# Global Value Chains and Effective Exchange Rates at the Country-Sector Level

Nikhil Patel* Zhi Wang† Shang-Jin Wei‡

April 20, 2014

**Abstract**

The real effective exchange rate (REER) is one of the most cited statistical constructs in open-economy macroeconomics. The conventional measures of the REER assume a world in which every country exports only final goods. With rising importance of offshoring and cross-border trade in intermediate goods, such measures are increasingly flawed. Taking into account international value chains, we develop a theoretical framework to compute REER at the both sector and country levels. The framework nests all existing measures in the literature and addresses their shortcomings. We exploit detailed trade data from the recently available World Input-Output Database(WIOD) spanning the period 1995-2011 to compute the REER for gross output as well as value added for 40 countries and 1435 country-sectors.

*Columbia University

†United States International Trade Commission

‡Columbia University, NBER, CEPR, and CIER

# 1 Introduction

The Real Effective Exchange Rate (REER) is one of the the most quoted indices in international economics among academics, policy makers and financial market participants.[1]The underlying purpose behind the construction of REER is to have a statistic that measures competitiveness by quantifying the sensitivity of demand for output originating form a particular country as a function of changes in world prices. It is a partial equilibrium construct which takes observed price changes as given and does not require modeling of primitive shocks that lead to those price changes, or any other general equilibrium constraints like balanced trade.[2]

In addition to being a gauge of competitiveness, it also finds use as a measure of relative price to summarize an economy's price level vis -a-vis the rest of the world. It is also used as an indicator to draw inferences regarding currency manipulation, currency misalignment and vulnerability to crises-see Chinn (2000) , Goldfajn and Valdés (1999) and Gagnon (2012).

The importance of REER is further evident from the time, effort and resources devoted to computing REER indices by leading organizations like the International Monetary Fund(IMF), Bank of International Settlements(BIS), OECD as well as various central banks around the world.[3]

Standard REER measures make a number of simplifying assumptions which we evaluate in this paper. For instance, they assume that every country exports only final goods which are produced without using imported intermediate goods. The first point we wish to emphasize is that this assumption is not innocuous, as the following example shows.[4]

Consider a stylized world with three countries involved in a global value chain-China, Japan and the U.S. Suppose Japan manufactures raw materials for the production of a mobile phone and ships it to China which acts as an assembly point. China in turn exports the finished product to the U.S which is then consumed by US consumers. According to the traditional REER measures like the one used at the IMF, the phone would be classified as China's "product" and China would be assumed to be competing with other providers of phones. Consequently these models would conclude that an increase in the price of mobile phone in Japan would lead to an increase in the demand for China's mobile phone and hence increase its competitiveness. In reality however, China is not producing the entire phone but is the producer of assembly services which accounts for only a small fraction of the total value of the product. It therefore competes with other providers of such processing services and not phone manufacturers. Once we recognize this, it becomes clear that an increase in Japanese prices of mobile phone components could very well lead to a decline in

[1]A google search on "effective exchange rates" yields close to 58 million hits,higher than terms like "inflation targeting "(~5 million) and "comparative advantage"(~9 million).

[2]See Chinn (2006) for a primer on the concept of REER and Rogoff (2005) for an application and discussion.

[3]For the IMF REER computation see McGuirk (1986) and Bayoumi et al. (2005). For the Federal reserve's REER measure see Loretan (2005). The BIS's REER methodology is summarized in Klau et al. (2008)

[4]Among the exceptions to this criticism are recent papers by Bems and Johnson (2012) and Bayoumi et al. (2013). These will be discussed later on in the paper where we argue how our measure improves upon theirs.

demand for China's services and hence a decline in its competitiveness.

This example shows that REER computed by organizations like the IMF is not only inaccurate in terms of magnitude, but may also have the wrong sign[5]. Given that trade in intermediate goods can potentially have a major influence on the REER, it is puzzling that this feature continues to be ignored in the mainstream, especially given the prominence and rising trend in intermediate goods trade in the last two decades.[6]

Another limitation common to all previous work in this literature is that REER is defined at the level of individual countries. With increasing specialization and trade in intermediate inputs, inter-sectoral linkages between countries differ substantially from aggregate country level relationships. Wang et al. (2013) have document this heterogeneity substantially. In particular, they find that total foreign content (VS) sourced from manufacturing and services sectors used in world manufacturing goods production has increased by 8.3 percentage points from 1995 to 2011 (from 22.5% in 1995 to 30.8% in 2011). Moreover, they show that this increase is primarily accounted for by the increase in foreign double counted terms(FDC) which are a result of back and fourth trading between countries. With this development of internationalization in production, individual sectors within countries are likely to show substantial heterogeneity in their competitiveness. An aggregate country level measure is incapable of capturing these. Indeed, in section 10 we document several instances where the REERs move in opposite directions for different sectors within a country. In these cases where the country level REER moves in the opposite direction to certain key sectors within the country, lack of information on the latter can lead to false conclusions and inefficient or even counter-productive policy measures. For example, we document in figure 10.2 that Mexico as a whole has experienced a loss in competitiveness(appreciation) starting in 1995. One policy measure this may instigate is (implicit or explicit) subsidies, especially to exporters. However applying these measures uniformly across all sectors would be erroneous since there are sectors like the financial intermediation sector that are already experiencing a gain in competitiveness. This illustrates the usefulness of sector level exchange rates in enabling countries to better target and manage their producers and exporters. Although lack of data and absence of the global value chain phenomenon had prevented the feasibility and need for such measures in the past, in this paper we emphasize that this is no longer the case.

Recognizing all these shortcomings, this paper proposes a concept of REER that improves upon the existing REER measures in the literature along four dimensions. Firstly, by explicitly allowing for trade in intermediate inputs and distinguishing trade by end use category, our model recognizes that value added and gross output are not the same. We therefore compute different REER indices

[5]This example is discussed in more detail quantitatively in section 7

[6]For OECD countries Miroudot et al. (2009) find the share of trade in intermediate goods and services to be 56% and 73% respectively.As emphasized in Baldwin and Lopez-Gonzalez (2012), intermediate goods trade and vertical specialization has grown many fold in developing countries starting in the 1980s(see alsoWang et al. (2013)) Also important is the import content of exports, epitomized by the prevalence of processing trade involving Asian economies, especially China. Koopman et al. (2014) find that the import content of exports is as high as 90 percent for some sectors in China.

for value added(GVC-REER) and gross output(Q-REER). Secondly, we start at the level of sectors within countries and build our way up, allowing us to define and compute not just country REERs but also REERs for sectors within countries.Given our data source(to be discussed in detail later on in the paper) we can compute REER indices for 35 sectors within each of the 40 countries in the sample. We find substantial heterogeneity in the REER across sectors within countries and are in a position to talk about competitiveness of individual sectors within the same countries. Thirdly, we estimate and explicitly incorporate different elasticities of substitution across different groups of goods in our REER measure. This is a significant improvement relative to other attempts to take GVCs into account which continue to work with a simplified Cobb douglas case in which all elasticities are identically equal to one. Lastly, we compute sector level price indices and use these in our REER measure instead of the more coarse country level price indices(GDP deflator or CPI) which have been used in the literature so far.

It is worth emphasizing that our objective is to model relatively short term and small scale movements in competitiveness. We therefore take the nature of GVC and trade patterns across countries and sectors as given and do not consider the issue of endogenous off-shoring and production sharing decisions [7]. Moreover, due to the complex nature of the model we solve it using log linearization techniques. This further reinforces the view that our REER indices are best suited for short term movements resulting from shocks that are not too large so as to affect organization of GVCs.

This paper is related to two different strands of the literature. First and foremost the paper is a contribution to the literature on international trade and finance. The most prominent and commonly cited REER measure today is the IMF's REER measure. This along with other commonly used REER indices like the ones by the Federal Reserve and BIS do not distinguish between trade in intermediate and final goods and consider all trade flows to be in the latter category. The consequence of making this assumption could be quite detrimental as we have discussed above.

A few recent papers have recognized this drawback and made attempts to address them. Bems and Johnson (2012)(henceforth BJ) allow for trade in intermediates and compute the REER weighting matrix at the country level. Bayoumi et al. (2013) propose a measure of competitiveness in which they borrow the weighting matrix from the IMF but adjust the price indices to acknowledge the presence of imported inputs. But these papers work with the constant elasticity(Cobb Douglas) assumption and country level (instead of more detailed sector level) price indices. Our attempt to incorporate sector level price indices and build sector level exchange rates has a precedent in the work of Bennett and Zarnic (2009). But their work does not incorporate trade in intermediate goods and use an IMF-like weighting matrix. Moreover, they use unit labor costs to proxy for price of value added, whereas we have a more comprehensive measure of value added price index at the sector level which includes not only labor but also capital.

In Table 1 we summarize our contribution to this literature by drawing comparisons across the

[7] There is a growing literature on organization of global value chains that looks into these questions. See for instance Costinot et al. (2013) and Johnson and Moxnes (2012).

most influential as well as the most advanced REER computation frameworks available. A single check mark ($\surd$) indicates that the paper attempts to address the particular attribute, irrespective of whether they achieve the desired objective in our opinion.In fact, in most cases where there are check marks in the table, we show drawbacks that are addressed by our framework.As will be shown in the sections to follow, our measure is not only more comprehensive, but nests most of the common measures in the literature.

***Table 1*** *– Comparison with the literature on computing REER*

| | IMF | FED | BIS | BJ | BST | BZ | This paper |
|---|---|---|---|---|---|---|---|
| Value added competitiveness | | | | $\surd$ | | | $\surd$ |
| Sector level exchange rates/prices | | | | | | $\surd$ | $\surd$ |
| Trade in intermediate goods | | | | $\surd$ | $\surd$ | | $\surd$ |
| Heterogenous elasticities | | | | | | | $\surd$ |

Key: BJ: Bems and Johnson (2012); BST:Bayoumi et al. (2013);BZ:Bennett and Zarnic (2009); A check mark ($\surd$) indicates that the paper attempts to address the particular attribute, irrespective of whether they achieve the desired objective in our opinion

More broadly, the paper is motivated by and is linked to new but rapidly expanding literature on global value chains and vertical specialization in trade(see Hummels et al. (2001) and Baldwin and Lopez-Gonzalez (2012) among many others) as well as the literature on trade statistics and export accounting in the presence of intermediate goods trade(Koopman et al. (2012) and Wang et al. (2013)).

We start by presenting a simplified version of our framework in section 3 to illustrate some of the features of our exchange rate measure before moving on to the general model in section 4 which is discussed in greater detail and applied to the data.

## 2 The concept of REER as a measure of competitiveness

The real effective exchange rate measures change in competitiveness by quantifying changes in the demand for goods produced by a country as a function of changes in relative prices. To be more precise, if $V_J$ is the demand for the goods produced(or alternatively, value added) by country $J$, then the effective exchange rate of country $J$ is defined as:

$$\triangle REER_J = \triangle V_J = G_J\left(\{\triangle p\}_{i=1}^n\right) \tag{2.1}$$

Where $\{\triangle p\}_{i=1}^n$ is a vector of price changes in all countries including the home country. Note that no other variables except the prices explicitly enter the function $G(.)$. Hence by construction REER is a partial equilibrium construct where the primitive shocks that lead to the observed price changes are not modeled. Moreover the demand side of the economy is assumed to be exogenous and the aggregate final demand is assumed to be constant (although relative demands are affected when prices change).

The function $G_J(.)$ is homogenous of degree zero, so that the model satisfies neutrality in the sense that if all prices (including the home price) double, then the relative demands remain unchanged(and since by construction aggregate demand is held fixed, the absolute demand for each good also remains unaffected.)

It is worth pointing out that REER models like ours do not assume balanced trade or any restrictions on the trade balance. Trade balances are allowed to be non zero in the steady state and are calibrated to their observed counterparts in the data. Since this is a partial equilibrium model in which the demand side is exogenous, this assumption is not unsuitable.

## 3 A Stylized Three Country Global Value Chain

We start with a simplified 3 country, 2 sector model capturing the basic Global Value Chain(GVC) features. There are three countries (J, C and U) and two sectors indexed by {1,2}.Table 2 displays the input output table. All boxes with "X" denote non zero entries while the remaining entries are zero. Note that the IO matrix is sparse and contains only one (out of a possible 36) non zero entries. The Global value chain is modeled across two sectors in three different countries . The upstream sector J1(sector 1 in country J) produces raw materials that are exported to country C. Sector 2 in country C combines these intermediate inputs from J along with its own value added to produce final goods that are then exported to countries J and U in addition to being consumed internally by country C. All other sectors (i.e J2, C1, U1 and U2) only produce goods using own value added (i.e no intermediate inputs) and sell them as final demand in the home country. Sector 2 can be interpreted as the electronics sector and sector 1 can be interpreted as a (raw) materials sector.

***Table 2*** – *A stylized 3 country 2 sector Global value chain set up*

| | | J | | C | | U | | JFinal | CFinal | Ufinal | total output |
|---|---|---|---|---|---|---|---|---|---|---|---|
| | | J1 | J2 | C1 | C2 | U1 | U2 | | | | |
| J | J1 | 0 | 0 | 0 | X | 0 | 0 | X | 0 | 0 | X |
| | J2 | 0 | 0 | 0 | 0 | 0 | 0 | X | 0 | 0 | X |
| C | C1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | X | 0 | X |
| | C2 | 0 | 0 | 0 | 0 | 0 | 0 | X | X | X | X |
| U | U1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | X | X |
| | U2 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | X | X |
| VA | | X | X | X | X | X | X | | | | |
| total output | | X | X | X | X | X | X | | | | |

**Specifying the model:**

We use $Q_l^h$ to denote the gross output of sector $l$ is country $h$ [8]. $X_{sl}^{fh}$ denotes the intermediate input from country $f$ sector $s$ used in production by $(h, l)$. $V_l^h$ denotes the value added by $(h, l)$.

[8]Throughout this paper, superscripts will be used for countries and subscripts for sectors.

We assume a constant elasticity of substitution which is allowed to differ across consumption and production aggregators as clarified below.

**Production:**

The production function of $(C, 2)$ is given by

$$Q_2^C = \left[(w^V)^{\frac{1}{\sigma}}(V_2^C)^{\frac{\sigma-1}{\sigma}} + (w^X)^{\frac{1}{\sigma}}(X_{12}^{JC})^{\frac{\sigma-1}{\sigma}}\right]^{\frac{\sigma}{\sigma-1}} \tag{3.1}$$

where $\sigma$ is the elasticity of substitution between the two inputs and $w^V$ and $w^X$ are weights that can be mapped to the shares of the two inputs.

The associated price index is given by:

$$p_2^C = \left[(w^V)(p_2^{Vc})^{1-\sigma} + (w^X)(p_1^J)^{1-\sigma}\right]^{\frac{1}{1-\sigma}} \tag{3.2}$$

Since all other production comprises entirely of own value added, the remaining production functions are of the form:

$$Q_l^h = V_l^h \forall (h, l) \neq (C, 2) \tag{3.3}$$

**Consumption:**

We use $F_l^{hf}$ to denote output of $(h, l)$ that is absorbed in country $f$ as final demand. Based on table 2, the consumption aggregators for the three countries are given as follows:

$$F^J = [(\kappa_1^{JJ})^{\frac{1}{\theta}}(F_1^{JJ})^{\frac{\theta-1}{\theta}} + (\kappa_2^{JJ})^{\frac{1}{\theta}}(F_2^{JJ})^{\frac{\theta-1}{\theta}} + (\kappa_2^{CJ})^{\frac{1}{\theta}}(F_2^{CJ})^{\frac{\theta-1}{\theta}}]^{\frac{\theta}{\theta-1}} \tag{3.4}$$

$$F^C = [(\kappa_1^{CC})^{\frac{1}{\theta}}(F_1^{CC})^{\frac{\theta-1}{\theta}} + (\kappa_2^{CC})^{\frac{1}{\theta}}(F_2^{CC})^{\frac{\theta-1}{\theta}}]^{\frac{\theta}{\theta-1}} \tag{3.5}$$

$$F^U = [(\kappa_1^{UU})^{\frac{1}{\theta}}(F_1^{UU})^{\frac{\theta-1}{\theta}} + (\kappa_2^{UU})^{\frac{1}{\theta}}(F_2^{UU})^{\frac{\theta-1}{\theta}} + (\kappa_2^{CU})^{\frac{1}{\theta}}(F_2^{CU})^{\frac{\theta-1}{\theta}}]^{\frac{\theta}{\theta-1}} \tag{3.6}$$

The associated price indices for final goods consumption (CPIs) can be computed as follows:

$$P^J = [(\kappa_1^{JJ})(p_1^J)^{1-\theta} + (\kappa_2^{JJ})(p_2^J)^{1-\theta} + (\kappa_2^{CJ})(p_2^C)^{1-\theta}]^{\frac{1}{1-\theta}} \tag{3.7}$$

$$P^C = [(\kappa_1^{CC})(p_1^C)^{1-\theta} + (\kappa_2^{CC})(p_2^C)^{1-\theta}]^{\frac{1}{1-\theta}} \tag{3.8}$$

$$P^U = [(\kappa_1^{UU})(p_1^U)^{1-\theta} + (\kappa_2^{UU})(p_2^U)^{1-\theta} + (\kappa_2^{CU})(p_2^C)^{1-\theta}]^{\frac{1}{1-\theta}} \tag{3.9}$$

Here, large case $P$ is used to denote CPI, small case $p$ is used to denote price of gross output; $p^V$ will be used to denote price of value added. $\theta$ is the elasticity of substitution between different goods in the final consumption good bundle and is assumed to be the same across all countries[9]. $\kappa$ s are weights that denote the shares of the different components in the aggregators.

[9]This and many other restrictions imposed so far will be relaxed in the general model presented later.

**Market clearing conditions:**

Output of all entities except $(C, 2)$ is sold only as final good. Table 2 implies the following market clearing conditions:

$$
\begin{aligned}
Q_1^J &= X_{12}^{JC} + F_1^{JJ} &(3.10)\\
Q_2^J &= F_2^{JJ} &(3.11)\\
Q_1^C &= F_1^{CC} &(3.12)\\
Q_2^C &= F_2^{CC} + F_2^{CJ} + F_2^{CU} &(3.13)\\
Q_1^U &= F_1^{UU} &(3.14)\\
Q_2^U &= F_2^{UU} &(3.15)
\end{aligned}
$$

**Solving the model:**

We solve the model by combining the log linearized first order conditions with the market clearing conditions.The first order condition for final good can be written as:

$$F_s^{fh} = \kappa_s^{fh} \left(\frac{p_s^f}{P^h}\right)^{-\theta} F^h, h, f \in \{J, C, U), s \in \{1, 2\} \tag{3.16}$$

(note that only 8 out of the 18 values of $F_s^{fh}$ are positive, as denoted in table 2 ).
We will work with the following log linearized version:

$$\hat{F}_s^{fh} = -\theta \hat{p}_s^f + \theta \hat{P}^h + \hat{F}^h \tag{3.17}$$

Linearizing the expressions for the CPIs in the three countries we get:

$$
\begin{aligned}
\hat{P}^U &= \left(\frac{p_2^C F_2^{CU}}{P^U F^U}\right) \hat{p}_2^C + \left(\frac{p_1^U F_1^{UU}}{P^U F^U}\right) \hat{p}_1^{VU} + \left(\frac{p_2^U F_2^{UU}}{P^U F^U}\right) \hat{p}_2^{VU} &(3.18)\\
\hat{P}^C &= \left(\frac{p_1^C F_1^{CC}}{P^C F^C}\right) \hat{p}_1^C + \left(\frac{p_2^C F_2^{CC}}{P^C F^C}\right) \hat{p}_2^C &(3.19)\\
\hat{P}^J &= \left(\frac{p_1^J F_1^{JJ}}{P^J F^J}\right) \hat{p}_1^J + \left(\frac{p_2^J F_2^{JJ}}{P^J F^J}\right) \hat{p}_2^J + \left(\frac{p_2^C F_2^{CJ}}{P^J F^J}\right) \hat{p}_2^C &(3.20)
\end{aligned}
$$

The first order conditions for production are as follows:

$$
\begin{aligned}
X_{12}^{JC} &= w^X \left(\frac{p_1^J}{p_2^C}\right)^{-\sigma} Q_2^C\\
V_2^C = &= w_2^V \left(\frac{p_2^{VC}}{p_2^C}\right)^{-\sigma} Q_2^C
\end{aligned}
$$

These along with the production function (3.1) and its associated price index can be linearized as follows:

$$\hat{X}_{12}^{JC} = -\sigma\hat{p}_1^J + \sigma\hat{p}_2^C + \hat{Q}_2^C \tag{3.21}$$

$$\hat{V}_2^C = -\sigma\hat{p}_2^{VC} + \sigma\hat{p}_2^C + \hat{Q}_2^C \tag{3.22}$$

$$\hat{Q}_2^C = \left(\frac{p_2^{VC}V_2^C}{p_2^CQ_2^C}\right)\hat{V}_2^C + \left(\frac{p_1^{VJ}X_{12}^{JC}}{p_2^CQ_2^C}\right)\hat{X}_{12}^{JC} \tag{3.23}$$

$$\hat{p}_2^C = \left(\frac{p_2^{VC}V_2^C}{p_2^CQ_2^C}\right)\hat{p}_2^{VC} + \left(\frac{p_1^{VJ}X_{12}^{JC}}{p_2^CQ_2^C}\right)\hat{p}_1^{VJ} \tag{3.24}$$

Next, the non trivial market clearing conditions (3.13) and (3.10) can be linearized as follows:

$$\hat{Q}_2^C = \left(\frac{p_2^CF_2^{CC}}{p_2^CQ_2^C}\right)\hat{F}_2^{CC} + \left(\frac{p_2^CF_2^{CJ}}{p_2^CQ_2^C}\right)\hat{F}_2^{CJ} + \left(\frac{p_2^CF_2^{CU}}{p_2^CQ_2^C}\right)\hat{F}_2^{CU} \tag{3.25}$$

$$\hat{Q}_1^J = \left(\frac{p_1^JX_{12}^{JC}}{p_1^JQ_1^J}\right)\hat{X}_{12}^{JC} + \left(\frac{p_1^JF_1^{JJ}}{p_1^JQ_1^J}\right)\hat{F}_1^{JJ} \tag{3.26}$$

$$\tag{3.27}$$

Appendix A shows how the linearized version of the model can be solved to write the demand for value added for each entity as a function of all the value added prices assuming that total final demand remains constant(i.e $\hat{F}^h = 0\forall h$), or what we define as the real exchange rate(GVC-REER) following this partial equilibrium literature.:

$$\triangle GVC{-}REER_s^h = \hat{V}_s^h = w(v)_{s1}^{hJ}\hat{p}_1^{vJ} + w(v)_{s2}^{hJ}\hat{p}_2^{vJ} + w(v)_{s1}^{hC}\hat{p}_1^{vC} + w(v)_{s2}^{hC}\hat{p}_2^{vC} + w(v)_{s1}^{hU}\hat{p}_1^{vU} + w(v)_{s2}^{hU}\hat{p}_2^{vU} \tag{3.28}$$

Where $h \in \{J, C, U\}, s \in \{1, 2\}$

To illustrate the properties of the weighting matrix, we will focus on the weight assigned by $(C, 2)$ to the six production entities(including itself).

$$\begin{aligned}\hat{V}_2^C = {} & w(v)_{21}^{CJ}\hat{p}_1^{vJ} + w(v)_{22}^{CJ}\hat{p}_2^{vJ} + w(v)_{21}^{CC}\hat{p}_1^{vC} + w(v)_{22}^{CC}\hat{p}_2^{vC} \\ & + w(v)_{21}^{CU}\hat{p}_1^{vU} + w(v)_{22}^{CU}\hat{p}_2^{vU}\end{aligned} \tag{3.29}$$

In particular, we focus on the weight assignment between the two sectors that are involved in the the GVC. The appendix shows that the weight assigned by sector 2 in country $C$ to sector 1 in country $J$ (its input supplier) is given by

$$
\begin{aligned}
w(v)_{21}^{CJ} &= \left(\frac{p_1^J X_{12}^{JC}}{p_2^C Q_2^C}\right)(\sigma-\theta) + \underbrace{\theta\left(\frac{p_1^J X_{12}^{JC}}{p_2^C Q_2^C}\right)\left(\frac{p_2^C F_2^{CJ}}{p_2^C Q_2^C}\right)\left(\frac{p_2^C F_2^{CJ}}{P^J F^J}\right)}_{term2} + \underbrace{\theta\left(\frac{p_2^{VC} V_2^C}{p_2^C Q_2^C}\right)\left(\frac{p_2^C F_2^{CC}}{p_2^C Q_2^C}\right)\left(\frac{p_2^C F_2^{CC}}{P^C F^C}\right)}_{term3} \\
&+ \underbrace{+\theta\left(\frac{p_2^C F_2^{CJ}}{p_2^C Q_2^C}\right)\left(\frac{p_1^J F_1^{JJ}}{P^J F^J}\right)}_{term4} + \underbrace{\theta\left(\frac{p_1^J X_{12}^{JC}}{p_2^C Q_2^C}\right)\left(\frac{p_2^C F_2^{CU}}{p_2^C Q_2^C}\right)\left(\frac{p_2^C F_2^{CU}}{P^U F^U}\right)}_{term5} \qquad (3.30)
\end{aligned}
$$

We can interpret the different terms on the right hand side of the above equation as follows:

$$
\begin{aligned}
term2 + term4 &= \theta\left(\frac{p_1^J X_{12}^{JC}}{p_2^C Q_2^C}\right)\left(\frac{p_2^C F_2^{CJ}}{p_2^C Q_2^C}\right)\left(\frac{p_2^C F_2^{CJ}}{P^J F^J}\right) + \theta\left(\frac{p_2^C F_2^{CJ}}{p_2^C Q_2^C}\right)\left(\frac{p_1^J F_1^{JJ}}{P^J F^J}\right) \\
&= \theta\left[\left(\frac{p_2^{VC} V_2^C}{p_2^C Q_2^C}\right)\left(\frac{p_2^C F_2^{CJ}}{p_2^{VC} V_2^C}\right)\right]\left[\left(\frac{p_1^J X_{12}^{JC}}{p_2^C Q_2^C}\right)\left(\frac{p_2^C F_2^{CJ}}{P^J F^J}\right) + \left(\frac{p_1^J F_1^{JJ}}{P^J F^J}\right)\right] \qquad (3.31)
\end{aligned}
$$

$\left(\frac{p_2^{VC} V_2^C}{p_2^C Q_2^C}\right) p_2^C F_2^{CJ}$ is the value added created by $(C,2)$ that is ultimately absorbed in country $J$. Similarly $\left(\frac{p_1^J X_{12}^{JC}}{p_2^C Q_2^C}\right) p_2^C F_2^{CJ} + p_1^J F_1^{JJ}$ is the value added created in $(J,1)$ that is ultimately absorbed in country $J$. Therefore we can simplify (3.31) to write:[10]

$$
term2 + term4 = \theta\left(\frac{p_2^{VC} V_2^{CJ}}{p_2^{VC} V_2^C}\right)\left(\frac{p_1^{VJ} V_1^{JJ}}{P^J F^J}\right) \qquad (3.32)
$$

where we use the notation $p_l^{Va} V_l^{ab}$ to denote the total value added created in $(a,l)$ that is ultimately absorbed as country $b$'s final demand. Similar simplifications can be applied to $term3$ and $term5$ to reduce (3.30) to the following:

$$
\begin{aligned}
w(v)_{21}^{CJ} &= \theta\left[\left(\frac{p_2^{VC} V_2^{CJ}}{p_2^{VC} V_2^C}\right)\left(\frac{p_1^{VJ} V_1^{JJ}}{P^J F^J}\right) + \left(\frac{p_2^{VC} V_2^{CC}}{p_2^{VC} V_2^C}\right)\left(\frac{p_1^{VJ} V_1^{JC}}{P^C F^C}\right) + \left(\frac{p_2^{VC} V_2^{CU}}{p_2^{VC} V_2^C}\right)\left(\frac{p_1^{VJ} V_1^{JU}}{P^U F^U}\right)\right] \\
&+ \left(\frac{p_1^J X_{12}^{JC}}{p_2^C Q_2^C}\right)(\sigma-\theta) \qquad (3.33)
\end{aligned}
$$

Several aspects of the weighting schemes now become evident from (3.33). Firstly, under the constant elasticity assumption $(\theta = \sigma = 1)$the weight reduces to:

$$
w(v)_{21}^{CJ} = \sum_{k=J,C,U}\left[\frac{\left(p_2^{vC} V_2^{Ck}\right)\left(p_1^{vJ} V_1^{Jk}\right)}{\left(p_2^{vC} V_2^C\right)\left(P^K F^k\right)}\right] \geq 0 \qquad (3.34)
$$

This is the same as the sum of the three terms inside the bracket in (3.33)[11].Each of these terms can be interpreted as capturing the intensity of competition between $(C,2)$ and $(J,1)$ in the three

---

[10]the derivation follows by multiplying and dividing both terms by $\left(\frac{p_2^{VC} V_2^C}{p_2^C Q_2^C}\right)$ and rearranging.

[11]In fact, under the assumption of constant elasticity, all weights can be represented by a formula

final goods markets, namely $C$,$J$ and $U$. The higher is the intensity of competition as measured by these terms, the higher would be the weight assigned by $(C,2)$ to $(J,1)$, since in that case a fall in the price of $(J,1)$'s value added would hurt $(C,2)$ more. As an example, if the last term in (3.33) $\left[\left(\frac{p_2^{VC}V_2^{CU}}{p_2^{VC}V_2^{C}}\right)\left(\frac{p_1^{VJ}V_1^{JU}}{P^U F^U}\right)\right]$ is high, it conveys that value added by $(C,2)$ and $(J,1)$ are competing intensively in order to satisfy country $U$'s final demand, so a fall in $(J,1)$'s price will hurt $(C,2)$ to a greater extent.

Secondly, note that $w(v)_{21}^{CJ}$ is strictly increasing in $\sigma$. Intuitively, the lower is the elasticity of substitution between own value added and Japanese imports in $(C,2)$'s production, the higher will be the co-movement between the value added by $(C,2)$ and $(J,1)$. In this case, the weight assigned by $(C,2)$ to $(J,1)$ will be lower, since a fall in the price of $(J,1)$ 's value added, which increases demand for output of $(J,1)$ , will also end up exerting a positive effect on the demand for value added by $(C,2)$.

Thirdly, unlike $\sigma$, the effect of an increase in $\theta$ is ambiguous. On the one hand an increase in $\theta$ exerts a positive effect on $w(v)_{21}^{CJ}$ via the standard expenditure switching effect-if consumers are more willing to substitute between different goods in their consumption bundle, a fall in the price of a substitute will decrease demand for the own good to a greater extent. On the other hand, to the extent that the value added by $(C,2)$ and $(J,1)$ are complementary, expenditure switching towards $(J,1)$'s value added indirectly also implies a shift in expenditure towards $(C,2)$'s output and hence towards $(C,2)$'s value added which is embodied in its own output. In this example the two effects run in opposite directions.

Lastly, (3.33) and (3.34)also illustrate the restrictive nature of the weighting scheme in the constant elasticity case. In particular the complementarity effect discussed above is not present, and as is evident from (3.34) the weighting scheme is not flexible enough to accommodate negative weights.

**Exchange rate weights for gross output competitiveness:**

With trade in intermediate inputs, competitiveness in gross output and value added can be delinked. Similar to (3.29) we can compute a log linear approximation of the change in $(C,2)$'s gross output as follows(derivations are in the appendix). We label this measure of gross output competitiveness "Q-REER".

$$\triangle Q{-}REER_2^c = \hat{Q}_2^c = w(Q)_{21}^{CJ}\hat{p}_1^{vJ} + w(Q)_{22}^{CJ}\hat{p}_2^{vJ} + w(Q)_{21}^{CC}\hat{p}_1^{vC} + w(Q)_{22}^{CC}\hat{p}_2^{vC} + w(Q)_{21}^{CU}\hat{p}_1^{vU} + w(Q)_{22}^{CU}\hat{p}_2^{vU} \tag{3.35}$$

---

mimicking (3.34), i.e.

$$w(v)_{ls}^{hk} = \sum_{k=J,C,U}\left[\frac{\left(p_l^{vh}V_l^{hk}\right)\left(p_s^{vc}V_s^{ck}\right)}{\left(p_l^{vh}V_l^{h}\right)\left(P^K F^k\right)}\right] \forall h,k,s,l$$

This is the same as 5.27 in the case of the general model.

