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The Determination of Prices of Production

Since the turn of the century a multitude of criticisms have been directed against the so-called transformation of quantities of commodity value into quantities of price, on the assumption of a uniform profit rate for all industries, as set forth in chapter 9 of Marx's *Capital*, volume 3. Common to all these criticisms is the idea that Marx's procedure lacks a transformation of the quantities of value of the means of production and of labor (the *inputs*). If this dilemma is to be resolved—according to all the critics—the two fundamental equations of the Marxian transformation (total price = total value; total profits = total surplus value) can obviously no longer be simultaneously valid. The result of this is that the general profit rate can no longer be calculated from Marx's law of value, and the value quantities themselves cease to play any role in the calculation of prices of production.

This result stems directly from the assumption that the prices of the products and those of the means of production must be determined *simultaneously*, an assumption that has always been accepted uncritically by almost everyone. This has been true even of Marxists, especially given the acceptance by "left-wing" economists of Sraffa's theory of prices and distribution of revenue. Yet this result is much less obvious than it may seem. Once the assumption (which is *not* found in Marx) is proved to be irrational and is therefore discarded, the criticisms directed against the Marxian determina-

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tion of the prices of production and the rate of profit lose all their force.

1. The simultaneous determination of prices

None of the discussions of the prices of production that have appeared so far, and certainly not Sraffa's 1960 treatment,[1] has taken the trouble to provide a theoretical justification for the procedure of simultaneous calculation of input and output prices. Nevertheless, there is a kind of reasoning prevailing among the so-called experts, according to which "prices must necessarily be simultaneous prices, since the prices of all commodities are based not on the costs of production (or on the labor) sustained in the past, at the time when the commodities were produced, but on the *current* costs (or the quantity of labor)." The astute observer will easily perceive that this is an entirely circular argument. If prices are determined by current costs, it is necessary to establish what these current costs are, but since the current costs are the prices themselves (of the various inputs), the whole thing comes down to saying that (current) prices are determined by (current) prices.

Another idea, also expounded by Joan Robinson, is much more elementary—almost trivial. Since the commodities of the same type have the same price as output and as input, this argument goes, the same price for those commodities is placed on the left and on the right side of the equations used for price calculation. Unfortunately, as we shall see, this argument forgets that a given commodity cannot operate as an input and as an output at the same time. In reality, the procedure of simultaneous calculation of prices cannot be given any theoretical foundation; it is simply an axiom, which works only so long as it is not shown to produce consequences that are absurd and/or in plain contradiction with the facts. To find out whether this is indeed so, let us imagine a system of production composed of three industries (A, B, C), each of which uses only productive inputs coming from the other two, and in which sector C produces the good serving as the real wages consumed by all the workers. The material structure of this system is represented by means of the following schema:

(1)
$$B_a + L_a \rightarrow A$$
$$A_b + L_b \rightarrow B$$
$$A_c + B_c + L_c \rightarrow C$$
$$C = C_a + C_b + C_c \rightarrow L_a + L_b + L_c = L\,.$$

If, for convenience, we now suppose that the system is reproduced on an unaltered scale (simple reproduction), it should turn out that sector A exchanges its total product A against a portion of the product of B and C, specifically, against the quantity B_a + C_a, while the commodity B should be exchanged against $A_b + C_b$, and the commodity $C–C_c$ with $A_c + B_c$. We therefore should have the following series of equations (the last being a simple identity, since the quantity C_c is consumed within the same industry in which it is produced):

(2)
$$A = B_a + C_a$$
$$B = A_b + C_b$$
$$C - C_c = C_a + C_b\,.$$

As can be seen, the commodities that enter into the exchanges among the various productive sectors are only the outputs, that is, what is found on the *right* side of expressions (1), while the inputs (the left side) have been exchanged in the *previous* period. It is quite obvious that no output can be exchanged against an input, since there cannot be use values that function simultaneously as inputs *and* as commodities. If a particular productive use value (e.g., oil) is playing the role of input, this means that it is in the form of *productive* capital and therefore *cannot* at that time be found in the (capital) commodity form.

