilibrium growth-rate, or the rate of profit on capital. But it will have an influence on the share of profits, and hence on the savings coefficient, s, for any given rate of growth, since λ and $\phi(\lambda)$ are inversely related to one another: the higher the value of λ, the lower the equilibrium value $\phi(\lambda)$. In the special case where the function $\phi(\lambda)$ is one of constant unit elasticity (i.e. when doubling the rate of growth and the rate of profit involves halving the capital-output ratio, etc.) the investment coefficient, $\lambda\phi(\lambda)$, will be invariant with respect to any change in the rate of growth and the rate of profit on capital, and, in that sense, the share of profits and wages can be said to be uniquely determined by the coefficients of the production function. But the assumption of constant unit elasticity for the ϕ function is by no means implicit in the assumption of homogeneous and linear

production functions, and indeed it cannot hold in all cases where there are limits to the extent to which any one factor can be dispensed with. If, in the relevant range, the elasticity of this function is appreciably smaller than one, the share of profit will predominantly depend on the rate of economic growth (and on the propensities to save out of profits and wages discussed below) and only to a minor extent on the technical factors, the marginal rates of substitution between capital and labour (which determine the elasticity of the ϕ function).[1]

VI. THE PROPENSITIES TO SAVE

We can now relax our fourth assumption, the one implicit in all 'classical' models, that there is no consumption out of profits and no saving out of wages. We can allow both for the fact that profits are a source of consumption expenditure and that wages may be a source of savings — provided that we assume that the proportion of profits saved is considerably greater than the proportion of wages (and other contractual incomes) saved.[2] This assumption can be well justified both by empirical evidence and by theoretical considerations. Thus, on U.S. data, gross savings out of gross (company) profits can be put at 70 per cent, whereas savings out of personal incomes (excluding unincorporated businesses) are only around 5 per cent. Statistical evidence from other countries yields very similar results. On theoretical grounds one can expect the propensity to save out of business profits to be greater than that of wage and salary incomes (i) because residual incomes are much more uncertain, and subject to considerable fluctuations, year by year; (ii) because the accumulation of capital by the owners of the individual firms is closely linked to the *growth* of the firms: since a firm's borrowing power is limited to some proportion of its equity capital, the growth of the latter is a necessary pre-condition of the growth in its scale of operations. Apart from this, it could be argued on Keynesian considerations that it is precisely this difference in savings-ratios which lends stability to a capitalist system, under full employment or near-full employment

[1] Empirical evidence, such as it is, lends little support to the supposition that the capital-output ratio is smaller in fast-growing economies than in slow-growing economies, or in economies where the amount of capital per head is relatively small as against those where it is large. But the reason for this, as we shall argue later, is not the lack of substitutability between capital and labour, but the unreality of the postulate of a ϕ function which abstracts from all technical progress.

[2] I am assuming here, purely for simplicity, that the savings functions for both profits and wages are linear (with a zero constant) so that the average and marginal propensities are identical. If this were not so, it would be the difference in marginal propensities which was critical to the theory.

conditions. For if these differences did *not* exist, any chance increase in demand which raised prices would bring about a cumulative tendency: a rise in prices is only capable of eliminating the disequilibrium in so far as the transfer of purchasing power from 'contractual' to 'residual' incomes which it represents reduces effective demand in real terms.

If we denote by α the proportion of profits saved and β the proportion of wages saved,

$$I = \alpha P + \beta W, \text{ where } 1 > \alpha > \beta > 0 \tag{10}$$

$$s \equiv \frac{I}{Y} = (\alpha - \beta)\frac{P}{Y} + \beta \tag{10a}$$

and

$$\frac{P}{Y} = \frac{1}{\alpha - \beta}\,\frac{I}{Y} - \frac{\beta}{\alpha - \beta} \tag{10b}$$

If, in the first approximation, we assumed that βW is zero the equilibrium relationships will remain the same as in Model III, with the exception of (iii) which becomes

$$s(t) = \alpha \frac{P(t)}{Y(t)}.$$

This modification implies that in equilibrium

$$\frac{P}{K} = \frac{\lambda}{\alpha}$$

$$\frac{P}{Y} = \frac{\lambda}{\alpha} \cdot \phi\left(\frac{\lambda}{\alpha}\right). \tag{IV}$$

In other words, the rate of profit on capital will now exceed the rate of growth by the reciprocal of the proportion of profits saved. Similarly, the share of profit in income will also be raised, except in so far as the rise in $\frac{P}{K}$ will reduce v, and hence the investment-output ratio at any given rate of growth.

VII. COMPETITION AND FULL EMPLOYMENT

Before examining the implications of assumption (3), the general rule of competition, I should like to translate our results into terms that are in accord with the Keynesian techniques of analysis. So far

we have assumed that the level of production at any one time is limited not by effective demand but by the scarcity of resources available; which meant in the case of Model I that it was limited by the amount of capital (i.e. physical capacity) and in the case of Model II by the available supply of labour. In the 'Keynesian' sense, therefore, the equilibrium in both cases is one of 'full employment'. This is ensured, in the case of Model I, through the assumption, implicit in the model, that it is the 'surplus' remaining after the payment of subsistence wages which determines the rate of accumulation. In the case of Model II, where investment demand per unit of time is independently determined by the accrual of new investment opportunities resulting from the given rate of increase in the labour supply, it is ensured through the fact that the level of wages in real terms, and thus the share of profits, is assumed to settle at the point where savings out of profits are just equal to the required rate of investment. This latter presumes in effect a 'Keynesian' model where investment is the independent variable, and savings are the dependent variable: but the process of adjustment is assumed to take place not in a Keynesian but in a classical manner through forces operating in the labour market. An excess of savings over investment manifests itself in an excess of the demand for labour over the supply of labour; this leads to a rise in wages which reduces profits, and thus savings, and hence diminishes the rate of increase in the demand for labour. There is therefore some particular real wage at which the rate of increase in the demand for labour, resulting from capital accumulation, keeps in step with the rate of increase in the supply of labour, and which therefore is alone capable of maintaining the labour market in equilibrium.

But we are not *obliged* to look upon the equilibrating mechanism in this way; we could equally describe the equilibrating process in the 'Keynesian' manner, through the forces of adjustment operating not in the labour market, but in the commodity markets. In the Keynesian system an excess in the demand for labour in the labour market can only cause a rise in *money* wages, not of *real* wages, since a rise in *money* wages, *ceteris paribus*, will raise monetary demand, and thus prices, in the same proportion. To explain movements in *real* wages (output per man being assumed as given) we need to turn to the commodity markets and examine the conditions of equilibrium for the demand and supply of commodities. It is the most significant feature of Keynes' theory to have shown that equilibrium between savings (*ex ante*) and investment (*ex ante*) is secured through forces operating in the commodity markets. When investment exceeds savings, the demand for commodities

will exceed the supply. This will lead *either* to an expansion of supply (assuming the prevalence of 'Keynesian' unemployment and hence a state of affairs where production is less than the short-period maximum) *or* to a rise in prices relatively to costs (assuming 'full employment' in the Keynesian sense, i.e. that supply is limited by physical bottlenecks). In both cases an increase in the demand for commodities will lead to an increase in savings; in the first case, because savings are an increasing function of real income, at any *given* relationship of prices to costs (or of profits to wages); in the second case, because the rise in prices relative to costs implies a rise in profits and a fall in wages (in *real* terms) which increases savings. Keynes, in the *General Theory*, writing in the middle of the big slump of the 1930s, concentrated on the under-employment case, and conceived of the mechanism which equates savings with investment as one which operates through variations in the general level of employment. But in his previous book, *A Treatise on Money* (written in the late 1920s), he described essentially the same mechanism as determining the relationship of prices to costs, with output and employment as given.[1]

To illustrate the nature of this process and to analyse the conditions under which the forces equalizing savings and investment determine the price-cost relationship at full employment, rather than the level of employment at some *given* relationship of prices to costs, I should like to make use of the time-honoured device of the 'representative firm' which is assumed to behave like a small-scale replica of the economy as a whole. I shall assume, in other words, that variations in the output of the 'representative firm' reflect equivalent variations in total production, and that the firm employs a constant fraction of the total employed labour force.