Where

$$
\begin{aligned}
w(Q)_{21}^{CJ} &= -\theta\left(\frac{p_1^J X_{12}^{JC}}{p_2^C Q_2^C}\right)+\theta\left[\left(\frac{p_2^{VC}V_2^{CJ}}{p_2^{VC}V_2^C}\right)\left(\frac{p_1^{VJ}V_1^{JJ}}{P^J F^J}\right)+\left(\frac{p_2^{VC}V_2^{CC}}{p_2^{VC}V_2^C}\right)\left(\frac{p_1^{VJ}V_1^{JC}}{P^C F^C}\right)+\left(\frac{p_2^{VC}V_2^{CU}}{p_2^{VC}V_2^C}\right)\left(\frac{p_1^{VJ}V_1^{JU}}{P^U F^U}\right)\right] \\
&= w(v)_{21}^{CJ}-\sigma\left(\frac{p_1^J X_{12}^{JC}}{p_2^C Q_2^C}\right) \qquad (3.36)
\end{aligned}
$$

The idea behind the "Goods-REER" measure of Bayoumi et al. (2013) is to measure competitiveness of gross output as opposed to value added. The analogous expression in our framework is given by (3.35), but as will be shown later, the two measures do not coincide expect in very restrictive and/or knife edge cases.

Two differences between the value added weight $w(v)_{21}^{CJ}$ and gross output weight $w(Q)_{21}^{CJ}$ are worth highlighting. First, note that as long as the production function is not Leontief (i.e $\sigma \neq 0$) , the gross output competitiveness weight is always lower then the value added weight $(w(Q)_{21}^{CJ} < w(V)_{21}^{CJ})$. This is a consequence of the fact that substitutability in the production function which causes the weight $w(V)_{21}^{CJ}$ to increase because of the possibility of a shift occurring from $V_2^C$ to $(J,1)$'s value added(embodied in $X_{12}^{JC}$) does not affect the gross output weight $w(Q)_{21}^{CJ}$, for as far as gross output is concerned the substitution between different inputs in production is irrelevant as long as the final demand for the good increases.

Secondly note that when $X_{12}^{JC} = 0$, the two weights are equivalent. As will be shown in the paper later on,this is a general result- that in the absence of intermediate inputs the gross output and value added weighting matrices are identical.

### Defining Aggregate Real Effective Exchange Rates for Countries

We compute aggregate country level REERs by exploiting information on sector level trade flows. For comparison we also compute an alternate measure that aggregates all trade flows within a country as is commonly done in the literature.

To derive the expression for country-level value added weights, we start with the following decomposition of the nominal GDP of country $h$ into its different sectoral components:

$$p^{vh}V^h = p_1^{vh}V_1^h + p_2^{vh}V_2^h \qquad (3.37)$$

Where $p^{vh}$ is the GDP deflator of country $h$. Log linearizing this equation we get:

$$\hat{p}^{vh} + \hat{V}^h = \left(\frac{p_1^{vh}V_1^h}{p^{vh}V^h}\right)\left[\hat{p}_1^{vh} + \hat{V}_1^h\right] + \left(\frac{p_2^{vh}V_2^h}{p^{vh}V^h}\right)\left[\hat{p}_2^{vh} + \hat{V}_2^h\right] \qquad (3.38)$$

Since (up to a first order approximation) the change in GDP deflator is a weighted sum of changes in the different sector level prices, the above equation reduces to

*Table 3 – Single sector version of the 3 by 2 model*

| | J | C | U | JFinal | CFinal | Ufinal | total output |
|---|---|---|---|---|---|---|---|
| J | 0 | X | 0 | X | 0 | 0 | X |
| C | 0 | 0 | 0 | X | X | X | X |
| U | 0 | 0 | 0 | 0 | 0 | X | X |
| VA | X | X | X | | | | |
| total output | X | X | X | | | | |

$$\begin{aligned}
\hat{V}^h &= \left(\frac{p_1^{vh}V_1^h}{p^{vh}V^h}\right)\left[\hat{V}_1^h\right] + \left(\frac{p_2^{vh}V_2^h}{p^{vh}V^h}\right)\left[\hat{V}_2^h\right] \\
&= \left(\frac{p_1^{vh}V_1^h}{p^{vh}V^h}\right)\left[\sum_{f\in\{J,C,F\}}\sum_{k\in\{1,2\}} w(v)_{1k}^{hf}\hat{p}_k^{vf}\right] + \left(\frac{p_2^{vh}V_2^h}{p^{vh}V^h}\right)\left[\sum_{f\in\{J,C,F\}}\sum_{k\in\{1,2\}} w(v)_{2k}^{hf}\hat{p}_k^{vf}\right] \\
&= \sum_{f\in(J,C,U)}\sum_{k\in\{1,2\}}\left[\left(\frac{p_1^{vh}V_1^h}{p^{vh}V^h}\right) w(v)_{1k}^{hf} + \left(\frac{p_2^{vh}V_2^h}{p^{vh}V^h}\right) w(v)_{2k}^{hf}\right]\hat{p}_k^{vf} \qquad (3.39)
\end{aligned}$$

If sector level price indices are available, then (3.39) provides the most accurate measure of competitiveness for a country as a whole. Typically however, country level REER is computed using an aggregate country-wide price index(like the GDP deflator, CPI or some measure of unit labor cost). To define an analogous measure in our framework we need to make an assumption regarding the link between sector level prices and the aggregate GDP deflator of a country. We choose to make the simplest possible assumption(which is also made implicitly throughout this literature), namely that all sector level prices change by the same proportion as the change in the aggregate GDP deflator. In particular,

$$\hat{p}^{vh} = \hat{p}_1^{vh} = \hat{p}_2^{vh} \qquad (3.40)$$

With this assumption we can simplify (3.39) to write:

$$\hat{V}^h = \sum_{f\in(J,C,U)}\underbrace{\left[\left(\frac{p_1^{vh}V_1^h}{p^{vh}V^h}\right)\left(w(v)_{11}^{hf} + w(v)_{12}^{hf}\right) + \left(\frac{p_2^{vh}V_2^h}{p^{vh}V^h}\right)\left(w(v)_{21}^{hf} + w(v)_{21}^{hf}\right)\right]}_{w_A(v)^{hf}}\hat{p}^{vf} \qquad (3.41)$$

Here $w_A(v)^{hf}$ denotes the aggregate(i.e country level) weight assigned by country $h$ to country $f$ in the real exchange rate of country $h$. As is evident from (3.41), this weight is itself a weighted sum of the weights assigned by each sector in $h$ to each sector in $f$, with the weights given by the value added shares of the sectors in $h$.

For comparison to the common approach in the literature we first reduce table 2 to a single sector version by aggregating across sectors within each country in table 3.

The production functions, price indices and final demands and market clearing conditions are now given as follows:

$$Q^c = \left[(w^{CV})^{\frac{1}{\sigma}}(V^c)^{\frac{\sigma-1}{\sigma}} + (w^{CX})^{\frac{1}{\sigma}}(X^{Jc})^{\frac{\sigma-1}{\sigma}}\right]^{\frac{\sigma}{\sigma-1}} \tag{3.42}$$

$$p^c = \left[(w^{CV})(p^{Vc})^{1-\sigma} + (w^{CX})(p^J)^{1-\sigma}\right]^{\frac{1}{1-\sigma}} \tag{3.43}$$

$$Q^h = V^h, h \in \{J, U\} \tag{3.44}$$

Consumption

$$\begin{aligned} F^J &= [(\kappa^{JJ})^{\frac{1}{\theta}}(F^{JJ})^{\frac{\theta-1}{\theta}} + (\kappa^{CJ})^{\frac{1}{\theta}}(F^{CJ})^{\frac{\theta-1}{\theta}}]^{\frac{\theta}{\theta-1}} && (3.45)\\ F^C &= F^{CC} && (3.46)\\ F^U &= [(\kappa^{UU})^{\frac{1}{\theta}}(F^{UU})^{\frac{\theta-1}{\theta}} + (\kappa^{CU})^{\frac{1}{\theta}}(F^{CU})^{\frac{\theta-1}{\theta}}]^{\frac{\theta}{\theta-1}} && (3.47) \end{aligned}$$

$$\begin{aligned} P^J &= [(\kappa^{JJ})(p^J)^{1-\theta} + (\kappa^{CJ})(p^C)^{1-\theta}]^{\frac{1}{1-\theta}} && (3.48)\\ P^C &= p^C && (3.49)\\ P^U &= [(\kappa^{UU})(p^U)^{1-\theta} + (\kappa^{CU})(p^C)^{1-\theta}]^{\frac{1}{1-\theta}} && (3.50) \end{aligned}$$

$$\begin{aligned} Q^J &= X^{JC} + F^{JJ}\\ Q^C &= F^{CC} + F^{CJ} + F^{CU}\\ Q^U &= F^{UU} \end{aligned}$$

The appendix shows that the weight assigned by country $C$ to country $J$ in this case is given by:

$$\hat{V}^C = w(v)^{CJ}\hat{p}^{VJ} + w(v)^{CC}\hat{p}^{VC} + w(v)^{CU}\hat{p}^{VU} \tag{3.51}$$

$$
\begin{aligned}
w(v)^{CJ} &= \sigma\left(\frac{p^{vJ}X^{JC}}{p^{C}Q^{C}}\right) - \theta\left(\frac{p^{vJ}X^{JC}}{p^{C}Q^{C}}\right) + \theta\left(\frac{p^{vC}F^{CC}}{p^{C}Q^{C}}\right)\left(\frac{p^{vC}F^{CC}}{p^{C}F^{C}}\right) && (3.52)\\
&\quad +\theta\left(\frac{p^{vJ}X^{JC}}{p^{C}Q^{C}}\right)\left(\frac{p^{vC}F^{CJ}}{p^{C}Q^{C}}\right)\left(\frac{p^{vC}F^{CJ}}{p^{J}F^{J}}\right) + \theta\left(\frac{p^{vC}F^{CJ}}{p^{C}Q^{C}}\right)\left(\frac{p^{vC}F^{CJ}}{p^{J}F^{J}}\right)\\
&\quad +\theta\left(\frac{p^{vJ}X^{JC}}{p^{C}Q^{C}}\right)\left(\frac{p^{vC}F^{CU}}{p^{C}Q^{C}}\right)\left(\frac{p^{vC}F^{CU}}{p^{U}F^{U}}\right) && (3.53)\\
w(v)^{CC} &= -\sigma\left(\frac{p^{vJ}X^{JC}}{p^{C}Q^{C}}\right) - \theta\left(\frac{p^{vC}V^{C}}{p^{C}Q^{C}}\right) + \theta\left(\frac{p^{vC}V^{C}}{p^{C}Q^{C}}\right)\left(\frac{p^{vC}F^{CC}}{p^{C}Q^{C}}\right)\left(\frac{p^{vC}F^{CC}}{P^{C}F^{C}}\right)\\
&\quad +\theta\left(\frac{p^{vC}V^{C}}{p^{C}Q^{C}}\right)\left(\frac{p^{vC}F^{CJ}}{p^{C}Q^{C}}\right)\left(\frac{p^{vC}F^{CJ}}{p^{J}F^{J}}\right) + \theta\left(\frac{p^{vC}V^{C}}{p^{C}Q^{C}}\right)\left(\frac{p^{vC}F^{CU}}{p^{C}Q^{C}}\right)\left(\frac{p^{vC}F^{CU}}{p^{U}F^{U}}\right)\\
w(v)^{CU} &= \theta\left(\frac{p^{vC}V^{C}}{p^{C}Q^{C}}\right)\left(\frac{p^{vC}F^{CU}}{p^{vC}V^{C}}\right)\left(\frac{p^{vU}F^{UU}}{p^{U}F^{U}}\right) && (3.54)
\end{aligned}
$$

It is evident from 3.52 and 3.41 that $w(v)^{CJ}$ and $w_A(v)^{CJ}$ are not equal. The issue of the non-equivalence of the two weighting matrices will be discussed after we have specified the general model. For now we just want to emphasize that our weighting matrix which exploits sector level information is unique in the literature and so a starting point to begin a comparison with other measures in the literature is to consider the case where there is only one sector within each country.

We next move on to our general model which builds on the intuition developed from this 3 by 2 setting. After discussing the relationship to other measures in the literature based on the general model we come back to the 3 by 2 model to show some illustrative examples.

## 4 The General Model

Consider a world economy comprising of $n$ countries. There are $m$ sectors within each country.Each country-sector is called an "entity" so that there are a total of $nm$ entities in the world economy.Each entity uses a production function with its own value added and a composite intermediate input which can contain intermediate inputs from all $mn$ entities including itself. The output of each entity can be used either as a final good (consumed in any of the $n$ countries) or as an input by another entity.Thus there are a total of $nm$ producers and $nm + n$ consumers ($nm$ entities plus $n$ final goods consumers) in the economy. Both the production function and final goods consumption aggregators are nested CES(constant elasticity of substitution) aggregators which are described in detail next.

### 4.1 Production

Consider the production process for entity $(c, l)$. The production process is assumed to follow the following three stage hierarchy:

**Stage 1**

**Stage 1.1** First, for each sector, inputs from all foreign countries from that sector are aggregated(with a constant elasticity of substitution) to form sectoral intermediate inputs $\{X(f)^c_{sl}\}^m_{s=1}$. In other words, $X(f)^c_{sl}$ is the aggregate sector $s$ foreign intermediate input used in production by country $c$ sector $l$

$$X(f)^c_{sl} = \left[\sum_{i=1,i\neq c}^{n} (w^{ic}_{sl})^{1/\sigma^1_s(c,l)} (X^{ic}_{sl})^{\frac{\sigma^1_s(c,l)-1}{\sigma^1_s(c,l)}}\right]^{\frac{\sigma^1_s(c,l)}{\sigma^1_s(c,l)-1}}, s = 1, 2, ..m \tag{4.1}$$

Here $X^{ic}_{sl}$ denotes inputs from country $i$ sector $s$ used in production by country $c$ sector $l$, the $w'$s are aggregation weights and $\sigma^1_s(c,l)$ is the (constant) elasticity of substitution between different foreign varieties of the sector $s$ output in the production function of entity $(c,l)$

**Stage 1.2** The Sector $s$ import bundle is then combined with the domestic sector $s$ input to form the aggregate sector $s$ input. The elasticity of substitution between these two inputs is $\sigma^{1h}_s(c,l)$.

$$X^c_{sl} = \left[(w^{cc}_{sl})^{1/\sigma^{1h}_s(c,l)} (X^{cc}_{sl})^{\frac{\sigma^{1h}_s(c,l)-1}{\sigma^{1h}_s(c,l)}} + (w(f)^c_{sl})^{1/\sigma^{1h}_s(c,l)} (X(f)^c_{sl})^{\frac{\sigma^{1h}_s(c,l)-1}{\sigma^{1h}_s(c,l)}}\right]^{\frac{\sigma^{1h}_s(c,l)}{\sigma^{1h}_s(c,l)-1}} \tag{4.2}$$

With this two step framework we are allowing for a distinction between "macro"($\sigma^{1h}_s(c,l)$) and "micro"($\sigma^1_s(c,l)$) elasticities for each sector, which is a feature of the data documented in the literature(seeFeenstra et al. (2010) ).

**Stage 2:**

Next, these $m$ sectoral aggregates are combined to form the aggregate intermediate input $X^c_l$

$$X^c_l = \left[\sum_{s=1}^{m} (w^c_{sl})^{1/\sigma^2(c,l)} (X^c_{sl})^{\frac{\sigma^2(c,l)-1}{\sigma^2(c,l)}}\right]^{\frac{\sigma^2(c,l)}{\sigma^2(c,l)-1}} \tag{4.3}$$

**Stage 3:**

Finally, the aggregate intermediate input is combined with the entity's own value added to form the gross output for entity $(c,l)$ which is used both as intermediate and final good. This is denoted by $Q^c_l$. The elasticity of substitution between value added and aggregate intermediate is $\sigma^3(c,l)$

$$Q^c_{.l} = \left[(w^{vc}_l)^{1/\sigma^3(c,l)} (V^c_l)^{\frac{\sigma^3(c,l)-1}{\sigma^3(c,l)}} + (w^{Xc}_l)^{1/\sigma^3(c,l)} (X^c_l)^{\frac{\sigma^3(c,l)-1}{\sigma^3(c,l)}}\right]^{\frac{\sigma^3(c,l)}{\sigma^3(c,l)-1}} \tag{4.4}$$

## 4.2 Preferences

A country specific final good is obtained by aggregating goods form all $nm$ production entities in two stages.

**Stage 1**

**Stage 1.1** Firstly, for each sector $s$, goods from all foreign countries are aggregated to form a aggregate sector $s$ final imported good for consuming country $c$. The elasticity of substitution for each aggregate is $\theta_s^1(c)$

$$F_s^c(f) = \left[ \sum_{i=1, i\neq c}^{n} (\kappa_s^{ic})^{1/\theta_s^1(c)} (F_s^{ic})^{\frac{\theta_s^1(c)-1}{\theta_s^1(c)}} \right]^{\frac{\theta_s^1(c)}{\theta_s^1(c)-1}} \tag{4.5}$$

**Stage 1.2** Each of these imported goods are then combined with the domestic goods from their respective sector to form an aggregate sector $s$ consumption good for country $c$.

$$F_s{}^c = \left[ (\kappa_s^{cc})^{1/\theta_s^{1h}(c)} (F_s^{cc})^{\frac{\theta_s^1(c)-1}{\theta_s^1(c)}} + (\kappa(f)_s^c)^{1/\theta_s^{1h}(c)} (F(f)_s^c)^{\frac{\theta_s^{1h}(c)-1}{\theta_s^{1h}(c)}} \right]^{\frac{\theta_s^{1h}(c)}{\theta_s^{1h}(c)-1}} \tag{4.6}$$

**Stage 2**

Next, these $s$ sectoral aggregates are combined(with constant elasticity $\theta^2(c)$) to form the aggregate consumption good for country $c$

$$F^c = \left[ \sum_{s=1}^{m} (\kappa_s^c)^{1/\rho^2(c)} (F_s^c)^{\frac{\theta^2(c)-1}{\theta^2(c)}} \right]^{\frac{\theta^2(c)}{\theta^2(c)-1}} \tag{4.7}$$

### 4.3 Market clearing:

Gross output from an entity is absorbed either as an intermediate input or a final good(we do not allow for inventory accumulation or any inter temporal effects). Thus the following market clearing condition holds $\forall (c, l)$

$$Q_l^c = \sum_{i=1}^{n} F_l^{ci} + \sum_{j=1}^{m} \sum_{k=1}^{n} X_{lj}^{ck} \tag{4.8}$$

## 5 Computation of effective exchange rate weighting matrices

In order to define the exchange rates we take prices and final demands in all countries as exogenous and compute the demand for value added and gross output of different entities as functions of prices. This is a partial equilibrium set up is common in the literature and requires only one market clearing condition along with the different optimality conditions for production and consumption. We start by deriving these optimality conditions. The expression for the exchange rates is then computed using a log linear approximation of these optimality conditions. For brevity the log linearization and derivations are relegated to the appendix.

## 5.1 First order conditions

### 5.1.1 first order conditions for production:

$$
\begin{aligned}
V_l^c &= w_l^{vc} \left(\frac{p_l^{vc}}{p_l^c}\right)^{-\sigma^3(c,l)} Q_l^c && (5.1)\\
X_l^c &= w_l^{Xc} \left(\frac{q_l^c}{p_l^c}\right)^{-\sigma^3(c,l)} Q_l^c && (5.2)\\
X_{sl}^c &= w_{sl}^c \left(\frac{q_{sl}^c}{q_l^c}\right)^{-\sigma^2(c,l)} X_l^c && (5.3)\\
X_{sl}^{ic} &= w_{sl}^{ic} \left(\frac{p_s^i}{q(f)_{sl}^c}\right)^{-\sigma_s^1(c,l)} X(f)_{sl}^c && (5.4)\\
X_{sl}^{cc} &= w_{sl}^{cc} \left(\frac{p_s^c}{q_{sl}^c}\right)^{-\sigma_s^{1h}(c,l)} X_{sl}^c && (5.5)\\
X_{sl}^c(f) &= w(f)_{sl}^c \left(\frac{q(f)_{sl}^c}{q_{sl}^c}\right)^{-\sigma_s^{1h}(c,l)} X_{sl}^c && (5.6)
\end{aligned}
$$

here $q_l^c$ and $q_{sl}^c$ are price indices corresponding to $X_l^c$ and $X_{sl}^c$ respectively and are given by:

$$
\begin{aligned}
q_l^c &= \left[\sum_{s=1}^{m} (w_{sl}^c)(q_{sl}^c)^{1-\sigma^2(c,l)}\right]^{\frac{1}{1-\sigma^2(c,l)}} && (5.7)\\
q(f)_{sl}^c &= \left[\sum_{i=1, i\neq c}^{n} (w_{sl}^{ic})(p_s^i)^{1-\sigma_s^1(c,l)}\right]^{\frac{1}{1-\sigma_s^1(c,l)}} && (5.8)
\end{aligned}
$$

$$
q_{sl}^c = \left[(w_{sl}^{cc})(p_{sl}^{cc})^{1-\sigma^{1h}(c,l)} + (w_l^{Xc})(q(f)_{sl}^c)^{1-\sigma^{1h}(c,l)}\right]^{\frac{1}{1-\sigma^{1h}(c,l)}} \qquad (5.9)
$$

and price of gross output is given by:

$$
p_l^c = \left[(w_l^{vc})(p^{vc}_l)^{1-\sigma^3(c,l)} + (w_l^{Xc})(q_l^c)^{1-\sigma^3(c,l)}\right]^{\frac{1}{1-\sigma^3(c,l)}} \qquad (5.10)
$$

Where $p_l^{vc}$ is the price of value added(i.e price of factor of production) of country $c$ sector $l$

### 5.1.2 First order conditions for final consumption:

$$
\begin{aligned}
F_s^{ic} &= \kappa_s^{ic}\left(\frac{p_s^i}{P(f)_s^c}\right)^{-\theta_s^1(c)} F(f)_s^c && (5.11)\\
F_s^{cc} &= \kappa_s^{cc}\left(\frac{p_s^c}{P_s^c}\right)^{-\theta_s^{1h}(c)} F_s^c && (5.12)\\
F(f)_s^c &= \kappa(f)_s^c\left(\frac{P(f)_s^c}{P_s^c}\right)^{-\theta_s^{1h}(c)} F_s^c && (5.13)
\end{aligned}
$$

$$
F_s^c = \kappa_s^c\left(\frac{P_s^c}{P^c}\right)^{-\theta^2(c)} F^c \qquad (5.14)
$$

Here $P_s^c$ and $P^c$ are price indices for sector $s$ good and aggregate good consumed by country $c$, respectively and are given by

$$
\begin{aligned}
P_s^c(f) &= \left[\sum_{i=1, i\neq c}^{n} (\kappa_s^{ic})(p_s^i)^{1-\theta_s^1(c)}\right]^{\frac{1}{1-\theta_s^1(c)}} && (5.15)\\
P_s^c &= \left[(\kappa_s^{cc})(p_s^{cc})^{1-\theta_s^{1h}(c)} + (\kappa(f)_l^c)(P(f)_s^c)^{1-\theta_s^{1h}(c)}\right]^{\frac{1}{1-\theta_s^{1h}(c)}} && (5.16)
\end{aligned}
$$

$$
P^c = \left[\sum_{s=1}^{m} (\kappa_s^c)(P_s^c)^{1-\theta^2(c)}\right]^{\frac{1}{1-\theta^2(c)}} \qquad (5.17)
$$

Let $[A]_{nmXnm}$ be the input-output coefficient matrix at the country-sector level, i.e the $(i,j)^{th}$block which has dimension $mXm$ is given by

$$
[A]_{mXm}^{ij} = \begin{pmatrix} a_{11}^{ij} & a_{12}^{ij} & .. & a_{1m}^{ij} \\ a_{21}^{ij} & a_{22}^{ij} & .. & a_{2m}^{ij} \\ \vdots & \vdots & \vdots & \vdots \\ a_{m1}^{ij} & a_{m2}^{ij} & .. & a_{mm}^{ij} \end{pmatrix} \qquad (5.18)
$$

where $a_{sl}^{ij}$ denotes the output of $(i,s)$ used in the production of one unit of $(j,l)$, i.e

$$
a_{sl}^{ij} = \frac{p_s^i X_{sl}^{ij}}{p_l^j Q_l^j} \qquad (5.19)
$$

Let $[B]_{nmXnm}$ be the corresponding total requirement matrix given by

$$[B]_{nmXnm} \quad = \quad (I_{nm} - [A])^{-1} \tag{5.20}$$

Also, define the matrix $[D_Q]_{nmxnm}$ to be a diagonal matrix with the $(cl)^{th}$ diagonal entry given by $\frac{1}{p_l^c Q_l^c}$

### 5.2 Demand for value added as a function of price of value added:(GVC-REER)

The appendix shows that the demand for value added can be written as

$$vec\left(\hat{V}_l^c\right) = W_V vec\left(\hat{p}_l^{vc}\right) + W_{FV} vec\left(\hat{F}^c\right) \tag{5.21}$$

Here $\left(vec\left(\hat{V}_l^c\right)\right)_{nmX1}$ is the vector of changes in value added stacked across all countries and sectors, and $W_V$ and $W_F$ are $nm$ by $nm$ matrices derived in the appendix. Putting the change in final demand $vec\left(\hat{F}^c\right)$ to zero, the $nm$ by $nm$ matrix premultiplying $vec\left(\hat{p}_l^{vc}\right)$ can be interpreted as a matrix of weights for the real effective exchange rate, as it measures how the demand for value added originating in a country-sector changes when price of value added changes in any other entity.

**Interpretation in the case with constant elasticity:**

Under the assumption that all elasticities(both in production and consumption) are the same, we can interpret the country-sector level weights purely in terms of value added trade flows. Suppose the common elasticity is $\eta$ . Without loss of generality we can assume $\eta$ to be unity since it factors out. Then the weighting matrix $W$ can be written as(see appendix D for proof):

$$W_V = -I_{nm} + M_1 M_2 \tag{5.22}$$

The matrix $M_1$ is an $nm$ by $n$ matrix with each row corresponding to a unique production entity. Along this row, the $n$ columns give the value added created by the production entity that is finally absorbed by each country. As an example, the entry corresponding to row $(i, l)$ and column $j$ gives the value added created by production entity $(i, l)$ that is eventually absorbed in country $j$ as a fraction of total value added created by the production entity $(i, l)$. Entries in this matrix can thus be interpreted as export shares in value added terms.The corresponding mathematical expression is[12]

$$M_1((i,l),j) = \frac{v_l^i \sum_{c=1}^n \sum_{s=1}^m b_{ls}^{ic}\left(p_s^c F_s^{cj}\right)}{p_l^{vi} V_l^i} \tag{5.23}$$

[12]the raw expression form the matrix $M_1$ is $\frac{\sum_{c=1}^n \sum_{s=1}^m b_{ls}^{ic} p_s^c F_s^{cj}}{p_l^i Q_l^i}$. Multiplying and dividing by $v_l^i = \frac{p_l^{vi} V_l^i}{p_l^i Q_l^i}$ yields the expression below.

Where $v_l^i = \frac{p_l^{vi} V_l^i}{p_l^i Q_l^i}$ .For later, it is convenient to write this expression compactly as:

$$M_1((i,l),j) = \frac{p_l^{vi} V_l^{ij}}{p_l^{vi} V_l^i} \tag{5.24}$$

where $p_l^{vi} V_l^{ij}$ is the value added created by production entity $(i,l)$ that is finally absorbed in country $j$.

Matrix $M_2$ is an $n$ by $nm$ matrix with each column corresponding to a unique production entity and each row containing the value added created by the entity corresponding to the column that is absorbed in each country, as a fraction of the total final demand in that country. As an example, the entry corresponding to column $(i,l)$ and row $j$ gives the value added created by production entity $(i,l)$ that is ultimately absorbed in country $j$ as a fraction of total final demand of country $j$. The corresponding mathematical expression is :

$$M_2(j,(i,l)) = \frac{v_l^i \sum_{c=1}^{n} \sum_{s=1}^{m} b_{ls}^{ic} \left(p_s^c F_s^{cj}\right)}{P^j F^j} \tag{5.25}$$

As above, it turns out to be more convenient to rewrite the above expression in short-hand notation as follows:

$$M_2(j,(i,l)) = \frac{p_l^{vi} V_l^{ij}}{P^j F^j} \tag{5.26}$$

Using the generic terms from (5.24) and (5.26)we can write the weight assignment by country sector $(h,l)$ to country-sector $(c,s)$ where $(h,l) \neq (c,s)$ as follows:

$$w_{ls}^{hc} = \sum_{k=1}^{n} \left[ \frac{\left(p_l^{vh} V_l^{hk}\right)\left(p_s^{vc} V_s^{ck}\right)}{\left(p_l^{vh} V_l^{h}\right)\left(P^K F^k\right)} \right], (h,l) \neq (c,s) \tag{5.27}$$

Where we use lower case $w$ to denote constant elasticity weights. This is a generalized form of equation 3.34 which was derived in the context of a simplified model and the intuition is similar. In particular, the weight assigned by country sector $(h,l)$ to country-sector $(c,s)$ where $(h,l) \neq (c,s)$ is a weighted sum of the value added created by country-sector $(c,s)$ and absorbed by each of the countries $k(=1,..,n)$, where the weights are given by the value added created by $(h,l)$ that is absorbed in the same country $k$. This captures both mutual and third country competition, because the weight is high if both $\left(p_l^{vh} V_l^{hk}\right)$ and $\left(p_s^{vc} V_s^{ck}\right)$ are high, which happens when both $(h,l)$ and $(c,s)$ have a high share of value added exports to country $k$.

### Relaxing the constant elasticity Assumption

Since a full analytical characterization of the role played by different elasticities is infeasible given the complex nature of the model, we will illustrate the role played by the different elasticities in a series of illustrative examples in section 7. However, in order to provide some intuition the following proposition shows the effect of a small change in elasticity on the REER weights in the neighborhood of the constant elasticity equilibrium.

**Proposition 5.1.** *Suppose all production and consumption elasticities are constant and equal to $\sigma$ and $\theta$ respectively*[13]*. Then starting at the constant elasticity equilibrium, the effect of a change in elasticity on the weight assigned by entity $(h,l)$ to entity $(c,s)$ is given by:*

$$\frac{\partial w_{ls}^{hc}}{\partial \theta} = w_{ls}^{hc} - \frac{v_l^h v_s^c \sum_{c_1=1}^{n} \sum_{c_2=1}^{n} \sum_{k=1}^{m} b_{lk}^{hc_1} b_{sk}^{cc_1} (p_k^{c_1} F_k^{c_1c_2})}{p_l^{vh} V_l^h}, (h,l) \neq (c,s) \tag{5.28}$$

*Proof(sketch): See appendix D.*

(5.28) shows that an increase in elasticity of substitution of consumption holding everything else constant(including the production elasticity) has two opposing effects on the weight assigned by home entity $(h,l)$ to the foreign entity $(c,s)$. The two terms correspond to the expenditure switching and complementarity effect illustrated earlier with the stylized model. In particular, the first effect (expenditure switching) is positive and is given by the constant elasticity weight $w_{ls}^{hc}$, which recall is always positive in the constant elasticity case. In addition, there is the countervailing complementarity effect which comes from the second term on the right hand side. This term is high when the products $b_{lk}^{hc_1} b_{sk}^{cc_1}$ are high for various entities indexed by $(c_1,k)$, which in turn happens if the outputs of the two entities are used together in production (i,e entities such as $(c_{1,}k)$ which use the output of $(c,s)$ as an input, also uses the output of $(h,l)$ as an input ) .

Intuitively, when price of $(c,s)$ decreases, its quantity demanded increases. This effect is greater the greater is the elasticity of substitution between goods($\theta$). Moreover, an increase in demand for $(c,s)$ will end up increasing the output of $(h,l)$ if it is highly complementary with $(c,s)$.

The corresponding expression for the two GVC sectors in section 3 is the following:

$$\frac{\partial w_{21}^{CJ}}{\partial \theta}|_{\theta=\sigma=1} = w_{21}^{CJ} - \left(\frac{p_1^J X_{12}^{JC}}{p_2^C Q_2^C}\right) \tag{5.29}$$

We will elaborate more on these mechanisms by the use of stylized examples.