Equations (2) represent direct exchanges of commodities for commodities; if we transform these barter exchanges into exchanges for commodity-money, the situation is worsened. The outputs $B_a + C_a$, $A_b + C_b$, $C_a + C_b$ must now all be computed first and then realized in money, that is, say, a *fourth* commodity, before they can be used as productive capital. The assumption of the simultaneous calculation of input and output prices in fact implies the elimination of this necessity, something justified at times by the pretext that "purchases, sales, and production always occur simultaneously for all sectors and individual capitals, without interruptions making possible the passage from one phase to the next." The objection is clearly irrelevant, since if we are moving from a *particular* group of inputs to a *particular* group of outputs, as in the so-called linear models of production, the phases of sale/purchase and of production for *these* groups obviously *cannot* occur at the same time. The various production processes must take place at the same time with respect to each other in the various sectors, and *after this* the products can all be exchanged.[2] To do away with this "minor" detail is basically equivalent to assuming that circulation is not a constituent element of the market economy, or (analogously to Ricardo's quantitative theory of money) that inputs enter the production process *without* prices and leave it *with* prices, that is, the products do not have the form of commodities.

2. The simultaneous linear equations

The modern theory of prices and distribution is clearly forced by its own premises to assume the simultaneous determination of the prices of inputs and outputs. If one wishes to obtain the prices of production from the known data concerning the production technology, together with knowledge of one of the so-called distributive variables (e.g., the wage rate), without the aid of other unwelcome quantities (among which the "value" of the commodities is obviously paramount), it is necessary to adopt the same temporal index for the prices of the inputs and the outputs, or else it will no longer be possible to determine the rate of profit.

To illustrate this elementary fact, let us imagine a system of two industries, A and B, each of which produces a productive input, with the commodity *B* also being used as the wage good. With a time spread between the price of the inputs and the price of the outputs (a spread indicated by the indices *t* and *t* + 1), the system of production prices would assume the following form:

$$(3) \qquad \left[B_a + B_{\ell_a}\right] p_{b_t} \; (1+r) = A \; p_{a_{t+1}}$$

$$\left[A_b \; p_{a_t} + B_{\ell_b} p_{b_t}\right] \; (1+r) = B \; p_{b_{t+1}} \,.$$

The expressions in equation (3) form a system of two linear first-order difference equations, whose solution requires preliminary knowledge of the quantity r (the rate of profit), which will enter into the coefficient $1 + r$. Alternatively, if we treat r as an unknown function of t (i.e., r_{t+1}), a further equation for the determination of r_{t+1} is required, an equation that cannot be constructed in the framework of the Sraffian theory, that is, without the addition of other quantities.

3. The physical surplus

Economists of the Sraffian school rarely miss an opportunity to assert that the category of surplus value is unnecessary for the determination of the rate and the volume of monetary profits, the simple surplus in physical terms being sufficient for this. Not without reason, they like to designate their own theoretical orientation as "one based on the *surplus approach.*" It is therefore appropriate to investigate whether this concept of material surplus is indeed defined sufficiently to play the role that its supporters assign to it.

Even children know that the strict conception of a surplus in purely physical terms is an absurdity, since the quantity of matter (and/or energy) in the universe is constant, or at least can be changed by there is no known force. One must therefore employ a *different* concept of surplus, a surplus that is a quantity of use values that can be utilized by human beings, in excess of those consumed either by individuals or by production. But how can this excess be measured? The matter seems simple. On one side, write the total of use values that are productively consumed during a given period, and on the other side, write the total use values produced during the same period: the difference between the latter quantity and the former is the physical surplus obtained. An attempt to carry out this procedure, however, would quickly show that it is impossible: the subtrahend elements are not found in the minuend, namely, all those natural forces and entities (the elements hidden in the earth's crust, for example) that are utilizable use values, to all intents and purposes, and in fact are absolutely indispensable, yet which cannot have a price because they are not commodities. This circumstance is enough to explain why the concept of a surplus of use values is in and of itself chimerical. It is therefore necessary to have recourse to a *third* and different concept of surplus or net product.