I shall ignore falling average prime costs in the short period and shall assume that average and marginal prime costs are constant up to the point where the optimum utilization of capacity is reached and begin to rise afterwards, as shown by the curves *APC* and *MC* in Fig. 13. I shall assume that our representative firm is fully integrated vertically, so that its average and marginal prime costs consist only of labour cost. (The rate of money-wages is assumed to be given.) And I shall further assume, as is appropriate for a 'developed' economy under conditions of imperfect competition, that the effective bottleneck setting an upper limit to production is labour rather than physical capacity: there is more than enough capacity to employ the available labour force. Hence, since our firm accounts for a constant fraction of total employment, it cannot produce at a rate

[1] *A Treatise on Money* (London, 1930), vol. i, p. 139.

higher than that indicated by the full-employment position (as shown by the dashed line in Fig. 13.) [1]

Finally, I shall assume that whatever the state of demand, our firm will not be forced to reduce prices to the bare level of prime costs ; there is a certain *minimum margin of profit* which competition cannot succeed in eliminating. We can call this minimum profit margin the 'degree of monopoly', or the 'degree of market imperfection', remembering, however, that it does not necessarily *set* the price (in relation to costs), it merely sets a rock-bottom to prices. (In Fig. 13 the dot-and-dash line indicates the minimum price at the given level of prime cost per unit of output.) The greater the intensity of competition the lower will be this minimum margin of profit.

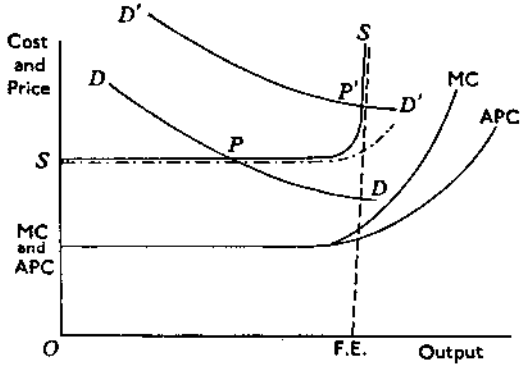

FIG. 13

The assumption that prices cannot fall below some minimum determined by the degree of market imperfection, and that production cannot exceed a certain maximum determined by full employment, yields a short-period supply curve (the curve *S–S* in Fig. 13) which exhibits the familiar reverse L-shaped feature : the curve is horizontal up to a certain point (when the supply price is set by the minimum profit margin) and well-nigh vertical afterwards (when production is limited by full employment).

We can now introduce the Keynesian demand function which shows the demand price for each level of output — i.e. it shows for

[1] The assumption that physical capacity is more than sufficient for the employment of the available labour force in 'developed' capitalist economies is empirically supported by the fact that even in times of very low unemployment, double or treble shift utilization of capacity is fairly rare. And it is the existence of considerable spare capacity under conditions of imperfect competition which alone explains the absence of diminishing productivity to labour with increasing employment in the short period, despite the co-existence of physical equipment of varying degrees of efficiency.

any particular output (and employment) that excess of price over prime cost which makes the effective demand in real terms equal to that output. (The excess of price over prime cost is of course the same thing, on our assumptions, as the share of profits in output.) Assuming that investment, I, is an independent variable invariant with respect to changes in output, this demand curve will be falling from left to right, much like the Marshallian demand curve, and its equation, according to the well-known multiplier formula, will be

$$D = \frac{1}{(\alpha - \beta)\frac{p - c}{p} + \beta} I, \tag{11}$$

where D represents aggregate demand in real terms, $\frac{p-c}{p}$ the margin of profit over selling price (which, for the representative firm, is the same as $\frac{P}{Y}$, the share of profits in income) and I the amount of investment (also in real terms), and α and β the coefficients of savings for profits and wages respectively. The higher is I, and the lower are the coefficients α and β, the higher the position of the curve; the greater the difference, $\alpha - \beta$, the greater elasticity of the curve. If $\beta = 0$, the curve approaches the APC curve asymptotically; if $\alpha = \beta$ the curve becomes a vertical straight line.

Depending on the relative position of the two curves, this intersection can yield either an under-employment equilibrium (when the demand curve cuts the supply curve in the horizontal segment of the latter, as shown by $D - D$, with the point of intersection P) or a full-employment equilibrium (as shown by $D' - D'$, with the point of intersection P'). In the former case the price-cost relationship (the distribution of income) will be independently given by the degree of market imperfection (marginal productivity plays no rôle in this case since the average productivity of labour is assumed to be constant) whilst the level of output is determined by the parameters of the demand function (the savings-investment relationship). In the latter case, output is independently given, and it is the price-cost relationship which will be determined by the demand function, i.e. by the savings-investment relationship.[1]

However, our demand curve has so far been based on the postulate that the rate of investment is invariant with respect to changes in

[1] It follows also that in so far as β (savings out of wages) is zero or negligible, under-employment equilibrium necessarily presupposes some degree of market imperfection; for if competition were perfect and the minimum profit margin were zero, the intersection of the demand curve with the supply curve would necessarily fall on the vertical section of the latter.

output. In fact, it is the rate of growth of output which governs investment demand ; and, in addition to the growth of output due to the natural rate of growth of the economy, investment in the short period will also vary with the change in output reflecting a change in the level of unemployment. Such 'induced' investment will only come into operation, however, when the degree of utilization of capacity permits a normal rate of profit to be earned ; in other words when receipts cover, or more than cover, *total* costs, including 'normal' profits on the capital invested.

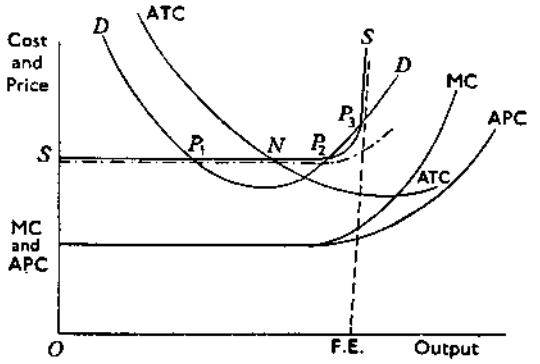

FIG. 14

In Fig. 14 the curve ATC indicates average total costs (including 'normal' profits) and the point N (where the curve ATC intersects the S–S curve) the level of production which yields a 'normal' profit on the existing capital equipment. Beyond N, any further increase in production will 'induce' investment in the shape of additions to productive capacity, and it is reasonable to suppose that the increase in investment associated with an increase in output will exceed the increase in savings for any *given* distribution of income. Hence the savings-investment relationship will yield a U-shaped demand curve ; the curve will be falling up to N (when induced investment is zero) [1] and will slope upwards to the right of N (when induced investment is positive). As shown in Fig. 14 this will yield multiple positions of equilibrium, P_1, P_2 and P_3, of which only P_1 and P_3 are stable positions whereas P_2 is unstable (since at P_2, where

[1] Up to point N, the position of the demand curve may be regarded as being determined by the existence of some 'autonomous' investment which is independent of the current level of activity, or else by a negative constant in the savings functions, which makes savings zero at some positive level of income and employment.

the demand curve cuts the supply curve from below, a small displacement in either direction will set up cumulative forces away from P_2 until either P_1 or P_3 is reached).

It follows that an under-employment equilibrium is only *stable* under slump conditions when induced investment is zero.