### 5.3 Gross competitiveness

We also derive the demand for aggregate output as a function of price of value added(this is the analogous to the "goods" REER measure proposed in Bayoumi et al. (2013).See appendix for steps of proof.)

$$vec\left(\hat{Q}_l^c\right) = W_Q vec\left(\hat{p_l^{vc}}\right) + W_{FQ} vec\left(\hat{F}^c\right) \tag{5.30}$$

Here $W_Q$ is an $nm$ by $nm$ weighting matrix derived in appendix C. Again putting the change in final demand $vec\left(\hat{F}^c\right)$ to be zero, the $nm$ by $nm$ matrix premultiplying $vec\left(\hat{p}_l^{vc}\right)$can be interpreted as a matrix of weights for the real effective exchange rate with regard to gross competitiveness, i.e it measures how the demand for output of a country-sector changes with changes in prices of other country-sectors .This is in contrast to the first measure defined above, which looks at change in demand for value added. (As is shown in 6.1,the two are the same in the special case where gross

[13]This is equivalent to assumption (A2) in section 6

output is the same as value added, as is assumed in most of the REER measures including IMF, FED and BIS).

Bayoumi et al. (2013) define the "Goods REER" as a measure of competitiveness of a country's gross output.

$$\triangle log(GoodsREER^i) = \sum_{j \neq i} W_{imf}^{ij} \left( \hat{p^i} - \hat{p^j} \right) \tag{5.31}$$

where $W_{imf}^{ij}$ are the IMF weights. The analogous expression in the context of the framework proposed in this paper(and BJ) can be obtained by setting $m = 1$, $\hat{F}^c = 0 \forall c$ and taking the $i^{th}$ row of the following expression (5.30)

However in general (5.30) and(5.31) are not the same since as shown in the next section, the IMF weights coincide with the weights obtained in the present framework only under fairly restrictive conditions.

The idea behind the Integrated real exchange rate (IRER) measure proposed in Thorbecke (2011)[14] is also similar to Bayoumi et al. (2013) with the analogous expression in the present model given by (5.30) . However they too use the IMF weighting scheme, which means that their measure only coincides with the measure proposed in the present paper under fairly restrictive assumptions, which we discuss in detail below.

## 6 Relationship to other REER weighting matrices in the literature

This section shows the link between the two REER measures proposed in the previous section and some common REER measures in the literature with particular emphasis on whether and under what conditions the different measures in the literature can be recovered from the more general measures proposed here. We start by listing the various REER measures that are compared in this section. With some abuse of terminology, we refer to the weighting matrix by the same name as the name given to the associated REER measure by the authors(irrespective of the price index used).

1. GVC-REER and QREER are as defined in the previous section
2. IOREER(BJ):inout-output real effective exchange rates as defined by BJ in their model
3. VAREER(BJ) : value added real effective exchange rates as defined by BJ in their model. It is a special case of IOREER with all elasticities(production and consumption) set equal to each other.
4. GOODS-REER: as defined by Bayoumi et al. (2013) .
5. IRER: Integrated real exchange rate as proposed by Thorbecke (2011), but without lagged dependence

[14] Thorbecke (2011) define their IRER measure with lag dependence. However, I ignore this feature and refer to IRER as the Thorbecke (2011) measure without lag dependence in order to make the measure comparable to the other measures discussed here and in the following sections

6. IMF-REER

   As shown in Bayoumi et al. (2005) the weight assignment by country $i$ to country $j$ in the IMF's REER measure is given by:

$$W^{ij}_{imf} = (\alpha_m + \alpha_s)W^{ij}_{imfm} + (\alpha_c)W^{ij}_{imfc} + (\alpha_T)W^{ij}_{imfT} \tag{6.1}$$

   where $\alpha_m$,$\alpha_s$, $\alpha_c$, $and \alpha_T$ are shares of manufactures, (non-tourism) services, commodities, and tourism in overall trade.

**Assumptions:**

(A1)$m = 1$. i.e, each country has only one sector

(A2)Elasticities are the same across consumption and production entities

1. $\sigma^1_s(c, l) = \sigma_1 \forall s, c, l$
2. $\sigma^{1h}_s(c, l) = \sigma_1 \forall s, c, l$
3. $\sigma^2(c, l) = \sigma_2 \forall c, l$
4. $\sigma^3(c, l) = \sigma_3 \forall c, l$
5. $\theta^1_s(c) = \theta_1 \forall s, c$
6. $\theta^{1h}_s(c) = \theta_1 \forall s, c$
7. $\theta^2(c) = \theta_2 \forall c$

(A3)all elasticities(in both consumption and production) are the same

- $\sigma_1 = \sigma_2 = \sigma_3 = \theta_1 = \theta_2$

(A4)No intermediates in production and only final goods are traded.

(A5)All trade flows comprise of trade in manufacturers and non tourism services, i.e$\alpha_c = \alpha_T = 0$

**Proposition 6.1.**

1. *Under (A1) and (A2):*

   *GVC-REER =IOREER*

2. *Under (A1) ,(A2) and (A3):*

   *GVC-REER=IOREER=VAREER*

3. *Under (A1), (A2),(A3) and (A4):*

   *GVC-REER=Q-REER=IOREER=VAREER*

4. *Under (A1), (A2), (A3),(A4) and (A5)*

   *IMF-REER = GVC-REER=Q-REER=IOREER=VAREER=GOODSREER=IRER*[15]

15

proof: see appendix.

In general , the GVC-REER measure(or any of the measures defined by BJ) does not reduce to the common measures currently is use, such as those of the FED(Loretan (2005)), BIS(see Klau et al. (2008)) or IMF(Bayoumi et al. (2005)). Certain parallels can however be drawn between the IMF measure and the GVC-REER (and BJ) measure as shown in part 4 of proposition 6.1.

### Building Country-level REER From Ground Up:

### Value added weights at the country level

This section provides a method to aggregate the country-sector level weights derived above and defines country level weights analogous to the ones commonly discussed in the literature. We show that the aggregated weights so derived in general do not correspond to any of the ones proposed in the literature except in knife edge cases. This is attributable to the fact that our measure exploits inter-sectoral linkages between countries to provide a more comprehensive measure of competitiveness then what can be obtained by using just country level data.

To derive the expression for country-level value added weights, we start with the following decomposition of the nominal GDP of country $c$ into its different sectoral components:

$$p^{vc}V^{c} = \sum_{l=1}^{m} p_l^{vc}V_l^{c} \tag{6.2}$$

log linearizing this equation we get:

$$\hat{p}^{vc} + \hat{V}^{c} = \sum_{l=1}^{m} \left(\frac{p_l^{vc}V_l^{c}}{p^{vc}V^{c}}\right)\left[\hat{p}_l^{vc} + \hat{V}_l^{c}\right] \tag{6.3}$$

Stacking the $n$ equations in (6.3) we can write the system in matrix notation as:

$$vec\left(\hat{p}^{vc}\right)_{nX1} + vec\left(\hat{V}^{c}\right)_{nX1} = R_V\left[vec\left(\hat{p}_l^{vc}\right)_{nmX1} + vec\left(\hat{V}_l^{c}\right)_{nmX1}\right] \tag{6.4}$$

where

$$(R_V)_{nXnm} = \begin{pmatrix} S_1^V & 0_m^{'} & .. & 0_m^{'} \\ 0_m^{'} & S_2^V & & : \\ : & & .. & : \\ 0_m^{'} & 0_m^{'} & .. & S_n^V \end{pmatrix} \tag{6.5}$$

(a) *As mentioned before, note that the IMF uses CPI to compute REER, but in this section we will use IMF-REER to denote total effective exchange rates computed with IMF weights but using the GDP deflator, to make the measure comparable with other measures proposed here and in BJ* and $\left(S_i^V\right)_{1Xm} = \left(\frac{p_1^{vi}V_1^i}{p^{vi}V^i}, \frac{p_2^{vi}V_2^i}{p^{vi}V^i}, .., \frac{p_m^{vi}V_m^i}{p^{vi}V^i}\right)$ and $0_m$ is an $m$ by 1 matrix of zeros. By definition the change in the GDP deflator is the weighted sum of change in ints components and hence (6.4) reduces to

$$vec\left(\hat{V}^c\right)_{nX1} = R_V\left[vec\left(\hat{V}_l^c\right)_{nmX1}\right] \tag{6.6}$$

using (5.21) in (6.6) and imposing $vec\left(\hat{F}^c\right) = 0$ as before we get:

$$vec\left(\hat{V}_l^c\right) = R_V W_V vec\left(\hat{p}_l^{vc}\right) \tag{6.7}$$

**Defining the two measures of country level value added exchange rates:**

When sector level price indices are available, (6.7) defines the change in the country level GVC-REER(C), i.e

$$\triangle log(GVC - REER(C)) = W_V(C) vec\left(\hat{p}_l^{vc}\right) \tag{6.8}$$

Where the $n$ by $nm$ matrix $W_V(C) = R_V W_V$ is the weighting matrix which can be interpreted as follows: the weight assigned by country $i$ to country $j$ sector $l$ is itself a weighted sum of the weights assigned by each sector of country $i$ to $(j, l)$, with the weights being proportional to the country $i$ sector's share of value added as a fraction of total value added by country $i$

$$W_V(C)_l^{ij} = \sum_{s=1}^{m}\left(\frac{p_s^{vi}V_s^i}{p^{vi}V^i}\right)\left(W_V\right)_{sl}^{ij} \tag{6.9}$$

Sector level prices are often not available for many countries. In such cases we need a further approximation. In particular, we need to assume a mapping between sector level prices and GDP deflator i.e between $\hat{p}^{vc}$ and $\{\hat{p}_l^{vc}\}_{l=1}^{M}$. We make the relatively uninformed assumption that all sectoral level prices change in the same proportion as the aggregate GDP,i.e we make the following assumption[16]

Assumption (AP):

$$\hat{p^{vj}} = \hat{p_l^{vj}} \forall l \forall j \tag{6.10}$$

Using this assumption we can write the relationship between the vector of GDP deflators and sector level price indices as:

$$vec\left(\hat{p_l^{vj}}\right) = R_g vec\left(\hat{p^{vj}}\right) \tag{6.11}$$

[16]note that in a world with price rigidity and producer currency pricing this assumption is satisfied automatically.

where $R_g = I_n \otimes 1_m$. This constitutes our second measure of country level value added exchange rate, GVC-REER(CG)

$$\triangle log(GVC - REER(CG)) = W_V(CG) vec\left(\hat{p}^{vc}\right) \tag{6.12}$$

where $W_V(CG) = R_V W_V R_g$ is an $n$ by $n$ matrix of weights

**Link to other measures in the literature:**

Our second measure of country level exchange rates which uses only the GDP deflator (GVC-REER(CG)) has an $n$ by $n$ weighting matrix as all other measures in the literature and we can hence make a comparison with them.

Given the country-sector level weights($W_V$), the country level weights ($W_V(CG)$) have an intuitive interpretation. The weight assigned by country $i$ to country $j$ is a weighted sum of the weights assigned by each sector of country $i$ to each sector of country $j$, with the weights being proportional to the home sector's share of value added as a fraction of total home value added.

$$W_V(CG)^{ij} = \sum_{s=1}^{m} \left(\frac{p_s^{vi} V_s^i}{p^{vi} V^i}\right) \left(\sum_{k=1}^{m} (W_V)_{sk}^{ij}\right) \tag{6.13}$$

These country level weights defined here are different from others proposed in the literature in several respects as will be discussed in the following sections. The closest to our measure is the one by Bems and Johnson (2012) who also take into account the input-output linkages in their measure and define weights in terms of value added, but do not exploit sector level linkages across countries. Because of the greater information used in our measure, it is in general different from their VAREER and IOREER, even under the assumption of all elasticities being the same. The following proposition shows that even under the constant elasticity assumption, GVC-REER(CG) and VAREER differ from each other except in special cases.[17]

**Condition 5.1**

$$v^i \sum_{c=1}^{n} b^{ic} F^{cj} = \sum_{l=1}^{m} v_s^i \sum_{c=1}^{n} \sum_{s=1}^{m} b_{ls}^{ic} F_s^{cj} \forall i, j \tag{6.14}$$

or, in stacked matrix notation:

$$diag[V^c]_{nXn} [B^c]_{nXn} [F^C]_{nXn} = (M_V)_{nXnm} diag[V_l^c]_{nmXnm} [B]_{nmXnm} [F_l^c]_{nmXn} \tag{6.15}$$

$M_V = I_n \otimes 1_m'$

[17]We make the comparison with Bems and Johnson (2012) because it is the closest to our framework.Although they do not allow for sector level linkages in their theoretical model, in the empirical implementation of their model they do use sector level linkages from Johnson and Noguera (2012) by making some simplifications.(However their simplification only works in the constant elasticity case.).

**Proposition 6.2.**

The country level weights ($W_V(CG)$) defined above reduces to VAREER (and IOREER) weights defined in Bems and Johnson (2012) if either of the 2 conditions below are satisfied.

1. (A2),(A3) and condition 5.1
2. (A3),(A4) and $\theta_1 = \theta_1^h = \theta_2$

The proof is given in appendix D. The first part of the proposition shows that outside of the knife-edge case in which the above condition is satisfied, the GVC-REER(CG) weights which exploit inter-sectoral linkages between countries will dominate the VAREER measure even under the constant elasticity assumption(they would of course differ if elasticities are different even if the first condition in the proposition is satisfied). Intuitively, condition (6.14) is satisfied if different sectors within a country are "symmetric" vis-a vis their input-output linkages with the rest of the world, for in that case aggregation across sectors within a country will be a closer approximation to the behavior of each individual sector. The next section will provide an example to illustrate the role played by the condition in aggregating weights at the country level.

The second part of the proposition shows that differences between GVC-REER and VAREER vanish when there is no trade in intermediates. This shows that if there is no trade in intermediates, then aggregating trade flows across sectors within a country does not lead to any loss of information as far as computation of real effective exchange rate is concerned. Intuitively, if all production by all entities involves only own value added and no intermediates, then there is no asymmetry between sectors within a country with regard to the foreign value added embodied in their output (which is zero in all cases) and hence aggregation does not lead to any loss of relevant information.

### Goods exchange rate at country level

Following a similar procedure to the one used for task exchange rates, we can define weights and exchange rates for goods at the country level:

$$\triangle log(QREER(C)) = W_Q(C)vec\left(\hat{p}_l^{vc}\right) \tag{6.16}$$

$$\triangle log(QREER(CG)) = W_Q(CG)vec\left(\hat{p}^{vc}\right) \tag{6.17}$$

where $W_V(C) = R_V W_V$ , $W_V(CG) = R_V W_V R_g$ and $R_Q$is defined analogous to $R_V$ as follows

$$R_Q = \begin{pmatrix} S_1^Q & 0_m' & .. & 0_m' \\ 0_m' & S_2^Q & & : \\ : & & .. & : \\ 0_m' & 0_m' & .. & S_n^Q \end{pmatrix} \tag{6.18}$$

*Table 4 – Input output table for 7.1*

| | J | C | U | J final | C final | U final | Total output |
|---|---|---|---|---|---|---|---|
| J | 0 | 1 | 0 | 1 | 0 | 0 | 2 |
| C | 0 | 0 | 0 | 0.1 | 0.1 | 1 | 1.2 |
| U | 0 | 0 | 0 | 0 | 0 | 1 | 1 |
| Value added | 2 | 0.2 | 1 | | | | |
| Total output | 2 | 1.2 | 1 | | | | |

and $\left(S_i^Q\right)_{1Xm} = \left(\frac{p_1^i Q_1^i}{p^i Q^i}, \frac{p_2^i Q_2^i}{p^i Q^i}, .., \frac{p_m^i Q_m^i}{p^i Q^i}\right)$[18]

# 7 Illustrative Examples

This section presents some examples to illustrate the different aspects of the weighting matrices derived above and compare them to other measures proposed in the the literature.

**Example 7.1.** Three country world with limited trade in intermediate inputs:

Consider the following 3 country one sector example where the input output linkages are restricted to just one non zero entry. Country C imports intermediates from country J, puts in own value added and sells the output to all the three countries as final output. Table 4 displays the associated input-output table.

In this simplified example only 2 elasticities are relevant, namely $\sigma_3$ (elasticity of substitution between C's value added and intermediate input from J in C's gross output) and $\theta_1$ (Elasticity of substitution between final goods in the final consumption basket of all countries. For simplicity, this elasticity is assumed to be common across countries).

Consider the weight assigned by country C to country J, $W_{CJ}$, which measures the change in demand for value added by C when price of value added by J changes. A decrease in $p_J^v$ affects the demand for C's value added via 2 channels. Firstly, with regard to final goods consumption, a decrease in $p_J^v$ leads to a shift towards J's value added(and goods containing value added by J, namely the gross output of C) in the final goods consumption bundle of all countries. The strength of this effect depends on $\theta_1$. A higher $\theta_1$ means that goods are more substitutable in the final goods consumption bundle of countries and hence the shift towards J's value added will be more pronounced when its price decreases.

Secondly,with regard to intermediate goods and production mix, a decrease in the price of J's value added leads to a shift towards J's value added and a shift away from C's value added in the production function of C. The strength of this effect depends on $\sigma_3$. The higher is this elasticity, the higher is the shift towards J's value added in C's production(at the expense of C's own value added) and hence higher is the fall in demand for C's value added.

As a result of these two effects $W_{CJ}$ is an increasing function of $\sigma_3$ and a decreasing function of $\theta_1$, as was pointed out in proposition (5.1). Interestingly, when $\theta_1$ is sufficiently high and $\sigma_3$ is

[18]here $p^i$ can be interpreted as the price index of gross output produced by country $i$.

*Table 5 – Comparison of weights under different measures for example 7.1*

| | GVC-REER=IOREER (PWW,BJ) | VAREER (BJ) | Q-REER (PWW) | GOODS-REER (BST) | IMF Weights (BLS) |
|---|---|---|---|---|---|
| $W_{JC}$ | **-0.04** | **0.19** | **-0.04** | **1.0** | **1.00** |
| $W_{JU}$ | 1.04 | 0.80 | 1.04 | 0 | 0 |
| $W_{CJ}$ | **-0.25** | **0.54** | **-4.07** | **-3.40** | **0.26** |
| $W_{CU}$ | 1.25 | 0.45 | 5.07 | 4.40 | 0.73 |
| $W_{UJ}$ | 0.83 | 0.83 | 0.83 | 0.83 | 0 |
| $W_{UC}$ | 0.16 | 0.16 | 0.16 | 0.16 | 1 |

| key | |
|---|---|
| PWW | Patel, Wang, Wei(2013) |
| BJ | Bems and Johnson (2012) |
| BST | Bayoumi et al. (2013) |
| BLS | Bayoumi et al. (2005) |

sufficiently low, $W_{CJ}$ may indeed be negative, something that the IMF weights or the Value aded weights in BJ do not allow for.

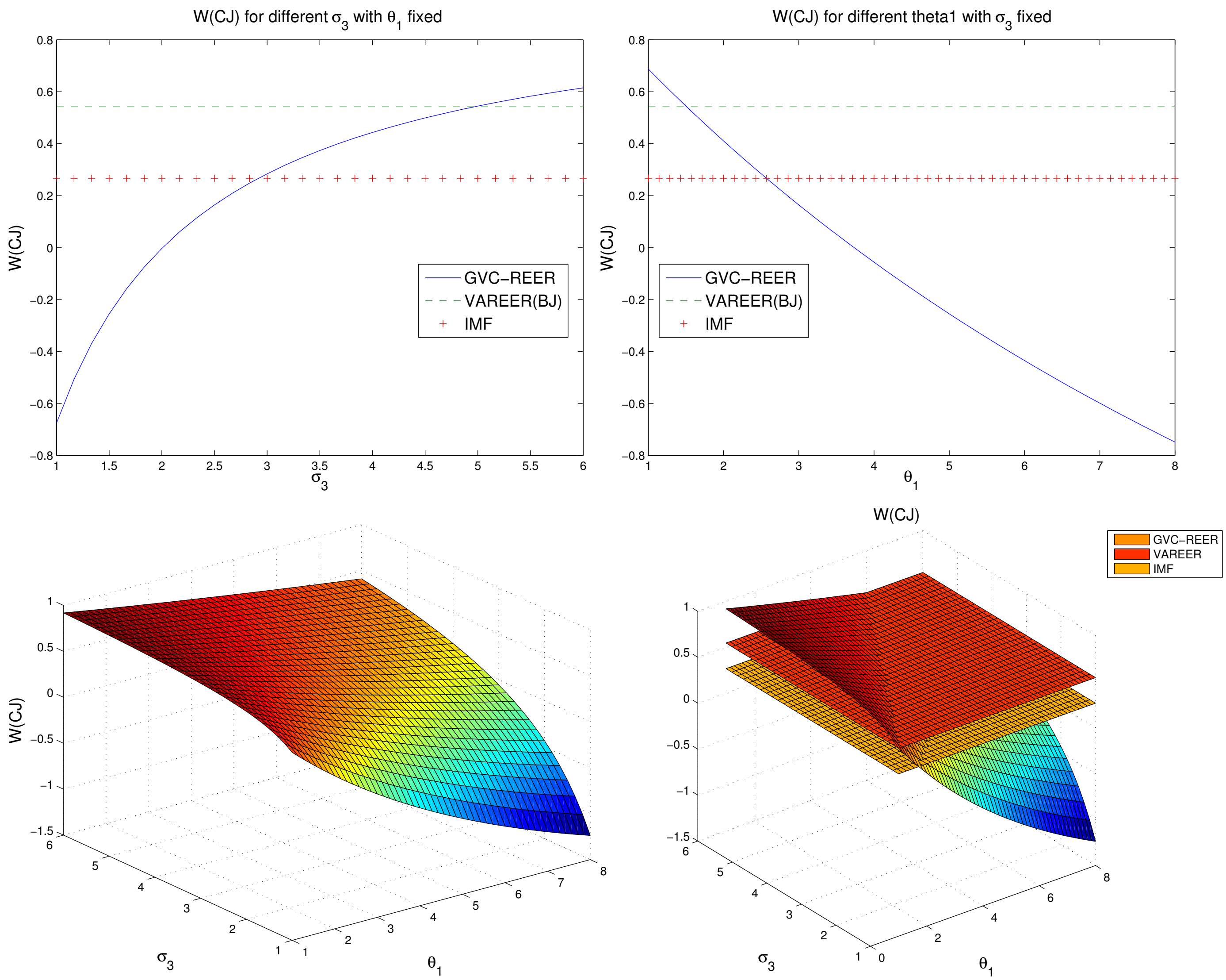
Table 5 presents weights based on different schemes for this example when $\sigma_3 = 1.5$, $\theta_1 = 5$. (as is done by the IMF and others, weights are normalized so that own weight is -1 and is not reported) .

Several aspects of the differences in the weighting schemes are noteworthy. Firstly, note that there are no negative weights in the IMF and the VAREER weighting matrix. In fact it can be easily shown that these weighting schemes are not flexible enough to accommodate negative weights under any circumstances. Next, note from column 1 that $W_{JC}$ and $W_{CJ}$ are negative in the GVC-REER measure. As discussed above, this is a consequence of the input output structure and a combination of a relatively high $\theta_1$ (=5) and low $\sigma_3$ (=1.5). Column 3 illustrates that as far as gross output is concerned, the magnitude of the negative weight assigned by country C to country J is much larger. This is because only the first effect discussed above(i.e shift in final demand) affects gross output, whereas the second effect(shift towards intermediate composition) does not affect the gross output measure.

The constant elasticity assumption is overly restrictive can also be noted from the observation that the VAREER(BJ) weight which does take into account trade in intermediates does worse than the IMF weight which ignores it,although both have the wrong sign.

Column 4 shows that the Goods-REER measure of Bayoumi et al. (2013) falls somewhere in between the GVC-REER and the Q-REER measures(columns 1 and 3) so that it measures neither gross output competitiveness nor value added competitiveness but some arbitrary combination of the two. Although the aim in Bayoumi et al. (2013) is to capture gross competitiveness, they fall short of doing so because their measure uses the IMF weighting scheme which does not account for trade in intermediates. This aspect is further illustrated by the fact that the GOODS-REER measure(which in turn inherits this property from the IMF measure) assigns a value of 0 to $W_{JU}$ because there is no direct trade between J and U. However, J's value added does reach U via C and so the correct weighting matrix must have $W_{JU} \neq 0$.

***Figure 7.1***




Lastly, note form the last two rows of table 5 that the weights assigned by country U to the remaining two countries are the same in all the measures except IMF. This is a consequence of the fact that the US trades in only final goods and all its production comprises entirely of its own value added.

Figure 7.1 shows how the weight assigned by $C$ to $J$ changes with the elasticities. The top left figure plots $W_{CJ}$ for three measure(GVC-REER, VAREER(BJ) and IMF) for different values of $\sigma_3$ with $\theta_1$ fixed at 1.5. Top right picture plots the same weights for different values of $\theta_1$ with $\sigma_3$ fixed at 5.Bottom left figure shows a 3D plot of $W_{CJ}$ for the GVC-REER measure for different values of $\sigma_3$ and $\theta_1$ while the bottom right augments this graph by adding a surface each for VAREER and IMF weights.

**Example 7.2.** A three country 2 sector world:

This example is an extension of example 7.1 which will be used to illustrate the role of aggregation

***Table 6** – IO table and elasticities for example 7.2*

| | | J | | C | | U | | JFinal | CFinal | Ufinal | total output |
|---|---|---|---|---|---|---|---|---|---|---|---|
| | | J1 | J2 | C1 | C2 | U1 | U2 | | | | |
| J | J1 | 0 | 0 | 0 | 2 | 0 | 0 | 1 | 0 | 0 | 3 |
| | J2 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 1 |
| C | C1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 2 | 0 | 2 |
| | C2 | 0 | 0 | 0 | 0 | 0 | 0 | 0.5 | 0.5 | 2.5 | 3.5 |
| U | U1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 2 | 2 |
| | U2 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 1 |
| VA | | 3 | 1 | 2 | 1.5 | 2 | 1 | | | | |
| total output | | 3 | 1 | 2 | 3.5 | 2 | 1 | | | | |

Elasticities:
$\sigma_1 = 2$, $\sigma_2 = 2$ ,$\sigma_3 = 2$, $\theta_1 = 5$, $\theta_2 = 5$

***Table 7** – GVC-REER weights at country-sector level(raw) for example7.2*

| | $J_1$ | $J_2$ | $C_1$ | $C_2$ | $U_1$ | $U_2$ |
|---|---|---|---|---|---|---|
| $J_1$ | -2.36 | 0.86 | 0.38 | -0.169 | 0.86 | 0.43 |
| $J_2$ | 2.57 | -3 | 0 | 0.43 | 0 | 0 |
| $C_1$ | 0.571 | 0 | -1 | 0.4 | 0 | 0 |
| $C_2$ | -0.338 | 0.28 | 0.571 | -2.4 | 1.3 | 0.65 |
| $U_1$ | 1.30 | 0 | 0 | 0.97 | -3.18 | 0.90 |
| $U_2$ | 1.30 | 0 | 0 | 0.97 | 1.81 | -4.0 |

and comment on the practice of normalization of weights.In each country we now have two distinct production sectors. The main object of attention will be country C which is assumed to have two sectors that are different with regard to their production function. Sector 1(C1) uses its own value added and produces only final goods absorbed at home. Sector 2 (C2) operates downstream and uses intermediates from a different country and produces only a final good. The elasticities and input-output table is given in table 6below.

Table 6 displays the full 6X6 country-sector level weighting matrix (which is not normalized to illustrate the mechanics of aggregation based on equation (6.13) later on). In line with the observations made in example 7.1, C2 and J1 are found to attach negative weights to each other. Table 8 shows the weights at the country level. As can be seen from the country level weighting matrix, the negative weights disappear when sectors are aggregated by country.To understand the intuition behind this, table 9 shows a 2 by 2 sub matrix from table 8 which contains weights assigned by sectors in C and J to each other, along with the value added and gross output shares of the respective sectors, derived from table 6.

Note that the aggregate country weight $W_{CJ}$ is a combination of the 4 sector level weights (in line with (6.13)). Since in value added terms the size of $C_1$ and $J_1$ is higher compared to $C_2$ and

***Table 8** – GVC-REER weights at country level for example7.2*

| | J | C | U |
|---|---|---|---|
| J | -1.24 | 0.26 | 0.97 |
| C | 0.30 | -1.14 | 0.83 |
| U | 1.30 | 0.97 | -2.27 |

***Table 9*** – *2X2 weighting matrix for example 7.2*

| | $J_1$ | $J_2$ | value added share | gross output share |
|---|---|---|---|---|
| $C_1$ | 0.571 | 0 | **0.57** | 0.36 |
| $C_2$ | -0.338 | 0.28 | 0.43 | **0.64** |
| value added share | **0.75** | 0.25 | | |
| gross output share | **0.75** | 0.25 | | |

***Table 10*** – *Summary of different weights at country-country(normalized) example 7.2*

| | GVC-REER (PWW) | IO-REER (BJ) | VAREER (BJ) | Q-REER (PWW) | GOODS-REER (BST) | IMF Weights (BLS) |
|---|---|---|---|---|---|---|
| $W_{JC}$ | 0.21 | 0.41 | 0.69 | 0.21 | 1 | 1 |
| $W_{JU}$ | 0.78 | 0.58 | 0.30 | 0.79 | 0 | 0 |
| $W_{CJ}$ | **0.26** | **0.29** | **0.56** | **-0.80** | **0.25** | **0.53** |
| $W_{CU}$ | 0.73 | 0.71 | 0.44 | 1.80 | 0.75 | 0.47 |
| $W_{UJ}$ | 0.57 | 0.36 | 0.36 | 0.57 | 0.36 | 0 |
| $W_{UC}$ | **0.42** | **0.63** | 0.63 | -0.42 | 0.63 | 1 |

$J_2$ respectively, the country level GVC-REER weight is likely to be dominated by $W_{J_1C_1}$, which is positive(0.57)[19]

Table 10 provides a summary of country-level weights assigned by different measures in the literature alongside the weighting matrices proposed in the preceding sections(now the weights are normalized to make the comparison easier). Unlike the case with GVC-REER weights, note that Q-REER which focuses on gross output does end up assigning negative weights (see row 3 containing $W_{CJ}$) even at the country level, the intuition for which is again clear form noting that in terms of gross output shares the dominant sectors are $J_1$ and $C_2$ so the country level weight $W_{CJ}$ is likely to be dominated by the weight assigned by $C_2$ to $J_1$, which is negative(-0.338) However as in the previous example since the GOODS-REER measure uses IMF weights, it does not completely account for the input-output linkages and hence offers a different weight from our Q-REER measure.

**More on aggregation:**

To focus exclusively on the role of aggregation we now set all elasticities equal to one.Table 11 shows the country-level input output table derived from the general country- sector level IO table. The country level IO table is what is used to compute weights when inter-sectoral flows are ignored, as is common in the literature. Table12 gives the weights under the two difference schemes, GVC-REER and the corresponding measure derived from a country level IO table, which we call A-REER(for aggregate).Note that in theory the A-REER measure is equivalent to VAREER in Bems and Johnson (2012).