Inevitably, there is no other alternative than the surplus formed by the difference between groups of goods that have a price and that are therefore the only goods that end up in the calculation of the difference between gross revenues pocketed by the capitalist producers of commodities and production costs which these same individuals must sustain. However, if both the outputs produced and the inputs used change their physical and technical characteristics over time, it will no longer be possible to calculate a surplus in material terms given only which goods have a price and which do not, since terms will appear on the right side of the production equations that are not homogeneous with those on the left side, whose sole function is to be replaced in successive production cycles. It follows that the material surplus cannot be used as a basis for determination of the prices and the rate of profit. This is a necessary consequence of the simultaneous calculation of the input and output prices, because (as we have seen) this type of calculation implicitly presupposes that the use values appear simultaneously in the two positions of commodity capital and productive capital, which is an impossible premise.

4. The most recent "solution" proposed

It is known that none of the solutions proposed thus far for the problem of the transformation of values into prices is compatible with the simultaneous validity of the two Marxian equalities (total value = total price, total surplus value = total profit). Nor have any of them succeeded in using value quantities as the essential basis for the determination of the prices of production. However, a new solution recently proposed by Dumenil, Foley, and others[3] seems to have accomplished an apparent quadrature of the circle, obeying the two Marxian equalities by means of new definitions of both the value of labor power and the group of commodities to which the total of prices and the total of values refer.

Dumenil and Foley oppose the traditional view of the value of labor power (the value of the consumer goods that form the real wage of the workers), preferring to define this value in terms of to the sum of money received by the workers (value of the nominal wages); they base this preference on the argument that the workers may spend this sum of money as they see fit. If the quantity, the type, and the quality of the commodities consumed by the wage laborers vary, then, according to the first definition of the value of labor power, the average rate of surplus value should also change, regardless of changes in productivity; according to the second definition of the value of labor power, the rate of surplus value should be constant.

Now, it is quite evident that, with this new definition of the value of labor power, the value of this particular commodity will not merely change its characteristics, but will end up by simply disappearing. Instead of the magnitude of value of a definite group of goods (the real wage consumed by the workers), we have a sum of money, established without any link to the value of the real wage or to anything else. Because the prices of production of the commodities are not yet known at the time when this quantity of money that is to represent the wages is fixed, there is no guarantee that, once the prices are set, this quantity will actually enable the workers to purchase the group of commodities comprising, at that time, the socially established level of consumer goods required by the wage laborers, and, therefore, make possible the reproduction of the economic system. This allegedly new definition of wages as simply a quantity of money is in no way new, since it is in fact exactly equivalent to the nominal wage that we find in Sraffa, which causes so many problems for his theory. The same problems cannot fail to appear in the formulation of Dumenil and Foley.

On the other hand, if this sum of money that is equivalent to the wages is fixed in relation to the price of the composite commodity consumed by the workers, we may wonder where this price could come from, since the prices are supposed to be the final result of the transformation and surely not its starting assumption. In this second case, we thus find ourselves in the simple Sraffian formulation of the system of prices with real wages, to which is added (quite arbitrarily) the statement that the total nominal wages received by the workers and the value of the commodities purchased by the latter are one and the same.

Yet the new definition of the wage is necessary if the circle is to be squared with respect to the two famous equations. In fact, it is enough to add that the total product, corresponding to the equation between the sum of values and the sum of prices, cannot be the gross product, as was believed until now, but rather the *net* product, if we are to avoid a possible double counting of commodities that appear as inputs in a different, later process of production. If the sum of the prices of the commodities that form the net product of the economic system is equated to the sum of the values of these same commodities, we obtain only one of the infinite possible numeraires of the Sraffian system of production prices, and if (as follows from the new definition of wages) the sum of the wages is necessarily equal to the value of the aggregate labor power, it is a trivial result that the sum of profits, or the total profit, should be equal to the sum of surplus values, or the total surplus value, since the net product is known to be divided between wages and profits.