It also follows that it is impossible to conceive of a *moving* equilibrium of growth being an under-employment equilibrium. Such an equilibrium is necessarily one where productive capacity is growing, and where therefore induced investment is positive, and hence the $D-D$ curve slopes upwards and not downwards. It therefore postulates the equilibrium of the P_3 type and not of the P_1 type. In that situation the profit margin must be above the minimum level, and the distribution of income will tend to be such as to generate the same proportion of income saved as the proportion of investment in output.

In a balanced-growth equilibrium, the level of investment must of course also correspond to the rate of accumulation appropriate to the rate of growth of the economy, in other words (in terms of Model II) to $(\lambda v)Y$. This is not necessarily the rate of investment reflected by our (short-period) demand curve at the point P_3; if it is not, the adjustment takes the form of a change in capacity in relation to output (a shift in point N in the diagram) and a consequent change in the investment 'induced' by the excess of actual output over N sufficient to make the volume of induced investment equal to $(\lambda v)Y$.

It further follows that a moving equilibrium of growth is only possible when, given the savings propensities, the profit margin resulting from the equilibrium rate of investment is higher than the minimum profit margin indicated by the height of the horizontal section of the S–S line; and there must be sufficient competition to ensure this. If this were not so, the point P_3 would lie below the S–S line, and the only equilibrium conceivable in that case would be that of the P_1 type at which, as we have seen, induced investment is zero, and the level of output remains stationary over time, irrespective of the growth in population. *It is only under conditions of 'Keynesian' full employment that the growth-potential of an economy (indicated by its 'natural' rate of growth) is exploited in terms of actual growth.*

We must therefore add a further restriction to our models which can be written (putting m for the minimum profit margin, reflecting the degree of market imperfection):

$$\frac{P}{Y} \geq m \qquad (12)$$

which, under the assumption of Model II where $\frac{P}{Y}=\lambda v$, can be written in the form

$$m<\lambda v. \tag{12a}$$

If this condition is not satisfied, the economy will lapse into stagnation.

So far we have not mentioned marginal productivity. Clearly, the equilibrium real wage cannot *exceed* the short-period marginal product of labour: for if it did, the position of full employment could not be reached. Under our present assumptions, where the full-employment position falls within the range of the horizontal section of the average prime cost curve (or very near it), this does not impose any further restriction. For when productivity is constant, the marginal product of labour is the same as the average product, and the condition therefore is necessarily satisfied, so long as the equilibrium wage is lower than output per head (i.e. so long as the equilibrium share of profits is positive). In order to generalize our results, however, to cover the case of diminishing (short-period) returns (i.e. when the full employment line in Figs. 13 and 14 cuts the average prime cost curve in the *rising* section of the latter and marginal costs exceed average prime costs), we need to introduce a further restriction to the effect that the share of wages cannot exceed the marginal product of labour. Writing for a *given* value of K,

$$Y=\Psi(L), \text{ where } \Psi'>0,\ \Psi''\leqslant 0$$

for the *short-period* relationship between output and employment (L denoting the amount of employment) the condition is

$$\frac{W}{Y}<\frac{L\Psi'(L)}{\Psi(L)} \tag{13}$$

Under conditions of our Model II, where $\frac{P}{Y}=\lambda v$, this could also be written in the form

$$\frac{\Psi(L)-L\Psi'(L)}{\Psi(L)}<\lambda v \tag{13a}$$

i.e. the equilibrium share of profits, as determined by the 'dynamic' conditions, cannot be less than the excess of the average product of labour over the marginal product. We can assume, however, that the system will tend to generate sufficient excess capacity (in relation to the labour supply) for this condition to be satisfied.

These two restrictions, (12) and (13), together with that given in

(5), are not additive but alternative, and only the higher of them will apply. For our minimum margin of profit in (12) is not the same thing as the 'optimum' monopoly profit of the text-books, which is the outcome of short-period profit-maximization with reference to some given marginal-revenue schedule to the individual firm. It is more akin to Marshall's notion of a minimum margin of profit on turnover below which producers refuse to go 'for fear of spoiling the market',[1] but which tends to be the lower, the more intense the competition among producers. As such it is related to the average cost of production and not to marginal cost; and as an obstacle to a fall in the profit-share, it overlaps with the technical barrier set by the excess of short-period marginal cost over average prime cost.

VIII. TECHNICAL PROGRESS

We must now proceed to remove the most important of our 'simplifying' assumptions, the absence of technical progress. A moving equilibrium of growth involves continued increase in the productivity of labour, and not only in the working population, *pari passu* with a continued increase in the amount of capital per worker; though in the absence of any reliable measure of the quantity of capital (in a world where the technical specification of capital goods is constantly changing, new kinds of goods constantly appear and others disappear) the very notion of the 'amount of capital' loses precision. The terms 'income' or 'capital' no longer have any precise meaning; they are essentially accounting magnitudes, which merely serve as the basis of calculations in business planning; the assumption that money has a stable value in terms of some price index enables us to think of 'income' and 'capital' as *real* magnitudes only in a limited, and not precisely definable, sense.[2]

Orthodox theory attempts to deal with these problems in terms of the traditional tools — the assumption of a linear and homogeneous production function, coupled with the assumption that with the changing state of knowledge this function is continually shifting upwards and outwards. As depicted in Fig. 15 at any one *point* of time, t, there is assumed to be a unique relationship between capital and output, which conforms to the general hypothesis of diminishing

[1] *Principles* (8th ed.), Book V, ch. 5, section 5, pp. 374-6.

[2] These problems do not appear in a von Neumann type of model of balanced-growth equilibrium with constant technical functions, precisely because the technical specification of goods, their relative composition and their relative values remain unchanged through time; everything remains the same, except for the actual quantities of goods, and there is no problem involved in aggregation.

productivity, but this relationship is constantly shifting with the passage of time. The assumption of 'neutral' technical progress means that the production curve shifts in such a manner that the slope of the tangents of the functions f_t, f_{t+1}, f_{t+2}, etc., remain unchanged along any radius from the origin. This hypothesis is necessary in order to make it possible for a constant rate of profit over time to be consistent with a constant rate of growth and a constant relationship between capital and output (since the rate of profit on capital is uniquely related to the slope of the production function).

There are, however, several basic faults in this procedure — quite apart from the inherent improbability that technical progress should obey any such rigid rules.

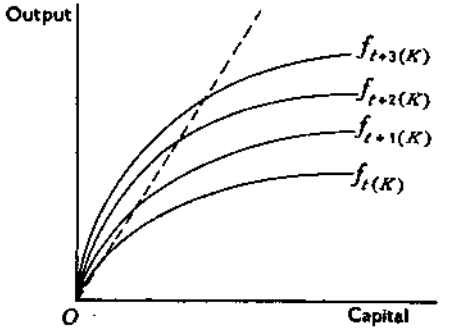

FIG. 15

(1) In the first place the production function assumes that the capital stock in existence at any one time is perfectly adapted to any given capital-labour ratio — that there is a particular assortment of equipment goods corresponding to each successive point of the production curve which is different from the assortment associated with any neighbouring point. (This will be true even in the absence of 'technical progress' so long as the substitution of capital for labour implies the use of different *kinds* of equipment, and not merely the use of relatively greater quantities of the *same* equipment.) Hence the successive points on this curve represent alternative states of long-period stationary equilibrium any one of which could be actually attained only when any given state of capital endowment (i.e., any given capital-output ratio) has obtained unchanged for a long enough period for the actual assortment of capital goods to have become optimally adapted to it. The production curve thus represents a kind of boundary indicating the maximum output corresponding to each particular 'quantity' of capital, a maximum which assumes that

the whole productive system is fully adapted to each particular state of accumulation. In an economy where capital accumulation is a continuous process this boundary is never attained — since the actual assortment of capital goods at any one time (even with a *constant* state of knowledge, whatever that assumption may be taken to mean) will consist of items appropriate to differing states of accumulation, and the output corresponding to any particular 'quantity' of capital will be less than the equilibrium (or maximum) output associated with that quantity. This is only another way of saying that in a society which is *not* in continuous long-run stationary equilibrium, output cannot be regarded as a unique function of capital and labour; and the slope of the production curve cannot be relevant to the pricing process, since the system does not move *along* the curve, but *inside* it.