The difference between the 2 weighting matrices can be illustrated using $W_{CU}$ as an example. $W_{CU}$ is an increasing function of value added by C that is ultimately absorbed in country $U$.By exploiting sector level information the GVC-REER measure recognizes that all the exports from

[19]the number 0.3 can be recovered from (6.13) as follows
$W^{CJ} = 0.57(W^{CJ}_{11} + W^{CJ}_{12}) + 0.43(W^{CJ}_{21} + W^{CJ}_{22})$=$0.57(0.571 + 0) + 0.43(-0.338 + 0.28) \simeq 0.3$

***Table 11*** – *Country level IO table for example 7.2*

| table 6.2.6 | J | C | U | J final | C final | U final | Total output |
|---|---|---|---|---|---|---|---|
| J | 0 | 2 | 0 | 2 | 0 | 0 | 4 |
| C | 0 | 0 | 0 | 0.5 | 2.5 | 2.5 | 5.5 |
| U | 0 | 0 | 0 | 0 | 0 | 3 | 3 |
| Value added | 4 | 3.5 | 3 | | | | |
| Total output | 4 | 5.5 | 3 | | | | |

***Table 12*** – *GVC-REER and VAREER weights (raw,constant elasticity)*

| | GVC-REERC | A-REER |
|---|---|---|
| $W_{JC}$ | 0.18 | 0.27 |
| $W_{JU}$ | 0.19 | 0.12 |
| $W_{CJ}$ | 0.20 | 0.32 |
| $W_{CU}$ | **0.16** | **0.25** |
| $W_{UJ}$ | 0.25 | 0.16 |
| $W_{UC}$ | 0.19 | 0.28 |

***Table 13*** – *Normalization and the role of aggregation*

| | Raw Weights | | Normalized weights | |
|---|---|---|---|---|
| | GVC-REER | A-REER(raw) | GVC-REER | A-REER |
| $W_{JC}$ | 0.18 | 0.27 | 0.48 | 0.69 |
| $W_{JU}$ | 0.19 | 0.12 | 0.51 | 0.37 |
| $W_{CJ}$ | 0.20 | 0.32 | 0.55 | 0.56 |
| $W_{CU}$ | **0.16** | **0.25** | **0.44** | **0.43** |
| $W_{UJ}$ | 0.25 | 0.16 | 0.57 | 0.36 |
| $W_{UC}$ | 0.19 | 0.28 | 0.43 | 0.63 |

C to U are associated with sector C2, which uses foreign (J) value added (and therefore less of its own value added)and hence it tends to reduce the weight assigned by C to U. The aggregate measure on the other hand looks at aggregate country -level data and as a result attributes a higher amount of value added by C in its exports to U (because C2 has a higher fraction of own value added). As a result, $W_{CU}$ is higher under the aggregate measure compared to GVC-REER.Table 13 shows a comparison between normalized and raw weights under the two measures. Note that now the ordering in $W_{CU}$ is reversed and the GVC-REER measure assigned a higher weight than A-REER.This undesirable feature is a consequence of the arbitrariness involved in normalization and highlights its drawbacks.

## 8 Estimation of Elasticities

The examples in the previous section have shown that the different elasticities of substitution are key parameters in computing the REER weights. In this section we take up the task of estimating these elasticities. The availability of input output tables at previous year prices in the World Input Output Database(WIOD) allow us to estimate elasticities used in the computation of weights instead of assuming all elasticities to be unity as is done in the literature .We use the framework pioneered in Feenstra (1994) and subsequently used in Broda and Weinstein (2006) and Soderbery (2013) to estimate the different elasticities used in the CES aggregators for production and consumption which also enter the expression for the real exchange rate. We start with a brief overview of the framework followed by a discussion of how the estimation in carried out in the context of out model.

### 8.1 Framework

The approach used here will be based on recent work by Soderbery (2013) which outlines certain drawbacks in the preceding two papers and proposes an estimator which outperforms them. Consider a generic CES Armington aggregator defined as follows:

$$D_t = \left[\sum_{k\in K}(w_k)^{1/\eta}(D_{kt})^{\frac{\eta-1}{\eta}}\right]^{\frac{\eta}{\eta-1}} \tag{8.1}$$

The objective is to estimate the demand elasticity $\eta$. The double differenced demand equation in terms of expenditure shares is given by[20]:

$$\triangle^r ln(s_{kt}) = -(\eta-1)\triangle^r ln(p_{kt}) + \epsilon^r_{kt} \tag{8.2}$$

where $\triangle^r ln(x_{kt}) = \triangle ln(x_{kt}) - \triangle ln(x_{rt})$ and $\triangle ln(x_{jt}) = ln(x_{jt}) - ln(x_{j(t-1)}), x = s, p$ $r$ is called a reference variety and is typically chosen to the the one with the largest share .$s_{kt}$ is the expenditure share of the $k^{th}$ variety and is given by:

[20] see Soderbery (2013) , Broda and Weinstein (2006) or Feenstra (1994) for further details including the actual derivation

$$s_{kt} = \frac{p_{kt}D_{kt}}{\sum_{k\in K} p_{kt}D_{kt}} \tag{8.3}$$

Next, given a supply curve with elasticity $\rho$, the supply curve in terms of differenced shares and prices can be written as:

$$\triangle^r ln(p_{kt}) = \left(\frac{\rho}{1+\rho}\right) \triangle^r ln(s_{kt}) + \delta^r_{kt} \tag{8.4}$$

If the demand and supply disturbances are independent across time, then the 2 equations can be multiplied and scaled to yield:

$$Y_{kt} = \theta_1 Z_{1kt} + \theta_2 Z_{2kt} + u_{kt} \tag{8.5}$$

where $Y_{kt} = (\triangle^r ln(p_{kt}))^2$ ,$Z_{1kt} = (\triangle^r ln(s_{kt}))^2$,$Z_{2kt} = (\triangle^r ln(p_{kt}))(\triangle^r ln(s_{kt}))$,and $u_{kt} = \frac{\epsilon^r_{kt}\delta^r_{kt}}{1-\phi}$.

Further, the parameters of this regression model can be mapped to the primitive parameters of the demand and supply system as follows:

$\phi = \frac{\rho(\eta-1)}{1+\rho\eta} \in [0, \frac{\sigma-1}{\sigma})$

$\theta_1 = \frac{\phi}{(\eta-1)^2(1-\phi)}$ $\theta_2 = \frac{2\phi-1}{(\eta-1)(1-\phi)}$

Consistent estimates of $\theta_1$ can be obtained by using the moment condition $E(u_{kt}) = 0$, where consistency relies on $T \to \infty$.[21] If standard procedures (2SLS or LIML) yields a value of $\theta_1$ that gives imaginary values for $\eta$ and $\rho$ or values with the wrong sign, then the grid search or the non-linear search method of Soderbery (2013) can be used.

### 8.2 Implementation:

We construct sectoral price indices for all cells in the WIOD input output table using the tables in previous year prices. For a fixed production entity(identified by the country-sector pair $(c, l)$) and a fixed sector $s$, the table 14 shows how the estimation of the different elasticities in the model maps onto the procedure outlined above.

***Table 14***

| | $D$ | $D_k$ | $p_k$ |
|---|---|---|---|
| Production elasticities | | | |
| $\sigma^1_s(c,l)$ | $(X(f)^c_{sl})$ | $(X^{kc}_{sl})$ | $p^k_s$ |
| $\sigma^{1h}_s(c,l)$ | $(X^c_{sl})$ | $(X^{cc}_{sl}), (X(f)^c_{sl})$ | $p^k_s$ |
| $\sigma^2(c,l)$ | $(X^c_l)$ | $(X^c_{kl})$ | $q^c_{kl}$ |
| $\sigma^3(c,l)$ | $(Q^c_l)$ | $(X^c_l, V^c_l)$ | $(p^c_l, p^{vc}_l)$ |
| Consumption elasticities | | | |
| $\theta^1_s(c)$ | $(F(f)^c_s)$ | $(F^{kc}_s)$ | $p^k_s$ |
| $\theta^{1h}_s(c)$ | $(F^c_s)$ | $(F^c_s), (F(f)^c_s)$ | $p^k_s$ |
| $\theta^1(c)$ | $(F^c)$ | $(F^c_k)$ | $P^c_k$ |

[21]Given the nature of the data, the value of T is typically very small. For example Soderbery (2013) uses an unbalanced panel with 15 years of data

## 9 Data

We use recently released data form the World inout-Output Database(WIOD) which was developed by a consortium of eleven European research institutions with funding from the European Commission. The database consists of a time series of input-output tables covering 40 countries and 35 sectors from 1995-2011[22]. The data is available in both current and previous year prices which enables us to compute price indices for different entries in the input-output table. A detailed description of this database can be found in Timmer and Erumban (2012). As documented by these authors,the database in more precise than previous attempts in the literature (for instance Johnson and Noguera (2012)) as it uses less approximations and more detailed trade data[23].

## 10 Results

### 10.1 Elasticity Estimation

To minimize the effect of outliers we winsorize the data at the 10th and 90th percentiles. Further, we propose to use the bootstrap median as our point estimate since we find this to be more stable than the MLE in our simulations. Table 20 in the appendix reports the moments of the sample bootstrap distribution obtained using 50 draws. In table 16 we report estimates across different country and sector groups.

#### 10.1.1 Benchmark Calibration:

As shown in table 16 we find substantial heterogeneity in the elasticities across country and sector groups. The p-values for the null of equality of medians across samples is often insignificantly different from zero. For our benchmark calibration of consumption elasticities we pool across sectors and split countries into two groups, OECD and non-OECD. For production elasticities we pool across countries and split sectors into primary, secondary and tertiary. thus we use 16 different elasticities in the benchmark calibration. These are highlighted in bold. (15 in table 16 and one in table 20, namely $\sigma^3$=1.015).

[22]The full set of countries and sectors is listed in appendix I

[23]for instance, unlike Johnson and Noguera (2012) who use a proportionality assumption to split intermediate and final goods imports, the WIOD uses detailed data from com-trade to distinguish trade flow into the different categories.

***Table 15*** – *Comparison of median elasticities across different country and sector groups*

| | Consumption Elasticity | | | Production Elasticity | | | |
|---|---|---|---|---|---|---|---|
| | $\theta^1$ | $\theta^{1h}$ | $\theta^2$ | $\sigma^1$ | $\sigma^{1h}$ | $\sigma^2$ | $\sigma^3$ |
| country group(size) | | | | | | | |
| OECD(28) | **7.75** | **3.38** | **1.35** | NA | 7.78 | 4.05 | 1.046 |
| Non-OECD(13) | **17.80** | **9.45** | **1.925** | NA | 7.02 | 3.38 | 0.99 |
| Europe(29) | 8.22 | 7.60 | 1.536 | NA | 7.30 | 4.03 | 1.00 |
| Asia(7) | 14.28 | 1.78 | 1.499 | NA | 7.75 | 3.16 | 1.076 |
| Americas(4) | 9.1035 | 37.75 | 1.1822 | NA | 7.81 | 2.80 | 1.16 |
| p(OECD,non-Oecd) | 0.00*** | 0.074* | 0.266 | NA | 0.22 | 0.074* | 0.3677 |
| observations | 1435 | 41 | 41 | | 1435 | 1453 | 41 |
| sectors | | | | | | | |
| primary(2) | 13.98 | NA | NA | **12.25** | **8.19** | **4.76** | NA |
| secondary(15) | 14.23 | NA | NA | **5.81** | **8.02** | **4.36** | NA |
| tertiary(18) | 7.75 | NA | NA | **9.14** | **7.29** | **3.22** | NA |
| p(secondary, tertiary) | 0.00*** | NA | NA | 0.00*** | 0.34 | 0.00*** | NA |

Notes: p(a,b) denotes the p-value for the null hypothesis that the medians are constant for a and b using Moods chi squared test.Tests for $\sigma^1$ are based only on 2 countries(i.e 2450(=2*35*35) observations). Agriculture and mining are classified as primary sectors while the rest are split into manufacturing(secondary, 15 sectors) and services(tertiary, 18 sectors).
For p values, "***","**" and "*" denote significance at 1%, 5% and 10% level respectively

## 10.2 Multilateral Exchange rates

In this section we illustrate the properties of our different REER indices using a series of figures, focussing mostly on country level exchange rates

Figure 10.1 illustrates the different kinds of REER indices that we generate using our framework taking the example of USA. The first plot displays 8 different country REER indices that can be constructed using the framework developed in this paper. Four of these(solid lines) are value added exchange rate indices and the other four are indices for gross output. The eight indices can also be seen as two groups of four indices each corresponding to the constant elasticity(ce) and benchmark elasticity(bm) calibration. Here we use normalized versions of the weighting matrices, the rationale for which we will discuss below. All indices are in logs and normalized to zero at the start of the sample period so the value on the y axis can be read as the percentage deviation from the start of the sample. The second plot illustrates how each of the 8 indices in the first plot can be split into 35 sectoral components using The benchmark GVC-REER as an example.

For reference , figure J.1 in appendix J shows the 8 indices for all 40 countries in the sample. Several interesting observations are worthy of mention. Firstly, note that there is substantial heterogeneity across the indices which speaks to the importance of incorporating trade in intermediate goods. Secondly, we note that the 8 indices, although different, show high comovement for most countries. The notable exception is China. Here a comparison of the red dotted line and the pink solid line for instance shows that although there was an appreciation in China's value added exchange rate(GVC-REER(BM)) in the initial part of the sample, if we impose the constant elasticity assumption and look at China's gross output instead of value added, the conclusion seems to be the opposite. China is a country that shows the most disparity across REER indices and we will discuss and illustrate them in the remainder of this section.

As mentioned before, our framework also allows us to compute exchange rates at the sector level to gauge competitiveness of individual sectors within a country. Figure 10.2 some sector level exchange rates for select countries. As can be seen in the figure, we find evidence of substantial heterogeneity across movements in competitiveness within countries. For Mexico for instance we find that although the aggregate exchange rate appreciates through the sample period, the REER for the financial intermediation sector indicates depreciation, implying an increase in its competitiveness even as the overall competitiveness of the economy falls.

We next illustrate the role of different aspects of our REER indices in isolation. Figure 10.3 Shows a comparison of constant elasticity and benchmark elasticity GVC_REER indices for 7 countries. The figure clearly shows the dramatic increase in the volatility of REER when moving from Cobb douglas case(constant elasticity) to the case where more realistic elasticities estimated from the data are incorporated [24]. This is the reason we chose to display indices based on normalized weights in figure 10.1.

Due to the high volatility of the REER with heterogenous elasticities, a mere visual comparison between the two indices is not informative. In order to illustrate the role played by heterogenous elasticities we therefore define a statistic to qualitatively capture the differences in REER based on constant and heterogenous elasticity. For each entity (e) and for each year, we create a variable $d_t^e$ which takes the value one if the GVC-REER constant elasticity and heterogenous elasticity (with benchmark calibration) indices move in opposite directions and zero otherwise.

$$d_t^e = 1\left(sign(\triangle GVC - REER(BM)_t) \neq sign(\triangle GVC - REER(CE)_t)\right) \tag{10.1}$$

We then compute the mean of $d_t^e$ for each $e$ across all time periods and to define the "Divergence index" for entity $e$ as follows:

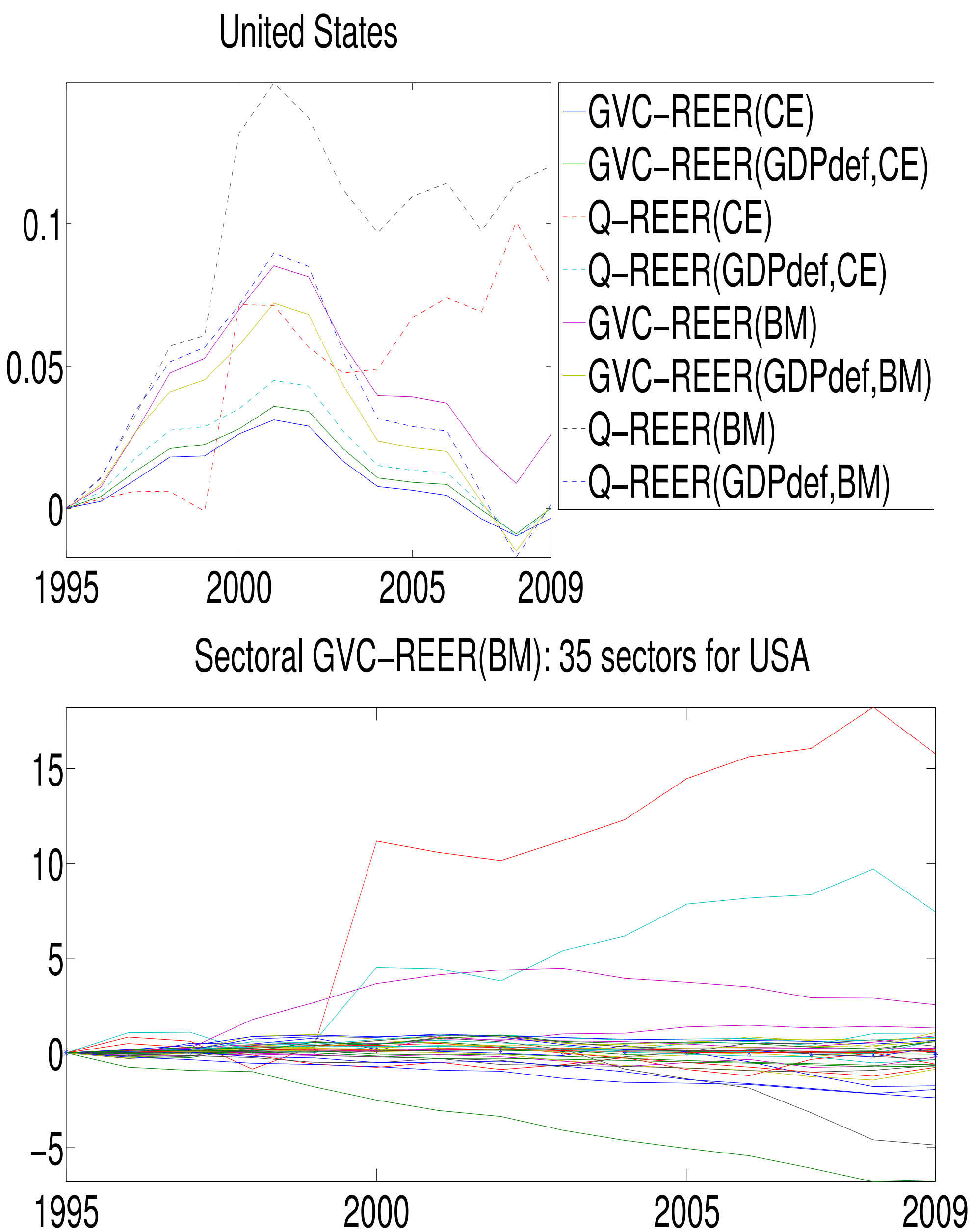
$$d^e = \frac{\sum_{t=2}^{T} d_t^e}{T-1} \tag{10.2}$$

Note that $d^e$ takes the value zero if the two REER measures always agree in their direction of movement and takes the value of 1 if they never agree, i.e always move in opposite directions.

Table 17 summarizes the distribution of the divergence index for country level exchange rates. A large fraction of countries (21) never see a divergence between the two measures. The maximum number of times the measures move in opposite directions is 3, which is still a small fraction (20%) of the total number of years. This happens for Slovenia. Our main takeaway from these statistics is that incorporation of heterogenous elasticity do not significantly alter the REER indices at the

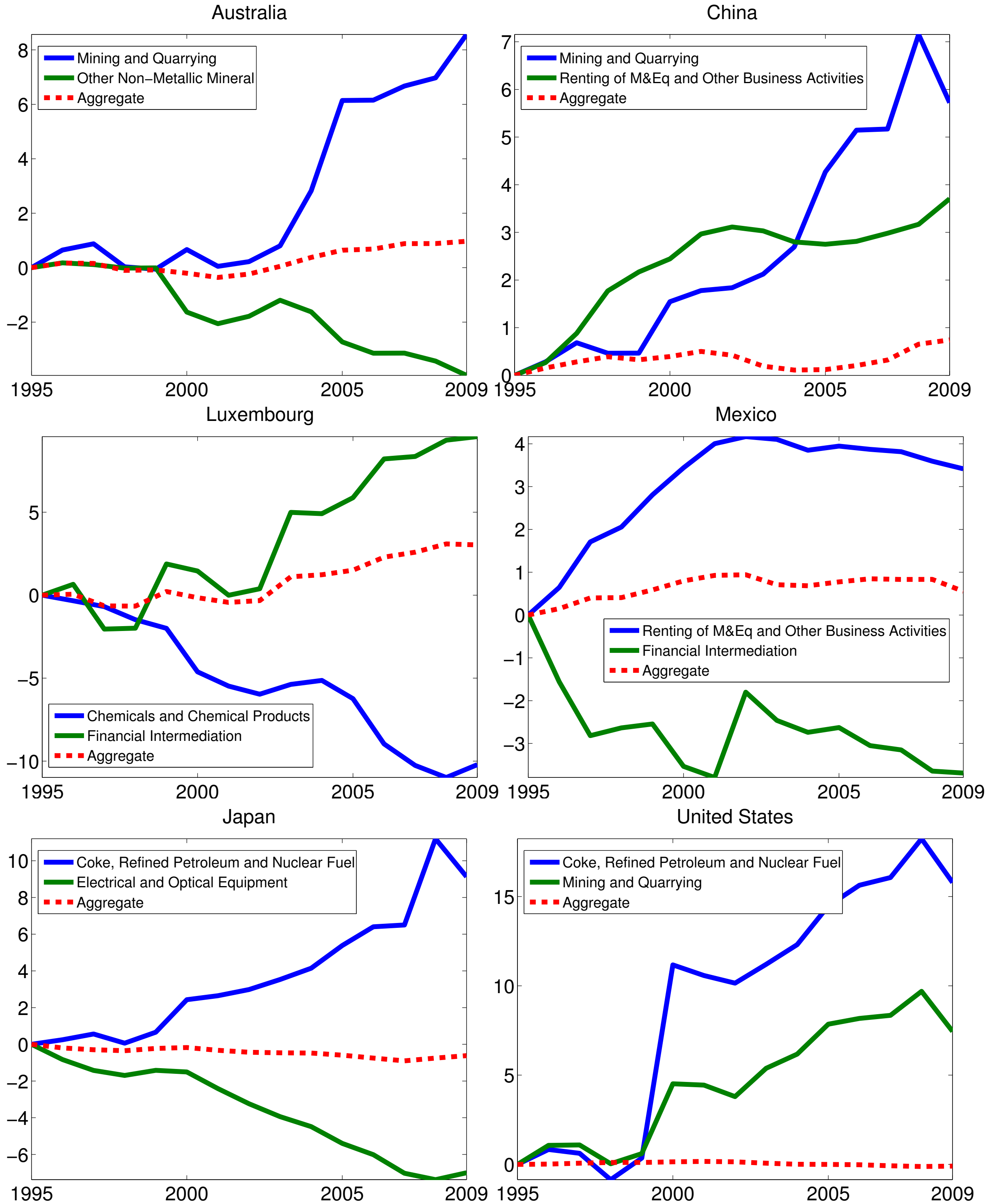
[24] The same pattern holds in a comparison of constant and heterogenous elasticities indices for Gross output competitiveness as well (These results are not reported.)

*Figure 10.1* – *REER indices for USA*



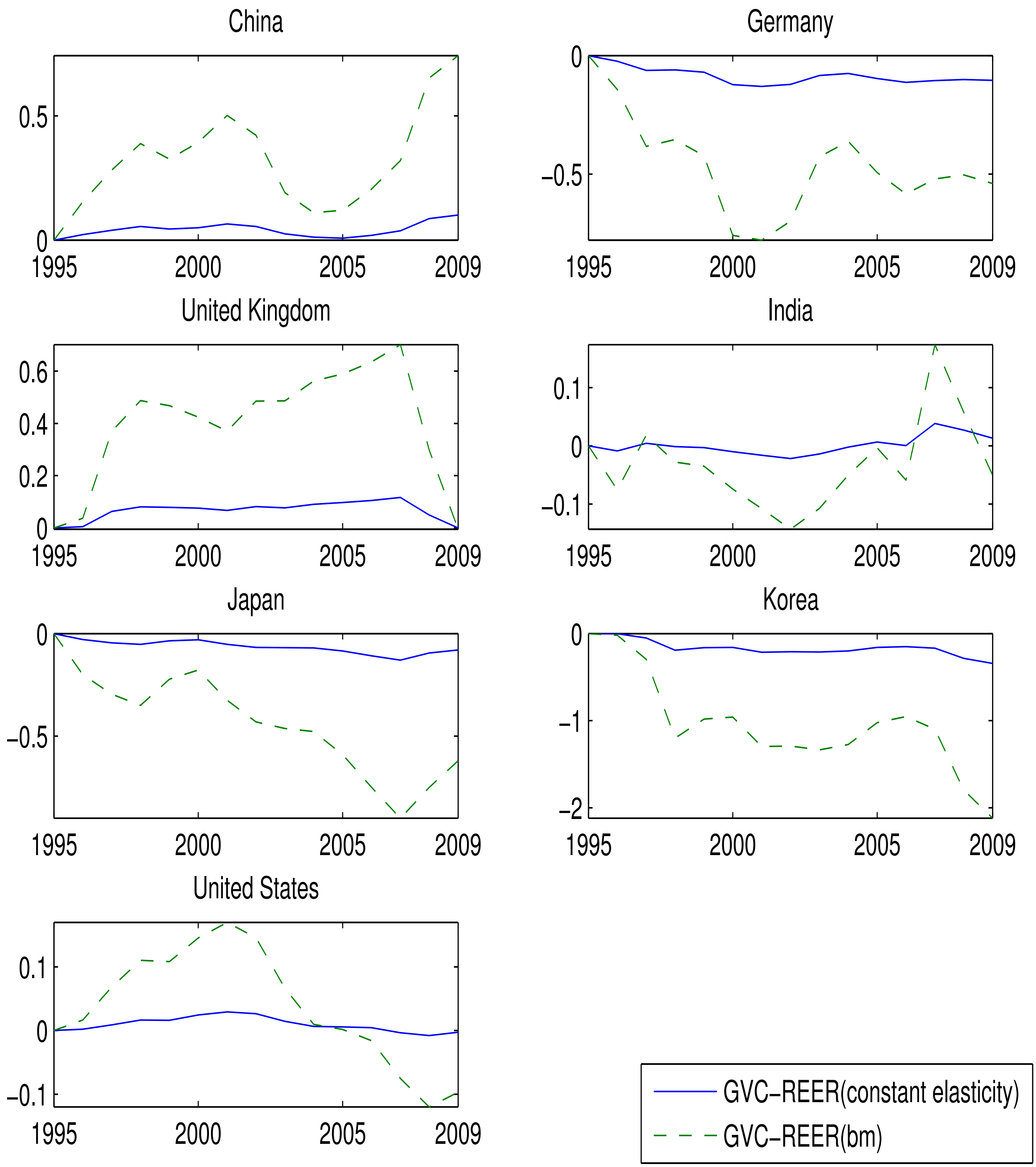

Notes: This figure illustrates all the different REER indices that we compute using our framework taking the case of USA as an example. This first plot shows the 8 indices at the aggregate(country) level and the second plot shows how each of the 8 can be further split into 35 sectoral components. All indices are in logs and normalized to zero at the start of the sample period so the reading on the value on the y axis can be read as the percentage deviation from the start of the sample.

*Figure 10.2 – Sector level Exchange rates along with Aggregate country REER for select countries*




Notes:All indices are in logs and normalized to zero at the start of the sample period so the reading on the value on the y axis can be read as the percentage deviation from the start of the sample.

*Figure 10.3* – *The role of heterogenous elasticities*

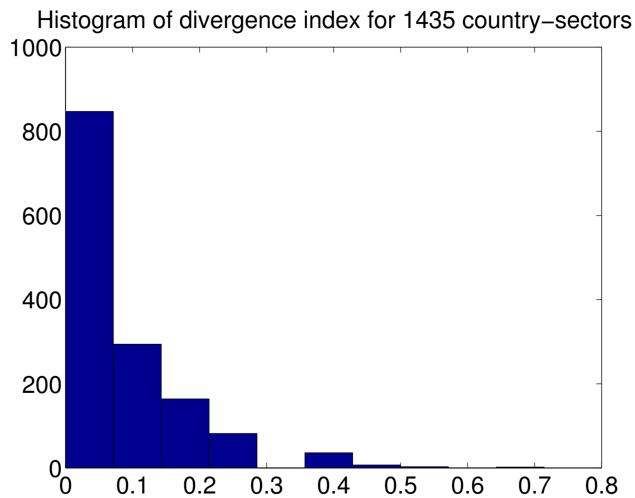



Notes:All indices are in logs and normalized to zero at the start of the sample period so the reading on the value on the y axis can be read as the percentage deviation from the start of the sample.

*Table 16 – Divergence index for Countries*

| $d^e$ | number of countries |
|---|---|
| 0.21 | 1 |
| 0.14 | 4 |
| 0.07 | 15 |
| 0 | 21 |
| | total=41 |

*Figure 10.4 – Divergence index at the country-sector level*

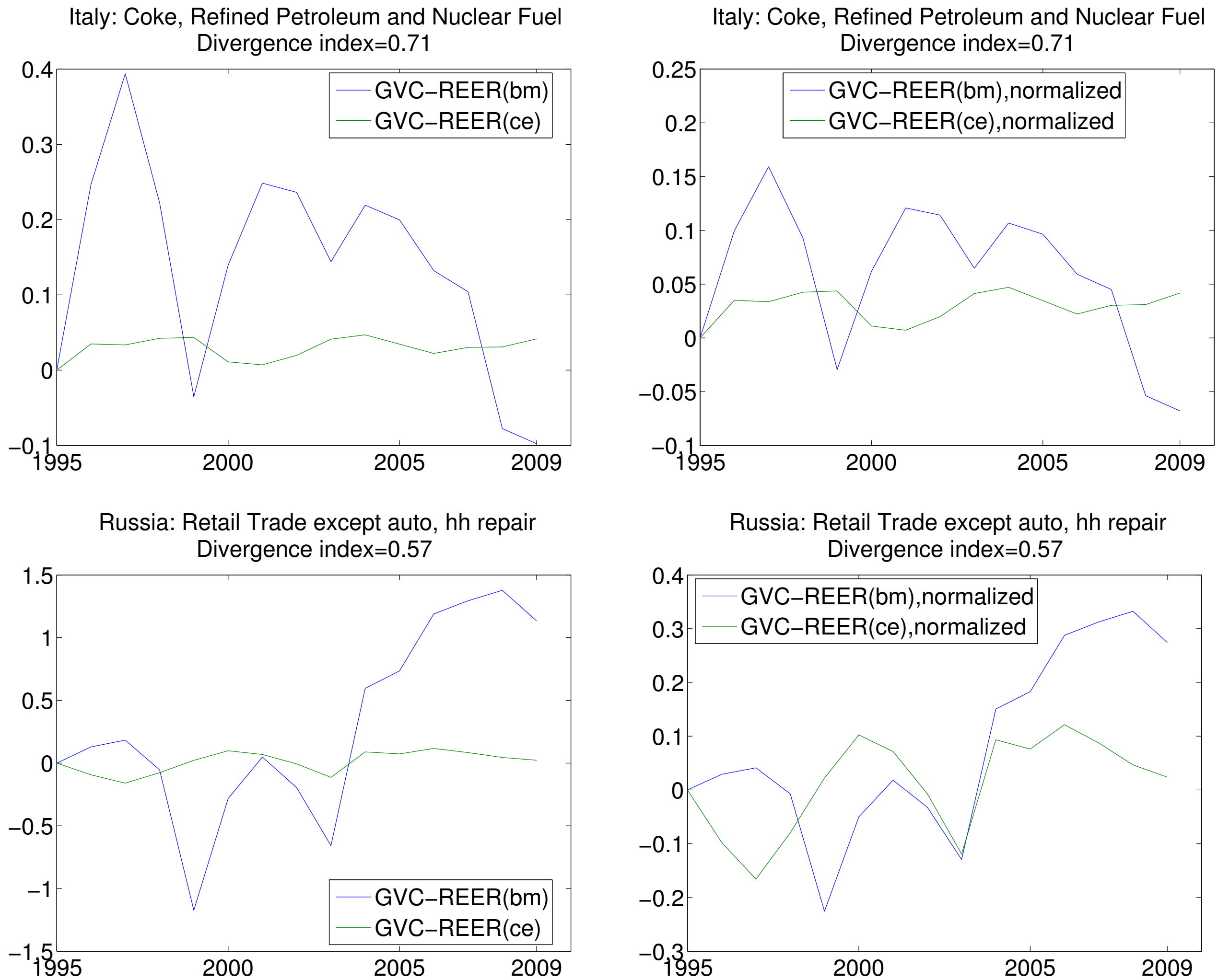



country level to the extent that one is interested in studying the direction of movements in REER.

The story however is different when we go to more disaggregated level of sectors within each country. Here we find examples where the 2 measures disagree on sign in as many as 10 (67%) of the time periods. Figure 10.5 displays a histogram plot of the divergence index for 1435 country-sector pairs, and figure 10.5 shows some examples where the two measures have the highest disparity.