Not only could all of this be obtained directly and easily from the Sraffian system by the calculation of the prices of production, using suitable ad hoc definitions, without having to present the facts as though a mystery cloaked in secrecy for centuries were finally being explained, but also this operation runs the risk of defining a numeraire that is unacceptable, since it is not neutral with respect to relative prices, as has been brilliantly shown by Stamatis (1990) in his demonstration that the numeraires comprised in a composite commodity cannot guarantee invariance of relative prices with respect to the choice of the same numeraire in so-called linear systems of production.

5. The determination of the prices of production

If it is necessary to introduce a time lag between the prices of production of the inputs and the prices of production of the outputs in order to take the circulation process into account—something that would be essential even in the case of a commodity circulation time equal to zero[4]—assuming input–output relations identical to those of equation (3), the system determining the prices of production would assume the following form:

(4)

$$\left[B_a + B_{\ell_a}\right] p_{b_t} (1 + r_{t+1}) = A\, p_{a_{t+1}}$$

$$\left[A_b p_{a_t} + B_{\ell_b} p_{b_t}\right] (1 + r_{t+1}) = B\, p_{b_{t+1}}$$

As we have already noted, this system of equations of itself does not have a solution; in order to be useful for something, it must be furnished with supplementary equations that give a precise meaning to the factor $1 + r_{t+1}$, that is, the general rate of profit. This is in fact the purpose of the Marxian theory of the exchange values of commodities as determined by the labor time necessary for their production.

The Marxian theory declares that the magnitude of the value of every commodity is determined by the sum of the value of the means of production consumed in its production (fixed constant capital consumed plus circulating constant capital consumed) and the value added by labor in the labor process (which is divided into the value of labor power and surplus value). In the terms of our formulation, this means that the value magnitudes of the commodities *A* and *B* (which we denote respectively λ_a and λ_b) are determined by the following system:

(5)

$$B_a \lambda_{b_t} + \ell_{a_t} = A \lambda_{a_{t+1}}$$

$$A_b \lambda_{a_t} + \ell_{b_t} = B \lambda_{b_{t+1}}.$$

Now, substituting the production prices at time *t* for the quantities of value at time *t* of the constant capital employed and consumed in the system (5), we obtain the system of what may be called *direct prices* (designated by the quantities π_a and π_b). This system is indispensable if we are to derive the equation for the general rate of profit:

(6)

$$B_a p_{b_t} + \ell_{a_t} = A \pi_{a_{t+1}}$$

$$A_b p_{a_t} + \ell_{b_t} = B \pi_{b_{t+1}}.$$

The direct prices express the quantity of total (gross) value created in each production sector, constituted by the price of the constant capital consumed and the new value added, but not yet redistributed among the various sectors in accordance with the criterion of the uniform temporal rate of profit on the total anticipated capital. These prices, far from being a fictitious quantity or a mere step in the procedure for calculating the prices of production, constitute the magnitudes to which the market prices of the commodities would tend if the mechanism by which money capital goods is continually transferred from one production sector to another in the direction of the highest rate of profit should cease.[5] Of course, the direct prices of the commodities would coincide with the value magnitudes if the prices of the commodities that appear as constant capital in (6) were equal to the values of these commodities.

At this point, it is quite easy to derive the equation for the general rate of profit. Given the starting prices of the commodities that form the aggregate real wages in the two sectors ($B_{l_a} p_{b_t} + B_{l_b} p_{b_t}$) and the quantities of the new values created, we obtain the equation for the uniform rate of profit:

(7)

$$r_{t+1} = \frac{\ell_{a_t} + \ell_{b_t} - \left(B_{\ell_a} + B_{\ell_b}\right) p_{b_t}}{A_b p_{a_t} + B_a p_{b_t} + B_{\ell_a} p_{b_t} + B_{\ell_a} p_{b_t}},$$

which we may insert into each of the two equations of the system (4) to obtain the prices of production $p_{a_{t+1}}$ and $p_{b_{t+1}}$.[6]