(2) In the second place (and quite independently of the first point) the assumption that there is a curve which continually shifts upwards means that technical progress is treated as a variable of the function in a manner perfectly analogous to a second factor of production, like labour (or land). This is evident from the consideration that if, instead of postulating rising technical knowledge and a constant labour force, we postulated a constant state of technical knowledge and a rising labour force, the nature of the shift of the curve (under the hypothesis of a homogeneous and linear function) would be exactly the same. A given rate of shift of the curve, along any radius from the origin, could equally well result from a given percentage increase in the labour supply as from the same percentage increase in the state of 'knowledge'. But unlike labour, the state of knowledge is not a quantifiable factor. A *given* or a *constant* state of knowledge is only capable of being defined implicitly: there is no possible way in which, comparing two different positions, at two different points of time, the change due to the movement along the curve could be isolated from the change due to the shift of the curve. The whole procedure by which this separation is attempted is purely circular: since the *slope* of the curve (under the additional hypothesis that the function is not only homogeneous and linear but a constant-elasticity function *à la* Cobb-Douglas !) is supposed to determine the share of profits in income, the share of profits is taken to be an indication of its slope, and the residual is then attributed to the shift of the curve! There could be no better example of *post hoc ergo propter hoc.*

(3) The hypothesis that the *slope* of the curve determines the share of profits, in accordance with the marginal productivity principle, despite the continued shift in the curve, presumes of course that the factor responsible for the shift is itself rewarded on the same principle, since it is the marginal product of *all* factors taken *together*

which exhausts the total product. This condition can be satisfied when the shift of the curve is due to, say, a certain rate of increase in the quantity of labour, since that part of the increase in the product which is due to the shift is definitely imputed to labour in the form of wages. But knowledge, just because it is not a quantifiable factor which can be measured, or brought under exclusive ownership, or bought and sold, cannot receive its *own* marginal product. It is like other scarce but unappropriated agents of production (like the sea in the case of the fishing industry) whose existence causes divergences between the private and the social product of the *other* factors. This is only another way of saying that we are not free to elevate to the rôle of a 'factor of production' anything we like; the variables of the production function must be true inputs, and not vague 'background elements', like the sun or the sea or the state of knowledge, any of which may be thought to cause the results to diverge from the hypothesis of the homogeneous-and-linear production function. In terms of the *true* variables, Capital and Labour, the production function will not be linear-homogeneous but will be a function of a higher order, when technical knowledge is increasing over time.[1] It is therefore illegitimate to assume that factor rewards are allocated in accordance with their marginal productivities, since the sum of the marginal products of the factors will exceed the total product. When, the quantity of labour being given, an increase in capital by a given proportion yields an increase in output in the same proportion, the 'true' marginal product of capital will *alone* exhaust the total product.[2] For this reason any postulate derived from the hypothesis of diminishing productivity (such as our $v=\phi\left(\frac{P}{K}\right)$ function, given in equation (8a) above) is illegitimate when productivity, for whatever reason, is *not* diminishing. Given the fact of constant or increasing productivity to capital accumulation, the share of profit must necessarily be *less* than the marginal product of capital, and there is no

[1] It is a well-known dodge that any function whatsoever in n variables can be converted into a homogeneous-and-linear function of $n+1$ variables by adding a further variable which is *implicitly* defined. But as Samuelson has pointed out (*Foundations of Economic Analysis*, p. 84), any such procedure is illegitimate, since factor rewards will not conform to the partial differentials of this wider function.

[2] Supporters of the neo-classical approach would argue that the increase in product in this case is not *due* to the change in the quantity of capital alone — it is the joint result of the change in the quantity of the 'factor' capital, and the shift in the 'state of knowledge' which is presumed to have occurred in the interval of time during which the increase in capital occurred. But this is precisely the point: since the accumulation of capital is necessarily a process in time, and cannot be conceived of in a timeless fashion, a movement *along* the curve cannot be isolated from the shift of the curve; indeed it is illegitimate to assume the existence of a 'curve' independently of its shift, since there is no conceivable operation by which the slope of this 'curve' could be identified.

reason why a given capital-output ratio should be associated with a particular rate of profit, or indeed, why the two should be functionally related to each other on account of any technical factor.

(4) Added to this is the further complication that the rate of shift of the production function due to the changing state of 'knowledge' cannot be treated as an independent function of (chronological) time, but depends on the rate of accumulation of capital itself. Since improved knowledge is, largely if not entirely, infused into the economy through the introduction of new equipment, the rate of shift of the curve will itself depend on the *speed of movement* along the curve, which makes any attempt to isolate the one from the other the more nonsensical.[1]

The most that one can say is that whereas the rate of technical improvement will depend on the rate of capital accumulation, any society has only a limited capacity to *absorb* technical change in a given period. Hence, whether the increase in output will be more or less than proportionate to the increase in capital will depend, not on the state of knowledge or the rate of progress in knowledge, but on the *speed* with which capital is accumulated, relatively to the capacity to innovate and to infuse innovations into the economic system. The more 'dynamic' are the people in control of production, the keener they are in search of improvements, and the readier they are to adopt new ideas and to introduce new ways of doing things, the faster production (per man) will rise, and the higher is the rate of accumulation of capital that can be profitably maintained.

These hypotheses can, in my view, be projected in terms of a 'technical progress function' which postulates a relationship between the rate of increase of capital and the rate of increase in output and which embodies the effect of constantly improving knowledge and

[1] None of the above strictures against the postulate of a 'production function' which continually shifts with technical progress invalidates the assumption of a *short-period* relationship between employment and output, which takes the character and composition of fixed equipment of all kinds as given. This short-period production function (as employed in equations (13) and (13a) above) implies that for any given volume of employment a definite 'marginal product' can be imputed to labour, which, as we have seen, sets an *upper limit* to the share of wages in output (the 'rents' to be imputed to capital being the residual, i.e. the difference between the average and the marginal product of labour). This limit, however, only becomes significant when diminishing returns prevail, so that an increase in production is associated with a more-than proportionate increase in employment — with constant or increasing returns, the marginal product of labour will equal to, or exceed, the average product, and the former *cannot* therefore be the governing factor determining distributive shares. Whether diminishing returns prevail or not will predominately depend on the output capacity represented by the existing capital stock and its degree of utilization when labour is fully employed. Under conditions of imperfect competition it is perfectly compatible with 'profit-maximizing behaviour' to suppose that the representative firm will maintain a considerable amount of spare capacity even in relation to the output attainable under full-employment conditions.

know-how, as well as the effect of increasing capital per man, without any attempt to isolate the one from the other.

It is the shape and position of this 'technical progress function' which will exhibit features of diminishing returns. If we plot percentage growth rate of output per head, $\dot{Y}/Y$, along Oy and percentage growth rate of capital per head, $\dot{K}/K$, along Ox (Fig. 16), the curve will cut the y-axis positively (since a certain rate of improvement would take place even if capital per head remained unchanged) but it will be convex upwards, and reach a maximum at a certain point — there is always a maximum beyond which a further increase in the rate of accumulation will not enhance further the *rate* of growth of output (Fig. 16). This means that the increase in capital (per head) will yield increasing or diminishing returns in terms of output according as the rate of accumulation is relatively small or large. If the rate of accumulation is less than Op, output will increase faster than capital, and vice versa.