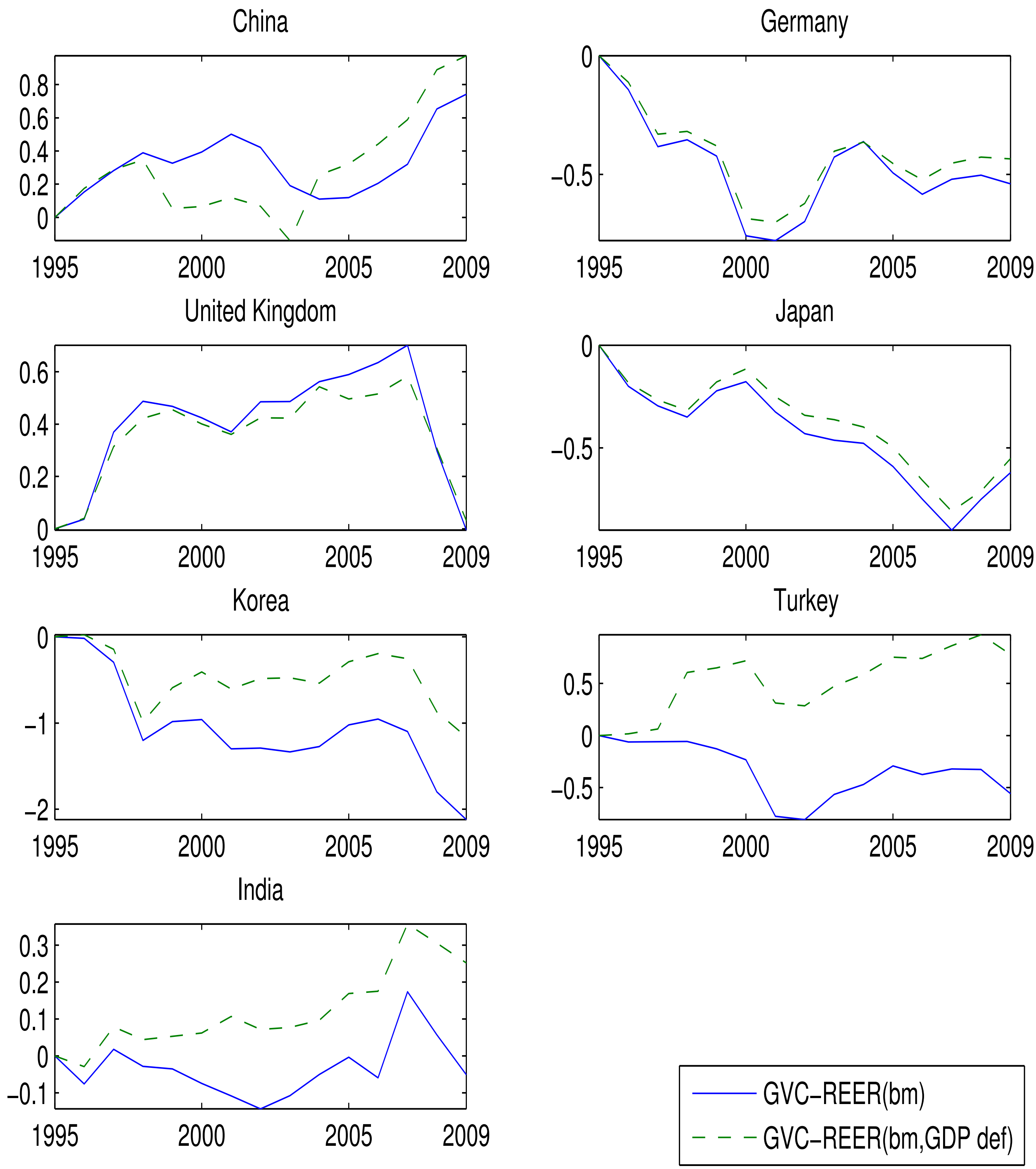
Next, we consider the role of using sector level price indices in isolation. Figure 10.6 plots the GVC-REER indices based on benchmark indices using sector level prices alongside the same indices constructed using an aggregate price index(namely the GDP deflator) for select countries.While there is very little difference between the two indices for some countries(notably Japan and the UK) the divergence is substantial for countries like China, Turkey and India. The difference is most stark when the indices move in opposite directions(as is the case for these three countries at various points in the sample), as it shows that ignoring sector level information can lead to an error in computing not only the magnitude but also the direction of exchange rate movement. For instance, in 2003, while the GDP deflator based REER indicates a depreciation, the more comprehensive REER based on sector level prices actually indicates an appreciation of the Chinese REER.Similar instances are also observed for other countries, most notably for India and Turkey as shown in figure 10.6.

***Figure 10.5*** – *Examples with high divergence between constant elasticity and Heterogenous elasticity GVC-REER*




Notes:All indices are in logs and normalized to zero at the start of the sample period so the reading on the value on the y axis can be read as the percentage deviation from the start of the sample.

***Figure 10.6*** – *The role of sector level price indices*




Notes:All indices are in logs and normalized to zero at the start of the sample period so the reading on the value on the y axis can be read as the percentage deviation from the start of the sample.

### 10.3 Application: Bilateral Real Exchange Rates:

Bilateral real exchange rate(RER) is commonly used to gauge competitiveness as well as cost of living differentials between countries. Based on the the insights gained from the previous sections we argue that if the goal is to measure competitiveness of one country vis-s-vis the other then the standard RER measures computed using an aggregate price index(such as those in Chinn (2006)) like the GDP deflator can be misleading since they ignore sector level linkages between the countries. In particular, the bilateral real exchange rate between countries $h$ and $f$ is defined as follows:

$$RER^{hf} = \hat{p}^f - \hat{p}^h \tag{10.3}$$

where $\hat{p}^f$ and $\hat{p}^h$ are changes in aggregate (country wide) price indices measured in a common currency. If one is interested in interpreting the RER as a measure of competitiveness, then this measure can be inaccurate as the following example shows

Consider a two country word where each country has two sectors. There is no trade in intermediate goods and production comprises entirely of own value added. Table 18 shows how the final demand is distributed across sectors.

***Table 17*** *– IO table for bilateral RER*

| | | C | | U | | CFinal | Ufinal | total output |
|---|---|---|---|---|---|---|---|---|
| | | C1 | C2 | U1 | U2 | | | |
| C | C1 | 0 | 0 | 0 | 0 | 1 | 1 | 2 |
| | C2 | 0 | 0 | 0 | 0 | 3 | 0 | 3 |
| U | U1 | 0 | 0 | 0 | 0 | 0 | 1 | 1 |
| | U2 | 0 | 0 | 0 | 0 | 0 | 1 | 1 |
| VA | | 2 | 3 | 1 | 1 | | | |
| total output | | 2 | 3 | 1 | 1 | | | |

Suppose in addition, $\hat{p^v}(c_1) = -0.01, \hat{p^v}(c_2) = 0.02, \hat{p^v}(u_1) = 0, \hat{p^v}(u_2) = 0$(all prices are in a common currency, so nominal exchange rate is already incorporated)

Change in China's GDP deflator is given by:

$$\hat{p^v}(c) = \frac{2}{5}\hat{p^v}(c_1) + \frac{3}{5}\hat{p^v}(c_2) = 0.008 \tag{10.4}$$

Change in US GDP deflator is zero, since

$$\hat{p^v}(u) = \frac{1}{2}\hat{p^v}(u_1) + \frac{1}{2}\hat{p^v}(u_2) = 0 \tag{10.5}$$

Based on the conventional RER definition using and aggregate country level price index,

$$\hat{RER}^{US-CH} = \hat{p^v}(c) - \hat{p^v}(u) = 0.008 \tag{10.6}$$

and hence the conventional RER measure would indicate an increase in competitiveness of the US. This however is misleading since the entire price change comes from China's sector 2 which does not compete with any of the US sectors. Moreover, the Chinese sector which does compete with the US

*Table 18* – *Weighting matrix for bilateral RER example*

| | $C_1$ | $C_2$ | $U_1$ | $U_2$ |
|---|---|---|---|---|
| $C_1$ | -3.54 | 1.88 | 0.83 | 0.83 |
| $C_2$ | 1.25 | -1.25 | 0 | 0 |
| $U_1$ | 1.67 | 0 | -3.33 | 1.67 |
| $U_2$ | 1.67 | 0 | 1.67 | -3.33 |

is C1, which actually experiences a decrease in its price, so the correct measure of competitiveness must signal an appreciation of the US exchange rate against China, not a depreciation as measured by the standard RER in 10.3.

Our framework to compute effective exchange rates can be easily modified to adjust these bilateral RERs to better reflect movements in competitiveness. We define our bilateral RER measure, the "GVC-RER" as follows

$$GVC - RER^{hf} = \sum_{i=1}^{m} v_i^h \left[ \sum_{j=1}^{m} w_{ij}^{hh} \hat{p}_j^{vh} + \sum_{j=1}^{m} w_{ij}^{hf} \hat{p}_j^{vf} \right] \tag{10.7}$$

here$v_i^h = \frac{p_i^{vh} V_i^h}{\sum_{j=1}^{m} p_j^{vh} V_j^h}$ is the share of sector $i$ in country $h$'s total value added, so that $\sum_{i=1}^{m} v_i^h = 1$. The $w$s are weights that are analogous to the GVC-REER weights.

Table 19 gives the GVC-REER weighting matrix . Based on our proposed measure, we have

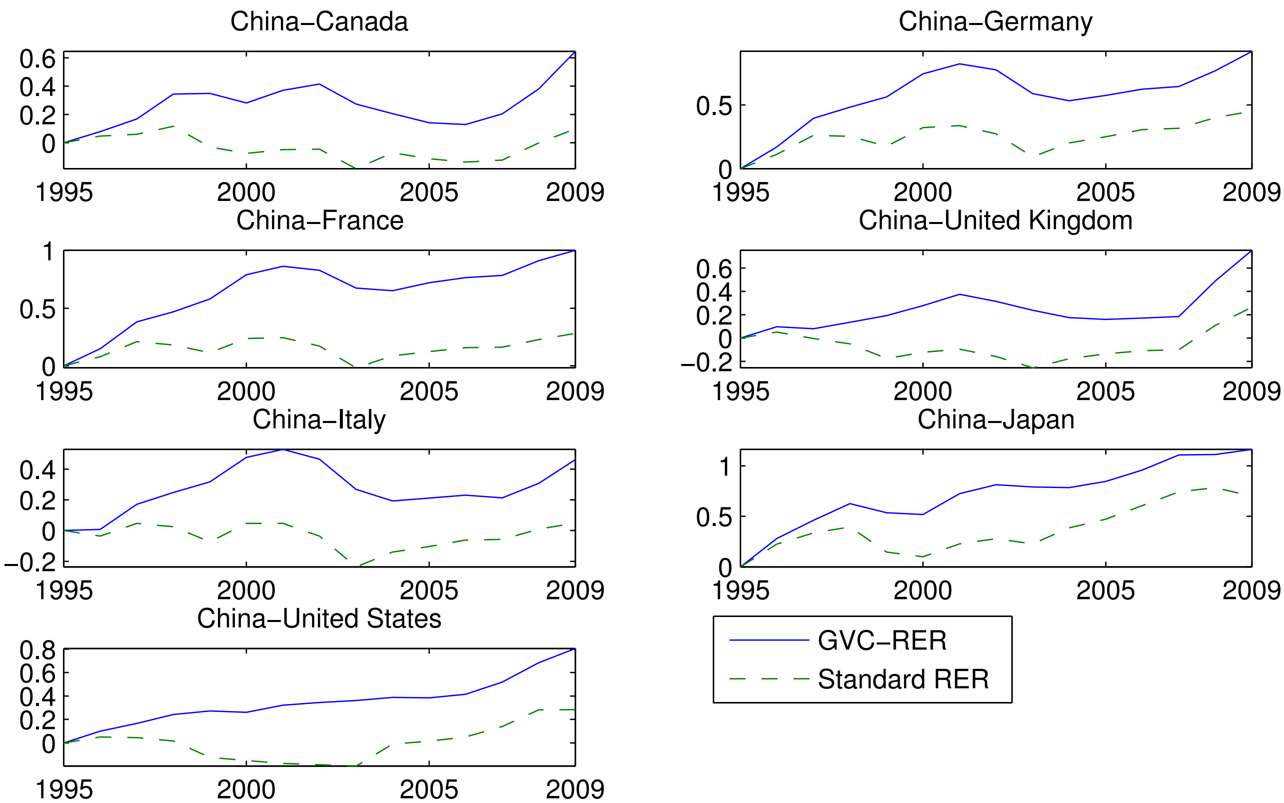
$$GVC - \hat{RER}^{US-CH} = \frac{1}{2}(-0.0167) + \frac{1}{2}(-0.0167) = -0.01 \tag{10.8}$$

Hence the two measures differ not just in magnitude but also the sign. The conventional measure shows a depreciation whereas the new measure shows an appreciation that is consistent with intuition.

Figure 10.7 Shows the comparison of the two RER measures for China against the G7 countries.In computing these indices, the weights are normalized so that the sum of the home country and foreign country weights are equal in magnitude, as is the case with the standard RER measure. Unlike in the GVC-REER effective exchange rate computation, here the normalization of weights cannot be avoided, since otherwise the GVC-RER measure would be dominated by home prices because home sectors(especially the own sector) on average carry much higher GVC-REER weights.

It can be seen that there are substantial differences between the two measures for some country pairs. For China's bilateral exchange rate against the US for instance, whereas the standard RER shows a U shaped pattern, the GVC-REER shows a secular appreciation during the sample period, indicating that price movements during this period have meant that China has lost competitiveness against the US steadily.

***Figure 10.7*** – *Comparison of GVC-RER and standard RER bilateral exchange rates for China*




Notes:All indices are in logs and normalized to zero at the start of the sample period so the reading on the value on the y axis can be read as the percentage deviation from the start of the sample.

## 11 Conclusion

This paper proposes a theoretical framework to compute real effective exchange rates(REER) as a measure of competitiveness by incorporating four features that have been typically overlooked in the literature and that we show are likely to lead to mis-measurement in competitiveness. Firstly, we distinguish between trade by end use category(i.e intermediate vs final). Recognizing that with trade in intermediate inputs, value added and gross output become delinked , we define and compute REER indices to quantify competitiveness both in terms of gross output(Q-REER) and value added(GVC-REER). Secondly, we go beyond aggregate REERs for countries and compute REERs for individual sectors within countries. We are able to do so by exploiting detailed sector level trade flows in the data and by specifying a general multi-country multi-sector model on the theoretical side. Thirdly, we construct sector level price indices and use these in our REER indices instead of relying on the more coarse country level price indices like CPI, GDP deflator or some measure of unit labor cost. Fourthly, we explicitly estimate and incorporate different elasticities of substitution in production functions and final demand aggregators in our REER indices, which is a significant improvement from the typical practice of assuming all elasticities to be unity as is done in the literature. We illustrate the importance of each of these additions using illustrative examples as well as actual REER indices computed using data from the World Input-Output Database

(WIOD) and outline the conditions under which our general framework nests the other measures in the literature.

We take our framework to the data by utilizing detailed input-output and final use tables from the World Input-Output Database(WIOD). We compute REER indices for 40 countries and 1435 country-sector pairs for the period 1995-2009 and display various aspects of our REER measures and contrast them with other measures in the literature.

In addition to addressing the issue of competitiveness in a comprehensive manner, we see two other important auxiliary contributions of the paper. Firstly, our modeling of the production and consumption aggregators is the most comprehensive in the literature and allows for features like intermediate inputs and several elasticities of substitution including a distinction between macro and micro elasticities that has been shown to be a feature of the data(see Feenstra et al. (2010)). Although we worked with a static partial equilibrium model in this paper in order to best address the primary question of interest, the model can be easily extended to a dynamic general equilibrium setting to study other important issues in international macroeconomics including international transmission of shocks. Secondly, this is the first paper to our knowledge that has taken up the task of estimating elasticities of substitution comprehensively by making a distinction between consumption and production elasticities on the one hand and micro and macro elasticities on the other. Since even the most advanced empirical estimates of elasticities available in the literature to date do not distinguish between production and consumption elasticities(see for instance Broda and Weinstein (2006), Soderbery (2013) or Feenstra et al. (2010)), DSGE models aiming to study the role of production sharing are often missing a key component in their calibration.[25]We believe our elasticity estimates provide a first step toward filling this void.

## References

Baldwin, R. and Lopez-Gonzalez, J. (2012). Supply-chain trade: A portrait of global patterns and several testable hypotheses. *HEI manuscript*.

Bayoumi, M. T., Saito, M., and Turunen, M. J. (2013). *Measuring Competitiveness: Trade in Goods or Tasks?* Number 13-100. International Monetary Fund.

Bayoumi, T., Lee, J., and Jayanthi, S. (2005). New rates from new weights.

Bems, R. and Johnson, R. C. (2012). Value-added exchange rates. Technical report, National Bureau of Economic Research.

Bennett, H. Z. and Zarnic, Z. (2009). International competitiveness of the mediterranean quartet: A heterogeneous-product approach. *IMF Staff Papers*, 56(4):919–957.

[25]The list of papers that would benefit from such estimates is too large to summarize comprehensively. But several recent examples are Burstein et al. (2008),Johnson (2012) and Di Giovanni and Levchenko (2010).

Broda, C. and Weinstein, D. E. (2006). Globalization and the gains from variety. *The Quarterly Journal of Economics*, 121(2):541–585.

Burstein, A., Kurz, C., and Tesar, L. (2008). Trade, production sharing, and the international transmission of business cycles. *Journal of Monetary Economics*, 55(4):775–795.

Chinn, M. D. (2000). Before the fall: were east asian currencies overvalued? *Emerging Markets Review*, 1(2):101–126.

Chinn, M. D. (2006). A primer on real effective exchange rates: determinants, overvaluation, trade flows and competitive devaluation. *Open economies review*, 17(1):115–143.

Costinot, A., Vogel, J., and Wang, S. (2013). An elementary theory of global supply chains. *The Review of Economic Studies*, 80(1):109–144.

Di Giovanni, J. and Levchenko, A. A. (2010). Putting the parts together: Trade, vertical linkages, and business cycle comovement. *American Economic Journal: Macroeconomics*, 2(2):95–124.

Feenstra, R. C. (1994). New product varieties and the measurement of international prices. *The American Economic Review*, pages 157–177.

Feenstra, R. C., Obstfeld, M., and Russ, K. N. (2010). In search of the armington elasticity. *University of California-Davis, unpublished mimeo*.

Gagnon, J. E. (2012). Combating widespread currency manipulation. *Policy Brief in International Economics*, pages 12–19.

Goldfajn, I. and Valdés, R. O. (1999). The aftermath of appreciations. *The Quarterly Journal of Economics*, 114(1):229–262.

Hummels, D., Ishii, J., and Yi, K.-M. (2001). The nature and growth of vertical specialization in world trade. *Journal of international Economics*, 54(1):75–96.

Johnson, R. C. (2012). Trade in intermediate inputs and business cycle comovement. Technical report, National Bureau of Economic Research.

Johnson, R. C. and Moxnes, A. (2012). Technology, trade costs, and the pattern of trade with multi-stage production.

Johnson, R. C. and Noguera, G. (2012). Accounting for intermediates: Production sharing and trade in value added. *Journal of International Economics*, 86(2):224–236.

Klau, M. et al. (2008). The new bis effective exchange rate indices. *BIS Quarterly Review*.

Koopman, R., Wang, Z., and Wei, S.-J. (2012). Tracing value-added and double counting in gross exports. Technical report, National Bureau of Economic Research.

Koopman, R., Wang, Z., and Wei, S.-J. (2014). Tracing value-added and double counting in gross exports. *The American Economic Review*, pages 459–494.

Loretan, M. (2005). Indexes of the foriegn exchange value of the dollar. *Fed. Res. Bull.*, 91:1.

McGuirk, A. (1986). Measuring price competitiveness for industrial country trade in manufactures.

Miroudot, S., Lanz, R., and Ragoussis, A. (2009). Trade in intermediate goods and services. Technical report, OECD Publishing.

Rogoff, K. (2005). Rethinking exchange rate competitiveness. *The Global Competitiveness Report*, 2006:99–105.

Soderbery, A. (2013). Estimating import supply and demand elasticities: Analysis and implications. *Available at SSRN 2272749*.

Thorbecke, W. (2011). Investigating the effect of exchange rate changes on china's processed exports. *Journal of the Japanese and International Economies*, 25(2):33–46.

Timmer, M. and Erumban, A. (2012). The world input-output database (wiod): Contents, sources and methods. *WIOD Background document available at www. wiod. org*.

Wang, Z., Wei, S.-J., and Zhu, K. (2013). Quantifying international production sharing at the bilateral and sector level. *working paper*.

## Appendix

## A Computation of linearized expression for $\hat{V}_2^C$ and $Q_2^C$ in section 3.

From (5.15) and (6.7) we get:

$$\hat{Q}_2^C = \hat{V}_2^C + \left(\frac{p_1^J X_{12}^{JC}}{p_2^{VC} V_2^C}\right)\left(-\sigma \hat{p}_1^{VJ} + \sigma \hat{p}_2^C\right) \tag{A.1}$$

Using the linearized first order conditions for final goods consumption (A.2) in the market clearing condition (B.1) we get:

$$\begin{aligned}
\hat{Q}_2^C &= \left(\frac{p_2^C F_2^{CC}}{p_2^C Q_2^C}\right)\hat{F}_2^{CC} + \left(\frac{p_2^C F_2^{CJ}}{p_2^C Q_2^C}\right)\hat{F}_2^{CJ} + \left(\frac{p_2^C F_2^{CU}}{p_2^C Q_2^C}\right)\hat{F}_2^{CU} \\
&= \left(\frac{p_2^C F_2^{CC}}{p_2^C Q_2^C}\right)(-\theta \hat{p}_2^C + \theta \hat{P}^C + \hat{F}^C) + \left(\frac{p_2^C F_2^{CJ}}{p_2^C Q_2^C}\right)(-\theta \hat{p}_2^C + \theta \hat{P}^J + \hat{F}^J) \qquad \text{(A.2)} \\
&\quad + \left(\frac{p_2^C F_2^{CU}}{p_2^C Q_2^C}\right)(-\theta \hat{p}_2^C + \theta \hat{P}^U + \hat{F}^U) \qquad \text{(A.3)}
\end{aligned}$$

Using the expressions for the linearized CPIs ((B.6) (B.7) and (B.22) )) as well as (B.15) we can write (A.2) as follows:

$$\hat{Q}_2^c = w(Q)_{21}^{CJ}\hat{p}_1^{vJ} + w(Q)_{22}^{CJ}\hat{p}_2^{vJ} + w(Q)_{21}^{CC}\hat{p}_1^{vC} + w(Q)_{22}^{CC}\hat{p}_2^{vC} + w(Q)_{21}^{CU}\hat{p}_1^{vU} + w(Q)_{22}^{CU}\hat{p}_2^{vU} \tag{A.4}$$

Where

$$\begin{aligned} w(Q)_{21}^{CJ} &= -\theta\left(\frac{p_1^J X_{12}^{JC}}{p_2^C Q_2^C}\right) + \theta\left(\frac{p_1^J X_{12}^{JC}}{p_2^C Q_2^C}\right)\left(\frac{p_2^C F_2^{CJ}}{p_2^C Q_2^C}\right)\left(\frac{p_2^C F_2^{CJ}}{P^J F^J}\right) + \theta\left(\frac{p_2^{VC} V_2^C}{p_2^C Q_2^C}\right)\left(\frac{p_2^C F_2^{CC}}{p_2^C Q_2^C}\right)\left(\frac{p_2^C F_2^{CC}}{P^C F^C}\right) \\ &+\theta\left(\frac{p_2^C F_2^{CJ}}{p_2^C Q_2^C}\right)\left(\frac{p_1^J F_1^{JJ}}{P^J F^J}\right) + \theta\left(\frac{p_1^J X_{12}^{JC}}{p_2^C Q_2^C}\right)\left(\frac{p_2^C F_2^{CU}}{p_2^C Q_2^C}\right)\left(\frac{p_2^C F_2^{CU}}{P^U F^U}\right) \end{aligned} \tag{A.5}$$

$$w(Q)_{22}^{CJ} = \theta\left(\frac{p_2^C F_2^{CJ}}{p_2^C Q_2^C}\right)\left(\frac{p_2^J F_2^{JJ}}{P^J F^J}\right) \tag{A.6}$$

$$w(Q)_{21}^{CC} = \theta\left(\frac{p_2^C F_2^{CC}}{p_2^C Q_2^C}\right)\left(\frac{p_1^C F_1^{CC}}{P^C F^C}\right) \tag{A.7}$$

$$\begin{aligned} w(Q)_{22}^{CC} &= -\theta + \theta\left(\frac{p_2^{VC} V_2^C}{p_2^C Q_2^C}\right)\left(\frac{p_2^C F_2^{CC}}{p_2^C Q_2^C}\right)\left(\frac{p_2^C F_2^{CC}}{P^C F^C}\right) + \theta\left(\frac{p_2^{VC} V_2^C}{p_2^C Q_2^C}\right)\left(\frac{p_2^C F_2^{CJ}}{p_2^C Q_2^C}\right)\left(\frac{p_2^C F_2^{CJ}}{P^J F^J}\right) \\ &+\theta\left(\frac{p_2^{VC} V_2^C}{p_2^C Q_2^C}\right)\left(\frac{p_2^C F_2^{CU}}{p_2^C Q_2^C}\right)\left(\frac{p_2^C F_2^{CU}}{P^U F^U}\right) \end{aligned}$$

$$\begin{aligned} w(Q)_{21}^{CU} &= \theta\left(\frac{p_2^C F_2^{CU}}{p_2^C Q_2^C}\right)\left(\frac{p_1^U F_1^{UU}}{P^U F^U}\right) \\ w(Q)_{22}^{CU} &= \theta\left(\frac{p_2^C F_2^{CU}}{p_2^C Q_2^C}\right)\left(\frac{p_2^U F_2^{UU}}{P^U F^U}\right) \end{aligned}$$

From (A.1) and (A.4) we can write the demand for value added by $(C, 2)$ as a function of prices as follows:

$$\hat{V}_2^c = w(v)_{21}^{CJ}\hat{p}_1^{vJ} + w(v)_{22}^{CJ}\hat{p}_2^{vJ} + w(v)_{21}^{CC}\hat{p}_1^{vC} + w(v)_{22}^{CC}\hat{p}_2^{vC} + w(v)_{21}^{CU}\hat{p}_1^{vU} + w(v)_{22}^{CU}\hat{p}_2^{vU} \tag{A.8}$$

where

$$
\begin{aligned}
w(V)_{21}^{CJ} &= w(Q)_{21}^{CJ} + \sigma\left(\frac{p_1^J X_{12}^{JC}}{p_2^C Q_2^C}\right) \\
w(V)_{22}^{CJ} &= w(Q)_{22}^{CJ} \\
w(V)_{21}^{CC} &= w(Q)_{21}^{CC} \\
w(V)_{22}^{CC} &= w(Q)_{22}^{CC} + \sigma\left(\frac{p_1^J X_{12}^{JC}}{p_2^C Q_2^C}\right) \\
w(V)_{21}^{CU} &= w(Q)_{21}^{CU} \\
w(V)_{22}^{CU} &= w(Q)_{22}^{CU}
\end{aligned}
$$

$$
\hat{V}_1^c = w(v)_{11}^{CJ}\hat{p}_1^{vJ} + w(v)_{12}^{CJ}\hat{p}_2^{vJ} + w(v)_{11}^{CC}\hat{p}_1^{vC} + w(v)_{12}^{CC}\hat{p}_2^{vC} + w(v)_{11}^{CU}\hat{p}_1^{vU} + w(v)_{12}^{CU}\hat{p}_2^{vU} \tag{A.9}
$$

where

$$
\begin{aligned}
w(V)_{11}^{CC} &= -\theta\left(1 - \frac{p_1^C F_1^{CC}}{P^C F^C}\right) \\
w(V)_{12}^{CC} &= \theta\left(\frac{p_2^C F_2^{CC}}{P^C F^C}\right)\left(\frac{p_2^{VC} V_2^C}{p_2^C Q_2^C}\right) \\
w(V)_{11}^{CJ} &= \theta\left(\frac{p_2^C F_2^{CC}}{P^C F^C}\right)\left(\frac{p_1^J X_{12}^{JC}}{p_2^C Q_2^C}\right) \\
w(V)_{12}^{CJ} &= w(V)_{11}^{CU} = w(V)_{12}^{CU} = 0
\end{aligned} \tag{A.10}
$$

other expressions for the weighting matrix:

$$
\hat{Q}_1^C = \left(-\theta + \theta\frac{p_1^c F_1^{cc}}{P^C F^C}\right)\hat{p}_1^{VC} + \theta\left(\frac{p_2^{VC} V_2^C}{p_2^C Q_2^C}\right)\left(\frac{p_2^C F_2^{CC}}{P^C F^C}\right)\hat{p}_2^{VC} + \theta\left(\frac{p_1^J X_{12}^{JC}}{p_2^C Q_2^C}\right)\left(\frac{p_2^C F_2^{CC}}{P^C F^C}\right)\hat{p}_1^{VJ} \tag{A.11}
$$

**Solution of the single sector version of the model:**

Final demand:

$$
\hat{F}^{fh} = -\theta\hat{p}_s^f + \theta\hat{P}^h + \hat{F}^h, (h, f) \in \{(C, C), (J, J), (U, U), (C, J), (C, U)\} \tag{A.12}
$$

Linearizing the expressions for the CPIs in the three countries((B.2)-(B.3)) we get:

$$\hat{P}^U = \left(\frac{p^C F^{CU}}{P^U F^U}\right)\hat{p}^C + \left(\frac{p^U F^{UU}}{P^U F^U}\right)\hat{p}^{VU} \tag{A.13}$$

$$\hat{P}^C = \hat{p}^C \tag{A.14}$$

$$\hat{P}^J = \left(\frac{p^J F^{JJ}}{P^J F^J}\right)\hat{p}^J + \left(\frac{p^C F^{CJ}}{P^J F^J}\right)\hat{p}^C \tag{A.15}$$

The first order conditions for production are as follows:

$$X^{JC}_{12} = w^X \left(\frac{p^J_1}{p^C_2}\right)^{-\sigma} Q^c_2$$

$$V^C_2 = = w^V_2 \left(\frac{p^{VC}_2}{p^C_2}\right)^{-\sigma} Q^c_2$$

These along with the production function (6.6) and its associated price index can be linearized as follows:

$$\hat{X}^{JC} = -\sigma\hat{p}^J + \sigma\hat{p}^C + \hat{Q}^C \tag{A.16}$$

$$\hat{V}^C = -\sigma\hat{p}^{VC}_2 + \sigma\hat{p}^C_2 + \hat{Q}^C_2 \tag{A.17}$$

$$\hat{Q}^C = \left(\frac{p^{VC}V^C}{p^C Q^C}\right)\hat{V}^C_2 + \left(\frac{p^{VJ}X^{JC}}{p^C Q^C}\right)\hat{X}^{JC} \tag{A.18}$$

$$\hat{p}^C = \left(\frac{p^{VC}V^C}{p^C Q^C}\right)\hat{p}^{VC} + \left(\frac{p^{VJ}X^{JC}}{p^C Q^C}\right)\hat{p}^{VJ} \tag{A.19}$$

Next, the market clearing conditions (4.8) and (5.14) can be linearlized as follows:

$$\hat{Q}^C = \left(\frac{p^C F^{CC}}{p^C Q^C}\right)\hat{F}^{CC} + \left(\frac{p^C F^{CJ}}{p^C Q^C}\right)\hat{F}^{CJ} + \left(\frac{p^C_2 F^{CU}_2}{p^C Q^C}\right)\hat{F}^{CU} \tag{A.20}$$

$$\hat{Q}^J = \left(\frac{p^J_1 X^{JC}}{p^J Q^J}\right)\hat{X}^{JC} + \left(\frac{p^J F^{JJ}}{p^J Q^J}\right)\hat{F}^{JJ} \tag{A.21}$$

Using these linearized first order and market clearing conditions we can derive an expression for change in demand for value added by country $C$(equation (D.13) and (D.1) )

# B Log Linearization:

### A note on notation:

- for any variable $Y^{ab}_{cd}$, $vec\left(Y^{ab}_{cd}\right)$ denotes a vector with all components of $Y^{ab}_{cd}$ stacked together
- The stacking order is as follows:$d, c, b, d$ i,e first the home sector index changes, followed by

foreign sector, followed by home country and finally foreign country

  - $vec\left(Y_{cd}^{b}\right), vec\left(Y_{c}^{ab}\right)$ etc are defined accordingly.