It is obvious that our system of equations (4)–(7) cannot yet have a complete solution, as it still lacks the establishment of an initial condition p_{a_0}, p_{b_0}. It may be thought that the initial condition is given by the value magnitudes of the commodities at the time $t = 0$, or at the moment of the passage from simple commodity production to capitalist production, or it may be thought as given in some other way. Whatever it is, it is irrelevant, since it is easy to see that the behavior of the solution of a system of type (4), supplemented by equation (7), *is entirely independent of the establishment of any initial condition.* This means, quite simply, that the prices of production can only be the redistribution of the quantities of value (which we have here designated l_a and l_b) newly created in the course of the continuous succession of labor processes over time.

Whatever the initial condition chosen for the system (4)–(7), this is necessarily equal to that which is adopted by the system of values (5). Whether one starts at the dawn of commodity production, or sets out from a hypothetical transition from simple commodity production (directly regulated by the value magnitudes) to production based on capital, the point of departure must be common to the three systems of values, direct prices, and prices of production—that is, systems (5), (6), and (4), respectively; hence, the sum of the prices in a given period will always necessarily be identical to that of the values in the same period, as is rather easy to verify from the solutions of the three systems. In each period, the quantity that is distributed among the various (two, in our case) capitals is given by:

(8)

$$\ell_{a_t} + \ell_{b_t} - \left(B_{\ell_a} + B_{\ell_b}\right) p_{b_t} ,$$

where $(B_{l_a} + B_{l_b})\, p_{b_t}$ comprises the quantity of value produced that goes to labor power. Probably it is exactly this last magnitude that will somewhat perplex those Marxists who strongly desire to respect everything that Marx said. But expression (8) is the only way to remain within Marx's own theory of the prices of production and of the value of labor power.

The quantity designated by $(B_{l_a} + B_{l_b})\, p_{b_t}$ is the price of production of the total of the commodities consumed by the workers (aggregate real wages) at time t, resulting (in turn) from a redistribution of quantity of labor performed in the preceding period on the basis of the criterion of the equal rate of profit for all types of capitalist production; as such, it is the price of production of labor power at time t. It is therefore appropriate to ask what role is played by the *magnitude of value* of labor power in the transformation process: does it not end up disappearing, *sic et simpliciter*? This value magnitude plays the same role as all the magnitudes of value in the general process of production and circulation of capital, being continuously and unceasingly redistributed among the various capitals. The value and the price of labor power must be considered not as the value and price of some metaphysical entity, but rather as the value and price of a particular group of use values. It is quite obvious that the total price of production of labor power, in the general case, is different, in each period, from the total value of labor power, as Marx himself notes quite carefully in his discussion of the transformation of values into prices:

> As for the variable capital, the average daily wage is certainly always equal to the value product of the number of hours that the worker must work in order to produce his necessary means of subsistence; but *this number of hours is itself distorted by the fact that the production prices of the necessary means of subsistence diverge from their values.* [Marx 1977c, p. 248; emphasis added]

> It is therefore possible that even the cost-price of commodities produced by capitals of average composition may differ from the sum of values of the elements which make up this component of their price of production. Suppose the average composition is 80c + 20v. Now, it is possible that in the actual capitals of this composition 80c may be greater or smaller than the value of c, i.e., the constant capital, because this c may be made up of commodities whose price of production differs from their value. In the same way, 20v might diverge from its value if the consumption of the wages includes commodities whose price of production diverges from their value; in which case *the labourer would work a longer, or shorter, time to buy them back (to replace them) and would thus perform more, or less, necessary labour than would be required of the price of production of such necessaities of life coincided with their value.* [Marx 1977c, p. 253; emphasis added]