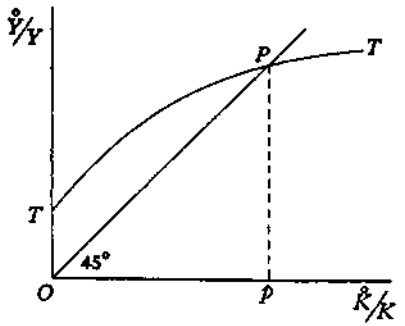

FIG. 16

The *height* of the curve expresses society's 'dynamism', meaning by this both inventiveness and readiness to change or to experiment. But the convexity of the curve expresses the fact that it is possible to utilize as yet unexploited ideas (whether old ideas or new ideas) more or less fully; and it is always the most profitable ideas (i.e. those that raise output most in relation to the investment which they require) which are exploited first. Some are old ideas; some are new ideas; most of the technical improvement that takes place embodies both. We cannot isolate the element of pure novelty in a world where knowledge is constantly improving, and where the actual techniques are constantly lagging behind the very latest techniques that would be selected if everything were started afresh. When capital is accumulated at a faster rate (and technical improve-

ment goes on at a faster rate), productivity will also increase at a faster rate, but the growth in the latter will lag behind the growth in the former, and beyond a certain point a further increase in the rate of accumulation ceases to be 'productive' — it is incapable of stepping up the rate of growth of productivity any further.

There is therefore no *unique* rate of technical progress — no *unique* rate at which alone a constant rate of growth can be maintained. There is a whole series of such rates, depending on the rate of accumulation of capital being relatively small or large.

On this analysis, it is the 'technical dynamism' of the economy, as shown by the height or position of our technical progress curve, which is responsible, in a capitalist economy, both for making the rate of accumulation of capital and the rate of growth of production relatively small or relatively large. It explains why there is no long-run tendency to a *falling* rate of profit, or for a continued increase in capital in relation to output, either in slow-growing or in fast-growing economies. In economies whose technical dynamism is low, both the rate of accumulation and the growth of production will be relatively low, but in either case, growth can go on at a steady rate, without any necessary tendency to diminishing returns and thus to a gradual approach to a stationary state.

On the assumption that this function cuts the y-axis positively (i.e. that there would be some positive rate of growth in output per man, even if capital-per-man remained unchanged — an assumption which is justified by the fact that even a zero rate of net investment implies a certain rate of infusion of new techniques or new designs, through the replacement of worn-out capital; and that there are always *some* improvements which may require no investment at all) and that the curve is convex upwards, there is necessarily a certain point on the curve at which it is intersected by a radius of 45 degrees from the origin — i.e. where the rate of growth of output is equal to the rate of growth of capital (P in Fig. 16). At that point *all* the conditions of 'neutral' technical progress are satisfied: the capital-output ratio will remain constant at a constant rate of growth, constant distributive shares, and a constant rate of profit on capital.

In order to 'close' our model — that is, to produce a model that would account for the empirical features of the growth process as summarized by our 'stylized facts' at the beginning — it is necessary to show, not merely that such a point exists, but that in a capitalist system there is a tendency to move towards this point, which thus represents a long-run equilibrium rate of growth, and which is also stable in the sense that displacements due to shifts in the curve, etc., set up forces to re-establish it.

The hypothesis that *given* the technical progress function, the system tends towards that particular rate of accumulation where the conditions of 'neutral progress' are satisfied, cannot of course be justified on *a priori* grounds ; it must be based on empirical evidence — at least in the sense that it can be shown to be consistent with facts which are more difficult to explain on any alternative hypothesis. Supposing that the statisticians were to agree that the capital-output ratio tends to be constant in periods in which the rate of growth of production is constant (in which therefore the rate of technical progress is neither increasing nor decreasing) whilst the capital-output ratio tends to decrease in periods of accelerating growth and vice versa. This would support the hypothesis that the system *tends* towards P : and variations in the rate of growth, and in the movements in the capital-output ratio, are then to be explained in terms of the unequal incidence of technical progress — i.e. in terms of shifts of our technical progress function. If, on the other hand, the statisticians were to agree that there is no correlation between these magnitudes, that periods of steady growth are just as likely to be associated with a steadily decreasing or a steadily increasing ratio of capital to output, this would support the hypothesis that the system tends towards some point on the curve — to some equilibrium rate of growth of output and of capital — which is not necessarily the one at which the two growth rates are *equal.*

IX. ASSUMPTIONS ABOUT INVESTMENT BEHAVIOUR

In either event, to obtain an equilibrium solution — to assert, in other words, that there is some particular equilibrium rate of growth of output and of capital towards which the system is tending — we need to introduce an 'investment function' based on entrepreneurial behaviour. Since we *cannot* say that the rate of capital accumulation depends on the community's propensity to save (since the latter is a dependent variable, depending on the share of profits, and thus on the share of investment) nor on the requirements of the 'natural rate of growth' (because one of the two constituents at least of the natural rate of growth, the rate of growth of productivity, is a dependent variable, depending on the rate of accumulation of capital and thus on the share of investment), we need to introduce, in order to close our model, an independent function governing the investment decisions of entrepreneurs. There are various alternative assumptions that can be made about investment behaviour which lead to divergent results ; and at the present stage we cannot say

that our knowledge of entrepreneurial behaviour is sufficient to rule out any particular assumption in preference to some other. Hence our final choice of assumption must be based on the admittedly weaker procedure of its yielding results that are more in conformity with the facts of experience than its alternatives.

(1) One hypothesis, originally advanced by Kalecki,[1] is that the subjective risks assumed by entrepreneurs are an increasing function of the rate of capital accumulation (or, as Kalecki put it, the rate of investment decisions is an increasing function of the gap between the prospective rate of profit and the rate of interest). This assumption, at any rate for a given market rate of interest, makes the rate of capital accumulation a single-valued function of the rate of profit

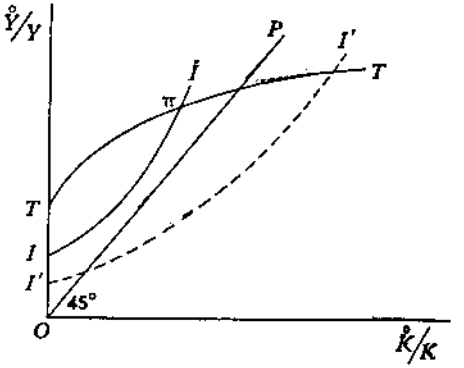

FIG. 17

on capital, and since the latter, in a state of balanced-growth equilibrium, is a single-valued function of the rate of growth, it makes the desired rate of accumulation a single-valued function of the rate of economic growth. Such an 'inducement to invest' function is shown by the curve *I–I* in Fig. 17. The height of this curve (i.e. the point at which it cuts the *y*-axis) reflects the market rate of interest, while the slope of the curve reflects increasing marginal risk. This postulate yields an equilibrium position at point π where the rate of economic growth resulting from the given rate of capital accumulation coincides with the rate of economic growth that is required in order to induce entrepreneurs to accumulate capital at that particular rate. On this hypothesis the equilibrium rate of growth can be anywhere on the *T–T* curve, depending only on the position of the risk preference function (governing the inducement to invest) relatively to the technical progress function (governing the rate of growth resulting

[1] 'The Principle of Increasing Risk', *Economica*, 1937, p. 440.

from varying rates of accumulation). Thus if π is to the left of P, the equilibrium rate of growth will involve a constantly falling capital-output ratio, and if it is to the right of P (as with the dotted line I'–I' in Fig. 17) it involves a constantly rising capital-output ratio. In both cases the rate of growth will be constant over time, but in the first case the equilibrium will involve a steadily falling share of profit in income and in the second case a steadily rising share of profit. On this hypothesis therefore the 'neutral' position at P will only be reached as a result of a coincidence — of the I–I curve cutting the T–T curve at that point.