- Examples in a 2 by 2 case($m = n = 2$)

  - $vec\left(Y_{cd}^{ab}\right) = \left(Y_{11}^{11}, Y_{12}^{11}, Y_{21}^{11}, Y_{22}^{11}, Y_{11}^{12}, Y_{12}^{12}, Y_{21}^{12}, Y_{22}^{12}, Y_{11}^{21}, Y_{12}^{21}, Y_{21}^{21}, Y_{22}^{21}, Y_{11}^{22}, Y_{12}^{22}, Y_{21}^{22}, Y_{22}^{22}\right)'$
  - $vec\left(Y_{cd}^{b}\right) = \left(Y_{11}^{1}, Y_{12}^{1}, Y_{21}^{1}, Y_{22}^{1}, Y_{11}^{2}, Y_{12}^{2}, Y_{21}^{2}, Y_{22}^{2}\right)'$

This appendix contains the log linearized first order and market clearing conditions and organizes them in stacked matrix notation which will be useful in deriving the results that follow. A variable with a "" denotes log deviation from steady state.

Log linearizing and stacking components of production function and price indices:

Raw expression $X(f)_{sl}^{c} = \left[\sum_{i=1,i\neq c}^{n} (w_{sl}^{ic})^{1/\sigma_s^1(c,l)} (X_{sl}^{ic})^{\frac{\sigma_s^1(c,l)-1}{\sigma_s^1(c,l)}}\right]^{\frac{\sigma_s^1(c,l)}{\sigma_s^1(c,l)-1}}$

Log linearized expression $\hat{X(f)}_{sl}^{c} = \sum_{i=1,i\neq c}^{n} \left(\frac{p_s^i X_{sl}^{ic}}{q(f)_{sl}^{c} X(f)_{sl}^{c}}\right) \hat{X}_{sl}^{ic}$

Stacked vector:

$$\left(vec(\hat{X}(f)_{sl}^{c})\right) = \underbrace{W_{1XXH}}_{nm^2Xn^2m^2} vec(\hat{X}_{sl}^{ic}) \tag{B.1}$$

Raw expression $X_{sl}^{c} = \left[(w_{sl}^{c})^{1/\sigma_s^{1h}(c,l)} (X_{sl}^{cc})^{\frac{\sigma_s^{1h}(c,l)-1}{\sigma_s^{1h}(c,l)}} + (w(f)_{sl}^{c})^{1/\sigma_s^{1h}(c,l)} (X(f)_{sl}^{c})^{\frac{\sigma_s^{1h}(c,l)-1}{\sigma_s^{1h}(c,l)}}\right]^{\frac{\sigma_s^{1h}(c,l)}{\sigma_s^{1h}(c,l)-1}}$

Log linearized expression $\hat{X}_{sl}^{c} = \sum_{i=1}^{n} \left(\frac{p_s^i X_{sl}^{ic}}{q_{sl}^{c} X_{sl}^{c}}\right) \hat{X}_{sl}^{ic}$

Stacked Vector:

$$\left(vec(\hat{X}_{sl}^{c})\right) = \underbrace{W_{1XX}}_{nm^2Xn^2m^2} vec(\hat{X}_{sl}^{ic}) \tag{B.2}$$

Raw expression $X_{l}^{c} = \left[\sum_{s=1}^{m} (w_{sl}^{c})^{1/\sigma^2(c,l)} (X_{sl}^{c})^{\frac{\sigma^2(c,l)-1}{\sigma^2(c,l)}}\right]^{\frac{\sigma^2(c,l)}{\sigma^2(c,l)-1}}$

Log linearized expression $\hat{X}_{l}^{c} = \sum_{s=1}^{m} \left(\frac{q_{sl}^{c} X_{sl}^{c}}{q_{l}^{c} X_{l}^{c}}\right) \hat{X}_{sl}^{c}$

Stacked vector:

$$vec(\hat{X}_{l}^{c}) = (W_{2XX})_{nmXnm^2} \, vec(\hat{X}_{sl}^{c}) \tag{B.3}$$

Raw expression $q(f)_{sl}^{c} = \left[\sum_{i=1,i\neq c}^{n} (w_{sl}^{ic})(p_s^i)^{1-\sigma_s^1(c,l)}\right]^{\frac{1}{1-\sigma_s^1(c,l)}}$

Log linearized expression $\hat{q}_{sl}^{c}(f) = \sum_{i=1,i\neq c}^{n} \left(\frac{p_s^i X_{sl}^{ic}}{q_{sl}^{c} X_{sl}^{c}}\right) \hat{p_s^i}$

Stacked vector:

$$vec(\hat{q}_{sl}^{c}(f)) = (W_{1XPH})_{nm^2Xnm} \, vec(\hat{p}_s^i) \tag{B.4}$$

Raw expression $q^c_{sl} = \left[(w^{cc}_{sl})(p^{cc}_{sl})^{1-\sigma^{1h}(c,l)} + (w^{Xc}_{l})(q(f)^c_{sl})^{1-\sigma^{1h}(c,l)}\right]^{\frac{1}{1-\sigma^{1h}(c,l)}}$

Log linearized expression $\hat{q}^c_{sl} = \sum_{i=1}^{n} \left(\frac{p^i_s X^{ic}_{sl}}{q^c_{sl} X^c_{sl}}\right) \hat{p^i_s}$

Stacked vector:

$$vec(\hat{q}^c_{sl}) = (W_{1XP})_{nm^2Xnm}\, vec(\hat{p}^i_s) \tag{B.5}$$

Raw expression $q^c_l = \left[\sum_{s=1}^{m}(w^c_{sl})(q^c_{sl})^{1-\sigma^2(c,l)}\right]^{\frac{1}{1-\sigma^2(c,l)}}$

Log linearized expression $\hat{q}^c_l = \sum_{s=1}^{m} \left(\frac{q^c_{sl} X^c_{sl}}{q^c_l X^c_l}\right) \hat{q^c_{sl}}$

Stacked vector:

$$vec(\hat{q}^c_l) = (W_{2Xp})_{nmXnm^2}\, vec(\hat{q^c_{sl}}) \tag{B.6}$$

Raw expression $P^c_s(f) = \left[\sum_{i=1, i\neq c}^{n}(\kappa^{ic}_s)(p^i_s)^{1-\theta^1_s(c)}\right]^{\frac{1}{1-\theta^1_s(c)}}$

Log linearized expression $P(\hat{f})^c_s = \sum_{i=1, i\neq c}^{n} \left(\frac{p^i_s F^{ic}_s}{P(f)^c_s F(f)^c_s}\right) \hat{p}^i_s$

Stacked vector:

$$vec\left(\hat{P}^c_s\right)_{nmX1} = (W_{1FPH})_{nmXnm} vec\left(p^i_s\right)_{nmX1} \tag{B.7}$$

Raw expression $P^c_s = \left[(\kappa^{cc}_s)(p^{cc}_s)^{1-\theta^{1h}_s(c)} + (\kappa(f)^c_l)(P(f)^c_s)^{1-\theta^{1h}_s(c)}\right]^{\frac{1}{1-\theta^{1h}_s(c)}}$

Log linearized expression $\hat{P}^c_s = \sum_{i=1}^{n} \left(\frac{p^i_s F^{ic}_s}{P^c_s F^c_s}\right) \hat{p}^i_s$

Stacked vector:

$$vec\left(\hat{P}^c_s\right)_{nmX1} = (W_{1FP})_{nmXnm} vec\left(p^i_s\right)_{nmX1} \tag{B.8}$$

Raw expression $P^c = \left[\sum_{s=1}^{m}(\kappa^c_s)(P^c_s)^{1-\theta^2(c)}\right]^{\frac{1}{1-\theta^2(c)}}$

Log linearized expression $\hat{P}^c = \sum_{s=1}^{m} \left(\frac{P^c_s F^c_s}{P^c F^c}\right) \hat{P}^c_s$

Stacked vector:

$$vec\left(\hat{P}^c\right)_{nX1} = (W_{2FP})_{nXnm} Vec\left(\hat{P}^c_s\right)_{nmX1} \tag{B.9}$$

Final goods consumption first order conditions:

$$\hat{F_s^{ic}} = -\theta_s^1(c)(\hat{p}_s^i - P(\hat{f})_s{}^c) + F(\hat{f})_s{}^c \tag{B.10}$$
$$\hat{F_s^{cc}} = -\theta_s^{1h}(c)(\hat{p}_s^c - \hat{P}_s{}^c) + \hat{F}_s{}^c \tag{B.11}$$
$$F(\hat{f})_s^c = -\theta_s^{1h}(c)(\hat{P}(f)_s^c - \hat{P}_s{}^c) + \hat{F}_s{}^c \tag{B.12}$$
$$\hat{F_s^c} = -\theta^2(c)(\hat{P}_s^c - \hat{P}^c) + \hat{F}^c \tag{B.13}$$

We can combine these 4 conditions to write:

$$\begin{aligned}
\hat{F}_s{}^{ic} &= -\theta_s^1(c)\hat{p}_s^i + \left(\theta_s^1(c) - \theta_s^{1h}(c)\right) P(\hat{f})_s^c + \left(\theta_s^{1h}(c) - \theta^2(c)\right) \hat{P}_s^c + \theta^2(c)\hat{P}^c + \hat{F}^c \\
\hat{F}_s{}^{cc} &= -\theta_s^{1h}(c)\hat{p}_s^c + \left(\theta_s^{1h}(c) - \theta^2(c)\right) \hat{P}_s^c + \theta^2(c)\hat{P}^c + \hat{F}^c
\end{aligned}$$

We can now stack the above $n^2m$ equations to write a single matrix equation as follows:

$$\begin{aligned}
vec\left(\hat{F_s^{ic}}\right)_{n^2mX1} &= J_F(i \neq c)\left[(Y_1)_{n^2mXnm} vec(\theta_s^1(c))_{nmX1}\right] \odot \left[(Y_2)_{n^2mXnm} vec(\hat{p}_s^i)_{nmX1}\right] \\
&- -J_F(i = c)\left[(Y_1)_{n^2mXnm} vec(\theta_s^{1h}(c))_{nmX1}\right] \odot \left[(Y_2)_{n^2mXnm} vec(\hat{p}_s^i)_{nmX1}\right] \\
&+ J_F(i \neq c)\left[Y_1\left(vec(\theta_s^1(c))_{nmX1} - vec(\theta_s^{1h}(c))_{nmX1}\right)\right] \odot \left[Y_1 vec(\hat{P}(f)_s^c)_{nmX1}\right] \\
&+ \left(Y_1 vec(\theta_s^{1h}(c))_{nmX1} - (Y_3)_{n^2mXn} vec(\theta^2(c))_{nmX1}\right) \odot \left(Y_1 vec(\hat{P}_s^c)_{nmX1}\right) \\
&+ \left[Y_3 vec(\theta^2(c))_{nmX1}\right] \odot \left[Y_3 vec(\hat{P}^c)_{nX1}\right] + Y_3\hat{F}^c
\end{aligned}$$

Where $Y_1 = 1_n \otimes I_{nm}$,$Y_2 = I_n \otimes 1_n \otimes I_m$,$Y_3 = 1_n \otimes I_n \otimes 1_m$, $\odot$ is the element by element multiplication operator for two vectors and $J_F(x)$ is an $n^2m$ by 1 vector with ones in all indices that satisfy the condition $x$ and zero elsewhere.

Combining this with (B.9) and (B.8),

$$vec\left(\hat{F_s^{ic}}\right)_{n^2mX1} = Z_F vec(\hat{p}_s^i)_{nmX1} + Y_3\hat{F}^c \tag{B.14}$$

where

$$\begin{aligned}
(Z_F)_{n^2mXnm} &= J_F(i \neq c)\left[(Y_1)_{n^2mXnm} vec(\theta_s^1(c))_{nmX1}\right] \odot [(Y_2)_{n^2mXnm}] \\
&- -J_F(i = c)\left[(Y_1)_{n^2mXnm} vec(\theta_s^{1h}(c))_{nmX1}\right] \odot [(Y_2)_{n^2mXnm}] \\
&+ J_F(i \neq c)\left[Y_1\left(vec(\theta_s^1(c))_{nmX1} - vec(\theta_s^{1h}(c))_{nmX1}\right)\right] \odot [Y_1 W_{FH}] \\
&+ \left(Y_1 vec(\theta_s^{1h}(c))_{nmX1} - (Y_3)_{n^2mXn} vec(\theta^2(c))_{nmX1}\right) \odot (Y_1 W_{1FP}) \\
&+ \left[Y_3 vec(\theta^2(c))_{nmX1}\right] \odot [Y_3 W_{2FP} W_{1FP}]
\end{aligned} \tag{B.15}$$

Log linearizing Production first order conditions:

$$V_l^c = w_l^{vc}\left(\frac{p_l^{vc}}{p_l^c}\right)^{-\sigma^3(c,l)} Q_l^c \tag{B.16}$$

$$X_l^c = w_l^{Xc}\left(\frac{q_l^c}{p_l^c}\right)^{-\sigma^3(c,l)} Q_l^c \tag{B.17}$$

$$X_{sl}^c = w_{sl}^c\left(\frac{q_{sl}^c}{q_l^c}\right)^{-\sigma^2(c,l)} X_l^c \tag{B.18}$$

$$X_{sl}^{ic} = w_{sl}^{ic}\left(\frac{p_s^i}{q(f)_{sl}^c}\right)^{-\sigma_s^1(c,l)} X(f)_{sl}^c \tag{B.19}$$

$$X_{sl}^{cc} = w_{sl}^{cc}\left(\frac{p_s^c}{q_{sl}^c}\right)^{-\sigma_s^{1h}(c,l)} X_{sl}^c \tag{B.20}$$

$$X_{sl}^c(f) = w(f)_{sl}^c\left(\frac{q(f)_{sl}^c}{q_{sl}^c}\right)^{-\sigma_s^{1h}(c,l)} X_{sl}^c \tag{B.21}$$

$$\begin{aligned}
\hat{X_{sl}^{ic}} &= -\sigma_s^1(c,l)\hat{p}_s^i + \sigma_s^1(c,l)q\hat{(f)}_{sl}^c + \hat{X}(f)_{sl}^c \\
\hat{X_{sl}^{cc}} &= -\sigma_s^{1h}(c,l)\hat{p}_s^c + \sigma_s^{1h}(c,l)\hat{q}_{sl}^c + \hat{X}_{sl}^c \\
X_{sl}^c\hat{(f)} &= -\sigma_s^{1h}(c,l)\hat{q}(f)_{sl}^c + \sigma_s^{1h}(c,l)\hat{q}_{sl}^c + \hat{X}_{sl}^c \\
\hat{X_{sl}^c} &= -\sigma^2(c,l)\hat{q}_{sl}^c + \sigma^{2h}(c,l)\hat{q}_l^c + \hat{X}_l^c
\end{aligned}$$

$$\begin{aligned}
\hat{X}_{sl}^{ic} &= -\sigma_s^1(c,l)\hat{p}_s^i + \left(\sigma_s^1(c,l) - \sigma_s^{1h}(c,l)\right) q\hat{(f)}_{sl}^c + \left(\sigma_s^{1h}(c,l) - \sigma^2(c,l)\right)\hat{q}_{sl}^c \\
&+ \left(\sigma^2(c,l) - \sigma^3(c,l)\right)\hat{q}_l^c + \sigma^3(c,l)\hat{p}_l^c + \hat{Q}_l^c \\
\hat{X}_{sl}^{cc} &= -\sigma_s^{1h}(c,l)\hat{p}_s^c + \left(\sigma_s^{1h}(c,l) - \sigma^2(c,l)\right)\hat{q}_{sl}^c \\
&+ \left(\sigma^2(c,l) - \sigma^3(c,l)\right)\hat{q}_l^c + \sigma^3(c,l)\hat{p}_l^c + \hat{Q}_l^c
\end{aligned}$$

These $n^2m^2$ equations can be stacked to write

$$\begin{aligned}
vec\left(\hat{X}_{sl}^{ic}\right)_{n^2m^2} &= -J_X(i \neq c)\left[C_1 vec\left(\sigma_s^1(c,l)\right)_{nm^2X1}\right] \odot \left[C_3 vec(\hat{p}_s^i)_{nmX1}\right] \\
&- J_X(i = c)\left[C_1 vec\left(\sigma_s^{1h}(c,l)\right)_{nm^2X1}\right] \odot \left[C_3 vec(\hat{p}_s^i)_{nmX1}\right] \\
&+ J_X(i \neq c)\left[C_1\left(vec\left(\sigma_s^1(c,l)\right)_{nm^2X1} - vec\left(\sigma_s^{1h}(c,l)\right)_{nm^2X1}\right)\right] \odot \left[C_1\hat{q}(f)_{sl}^c\right] \\
&+ \left[C_2\left(vec\left(\sigma^2(c,l)\right)_{nmX1} - vec\left(\sigma^3(c,l)\right)_{nmX1}\right)\right] \odot \left[C_2\hat{q}_l^c\right] \\
&+ \left[C_1 vec\left(\sigma_s^{1h}(c,l)\right)_{nm^2X1} - C_2 vec\left(\sigma^2(c,l)\right)_{nmX1}\right] \odot \left[C_1\hat{q}_{sl}^c\right] \\
&+ \left[C_2 vec\left(\sigma^3(c,l)\right)_{nmX1}\right] \odot \left[C_2 vec(\hat{p}_s^i)_{nmX1}\right] + C_2\hat{Q}_l^c
\end{aligned}$$

where $C_1 = 1_n \otimes I_{nm^2}$,$C_2 = 1_n \otimes I_n \otimes 1_m \otimes I_m$,$C_3 = I_n \otimes 1_n \otimes I_m \otimes 1_m$. $J_X(y)$ is an $n^2m$ by 1 vector with ones in all indices that satisfy the condition $y$ and zero elsewhere.

Combining this with (B.2) - (B.6) we get:

$$vec\left(\hat{X}_{sl}^{ic}\right)_{n^2m^2} = Z_X vec(\hat{p}_s^i)_{nmX1} + C_2 \hat{Q_l}^c \tag{B.22}$$

where

$$\begin{aligned} Z_X &= -J_X(i \neq c)\left[C_1 vec\left(\sigma_s^1(c,l)\right)_{nm^2X1}\right] \odot [C_3] \\ &- \;\; J_X(i = c)\left[C_1 vec\left(\sigma_s^{1h}(c,l)\right)_{nm^2X1}\right] \odot [C_3] \\ &+ \;\; J_X(i \neq c)\left[C_1\left(vec\left(\sigma_s^1(c,l)\right)_{nm^2X1} - vec\left(\sigma_s^{1h}(c,l)\right)_{nm^2X1}\right)\right] \odot [C_1 W_{XH}] \\ &+ \;\; \left[C_2\left(vec\left(\sigma^2(c,l)\right)_{nmX1} - vec\left(\sigma^3(c,l)\right)_{nmX1}\right)\right] \odot [C_2 W_{2XP} W_{1XP}] \\ &+ \;\; \left[C_1 vec\left(\sigma_s^{1h}(c,l)\right)_{nm^2X1} - C_2 vec\left(\sigma^2(c,l)\right)_{nmX1}\right] \odot [C_1 W_{1XP}] \\ &+ \;\; \left[C_2 vec\left(\sigma^3(c,l)\right)_{nmX1}\right] \odot [C_2] \end{aligned} \tag{B.23}$$

Next, linearizing the production function we have:

$$vec\left(\hat{Q}_l^c\right) = (D_v)_{nmXnm}\left(vec\left(\hat{V}_l^c\right)\right)_{nmX1} + (D_X)_{nmXnm}\, vec\left(\hat{X}_l^c\right) \tag{B.24}$$

$$vec\left(\hat{p_l^c}\right) = D_v vec\left(\hat{p}_l^{vc}\right) + D_X vec\left(\hat{q}_l^c\right) \tag{B.25}$$

(here $D_v$ and $D_X$ are $nmXnm$ diagonal matrices denoting the shares of value added and intermediate inputs in the production of different goods , i.e the $lc^{th}$ diagonal entry of $D_v$ is $\frac{p_l^{vc}V_l^c}{p_l^c Q_l^c}$ and that of $D_X$ is $\frac{q_l^c X_l^c}{p_l^c Q_l^c}$.We can use (B.5) and (B.6) in (B.25)to obtain the following expression linking price of gross output and price of value added:

$$vec\left(\hat{p_l^c}\right) = \left(I - D_X W_{2XP} W_{1XP}\right)^{-1} D_V vec\left(\hat{p_l^{vc}}\right) \tag{B.26}$$

Log-linearizing the market clearing condition:

the market clearing conditions (4.8) can be linearized as:

$$\hat{Q}_j^i = \sum_{h=1}^{n}\sum_{l=1}^{m}\frac{X_{jl}^{ih}}{Q_j^i}\hat{X}_{jl}^{ih} + \sum_{h=1}^{n}\frac{F_j^{ih}}{Q_j^i}\hat{F_j^{ih}} \tag{B.27}$$

As before, these can be written in stacked form by creating matrices $S_X$ and $S_F$ from the above equations to yield:

$$vec\left(\hat{Q}_l^c\right) = (S_F)_{nmXn^2m}\, vec\left(\hat{F_s^{fc}}\right) + (S_X)_{nmXn^2m^2}\, vec\left(\hat{X}_{sl}^{fc}\right) \tag{B.28}$$

## C Derivations of the expressions for GVC-REER and Q-TREER((5.21) and (5.30))

From (B.28) and (B.22) we get

$$vec\left(\hat{Q}_l^c\right)\left[I_{nm} - S_X C_2\right] = \left(S_X Z_X + S_F Z_F\right) vec\left(\hat{p}_l^c\right) + S_F Y_3 vec\left(\hat{F}^c\right) \tag{C.1}$$

using (B.26) in (C.1)and rearranging we get:

$$
\begin{aligned}
vec\left(\hat{Q}_l^c\right) &= \left[I_{nm} - S_X C_2\right]^{-1}\left(S_X Z_X + S_F Z_F\right)\left(I - D_X W_{2XP} W_{1XP}\right)^{-1} D_V vec\left(\hat{p_l^{vi}}\right) \\
&+ \left[I_{nm} - S_X C_2\right]^{-1} S_F Y_3 vec\left(\hat{F^i}\right)
\end{aligned}
\tag{C.2}
$$

this is equation (5.30) in the main text.

Next, starting from the linearized production function $vec\left(\hat{Q}_l^c\right) = D_v vec\left(\hat{V}_l^c\right) + D_X vec\left(\hat{X}_l^c\right)$ we first use (B.3) and (B.2)to get:

$$
vec\left(\hat{Q}_l^c\right) = D_v vec\left(\hat{V}_l^c\right) + D_X W_{2XX} W_{1XX} vec\left(\hat{X}_{sl}^{ic}\right) \tag{C.3}
$$

substituting (B.22) in (C.3) and rearranging we get:

$$
vec\left(\hat{Q}_l^c\right)\left[I - D_X W_{2XX} W_{1XX} C_2\right] = D_v vec\left(\hat{V}_l^c\right) + D_X W_{2XX} W_{1XX} Z_X vec\left(\hat{p}_l^c\right) \tag{C.4}
$$

It can be shown that $W_{2XX} W_{1XX} C_2 = I$ and hence $\left[I - D_X W_{2XX} W_{1XX} Z_4 Z_6\right] = D_v$ so that the above expression simplifies to:

$$
vec\left(\hat{Q}_l^c\right) = vec\left(\hat{V}_l^c\right) + D_V^{-1} D_X W_{2XX} W_{1XX} Z_X \left(I - D_X W_{2XP} W_{1XP}\right)^{-1} D_V vec\left(\hat{p_l^{vc}}\right) \tag{C.5}
$$

eliminating $vec\left(\hat{Q}_l^c\right)$ from (C.2) and (C.5) we get:

$$
\begin{aligned}
vec\left(\hat{V}_l^c\right) &= \left\{\left(I_{nm} - S_X C_2\right)^{-1}\left(S_F Z_F + S_X Z_X\right) - D_v^{-1} D_X W_{2XX} W_{1XX} Z_X\right\}\left(I - D_X W_{2XP} W_{1XP}\right)^{-1} D_V vec\left(\hat{p}_l^{vc}\right) \\
&+ \left(I - S_X Z_4 Z_6\right)^{-1} S_F Y_3 vec\left(\hat{F}^c\right)
\end{aligned}
\tag{C.6}
$$

It is easy to show the following identities:

$$
\left(I_{nm} - S_X C_2\right)^{-1} = D_Q^{-1} B D_Q \tag{C.7}
$$

$$
\left(I - D_X W_{2XP} W_{1XP}\right)^{-1} = B' \tag{C.8}
$$

Substituting (C.7)and (C.8)in (C.6) we get (5.21) in the main text, with:

$$
W_V = \left[D_Q^{-1} B D_Q (S_F Z_F + S_X Z_X) - D_V^{-1} D_X W_{2XX} W_{1XX} Z_X\right] B' D_V \tag{C.9}
$$

# D Proofs of Propositions

## D.1 Sketch of Proof of Proposition 5.1

In this appendix we sketch the proof of proposition 5.1. Since the underlying intuition is preserved in the case with $m = 1$, we will sketch the proof for this simplified case.

The expression for the weighting matrix is given by:

$$w = \left\{ D_Q^{-1} B D_Q \left( S_F Z_F + S_X Z_X \right) - D_v^{-1} D_X W_{2XX} W_{1XX} Z_X \right\} B' D_V \tag{D.1}$$

As shown in proposition (6.1), under the constant elasticity assumption and $m = 1$, the GVC-REER weighting matrix reduces to VAREER weighting matrix defined in Bems and Johnson (2012), which according to equation (18) in that paper is given by

$$w = -I + D_Q^{-1} B D_Q S_F M_2 B' D_v \tag{D.2}$$

define the matrices

$Z_1 = Z_4 = 1_n \otimes I_n \equiv M_2$

$Z_2 = Z_5 = I_n \otimes 1_n \equiv M_1$

Under the constant elasticity assumption, from (B.23) and (B.15) we have:

$$Z_X = \sigma(M_2 - M_1) \tag{D.3}$$

$$Z_F = \theta(M_2 W_{FP} - M_1) \tag{D.4}$$

Taking the partial derivative of (D.1) wrt $\theta$

$$\frac{\partial w}{\partial \theta} = D_Q^{-1} B D_Q S_F (M_2 W_{FP} - M_1) B' D_V \tag{D.5}$$

using (D.2) in (D.5) , the following relationship hold for the off diagonal elements of $w$

$$\frac{\partial w^{ij}}{\partial \theta} = w^{ij} - \left[ D_Q^{-1} B D_Q S_F M_1 B' D_V \right]_{ij}, i \neq j \tag{D.6}$$

Simplifying the last term in the above expression gives (5.28) in the main text.

## D.2 Proof of Proposition (6.1):

### Part 1.

the GVC-REER weighting matrix under (A2) is given by:

$$W_V = \left\{ (I - S_X Z_4 Z_6)^{-1} - D_v^{-1} D_X W_{2XX} W_{1XX} Z_X \right\} \left( I - D_X W_{2Xp} W_{1Xp} \right)^{-1} D_V \tag{D.7}$$

where

$Z_X = \sigma_1(Z_4 W_{1XP} - Z_5) + \sigma_2(Z_4 Z_6 W_{2XP} W_{1XP} - Z_4 W_{1XP}) + \sigma_3(Z_4 Z_6 - Z_4 Z_6 W_{2XP} W_{1XP})$

with

$Z_1 = 1_n \otimes I_{nm}$ , $Z_2 = I_n \otimes (1_n \otimes I_m)$, $Z_3 = I_n \otimes 1_m$, $(Z_4)_{n^2m^2Xnm^2} = 1_n \otimes I_{nm^2}$, $(Z_5)_{n^2m^2Xnm} = I_n \otimes 1_n \otimes I_m \otimes 1_m$ and $Z_6 = (I_n \otimes 1_m) \otimes I_m$

for $m = 1$, the different matrices in the above equation simplify as:

$Z_1 = Z_4 = 1_n \otimes I_n \equiv M_2$[26]

$Z_2 = Z_5 = I_n \otimes 1_n \equiv M_1$

[26]in this section the matrices $M_1$ and $M_2$ are as defined in Bems and Johnson (2012) and are different from the ones define earlier in this paper.

$Z_3 = Z_6 = I_n$

$W_{2FP} = W_{2XX} = W_{2XP} = I_n$
$D_X W_{1Xp} = \Omega'$, where $\Omega$ is the country level input output matrix with $\Omega_{ij} = \frac{p^i X_{ij}}{p^j Q_j}$

$$\begin{aligned} Z_X &= \sigma_1(Z_4 W_{1XP} - Z_5) + \sigma_2(Z_4 Z_6 W_{2XP} W_{1XP} - Z_4 W_{1XP}) + \sigma_3(Z_4 Z_6 - Z_4 Z_6 W_{2XP} W_{1XP}) \\ &= \sigma_1(M_2 W_{1XP} - M_1) + \sigma_2(M_2 W_{1XP} - M_2 W_{1XP}) + \sigma_3(M_2 - M_2 W_{1XP}) \\ &= \sigma_1(M_2 W_{1XP} - M_1) + \sigma_3(M_2 - M_2 W_{1XP}) \end{aligned} \tag{D.8}$$

$$\begin{aligned} Z_F &= \theta_1(Z_1 W_{1FP} - Z_2) + \theta_2 (Z_1 Z_3 W_{2FP} W_{1FP} - Z_1 W_{1FP}) \\ &= \theta_1(M_2 W_{FP} - M_1) \end{aligned}$$

substituting all these in the expression for $Z_{Vclp}$ we get

$$\begin{aligned} W_V &= -\theta_1 (I - S_X M_2)^{-1} S_F(M_1 - M_2 W_{FP})(I - \Omega')^{-1} D_V \\ &+ (I - S_X M_2)^{-1} S_X [\sigma_1(M_2 W_{1XP} - M_1) + \sigma_3(M_2 - M_2 W_{1XP})] (I - \Omega')^{-1} D_V \\ &- D_V^{-1} D_X W_X [\sigma_1(M_2 W_{1XP} - M_1) + \sigma_3(M_2 - M_2 W_{1XP})] (I - \Omega')^{-1} D_V \end{aligned}$$

This is the same as equation (33) in section 5 of Bems and Johnson (2012) IOREER-BJ. part 2 and 3 follow directly from Bems and Johnson (2012).