(2) An alternative hypothesis, which is a variant of the one put forward in my paper 'A Model of Economic Growth',[1] makes the principle of increasing risk applicable, not to the volume of investment decisions as such, but only to that part of investment which is in excess of that required to maintain a constant relationship between output capacity and prospective output. Whenever sales are rising, entrepreneurs will in any case increase the capital invested in the business by the amount necessary to enable them to increase their productive capacity in line with the growth of their sales — there are no greater risks involved in a larger business than a smaller one; and no greater risks are entailed in a higher rate of growth of employed capital, if this proceeds *pari passu* with a higher rate of growth of turnover. Hence if their actual sales are rising at the rate of g (where g may be any particular point on the T–T curve in Fig. 16) we may suppose, in accordance with the 'acceleration principle', that the growth in output in itself will 'induce' sufficient investment to enable that rate of growth of production to be maintained, without requiring a higher prospective rate of profit. As far as this 'induced investment' is concerned, any particular point on the curve could be an equilibrium point. But if a particular rate of growth of output and capital involves the expectation of a *rising* rate of profit in the minds of investors, it will induce an acceleration in the rate of accumulation and hence will cause the system to move to the right (on the curve); if it involves the expectation of a falling rate of profit, it will cause it to move to the left.

The *prospective* rate of profit in the minds of entrepreneurs is based on two things: on the amount of capital required per unit of output, and on the expected profit margin per unit of output. If we assume that all savings come out of profits (i.e. $\beta = 0$) then, given constant rates of accumulation and growth, the realized rate of profit

[1] *Economic Journal*, 1957, p. 604. The form of the 'investment function' given in that paper was justly criticized; the present version, I hope, meets the objections raised against the earlier version by Professor Meade, Mr. Hudson and others.

on capital will also be constant over time, irrespective of whether capital per unit of output is constant, rising or falling (since any reduction in the capital-output ratio will be matched by a corresponding reduction in the share of profits in output, and vice versa). But we cannot assume that the *prospective* rate of profit on current investment will be the same as the *realized* rate of profit on existing capital — the prospective rate of profit will be higher, precisely because the capital required for producing a unit-stream of future output is less than the amount of capital that was (historically) invested in producing a unit-stream of current output. Nor can it be assumed that the prospective rate of profit on new investment will be the same as the actually realized rate of profit in future periods, since the latter magnitude will itself depend on the investment decisions currently made by entrepreneurs. Thus if at some particular rate of accumulation the trend of progress causes a continued fall in the amount of capital required per unit of output,

$$\frac{P}{K}=\frac{P}{Y}\cdot\frac{Y}{K}$$

will remain constant if the rise in Y/K is offset by a corresponding fall in P/Y. This would occur if the fall in K/Y involved a corresponding reduction in I/Y; if, in other words, it left the rate of expansion of capacity unchanged. But if this consequential fall in profit margins is *not* foreseen, or not sufficiently foreseen, the rise in Y/K will involve the expectation of a higher prospective rate of profit, which by increasing the rate of investment may prevent the fall in P/Y from occurring at all. This is a case, therefore, where the movement of the economy, and the nature of the final equilibrium, cannot be predicted independently of the nature of the expectations of entrepreneurs. The assumption of 'static foresight' (i.e. the projection of existing prices, costs and output levels to the future) leads to a different result from the assumption of 'perfect foresight'; the latter assumption moreover leaves the situation indeterminate since the expectations that are capable of being actually realized are by no means unique. It is only in the 'neutral' equilibrium case (at point P) that the two kinds of assumptions (static foresight and perfect foresight) lead to consistent results.

Expectations are invariably based on past experience, and in that sense, are of the 'static' rather than of the 'perfect' kind. In addition, they can be defined as being more or less 'elastic' according as the projections into the future are based on the events of the very recent past, or on the average experience of a longer interval of elapsed time. Expectations are likely to be the more elastic the less past

experience justifies the assumption of some norm around which short-term movements fluctuate; the more, in other words, past movements have been subject to a trend. For that reason, business expectations are far more likely to be elastic with respect to volume of sales than with respect to the margin of profit on turnover; the future expectation concerning the margin of profit per unit of sales, which is taken as the basis of business calculations, is far more likely to reflect some standard, or norm, than the experience of the most recent period alone. This provides a further reason for supposing that in situations in which production rises faster than the stock of capital, the prospective rate of profit will be rising relatively to the realized rate of profit; and if, in response to this, the rate of accumulation is accelerated, the rate of growth of production, and the realized rate of profit, will rise as well.

Hence the tendency of the system to move towards a position where output and capital both grow at the same rate, and where therefore the rate of profit on capital will remain constant at a constant margin of profit on turnover, can be justified by the suppositions (i) that the prospective rate of profit on investments will be higher than the currently realized rate of profit on existing capital whenever production is rising faster than the capital stock; (ii) that a rise in the prospective rate of profit causes an increase in the rate of investment, relative to the requirements of a state of steady growth, and vice versa.[1]

X. THE FINAL MODEL

The equilibrium relationships of this final model can thus be set out as follows. It is based on three functions: first, on a savings function on the lines of equation (10) above, which can be written in the form

$$\frac{S}{Y}=(\alpha-\beta)\frac{P}{Y}+\beta, \qquad (10a)$$

where $$1>\alpha>\beta\geqslant 0.$$

[1] In the first version of the present growth model (published in the *Economic Journal*, December 1957) I postulated an investment function which made current investment depend (*inter alia*) on the change in the *realized* rate of profit as compared with the previous period. This was unsatisfactory in that it failed to take into account the fact that the inducement to invest depends on the prospective rate of profit, and not on the actual profit earned on existing capital; and that quite apart from the question of expectations, the prospective rate of profit will differ from the currently realized rate whenever (owing to technical progress, etc.) the 'productivity' of capital on new investment (i.e. the amount of investment required per unit of future output capacity) differs from the existing capital-output ratio.

Second, on a technical progress function showing the relationship between the rate of growth of output per worker (G_0) and the rate of growth of capital per head ($G_K - \lambda$), and which (using a linear equation for the sake of convenience) [1] can be written in the form

$$G_0 = \alpha' + \beta'(G_K - \lambda), \text{ where } \alpha' > 0,\ 1 > \beta' > 0. \tag{14}$$

Third, on an investment function based on the assumptions already described, and which makes investment a combination of two terms. The first term of the equation relates to the amount of investment *induced* by the change in output the previous period, and assumes that this investment will be such as to make the growth in *output capacity* in period $(t+\theta)$ equal to the growth in output in period t. Since in view of (14), the rate of capital accumulation per worker $(G_K - \lambda)$, which is required to increase output capacity by G_0 will not (necessarily) be equal to G_0 but to

$$\frac{G_0 - \alpha'}{\beta'}$$

[1] It has been pointed out to me by Professor Meade, Mr. Hahn and others that whilst, in general, the technical progress function cannot be integrated in terms of a production function with a particular rate of time shift, a *linear* technical progress function as given in (14) can be integrated to obtain

$$Y_t = Be^{\alpha t}K_t^{\beta} \tag{14a}$$

which appears to be the same as the Cobb-Douglas function (remembering that Y_t and K_t refer to the output and the capital *per unit* of labour). However, as was pointed out to me by H. Uzawa of Stanford University, in integrating the technical progress function, the constant of the integral $B = B(Y_0, K_0)$ is a function dependent on the initial amount of capital K_0 and of output Y_0, whereas a production function of the type

$$Y_t = f(K_t, t) \tag{14b}$$

requires that the function should be independent of the initial conditions.