**Part 4:**

the IMF manufacturing weights are given by (Bayoumi et al. (2005))

$$W^{ij}_{imfm} = \frac{\sum_k w^{ik} s^{jk}}{\sum_k w^{ik}(1 - s^{ik})} \tag{D.9}$$

where $s^{jk} = \frac{sales^{jk}}{\sum_l sales^{lk}}$ and $w^{ik} = \frac{sales^{ik}}{\sum_n sales^{in}}$($sales^{ij}$ denotes gross sales from country $i$ to country $j$)

substituting the expressions for $s^{jk}$ and $w^{ik}$ in $W^{ij}$ and simplifying we get:

$$W^{ij}_{imfm} = \frac{1}{T_i^{imfm}} \sum_k \left( \frac{sales^{ik}}{\sum_n sales^{in}} \right) \left( \frac{sales^{jk}}{\sum_l sales^{lk}} \right) \tag{D.10}$$

where

$$T_i^{imfm} = 1 - \sum_k \left( \frac{sales^{ik}}{\sum_n sales^{in}} \right) \left( \frac{sales^{ik}}{\sum_l sales^{lk}} \right) \tag{D.11}$$

From parts 1-3 we know that under (A1),(A2) TEER and VAREER-BJ are equivalent and given by equation (24) in BJ which is reproduced below.

$$W_{BJ}^{ij} = \frac{1}{T_i^{BJ}} \sum_k \left(\frac{p^{iv}V^{ik}}{P^{iv}V^i}\right)\left(\frac{p^{jv}V^{jk}}{P^kF^k}\right) \tag{D.12}$$

with $T_i^{BJ} = \sum_k \left(\frac{p^{iv}V^{ik}}{P^{iv}V^i}\right)\left(\frac{p^{iv}V^{ik}}{P^kF^k}\right)$

under the assumption of no intermediates (A3)we have:

- $p^{iv} = p^i$,$Q^i = V^i$,$V^{ik} = F^{ik}$
- $sales^{ik} = p^{iv}V^{ik} = p^iV^{ik}$
- $\sum_n sales^{in} = \sum_n p^{iv}V^{in} = p^{iv}V^i$
- $\sum_l sales^{lk} = \sum_l p^{lv}V^{lk} = P^kF^k$

Substituting these in (D.10) and (D.11)

$W_{imfm}^{ij} = W_{BJ}^{ij}$

Finally, using $\alpha_c = \alpha_T = 0$ we have

$W_{imf}^{ij} = W_{BJ}^{ij}$

the equivalence of IMF-REER to GOOD-SREER and IRER follows in a straightforward manner from the respective papers(Bayoumi et al. (2013) and Thorbecke (2011))

### D.3 Proof of Proposition 6.2

#### Part 1

We start with the following expression for GVC-REER weights at the country-sector level (D.7). under the constant elasticity assumption:

$$\begin{aligned} Z_X &= -Z_5 + Z_4Z_6 && \text{(D.13)} \\ Z_F &= -Z_2 + Z_1Z_3W_{2FP}W_{1FP} && \text{(D.14)} \end{aligned}$$

Here, without loss of generality we can assume that the elasticity is 1.

$$\begin{aligned} (I - S_XZ_4Z_6)^{-1} &= D_Q^{-1}BD_Q \equiv \lambda && \text{(D.15)} \\ (I - D_XW_{2XP}W_{1XP})^{-1} &= B' && \text{(D.16)} \end{aligned}$$

substituting (D.13),(D.14),(D.15) and (D.16) in (D.7)

$$\begin{aligned} W_V &= \left[(\lambda(S_FZ_F + S_XZ_X) - D_V^{-1}D_XW_{2XX}W_{1XX}Z_X\right]B'D_V \\ &= \lambda S_FZ_1Z_3W_{2FP}W_{1FP}B'D_v + \left[\lambda S_XZ_4Z_6 - \lambda(S_FZ_2 + S_XZ_5) - D_V^{-1}D_XW_{2XX}W_{1XX}Z_X\right]B'D_V && \text{(D.17)} \end{aligned}$$

Using the identities $S_F Z_2 + S_X Z_5 = I$ and $D_V - D_X W_{2XX} W_{1XX} Z_X = (I - A)' = B'^{-1}$, we can show that the second term in (D.17) is the identity matrix, so that (D.17) reduces to:

$$\begin{aligned} W_V &= -I_{nm} + \lambda S_F Z_1 Z_3 W_{2FP} W_{1FP} [B_l^c]' D_v \\ &= -I_{nm} + M_{1m} M_{2m} \end{aligned} \tag{D.18}$$

where

$$\begin{aligned} M_{1m} &= \lambda S_F Z_1 Z_3 \\ M_{2m} &= W_{2FP} W_{1FP} [B_l^c]' D_v \end{aligned}$$

Next, the country level weights (which correspond to VAREER in Bems and Johnson (2012)) are given by:

$$W_V^1 = \left\{ \left(I - S_X^1 Z_4^1 Z_6^1\right)^{-1} \left(S_F^1 Z_F^1 + S_X^1 Z_X^1\right) - (D_v^1)^{-1} D_X^1 W_{1XX} Z_X^1 \right\} \left(I - D_X^1 W_{1Xp}\right)^{-1} D_V^1 \tag{D.19}$$

(where the superscript 1 on the matrices on the RHS of (D.19) indicates that the matrix corresponds to the case where $m = 1$)

Following steps similar to those used to derive (D.18) we can get an analogous expression:

$$\begin{aligned} W_V^1 &= -I_n + \lambda^1 S_F^1 Z_1^1 Z_3^1 W_{1FP}^1 [B^c]' D_v \\ &= -I_n + M_1 M_2 \end{aligned} \tag{D.20}$$

where

$$\begin{aligned} M_1 &= \lambda^1 S_F^1 Z_1^1 Z_3^1 \\ M_2 &= W_{1FP}^1 [B^c]' D_v^1 \end{aligned} \tag{D.21}$$

the 2 country level weights are equal iff

$$R_V W_V R_g = W_V(CG) \tag{D.22}$$

‘

Since $R_V R_g = I_n$, a necessary and sufficient condition for (D.22) to hold is :

$$(R_V M_{1m})(M_{2m} R_g) = M_1 M_2 \tag{D.23}$$

$$
\begin{aligned}
(R_V M_{1m})_{ij} &= \sum_{c=1}^{n}\sum_{l=1}^{m}\sum_{s=1}^{m}\left(\frac{v_s^i b_{sl}^{ic} F_l^{cj}}{p^{vi}V^i}\right)\\
(M_{2m}R_g)_{ij} &= \sum_{c=1}^{n}\sum_{l=1}^{m}\sum_{s=1}^{m}\left(\frac{v_s^j b_{sl}^{jc} F_l^{ci}}{P^i F^i}\right)\\
(M_1)_{ij} &= \sum_{c=1}^{n}\left(\frac{v^i b^{ic} F^{cj}}{p^{vi}V^i}\right)\\
(M_2)_{ij} &= \sum_{c=1}^{n}\left(\frac{v^j b^{jc} F^{ci}}{P^i F^i}\right)
\end{aligned}
$$

here $v_s^i = \left(\frac{p_s^{vi} V_s^i}{p_s^i Q_s^i}\right)$.
From these expressions it is clear that the condition (D.23) is satisfied for all values if and only if

$$
v^i \sum_{c=1}^{n} b^{ic} f^{cj} = \sum_{l=1}^{m} v_s^i \sum_{c=1}^{n}\sum_{s=1}^{m} b_{ls}^{ic} F_s^{cj} \forall i,j \tag{D.24}
$$

or stacking these conditions in matrix notation:

$$
diag[v^c]_{nXn}[B^c]_{nXn}[F^C]_{nXn} = (M_V)_{nXnm} diag[v_l^c]_{nmXnm}[B_l^c]_{nmXnm}[F_l^c]_{nmXn} \tag{D.25}
$$

which is the same as (6.14) in the main text.

**Part 2:**

Under (A3), (A4) and $\theta_1 = \theta_2$ (=1(wlog)) we have,
$diag[v^c]_{nXn} = [B^c]_{nXn} = I_n,$
$diag[v_l^c]_{nmXnm} = [B_l^c]_{nmXnm} = I_{nm}$
$(M_V)_{nXnm}[F_l^c]_{nmXn} = [F^C]_{nXn}$
with these simplifications condition (D.25) is automatically satisfied and hence GVC-REER(CG) is equivalent to VAREER.

(D.26)

# E Bootstrap moments of elasticities

***Table 19*** – *Summary of Bootstrap moments Winsorized estimates*

| | Consumption Elasticities | | | Production Elasticities | | | |
|---|---|---|---|---|---|---|---|
| | $\theta^1$ | $\theta^{1h}$ | $\theta^2$ | $\sigma^1$ | $\sigma^{1h}$ | $\sigma^2$ | $\sigma^3$ |
| 15th percentile | 1.774 | 1.174 | 1.04 | 1.65 | 1.782 | 1.008 | 0.867 |
| median | 9.876 | 7.438 | 1.527 | 7.88 | 7.7 | 3.816 | **1.015** |
| 85th percentile | 82.535 | 65.550 | 4.73 | 67.22 | 37.607 | 14.553 | 1.501 |
| sample size | 1435(=35*41) | 41 | 41 | 2450 | 1435 | 1435 | 41 |

Note: the table reports the percentiles of bootstrap medians, for example, for $\theta^1$, 1.774 is the 15th percentile of the distribution of medians of the 1435 $\theta^1$ bootstrap distributions. We propose this quantity as our point estimate as they are more stable than the LIML point estimate. The moments reported above are based on 50 iterations.
statistics for $\sigma^1$ in this and the next table are based only on observations for 2 countries(China and the US), i.e <5percent of the total number of possible observations. To estimate all observations it will take about 60 hours.

# F Description of matrices defined in the main text with examples for the case with $n = m = 2$

This appendix includes details of each of the matrices as they appear in the text, including dimension and content. As always, $n$ stands for the number of countries and $m$ for the number of sectors within each country. An example with $n = m = 2$ is provided in each case.

## F.1 Matrix $(W_{1XX})_{nm^2Xn^2m^2}$

$$(W_{1XX})_{nm^2Xn^2m^2} = [\left(W^1_{1XX}\right)_{nm^2Xnm^2}, \left(W^2_{1XX}\right)_{nm^2Xnm^2}, .., (W^n_{1XX})_{nm^2Xnm^2}] \tag{F.1}$$

$$(W^i_{1XX})_{nm^2Xnm^2} = \begin{pmatrix} (diag(W^{i1}_{1XX}))_{m^2Xm^2} & 0_{m^2Xm^2} & .. & 0_{m^2Xm^2} \\ 0_{m^2Xm^2} & (diag(W^{i2}_{1XX}))_{m^2Xm^2} & .. & 0_{m^2Xm^2} \\ : & : & : & : \\ 0_{m^2Xm^2} & 0_{m^2Xm^2} & .. & (diag(W^{in}_{1XX}))_{m^2Xm^2} \end{pmatrix} \tag{F.2}$$

$$(W^{ij}_{1XX})_{m^2X1} = \left[\left\{\left(\frac{p^1_1 X^{ij}_{11}}{q^j_{11} X^j_{11}}\right), \left(\frac{p^1_1 X^{ij}_{12}}{q^j_{12} X^j_{12}}\right), .., \left(\frac{p^1_1 X^{ij}_{1m}}{q^j_{1m} X^j_{1m}}\right)\right\}, .., \left\{\left(\frac{p^1_m X^{ij}_{m1}}{q^j_{m1} X^j_{m1}}\right), \left(\frac{p^1_m X^{ij}_{m2}}{q^j_{m2} X^j_{m2}}\right), .., \left(\frac{p^1_m X^{ij}_{mm}}{q^j_{mm} X^j_{mm}}\right)\right\}\right]$$

**Example with $n = m = 2$**

$$
\begin{aligned}
W^{11}_{1XX} &= \left[\left(\frac{p^1_1 X^{11}_{11}}{q^1_{11} X^1_{11}}\right), \left(\frac{p^1_1 X^{11}_{12}}{q^1_{12} X^1_{12}}\right), \left(\frac{p^1_2 X^{11}_{21}}{q^1_{21} X^1_{21}}\right), \left(\frac{p^1_2 X^{11}_{22}}{q^1_{22} X^1_{22}}\right)\right] \\
W^{12}_{1XX} &= \left[\left(\frac{p^1_1 X^{12}_{11}}{q^2_{11} X^2_{11}}\right), \left(\frac{p^1_1 X^{12}_{12}}{q^2_{12} X^2_{12}}\right), \left(\frac{p^1_2 X^{12}_{21}}{q^2_{21} X^2_{21}}\right), \left(\frac{p^1_2 X^{12}_{22}}{q^2_{22} X^2_{22}}\right)\right] \\
W^{21}_{1XX} &= \left[\left(\frac{p^2_1 X^{21}_{11}}{q^1_{11} X^1_{11}}\right), \left(\frac{p^2_1 X^{21}_{12}}{q^1_{12} X^1_{12}}\right), \left(\frac{p^2_2 X^{21}_{21}}{q^1_{21} X^1_{21}}\right), \left(\frac{p^2_2 X^{21}_{22}}{q^1_{22} X^1_{22}}\right)\right] \\
W^{22}_{1XX} &= \left(\frac{p^2_1 X^{22}_{11}}{q^2_{11} X^2_{11}}\right), \left(\frac{p^2_1 X^{22}_{12}}{q^2_{12} X^2_{12}}\right), \left(\frac{p^2_2 X^{22}_{21}}{q^2_{21} X^2_{21}}\right), \left(\frac{p^2_2 X^{22}_{22}}{q^2_{22} X^2_{22}}\right)
\end{aligned}
$$

$$
W^1_{1XX} = \begin{pmatrix}
\left(\frac{p^1_1 X^{11}_{11}}{q^1_{11} X^1_{11}}\right) & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & \left(\frac{p^1_1 X^{11}_{12}}{q^1_{12} X^1_{12}}\right) & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & \left(\frac{p^1_2 X^{11}_{21}}{q^1_{21} X^1_{21}}\right) & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & \left(\frac{p^1_2 X^{11}_{22}}{q^1_{22} X^1_{22}}\right) & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & \left(\frac{p^1_1 X^{12}_{11}}{q^2_{11} X^2_{11}}\right) & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & \left(\frac{p^1_1 X^{12}_{12}}{q^2_{12} X^2_{12}}\right) & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & \left(\frac{p^1_2 X^{12}_{21}}{q^2_{21} X^2_{21}}\right) & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & \left(\frac{p^1_2 X^{12}_{22}}{q^2_{22} X^2_{22}}\right)
\end{pmatrix}
$$

$$
W^2_{1XX} = \begin{pmatrix}
\left(\frac{p^2_1 X^{21}_{11}}{q^1_{11} X^1_{11}}\right) & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & \left(\frac{p^2_1 X^{21}_{12}}{q^1_{12} X^1_{12}}\right) & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & \left(\frac{p^2_2 X^{21}}{q^1_{21} X^1_{21}}\right) & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & \left(\frac{p^2_2 X^{21}_{22}}{q^1_{22} X^1_{22}}\right) & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & \left(\frac{p^2_1 X^{22}_{11}}{q^2_{11} X^2_{11}}\right) & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & \left(\frac{p^2_1 X^{22}_{12}}{q^2_{12} X^2_{12}}\right) & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & \left(\frac{p^2_2 X^{22}_{21}}{q^2_{21} X^2_{21}}\right) & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & \left(\frac{p^2_2 X^{22}_{22}}{q^2_{22} X^2_{22}}\right)
\end{pmatrix}
$$

$W_{XX} = [W^1_{1XX}, W^2_{1XX}]$

### F.2 Matrices $(W_{2XX})_{nmXnm^2}$ and $(W_{2Xp})_{nmXnm^2}$ $((W_{2XP})_{nmXnm^2}{=}(W_{2XX})_{nmXnm^2})$

$$
(W_{2Xp})_{nmXnm^2} = \begin{pmatrix}
\left(W^1_{2Xp}\right)_{mXm^2} & 0_{mXm^2} & .. & 0_{mXm^2} \\
0_{mXm^2} & \left(W^2_{2Xp}\right)_{mXm^2} & .. & 0_{mXm^2} \\
\vdots & \vdots & & \vdots \\
0_{mXm^2} & 0_{mXm^2} & .. & \left(W^n_{2Xp}\right)_{mXm^2}
\end{pmatrix} \tag{F.3}
$$

$$\left(W_{2Xp}^{i}\right)_{mXm^2} = \left(\left(W_{2Xp}^{i,1}\right)_{mXm}, \left(W_{2Xp}^{i,2}\right)_{mXm}, .., \left(W_{2Xp}^{i,m}\right)_{mXm}\right) \tag{F.4}$$

$$\left(W_{2Xp}^{i,k}\right)_{mXm} = diag\left(\frac{q_{k1}^{i}X_{k1}^{i}}{q_1^{i}X_1^{i}}, \frac{q_{k2}^{i}X_{k2}^{i}}{q_2^{i}X_2^{i}}, .., \frac{q_{km}^{i}X_{km}^{i}}{q_m^{i}X_m^{i}}\right) \tag{F.5}$$

Example with $n = m = 2$

$$\begin{aligned}
\left(W_{2Xp}^{1,1}\right)_{mXm} &= \begin{pmatrix} \frac{q_{11}^1X_{11}^1}{q_1^1X_1^1} & 0 \\ 0 & \frac{q_{12}^1X_{12}^1}{q_2^1X_2^1} \end{pmatrix} \\
\left(W_{2Xp}^{1,2}\right)_{mXm} &= \begin{pmatrix} \frac{q_{21}^1X_{21}^1}{q_1^1X_1^1} & 0 \\ 0 & \frac{q_{22}^1X_{22}^1}{q_2^1X_2^1} \end{pmatrix} \\
\left(W_{2Xp}^{2,1}\right)_{mXm} &= \begin{pmatrix} \frac{q_{11}^2X_{11}^2}{q_1^2X_1^2} & 0 \\ 0 & \frac{q_{12}^2X_{12}^2}{q_2^2X_2^2} \end{pmatrix} \\
\left(W_{2Xp}^{2,2}\right)_{mXm} &= \begin{pmatrix} \frac{q_{21}^2X_{21}^2}{q_1^2X_1^2} & 0 \\ 0 & \frac{q_{22}^2X_{22}^2}{q_2^2X_2^2} \end{pmatrix}
\end{aligned}$$

$$\begin{aligned}
\left(W_{2Xp}^{1}\right)_{mXm^2} &= \begin{pmatrix} \frac{q_{11}^1X_{11}^1}{q_1^1X_1^1} & 0 & \frac{q_{21}^1X_{21}^1}{q_1^1X_1^1} & 0 \\ 0 & \frac{q_{12}^1X_{12}^1}{q_2^1X_2^1} & 0 & \frac{q_{22}^1X_{22}^1}{q_2^1X_2^1} \end{pmatrix} \\
\left(W_{2Xp}^{1}\right)_{mXm^2} &= \begin{pmatrix} \frac{q_{11}^2X_{11}^2}{q_1^2X_1^2} & 0 & \frac{q_{21}^2X_{21}^2}{q_1^2X_1^2} & 0 \\ 0 & \frac{q_{12}^2X_{12}^2}{q_2^2X_2^2} & 0 & \frac{q_{22}^2X_{22}^2}{q_2^2X_2^2} \end{pmatrix}
\end{aligned}$$

$$\left(W_{2Xp}\right)_{nmXnm^2} = \begin{pmatrix} \frac{q_{11}^1X_{11}^1}{q_1^1X_1^1} & 0 & \frac{q_{21}^1X_{21}^1}{q_1^1X_1^1} & 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{q_{12}^1X_{12}^1}{q_2^1X_2^1} & 0 & \frac{q_{22}^1X_{22}^1}{q_2^1X_2^1} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{q_{11}^2X_{11}^2}{q_1^2X_1^2} & 0 & \frac{q_{21}^2X_{21}^2}{q_1^2X_1^2} & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{q_{12}^2X_{12}^2}{q_2^2X_2^2} & 0 & \frac{q_{22}^2X_{22}^2}{q_2^2X_2^2} \end{pmatrix}$$

**Matrix** $\left(W_{1XP}\right)_{nm^2Xnm}$

$$\left(W_{1XP}\right)_{nm^2Xnm} = \begin{pmatrix} \left(W_{1Xp}^{11}\right)_{m^2Xm} & \left(W_{1Xp}^{21}\right)_{m^2Xm} & .. & \left(W_{1Xp}^{n1}\right)_{m^2Xm} \\ \left(W_{1Xp}^{12}\right)_{m^2Xm} & \left(W_{1Xp}^{22}\right)_{m^2Xm} & .. & \left(W_{1Xp}^{n2}\right)_{m^2Xm} \\ \vdots & \vdots & \vdots & \vdots \\ \left(W_{1Xp}^{1n}\right)_{m^2Xm} & \left(W_{1Xp}^{2n}\right)_{m^2Xm} & & \left(W_{1Xp}^{nn}\right)_{m^2Xm} \end{pmatrix} \tag{F.6}$$

$$\left(W_{1Xp}^{ij}\right)_{m^2Xm} = \begin{pmatrix} \left(W_{1Xp}^{ij}\right)_{mX1}^{1} & 0_{mX1} & .. & 0_{mX1} \\ 0_{mX1} & \left(W_{1Xp}^{ij}\right)_{mX1}^{2} & .. & 0_{mX1} \\ : & : & : & : \\ 0_{mX1} & 0_{mX1} & .. & \left(W_{1Xp}^{ij}\right)_{mX1}^{m} \end{pmatrix} \tag{F.7}$$

$$\left(W_{1Xp}^{ij}\right)_{mX1}^{k} = \left(\frac{p_k^i X_{k1}^{ij}}{q_{k1}^j X_{k1}^j}, \frac{p_k^i X_{k2}^{ij}}{q_{k2}^j X_{k2}^j}, .., \frac{p_k^i X_{km}^{ij}}{q_{km}^j X_{km}^j}\right)' \tag{F.8}$$

Example with $n = m = 2$

$$\begin{aligned}
\left(W_{1Xp}^{11}\right)^1 &= \left(\frac{p_1^1 X_{11}^{11}}{q_{11}^1 X_{11}^1}, \frac{p_1^1 X_{12}^{11}}{q_{12}^1 X_{12}^1}\right)' \\
\left(W_{1Xp}^{11}\right)^2 &= \left(\frac{p_2^1 X_{21}^{11}}{q_{21}^1 X_{21}^1}, \frac{p_2^1 X_{22}^{11}}{q_{22}^1 X_{22}^1}\right)' \\
\left(W_{1Xp}^{21}\right)^1 &= \left(\frac{p_1^2 X_{11}^{21}}{q_{11}^1 X_{11}^1}, \frac{p_1^2 X_{12}^{21}}{q_{12}^1 X_{12}^1}\right)' \\
\left(W_{1Xp}^{21}\right)^2 &= \left(\frac{p_2^2 X_{21}^{21}}{q_{21}^1 X_{21}^1}, \frac{p_2^2 X_{22}^{21}}{q_{22}^1 X_{22}^1}\right)' \\
\left(W_{1Xp}^{12}\right)^1 &= \left(\frac{p_1^1 X_{11}^{12}}{q_{11}^2 X_{11}^2}, \frac{p_1^1 X_{12}^{12}}{q_{12}^2 X_{12}^2}\right)' \\
\left(W_{1Xp}^{12}\right)^2 &= \left(\frac{p_2^1 X_{21}^{12}}{q_{21}^2 X_{21}^2}, \frac{p_2^1 X_{22}^{12}}{q_{22}^2 X_{22}^2}\right)' \\
\left(W_{1Xp}^{22}\right)^1 &= \left(\frac{p_1^2 X_{11}^{22}}{q_{11}^2 X_{11}^2}, \frac{p_1^2 X_{12}^{22}}{q_{12}^2 X_{12}^2}\right)' \\
\left(W_{1Xp}^{22}\right)^2 &= \left(\frac{p_2^2 X_{21}^{22}}{q_{21}^2 X_{21}^2}, \frac{p_2^2 X_{22}^{22}}{q_{22}^2 X_{22}^2}\right)'
\end{aligned}$$

$$
\begin{aligned}
\left(W^{11}_{1XP}\right)_{4X2} &= \begin{pmatrix} \frac{p_1^1 X_{11}^{11}}{q_{11}^1 X_{11}^1} & 0 \\ \frac{p_1^1 X_{12}^{11}}{q_{12}^1 X_{12}^1} & 0 \\ 0 & \frac{p_2^1 X_{21}^{11}}{q_{21}^1 X_{21}^1} \\ 0 & \frac{p_2^1 X_{22}^{11}}{q_{22}^1 X_{22}^1} \end{pmatrix} \\
\left(W^{21}_{1XP}\right) &= \begin{pmatrix} \frac{p_1^2 X_{11}^{21}}{q_{11}^1 X_{11}^1} & 0 \\ \frac{p_1^2 X_{12}^{21}}{q_{12}^1 X_{12}^1} & 0 \\ 0 & \frac{p_2^2 X_{21}^{21}}{q_{21}^1 X_{21}^1} \\ 0 & \frac{p_2^2 X_{22}^{21}}{q_{22}^1 X_{22}^1} \end{pmatrix} \\
\left(W^{12}_{1XP}\right) &= \begin{pmatrix} \frac{p_1^1 X_{11}^{12}}{q_{11}^2 X_{11}^2} & 0 \\ \frac{p_1^1 X_{12}^{12}}{q_{12}^2 X_{12}^2} & 0 \\ 0 & \frac{p_2^1 X_{21}^{12}}{q_{21}^2 X_{21}^2} \\ 0 & \frac{p_2^1 X_{22}^{12}}{q_{22}^2 X_{22}^2} \end{pmatrix} \\
\left(W^{22}_{1XP}\right) &= \begin{pmatrix} \frac{p_1^2 X_{11}^{22}}{q_{11}^2 X_{11}^2} & 0 \\ \frac{p_1^2 X_{12}^{22}}{q_{12}^2 X_{12}^2} & 0 \\ 0 & \frac{p_2^2 X_{21}^{22}}{q_{21}^2 X_{21}^2} \\ 0 & \frac{p_2^2 X_{22}^{22}}{q_{22}^2 X_{22}^2} \end{pmatrix}
\end{aligned}
$$

$$
W_{1XP} = \begin{pmatrix}
\frac{p_1^1 X_{11}^{11}}{q_{11}^1 X_{11}^1} & 0 & \frac{p_1^2 X_{11}^{21}}{q_{11}^1 X_{11}^1} & 0 \\
\frac{p_1^1 X_{12}^{11}}{q_{12}^1 X_{12}^1} & 0 & \frac{p_1^2 X_{12}^{21}}{q_{12}^1 X_{12}^1} & 0 \\
0 & \frac{p_2^1 X_{21}^{11}}{q_{21}^1 X_{21}^1} & 0 & \frac{p_2^2 X_{21}^{21}}{q_{21}^1 X_{21}^1} \\
0 & \frac{p_2^1 X_{22}^{11}}{q_{22}^1 X_{22}^1} & 0 & \frac{p_2^2 X_{22}^{21}}{q_{22}^1 X_{22}^1} \\
\frac{p_1^1 X_{11}^{12}}{q_{11}^2 X_{11}^2} & 0 & \frac{p_1^2 X_{11}^{22}}{q_{11}^2 X_{11}^2} & 0 \\
\frac{p_1^1 X_{12}^{12}}{q_{12}^2 X_{12}^2} & 0 & \frac{p_1^2 X_{12}^{22}}{q_{12}^2 X_{12}^2} & 0 \\
0 & \frac{p_2^1 X_{21}^{12}}{q_{21}^2 X_{21}^2} & 0 & \frac{p_2^2 X_{21}^{22}}{q_{21}^2 X_{21}^2} \\
0 & \frac{p_2^1 X_{22}^{12}}{q_{22}^2 X_{22}^2} & 0 & \frac{p_2^2 X_{22}^{22}}{q_{22}^2 X_{22}^2}
\end{pmatrix} \tag{F.9}
$$

**Matrix** $(S_X)_{nmXn^2m^2}$

$$
(S_X)_{nmXn^2m^2} = \begin{pmatrix}
(S_X^1)_{mXnm^2} & 0_{mXnm^2} & .. & 0_{mXnm^2} \\
0_{mXnm^2} & (S_X^2)_{mXnm^2} & .. & 0_{mXnm^2} \\
\vdots & \vdots & \vdots & \vdots \\
0_{mXnm^2} & 0_{mXnm^2} & .. & (S_X^n)_{mXnm^2}
\end{pmatrix} \tag{F.10}
$$

$$(S_X^i)_{mXnm^2} = \left(\left(S_X^{i1}\right)_{mXm^2}, \left(S_X^{i2}\right)_{mXm^2}, ..., \left(S_X^{in}\right)_{mXm^2}\right) \tag{F.11}$$

$$(S_X^{ij})_{mXm^2} = \begin{pmatrix} \left(S_{X1}^{ij}\right)_{1Xm} & 0_{1Xm} & .. & 0_{1Xm} \\ 0_{1Xm} & \left(S_{X2}^{ij}\right)_{1Xm} & .. & 0_{1Xm} \\ \vdots & \vdots & \vdots & \vdots \\ 0_{1Xm} & 0_{1Xm} & .. & \left(S_{Xm}^{ij}\right)_{1Xm} \end{pmatrix} \tag{F.12}$$

$$\left(S_{Xk}^{ij}\right)_{1Xm} = \left(\frac{X_{k1}^{ij}}{Q_k^i}, \frac{X_{k2}^{ij}}{Q_k^i}, .., \frac{X_{km}^{ij}}{Q_k^i}\right) \tag{F.13}$$

Example with $n = m = 2$:

$(S_{X1}^{11})_{1X2} = \left(\frac{X_{11}^{11}}{Q_1^1}, \frac{X_{12}^{11}}{Q_1^1}\right)$

$(S_{X2}^{11}) = \left(\frac{X_{21}^{11}}{Q_2^1}, \frac{X_{22}^{11}}{Q_2^1}\right)$

$(S_{X1}^{12}) = \left(\frac{X_{11}^{12}}{Q_1^1}, \frac{X_{12}^{12}}{Q_1^1}\right)$

$(S_{X2}^{12}) = \left(\frac{X_{21}^{12}}{Q_2^1}, \frac{X_{22}^{12}}{Q_2^1}\right)$

$(S_{X1}^{21}) = (\frac{X_{11}^{21}}{Q_1^2}, \frac{X_{12}^{21}}{Q_1^2})$

$(S_{X2}^{21}) = \left(\frac{X_{21}^{21}}{Q_2^2}, \frac{X_{22}^{21}}{Q_2^2}\right)$

$(S_{X1}^{22}) = (\frac{X_{11}^{22}}{Q_1^2}, \frac{X_{12}^{22}}{Q_1^2})$

$(S_{X2}^{22}) = (\frac{X_{21}^{22}}{Q_2^2}, \frac{X_{22}^{22}}{Q_2^2})$

$S_X^{11} = \begin{pmatrix} \frac{X_{11}^{11}}{Q_1^1} & \frac{X_{12}^{11}}{Q_1^1} & 0 & 0 \\ 0 & 0 & \frac{X_{21}^{11}}{Q_2^1} & \frac{X_{22}^{11}}{Q_2^1} \end{pmatrix}$

$S_X^{12} = \begin{pmatrix} \frac{X_{11}^{12}}{Q_1^1} & \frac{X_{12}^{12}}{Q_1^1} & 0 & 0 \\ 0 & 0 & \frac{X_{21}^{12}}{Q_2^1} & \frac{X_{22}^{12}}{Q_2^1} \end{pmatrix}$

$S_X^{21} = \begin{pmatrix} \frac{X_{11}^{21}}{Q_1^2} & \frac{X_{12}^{21}}{Q_1^2} & 0 & 0 \\ 0 & 0 & \frac{X_{21}^{21}}{Q_2^2} & \frac{X_{22}^{21}}{Q_2^2} \end{pmatrix}$

$S_X^{22} = \begin{pmatrix} \frac{X_{11}^{22}}{Q_1^2} & \frac{X_{12}^{22}}{Q_1^2} & 0 & 0 \\ 0 & 0 & \frac{X_{21}^{22}}{Q_2^2} & \frac{X_{22}^{22}}{Q_2^2} \end{pmatrix}$

$S_X^1 = \begin{pmatrix} \frac{X_{11}^{11}}{Q_1^1} & \frac{X_{12}^{11}}{Q_1^1} & 0 & 0 & \frac{X_{11}^{12}}{Q_1^1} & \frac{X_{12}^{12}}{Q_1^1} & 0 & 0 \\ 0 & 0 & \frac{X_{21}^{11}}{Q_2^1} & \frac{X_{22}^{11}}{Q_2^1} & 0 & 0 & \frac{X_{21}^{12}}{Q_2^1} & \frac{X_{22}^{12}}{Q_2^1} \end{pmatrix}$

$S_X^2 = \begin{pmatrix} \frac{X_{11}^{21}}{Q_1^2} & \frac{X_{12}^{21}}{Q_1^2} & 0 & 0 & \frac{X_{11}^{22}}{Q_1^2} & \frac{X_{12}^{22}}{Q_1^2} & 0 & 0 \\ 0 & 0 & \frac{X_{21}^{21}}{Q_2^2} & \frac{X_{22}^{21}}{Q_2^2} & 0 & 0 & \frac{X_{21}^{22}}{Q_2^2} & \frac{X_{22}^{22}}{Q_2^2} \end{pmatrix}$

$$S_X = \begin{pmatrix} \frac{X_{11}^{11}}{Q_1^1} & \frac{X_{12}^{11}}{Q_1^1} & 0 & 0 & \frac{X_{11}^{12}}{Q_1^1} & \frac{X_{12}^{12}}{Q_1^1} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{X_{21}^{11}}{Q_2^1} & \frac{X_{22}^{11}}{Q_2^1} & 0 & 0 & \frac{X_{21}^{12}}{Q_2^1} & \frac{X_{22}^{12}}{Q_2^1} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{X_{11}^{21}}{Q_1^2} & \frac{X_{12}^{21}}{Q_1^2} & 0 & 0 & \frac{X_{11}^{22}}{Q_1^2} & \frac{X_{12}^{22}}{Q_1^2} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{X_{21}^{21}}{Q_2^2} & \frac{X_{22}^{21}}{Q_2^2} & 0 & 0 & \frac{X_{21}^{22}}{Q_2^2} & \frac{X_{22}^{22}}{Q_2^2} \end{pmatrix}$$