Apart from this, the aggregative production function of the type (14b), a special case of which is the Cobb-Douglas function, implies the assumption that at any given time t, the output Y_t is uniquely determined by the aggregates, K_t and L_t, irrespective of the age-and-industry composition of the capital stock. However, when the technical progress of an economy depends on its rate of capital accumulation (when, in other words, the improvements in techniques require to be embodied in new equipment before they can be taken advantage of), no such functional relationship exists. To describe the relationship between capital, labour and output we require a function in the form

$$Y_t = \phi(A_t) \tag{14c}$$

where A_t specifies the distribution of capital according to age as well as (in a multi-commodity world) the distribution of both capital and labour between industries and firms. In that case the postulate of a linear technical progress function is perfectly consistent with the ϕ function being neither homogeneous in the first degree nor of constant elasticity. In the short run the age-and-industry distribution is of course given as a matter of past history. But even in a long-run growth equilibrium with technical progress, A_t could not be treated as a unique function of K_t and L_t, since it will also depend on λ and (in view of the varying incidence of obsolescence at differing rates of progress) on γ', the equilibrium value of G_0.

and since $$G_K \equiv \frac{I}{K},$$

the rate of *induced* investment in period $(t+\theta)$ and which is the first term of our investment equation, will be equal to

$$(G_0(t) - \alpha') \frac{K(t)}{\beta'} + \lambda K(t).$$

The second term of our investment equation depends on the change in the prospective rate of profit which, on our assumptions concerning the expected margin of profit turnover (i.e., that the *expected* value of P/Y is based on an average past values), will be a rising function of the *change* in Y/K over time. Assuming this latter relationship to be linear for the sake of convenience the whole function can be expressed in the following form :

$$I(t+\theta) = (G_0(t) - \alpha')\frac{K(t)}{\beta'} + \lambda K(t) + \mu \frac{d}{dt}\left(\frac{Y(t)}{K(t)}\right), \tag{15}$$

where $\mu > 0$.

The first term of this equation gives rise to an amount of investment at any given rate of growth of output that is sufficient to maintain that rate of growth of output — i.e. sufficient to keep the system on any particular point on the T–T curve. It can also be seen immediately that when

$$G_Y > G_K,$$

the second term of the expression is positive, hence G_K will be rising over time. A rise in G_K, in accordance with (14), will raise G_Y but less than proportionately, and hence lead to a further rise in investment in accordance with the first term at the same time as it diminishes the second term. Hence, whatever initial position we start from (defined by given values of K, L, and O at some initial point $t = 0$), this process will gradually lead to a situation in which the second term of equation (15) dependent on $\frac{d}{dt}\left(\frac{Y(t)}{K(t)}\right)$ vanishes to zero and where therefore

$$\frac{dv(t)}{dt} = 0. \tag{16}$$

This implies that $$G_0 = \frac{\alpha'}{1-\beta'} = \gamma', \tag{17}$$

and $$G_Y = G_K = \lambda + \gamma'. \tag{18}$$

Hence this model, like the earlier ones, also yields a state of moving equilibrium, where the rate of growth, the capital-output ratio and the distributive shares are constant over time — the main difference being that the output-per-worker, capital-per-worker and wages-per-worker are now no longer constant but rising at the equilibrium rate of growth productivity, γ'. However, these assumptions are not yet sufficient to set out a full equilibrium model. The reason is that since we no longer have a technical equation for v on the lines of equation (8a) which was incorporated in Models III and IV, the actual value of v is here left undetermined. From this model it only follows that at the position of equilibrium v will be constant (since this is implicit in equation (15), as shown by (16)); but this is consistent with any particular value for v — or rather v could only be determined in this model historically, if we assumed that it had a certain initial value at some particular point of time, and followed its resulting movement through the successive steps to final equilibrium.

Hence, in order to close the model, we shall introduce two more variables and three additional relationships. These are strictly 'Keynesian' — since they are, on the one hand, necessary to ensure that the reaction-mechanism of the model follows the Keynesian system in which the inducement to invest is independent of the propensities to save; and on the other hand because they incorporate Keynesian notions of the rate of interest and the supply price for risk capital based on liquidity preference or the aversion to risk taking.

We have already argued in connection with (5) above [1] that the inequality

$$\frac{P}{K} > r + \rho$$

is a necessary boundary condition of the model in the sense that the continued accumulation of capital cannot go on unless the ruling rate of profit is *at least* as high as the necessary compensation for risk and illiquidity involved in the productive employment of wealth.[2] Further consideration shows that in order that the investment equation in (15) should hold, it is not enough to make equation (5) into a boundary condition; for so long as P/K is higher than the supply price of risk capital, there is no reason to suppose that

[1] P. 189 above.

[2] A more precise statement of this condition would break down $r+\rho$ further into its component elements, distinguishing between the expected average of short rates of interest and the premium of the long rate over the expected average short rate on the one hand, and the additional leaders', borrowers' and speculative risks, etc., involved in direct investment, on the other hand, but this is not necessary for our present purposes.

investment outlay will be confined to that necessary for the increase in output capacity (i.e. to that given by 'the acceleration principle') or to that resulting from a given increase in the prospective rate of profit in a particular period. Indeed, unless the rate of profit actually corresponds to the supply price of risk capital, one cannot assume that the investment of each period will be confined to the *new* investment opportunities accruing in that period — an assumption necessary for an equilibrium of steady growth. Hence equation (5) should be converted into an equilibrium condition

$$\frac{P}{K}=r+\rho. \tag{19}$$

The second relationship concerns the behaviour of the rate of interest, r, and here we shall follow orthodox Keynesian lines in assuming that the rate of interest is determined by the liquidity preference function and/or monetary policy (summarized in the function $\pi\left(\frac{M}{Y}\right)$, where $\pi'\leqslant 0$ and M is the real quantity of money), subject to the condition that there is a minimum ($\bar{r}$) determined by the risk premium associated with the holding of long-term financial assets, below which the rate of interest cannot fall. This relationship can therefore be expressed in two alternative forms

$$r\geqslant\bar{r}$$

when $r>\bar{r}$,
$$r=\pi\left(\frac{M}{Y}\right) \tag{20}$$

The third relationship concerns the behaviour of ρ, and though this equation can be fully supported on *a priori* grounds, it is put forward here more tentatively, as at present there is insufficient empirical evidence available to support it. It is based on the following considerations.

(1) First, as explained earlier in this paper,[1] it may be assumed that at any given rate of interest the minimum rate of profit necessary to provide inducement for any particular kind of investment will be higher the riskier (or the more 'illiquid') that investment is considered to be ;

(2) Second, as was also argued,[2] investment in 'fixed assets' (plant and equipment, etc.) is considered to be far more risky or illiquid than either investment in financial assets or in working capital ;

(3) Third, it may be assumed that the turnover-period of circulating capital is invariant (or practically invariant) with respect to

[1] P. 189 above. [2] *Ibid.*

changes in the techniques of production, so that circulating capital stands always in a linear relationship to output; hence any increase in the ratio of fixed to circulating capital involves an increase in the capital-output ratio.

It follows as a joint result of (2) and (3) that a higher capital-output ratio (including both fixed and circulating capital in the capital employed) requires for any given rate of interest a *higher* minimum rate of profit. Hence when the stage of accumulation is reached in which the actual rate of profit becomes *equal* to this minimum, the capital-output ratio will be uniquely related to the rate of profit; and, as we have seen, it is only under these conditions that the actual investment in each period is limited by the 'new' investment opportunities becoming available in that period (through λ and γ').

Writing F for fixed capital and C for circulating capital, k for the turnover-period of circulating capital, ρ_F and ρ_C for the marginal risk premium on the two types of investments respectively, and ρ for the marginal risk premium on investment in general, we thus have the following additional assumptions and relationships:

$$K \equiv F + C$$

$$C = kY$$

$$v \equiv \frac{K}{Y} \equiv \frac{F + kY}{Y}$$

$$\rho_F > \rho_C$$

$$\therefore \rho \equiv \frac{\rho_F F + \rho_C kY}{F + kY} = \xi_1\left(\frac{F}{Y}\right)$$

$$\rho = \xi_2(v), \text{ where } \xi_2' > 0. \tag{21}$$

It will be noted that the relationship expressed in (21) operates in a reverse manner to equation (8a) which determines v in the 'neo-classical' model; since in the case of (8a), ϕ' is negative, not positive.