**Matrix** $(S_F)_{nmXn^2m}$

$$(S_F)_{nmXn^2m} = \begin{pmatrix} (S_F^1)_{mXnm} & 0_{mXnm} & .. & 0_{mXnm} \\ 0_{mXnm} & (S_F^2)_{mXnm} & .. & 0_{mXnm} \\ : & : & : & : \\ 0_{mXnm} & 0_{mXnm} & .. & (S_F^n)_{mXnm} \end{pmatrix}$$

$$(S_F^i)_{mXnm} = ((S_F^{i1})_{mXm}, (S_F^{i2})_{mXm}, ..., (S_F^{in})_{mXm})$$

$$(S_F^{ij})_{mXm} = \begin{pmatrix} S_{F1}^{ij} & 0 & .. & 0 \\ 0 & S_{F2}^{ij} & .. & 0 \\ : & : & : & : \\ 0 & 0 & .. & S_{Fm}^{ij} \end{pmatrix}$$

$$\left(S_{Fk}^{ij}\right)_{1X1} = \left(\frac{F_k^{ij}}{Q_k^i}\right)$$

Example with $n = m = 2$:

$S_{F1}^{11} = \frac{F_1^{11}}{Q_1^1}$, $S_{F2}^{11} = \frac{F_2^{11}}{Q_2^1}$, $S_{F1}^{12} = \frac{F_1^{12}}{Q_1^1}$, $S_{F2}^{12} = \frac{F_2^{12}}{Q_2^1}$

$S_{F1}^{21} = \frac{F_1^{21}}{Q_1^2}$, $S_{F2}^{21} = \frac{F_2^{21}}{Q_2^2}$, $S_{F1}^{22} = \frac{F_1^{22}}{Q_1^2}$,$S_{F2}^{22} = \frac{F_2^{22}}{Q_2^2}$

$$S_F^{11} = \begin{pmatrix} \frac{F_1^{11}}{Q_1^1} & 0 \\ 0 & \frac{F_2^{11}}{Q_2^1} \end{pmatrix}, \; S_F^{12} = \begin{pmatrix} \frac{F_1^{12}}{Q_1^1} & 0 \\ 0 & \frac{F_2^{12}}{Q_2^1} \end{pmatrix}$$

$$S_F^{21} = \begin{pmatrix} \frac{F_1^{21}}{Q_1^2} & 0 \\ 0 & \frac{F_2^{21}}{Q_2^2} \end{pmatrix}, S_F^{22} = \begin{pmatrix} \frac{F_1^{22}}{Q_1^2} & 0 \\ & \frac{F_2^{22}}{Q_2^2} \end{pmatrix}$$

$$S_F^1 = \begin{pmatrix} \frac{F_1^{11}}{Q_1^1} & 0 & \frac{F_1^{12}}{Q_1^1} & 0 \\ 0 & \frac{F_2^{11}}{Q_2^1} & 0 & \frac{F_2^{12}}{Q_2^1} \end{pmatrix}, S_F^2 = \begin{pmatrix} \frac{F_1^{21}}{Q_1^2} & 0 & \frac{F_1^{22}}{Q_1^2} & 0 \\ 0 & \frac{F_2^{21}}{Q_2^2} & & \frac{F_2^{22}}{Q_2^2} \end{pmatrix}$$

$$S_F = \begin{pmatrix} \frac{F_1^{11}}{Q_1^1} & 0 & \frac{F_1^{12}}{Q_1^1} & 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{F_2^{11}}{Q_2^1} & 0 & \frac{F_2^{12}}{Q_2^1} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{F_1^{21}}{Q_1^2} & 0 & \frac{F_1^{22}}{Q_1^2} & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{F_2^{21}}{Q_2^2} & & \frac{F_2^{22}}{Q_2^2} \end{pmatrix}$$

**Matrix** $(W_{1FP})_{nmXnm}$

$$W_{1FP} = \begin{pmatrix} (W_{1FP}^1)\, mXnm \\ (W_{1FP}^2)\, mXnm \\ : \\ (W_{1FP}^n)\, mXnm \end{pmatrix}$$

where $\left(W_{1FP}^k\right) = \left[\left(W_{1FP}^{1k}\right)_{mXm} \left(W_{1FP}^{2k}\right)_{mXm} \cdots \left(W_{1FP}^{nk}\right)_{mXm}\right]$

$\left(W_{1FP}^{jk}\right)_{mXm} = diag\left(\frac{p_1^j F_1^{jk}}{P_1^k F_1^k}, \frac{p_2^j F_2^{jk}}{P_2^k F_2^k}, ..., \frac{p_m^j F_m^{jk}}{P_m^k F_m^k}\right)$
Example with $n = m = 2$

$$W_{1FP} = \begin{pmatrix} \frac{p_1^1 F_1^{11}}{P_1^1 F_1^1} & 0 & \frac{p_1^2 F_1^{21}}{P_1^1 F_1^1} & 0 \\ 0 & \frac{p_2^1 F_2^{11}}{P_2^1 F_2^1} & 0 & \frac{p_2^2 F_2^{21}}{P_2^1 F_2^1} \\ \frac{p_1^1 F_1^{12}}{P_1^2 F_1^2} & 0 & \frac{p_1^2 F_1^{22}}{P_1^2 F_1^2} & 0 \\ 0 & \frac{p_2^1 F_2^{12}}{P_2^2 F_2^2} & 0 & \frac{p_2^2 F_2^{22}}{P_2^2 F_2^2} \end{pmatrix}$$

**Matrix** $(W_{2FP})_{nXnm}$

$W_{2FP} = [(W_{2FP}^1)_{nXm} \, (W_{2FP}^2)_{nXm} \, , .., (W_{2FP}^n)_{nXm}]$
$(W_{2FP}^i)_{nXm}$ is a matrix with the $i^{th}$ row given by $\left(\frac{P_1^i F_1^i}{P^i F^i}, \frac{P_2^i F_2^i}{P^i F^i}, ..., \frac{P_m^i F_m^i}{P^i F^i}\right)$
Example with $n = m = 2$

$$W_{2FP} = \begin{pmatrix} \frac{P_1^1 F_1^1}{P^1 F^1} & \frac{P_2^1 F_2^1}{P^1 F^1} & 0 & 0 \\ 0 & 0 & \frac{P_1^2 F_1^2}{P^2 F^2} & \frac{P_2^2 F_2^2}{P^2 F^2} \end{pmatrix}$$

**Matrix** $(D_V)_{nmXnm}$

$$(D_V)_{nmXnm} = diag\left(\underbrace{\frac{p_1^{v1} V_1^1}{p_1^1 Q_1^1}, \frac{p_2^{v1} V_2^1}{p_2^1 Q_2^1}, ..., \frac{p_m^{v1} V_m^1}{p_m^1 Q_m^1}}_{mX1}, \underbrace{\frac{p_1^{v2} V_1^2}{p_1^2 Q_1^2}, \frac{p_2^{v2} V_2^2}{p_2^2 Q_2^2}, ..., \frac{p_m^{v2} V_m^2}{p_m^2 Q_m^2}}_{mX1}, .., \underbrace{\frac{p_1^{vn} V_1^n}{p_1^n Q_1^n}, \frac{p_2^{vn} V_2^n}{p_2^n Q_2^n}, ..., \frac{p_m^{vn} V_m^n}{p_m^n Q_m^n}}_{mX1}\right)$$

Example with $m = n = 2$

$$(D_V)_{4X4} = diag\left(\underbrace{\frac{p_1^{v1} V_1^1}{p_1^1 Q_1^1}, \frac{p_2^{v1} V_2^1}{p_2^1 Q_2^1}}_{2X1}, \underbrace{\frac{p_1^{v2} V_1^2}{p_1^2 Q_1^2}, \frac{p_2^{v2} V_2^2}{p_2^2 Q_2^2}}_{2X1}\right)$$

**Matrix** $(D_X)_{nmXnm}$

$$(D_X)_{nmXnm} = diag\left(\underbrace{\frac{q_1^1 X_1^1}{p_1^1 Q_1^1}, \frac{q_2^1 X_2^1}{p_2^1 Q_2^1}, ..., \frac{q_m^1 X_m^1}{p_m^1 Q_m^1}}_{mX1}, \underbrace{\frac{q_1^2 X_1^2}{p_1^2 Q_1^2}, \frac{q_2^2 X_2^2}{p_2^2 Q_2^2}, ..., \frac{q_m^2 X_m^2}{p_m^2 Q_m^2}}_{mX1}, .., \underbrace{\frac{q_1^n X_1^n}{p_1^n Q_1^n}, \frac{q_2^n X_2^n}{p_2^n Q_2^n}, ..., \frac{q_m^n X_m^n}{p_m^n Q_m^n}}_{mX1}\right)$$

Example with $n = m = 2$

$$(D_X)_{4X4} = diag\left(\underbrace{\frac{q_1^1 X_1^1}{p_1^1 Q_1^1}, \frac{q_2^1 X_2^1}{p_2^1 Q_2^1}}_{2X1}, \underbrace{\frac{q_1^2 X_1^2}{p_1^2 Q_1^2}, \frac{q_2^2 X_2^2}{p_2^2 Q_2^2}}_{2X1}\right)$$

# G General algebra and results wi

# H Algebra with Armington aggregators

$$M = \left[ \sum_g (w_g)^{1/\delta} (m_g)^{\frac{\delta-1}{\delta}} \right]^{\frac{\delta}{\delta-1}} \tag{H.1}$$

foc:

$$m_g = w_g \left( \frac{p_g}{p_M} \right)^{-\delta} M \tag{H.2}$$

price index:

$$P_M = \left[ \sum_g (w_g)(p_g)^{1-\delta} \right]^{\frac{1}{1-\delta}} \tag{H.3}$$

linearized version:

$$\hat{M} = \sum_g \left( \frac{p_g m_g}{P_M M} \right) \hat{m_g} \tag{H.4}$$

$$\hat{P_M} = \sum_g \left( \frac{p_g m_g}{P_M M} \right) \hat{p_g} \tag{H.5}$$

# I List of countries and sectors

| | Countries | | Sectors |
|---|---|---|---|
| 1 | 'Australia' | 1 | 'Agriculture, Hunting, Forestry and Fishing' |
| 2 | 'Austria' | 2 | 'Mining and Quarrying' |
| 3 | 'Belgium' | 3 | 'Food, Beverages and Tobacco' |
| 4 | 'Bulgaria' | 4 | 'Textiles and Textile Products' |
| 5 | 'Brazil' | 5 | 'Leather, Leather and Footwear' |
| 6 | 'Canada' | 6 | 'Wood and Products of Wood and Cork' |
| 7 | 'China' | 7 | 'Pulp, Paper, Paper , Printing and Publishing' |
| 8 | 'Cyprus' | 8 | 'Coke, Refined Petroleum and Nuclear Fuel' |
| 9 | 'Czech Republic' | 9 | 'Chemicals and Chemical Products' |
| 10 | 'Germany' | 10 | 'Rubber and Plastics' |
| 11 | 'Denmark' | 11 | 'Other Non-Metallic Mineral' |
| 12 | 'Spain' | 12 | 'Basic Metals and Fabricated Metal' |
| 13 | 'Estonia' | 13 | 'Machinery, Nec' |
| 14 | 'Finland' | 14 | 'Electrical and Optical Equipment' |
| 15 | 'France' | 15 | 'Transport Equipment' |
| 16 | 'United Kingdom' | 16 | 'Manufacturing, Nec; Recycling' |
| 17 | 'Greece' | 17 | 'Electricity, Gas and Water Supply' |
| 18 | 'Hungary' | 18 | 'Construction' |
| 19 | 'Indonesia' | 19 | 'Sale, Maintenance and Repair of Motor Vehicles and Motorcycles; Retail Sale of Fuel' |
| 20 | 'India' | 20 | 'Wholesale Trade and Commission Trade, Except of Motor Vehicles and Motorcycles' |
| 21 | 'Ireland' | 21 | 'Retail Trade, Except of Motor Vehicles and Motorcycles; Repair of Household Goods' |
| 22 | 'Italy' | 22 | 'Hotels and Restaurants' |
| 23 | 'Japan' | 23 | 'Inland Transport' |
| 24 | 'Korea' | 24 | 'Water Transport' |
| 25 | 'Lithuania' | 25 | 'Air Transport' |
| 26 | 'Luxembourg' | 26 | 'Other Supporting and Auxiliary Transport Activities; Activities of Travel Agencies' |
| 27 | 'Latvia' | 27 | 'Post and Telecommunications' |
| 28 | 'Mexico' | 28 | 'Financial Intermediation' |
| 29 | 'Malta' | 29 | 'Real Estate Activities' |
| 30 | 'Netherlands' | 30 | 'Renting of M&Eq and Other Business Activities' |
| 31 | 'Poland' | 31 | 'Public Admin and Defense; Compulsory Social Security' |
| 32 | 'Portugal' | 32 | 'Education' |
| 33 | 'Romania' | 33 | 'Health and Social Work' |
| 34 | 'Russia' | 34 | 'Other Community, Social and Personal Services' |
| 35 | 'Slovak Republic' | 35 | 'Private Households with Employed Persons' |
| 36 | 'Slovenia' | | |
| 37 | 'Sweden' | | |
| 38 | 'Turkey' | | |
| 39 | 'Taiwan' | | |
| 40 | 'United States' | | |

# J REER indices plots

*Figure J.1*

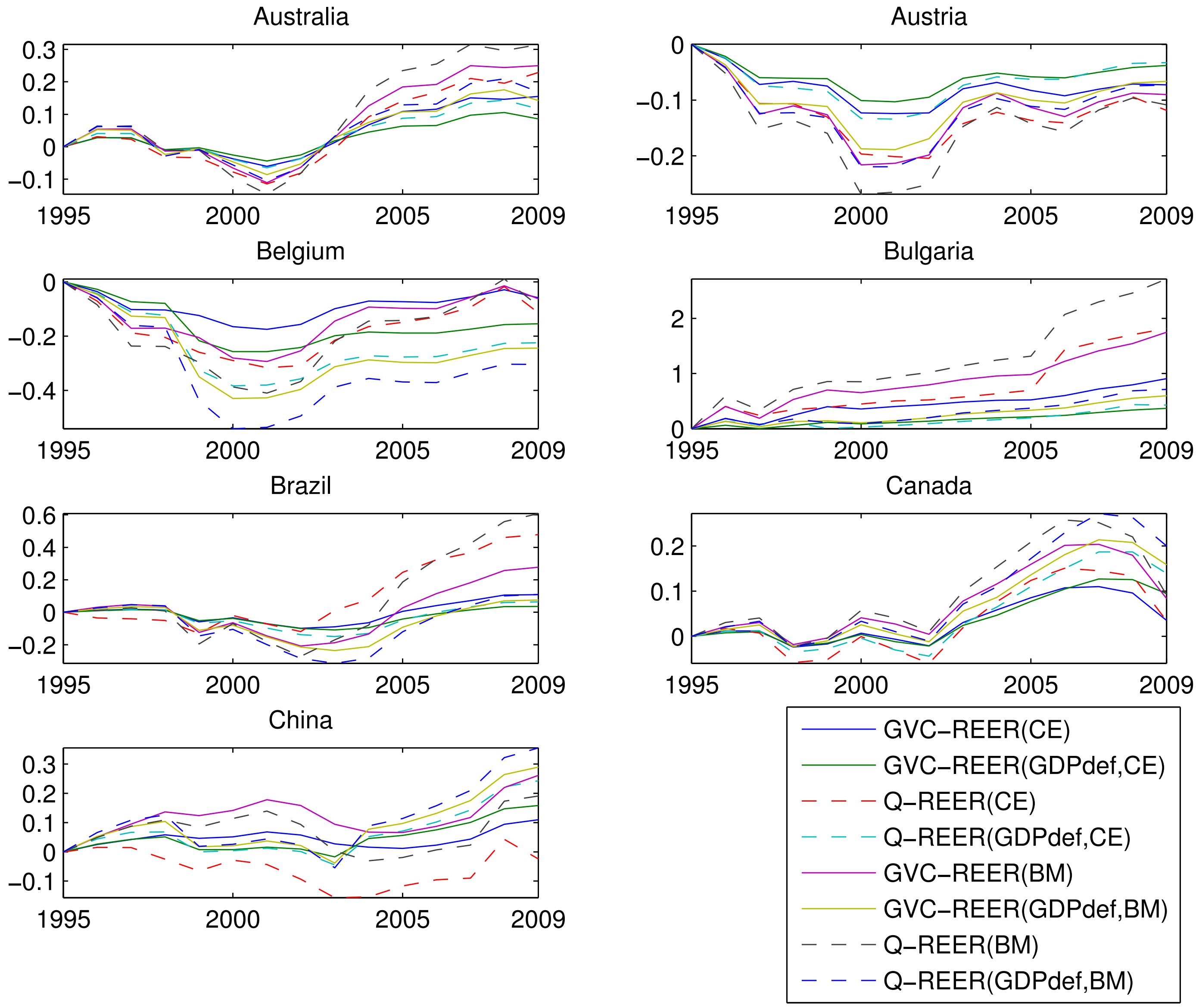


Figure J.1 cont.

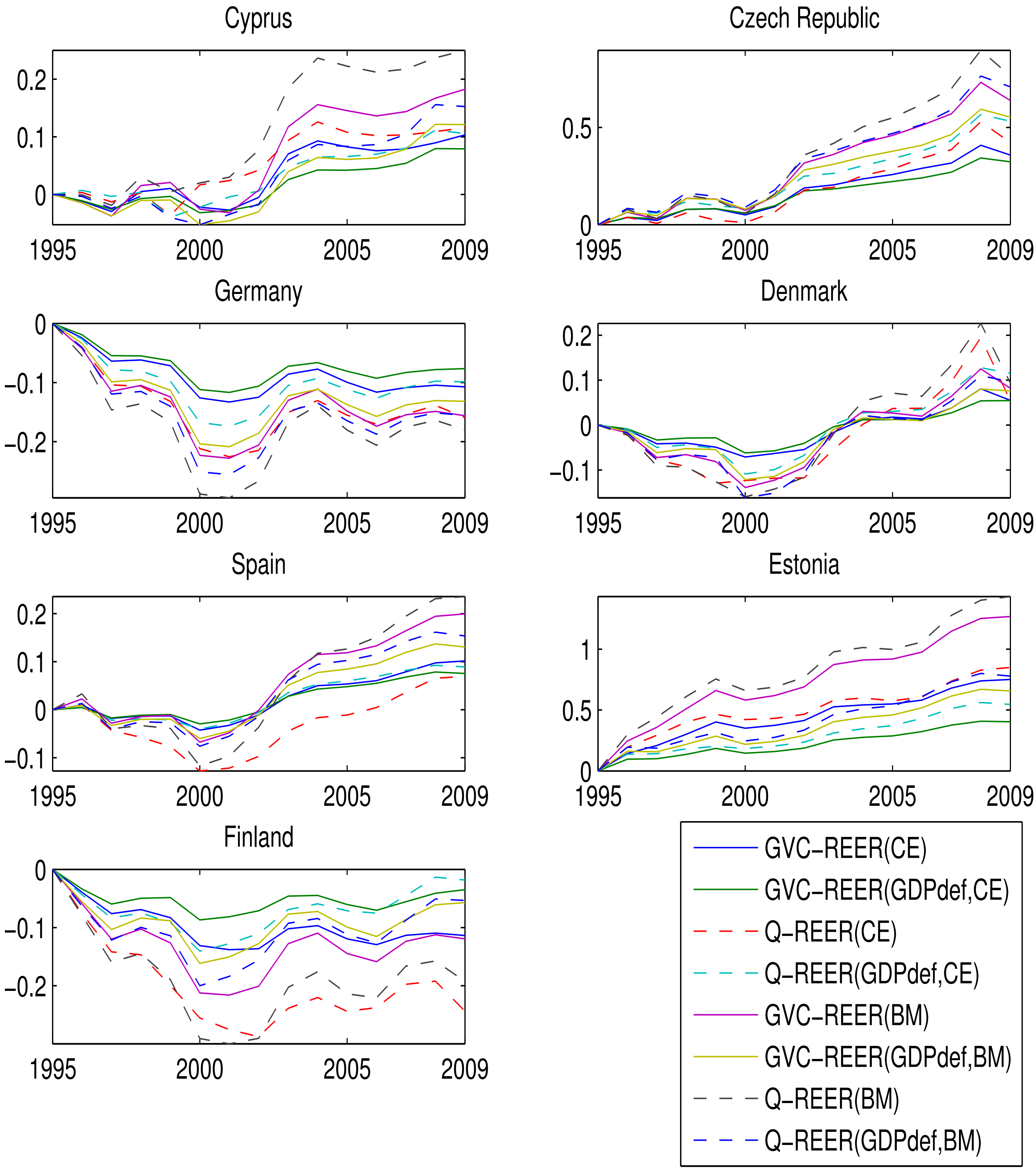


Figure J.1 cont.

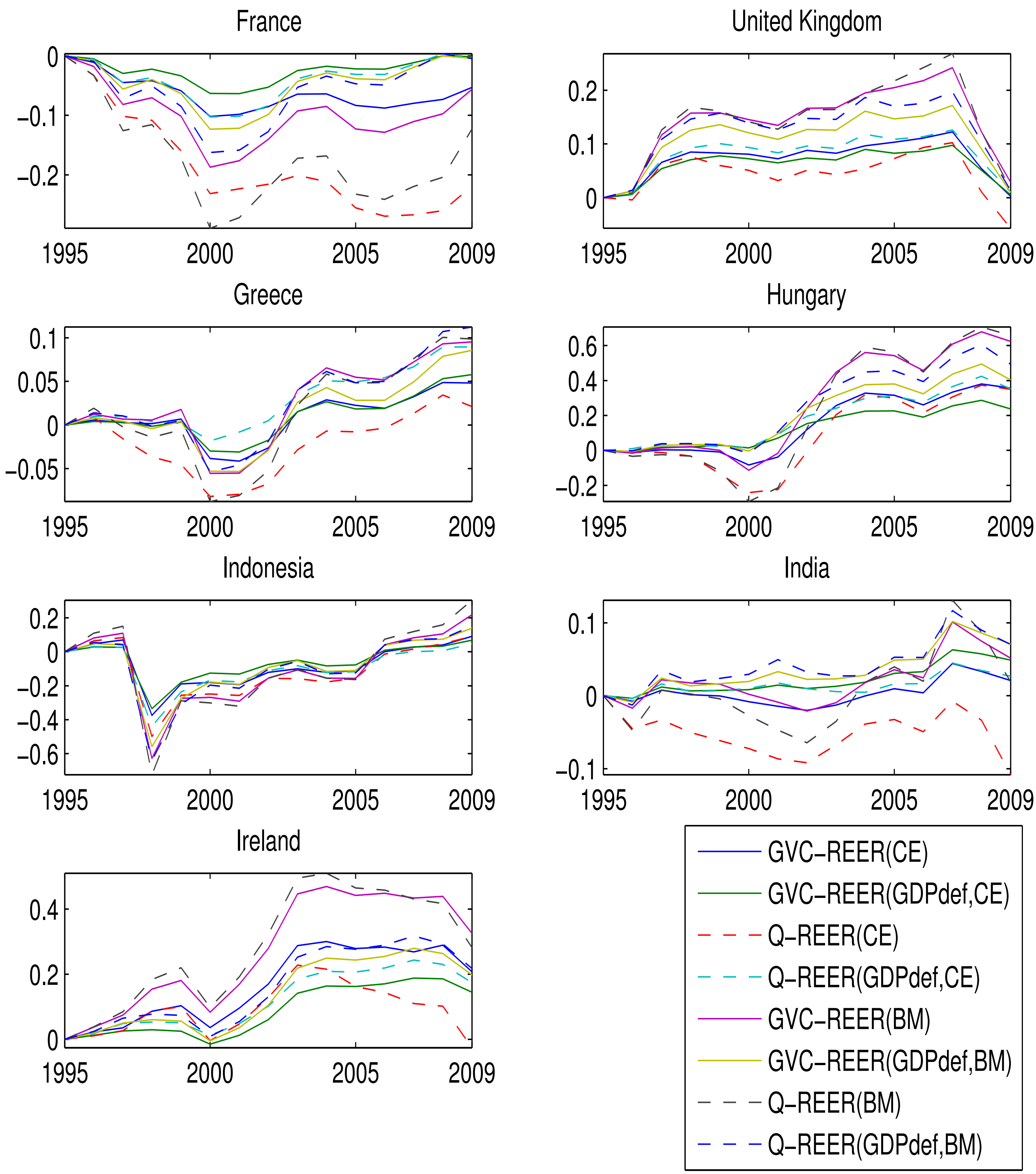


Figure J.1 cont.

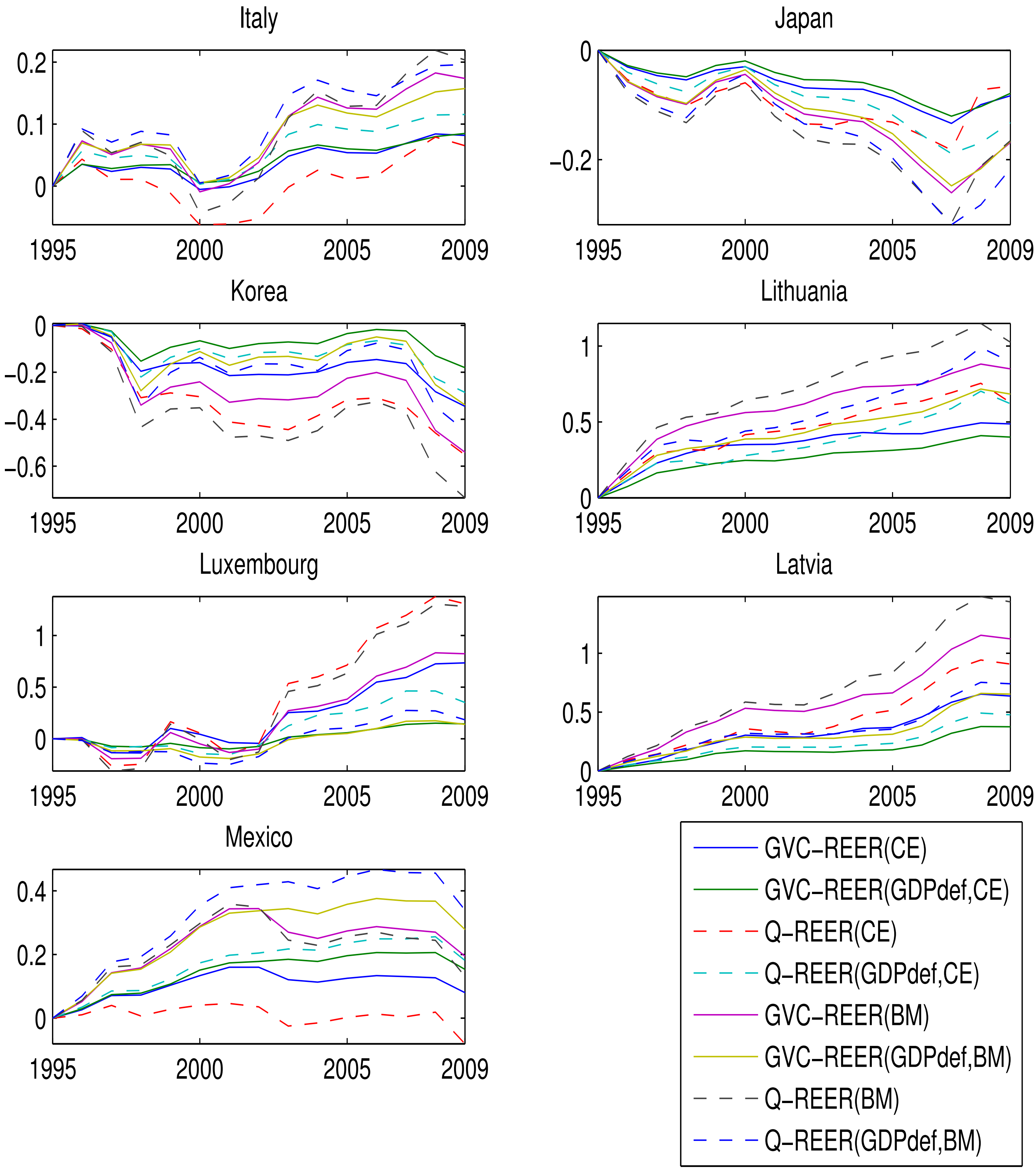


Figure J.1 cont.

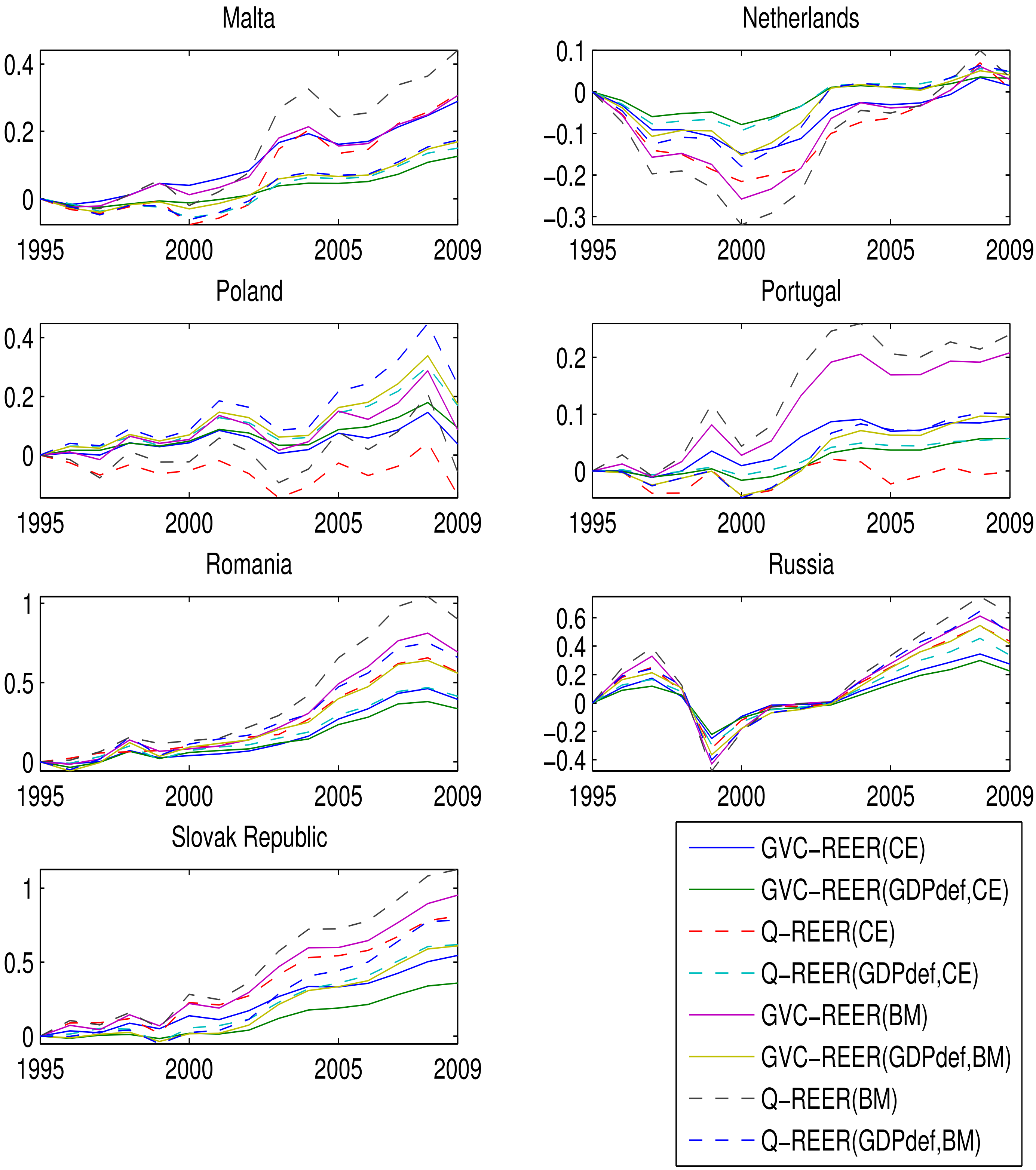


Figure J.1 cont.