We have argued at some length that equation (8a) can no longer be assumed to hold when technical progress is a continuing process and there is *no* unique function relating output to the capital stock, in which case, depending on the factors determining the rate of growth, varying shares of profit in income and varying rates of profit on capital can be associated with any *given* capital-output ratio. It is now seen that when equation (21) holds, equation (8a) *cannot* hold — at least not within the framework of a model which assumes that the money rate of interest is determined by 'monetary' factors and that

there is a minimum below which the rate of interest cannot fall.[1]

We can now set out our final Model V in a formal manner. It contains ten equations and ten variables — $Y(t)$, $O(t)$, $L(t)$, $P(t)$, $v(t)$, $s(t)$, $w(t)$, $G_o(t)$, $\rho(t)$ and $r(t)$. We shall continue to assume for simplicity that β is zero (there are no savings out of wages) and we shall take the simpler form of (20), treating the money rate of interest as a constant. We shall also bring together the various boundary conditions that emerged in the course of the analysis (cf. equations (4), (12) and (13) above), including a further one that is implicit in the relationship expressed in (21).

Assumptions

$$\left.\begin{aligned} L^*(t) &= L^*(o)e^{\lambda t} && \text{(i)}\\ G_o(t) &= \alpha' + \beta'(G_K(t) - \lambda) && \text{(ii)}\\ s(t) &= \alpha\frac{P(t)}{Y(t)} && \text{(iii)}\\ \frac{dv(t)}{dt} &= 0 && \text{(iv)}\\ r(t) &= \bar{r} && \text{(v)}\\ \rho(t) &= \xi(v(t)) && \text{(vi)}\\ \xi' &> 0 \end{aligned}\right\} \text{for all } t \geqslant 0$$

Identity

$$P(t) \equiv Y(t) - w(t)L(t) \qquad \text{(vii)}$$

Equilibrium Conditions

$$\left.\begin{aligned} s(t)Y(t) &= \frac{dK(t)}{dt} && \text{(viii)}\\ L(t) &= L^*(t) && \text{(ix)}\\ \frac{P(t)}{K(t)} &= r(t) + \rho(t) && \text{(x)} \end{aligned}\right\} \text{for all } t \geqslant 0$$

subject to the inequalities

(a) $$w(t) \geqslant w_{\min}$$

(b) $$\frac{P(t)}{Y(t)} \geqslant m$$

[1] It might be argued that the two equations could be made compatible with one another by an appropriate movement of the money price level which brought the 'real' rate of interest (*à la* Fisher) into an appropriate relationship with the other factors. But the movement of the price level depends on the behaviour of money wages (relatively to the change in productivity, γ') and this factor cannot, in turn, be treated as a function of the other variables.

(c) $$\frac{W(t)}{Y(t)} \leqslant \frac{\frac{dY(t)}{dL(t)}L(t)}{Y(t)}$$

(d) $$\rho_F + \bar{r} > \frac{\lambda + \gamma'}{\alpha} > \rho_C + \bar{r}. \qquad \text{(V)}$$

It is readily seen that the above yields a determinate system provided that the solutions fall within the limits indicated by the boundary conditions (a) – (d). By (ii) and (iv) we have

$$G_o = \frac{\alpha'}{1 - \beta'} \equiv \gamma' \text{ (say)}$$

Hence by (i) and (ix) $G_Y = \lambda + \gamma'$

But by (vii) $$G_Y(t) = \frac{s(t)}{v(t)} = \lambda + \gamma' \equiv N \text{ (say)}$$

By (iii), (v), (vi) and (x)

$$\frac{P(t)}{K(t)} = \frac{s(t)}{\alpha v(t)} = \frac{N}{\alpha} = \bar{r} + \xi(v(t)).$$

Hence by solving the last equality for $v(o)$, we can obtain all the remaining unknowns of the system.

If inequality (a) does not hold, $\frac{P}{Y}$ will be compressed below its equilibrium level, and hence the rate of accumulation and the rate of growth will be less than that indicated. As long, however, as we abstract from diminishing returns due to limited natural resources, and assume continuous technical progress, so that $G_o(t)$ rises over time, sooner or later the point must be reached where this inequality becomes satisfied.[1]

If, on the other hand, any one of the inequalities (b), (c) or (d) are not satisfied, $\frac{P}{Y}$ will be larger than its equilibrium value, and full-employment growth equilibrium becomes impossible. As regards (c) we may assume that there is always some degree of excess capacity (i.e., some relationship between output capacity and the full-employment labour supply) which satisfies this condition, and the system will tend to generate the required amount of excess capacity, if it did

[1] Allowing for diminishing returns, however, it is possible that (depending on the relative values of λ, α' and β') balanced growth equilibrium will necessarily settle at the point where the fall in $G_o(t)$ due to λ is precisely offset by the rise in $G_o(t)$ due to γ'; where, in other words, constancy of $G_o(t)$, and $w(t)$ over time, becomes a necessary condition of equilibrium. (This case seems to have application for many of the under-developed countries.)

not obtain initially.[1] It is possible, however, that the conditions (b) or (d) represent genuine obstacles to the attainment of balanced growth equilibrium.[2] In that case the system cannot grow at a steady rate. This does not mean, however, that the economy will lapse into permanent stagnation. As investment opportunities accumulate during periods of stagnation (owing to continued technical progress and population growth), it becomes possible for the system to grow, for a limited period, at a rate appropriately higher than $(\lambda+\gamma')$, thus generating the required value of $\frac{P(t)}{K(t)}$.

Finally, if condition (d) is not satisfied, a steady rate of growth is incompatible with the assumed rate of interest $\bar{r}$. Two cases are possible. If $\frac{\lambda+\gamma'}{\alpha}>\rho_F+\bar{r}$, equilibrium requires a higher money rate of interest. If $\frac{\lambda+\gamma'}{\alpha}<\rho_C+\bar{r}$, and the money rate of interest is already at its minimum level, it requires a rate of increase in money wages that would permit a rate of increase in the price level which reduced the real rate of interest to the appropriate figure.

Of all the relationships assumed in this model, that represented by (vi) and the inequality (d) are perhaps most open to doubt. Yet it can be shown that the assumption that ρ is a variable of v is the only one which makes the condition expressed in (x) — that the rate of profit is *equal* to the supply price of risk capital, consistent with the rate of profit being also determined by the growth factors, λ and γ' and by α. Equation (x) taken alone is incompatible with the rest of the model if the money rate of interest is assumed to be determined independently. But as indicated earlier, until there is more empirical evidence available to show that ρ_F is *appreciably* higher than ρ_C (or alternatively, that ρ_F itself is a rising function of the fixed-capital-output ratio, $\frac{F}{Y}$) and in consequence, the rate of profit is higher in industries and/or economies where the capital-output ratio is higher. I hesitate to put forward the relationship expressed in (vi) as more than a tentative suggestion, which I would be prepared to discard in favour of a better alternative, if such could be found.[3]

[1] Page 202 above. One may assume that the reaction mechanism here operates via the in- and out-flow of new firms as well as the investment behaviour of the representative firm.

[2] It is evident that these two restrictions are alternatives, of which only the higher one will apply.

[3] For the reasons given I regard Kalecki's assumption

$$\rho=\theta(G_K),\ \text{with}\ \theta'>0$$

as a worse alternative, apart from the fact that in the context of the present model it serves as a substitute for equation (15), not for equation (21), and hence is not sufficient for closing the model.