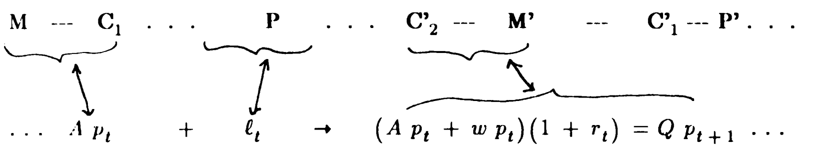
In practice, Marx's statement means that the difference between the magnitude of value and the price of production of the total real wage is quite irrelevant with respect to the transformation of values into prices. Since what the worker accomplishes in the labor process is not to transfer the value of the means of subsistence to the value of the product, but rather to *re-create ex novo* a value equal to that of the means of subsistence (plus, obviously, a surplus), the division of the working day between necessary labor and surplus value is adjusted in each period by respect to the price of production of the real wage, so that any quantitative difference between the value and the price of production of labor power loses all significance, a circumstance expressed in formula (8), which gives the total amount of the surplus value produced (and redistributable among the various capitals) in a given period. It follows automatically that in each individual period (circuit of capital) the sum of the surplus values produced in the various sectors can never differ from the sum of the profits received in the various sectors.

6. Characteristics of the solutions of the system of prices of production

Although the system of equations (4)–(7) is formally presented as a nonlinear system, it is easy to linearize and—if we hold constant the technical coefficients and the quantities of labor consumed—its solutions (for the two prices p_{a_t}, p_{b_t} and for the rate of profit r_t) are all absolutely stable and converge on a point of equilibrium reached from a certain *t*, whatever the initial conditions.[7]

The interesting fact is that the equilibrium solution of our system (4)–(7), except for its independence from the initial conditions, looks identical to the solution of the corresponding Sraffian system of simultaneous determination of the prices of inputs and the prices of outputs. This circumstance may easily lead one to think that the two types of system are, in the end, quite equivalent, and that it is not therefore worth the trouble to search so zealously for something new, in distinction to the already well-known Sraffian theory, which is so simple and convenient. But this would be a serious error. The two systems are equivalent (and this only under the assumption of fixed technical coefficients and quantity of labor) only after the fixed point of the discrete system (4)–(7) has been reached[8]; in the previous periods, the numerical values of the prices of production in the two systems differ. Without the assumption of the constancy of the technical coefficients and the labor inputs, and allowing the technologies adopted to vary, the two systems not only cease to be the same, and thus to provide identical numerical solutions, but actually reveal themselves to be qualitatively different. Our discrete system of the prices of production and of the rate of profit (4)–(7) provides a clear and reliable way of determining the quantities of interest, moving from one period to another, while the simultaneous system of the Sraffian type (intrinsically static in its mathematical nature) must ultimately break down into as many new systems as there are periods of technological change considered, without providing a way to move directly from one period to the next, or from one circuit of productive capital to another [$P \ldots P'$], a movement mediated by the circuit of commodity-capital [$C \ldots C'$] and by that of the money-capital [$M \ldots M'$].

This is clear from the following schema, which puts the process of capital circulation, as expounded by Marx in the second volume of *Capital*, in relation with our discrete system of equations of prices of production:



where M stands for money-capital; C_1 and C_2 indicate the commodity-capital serving as input and the commodity-capital that is the output of production, respectively; P is the production process; and the apostrophe (') denotes the quantitative increment with respect to the previous period. In the bottom row, A is an index for the productive inputs, and w is an index for the aggregate real wage, while Q indicates the aggregate output.

A numerical example of the effects of a continuous variation in the techniques and in labor productivity will serve much better than a general argument to illustrate this important fact. Keeping in mind the simultaneous (Sraffian) form of system (4)–(7), which is:

(9)
$$\left(B_a + B_{\ell_a}\right) p_b\,(1+r) = A\,p_a$$

$$\left(A_b p_a + B_{\ell_b}\,p_b\right)(1+r) = B\,p_b\;,$$

We may assign these initial numerical values to the various coefficients:

$$A_b = 10\;;\;\; B_a = 5\;;\;\; B_{\ell_a} = 10\;;\;\; B_{\ell_b} = 7\;;\;\; \ell_a + \ell_b = \ell = 20\,.$$

For these given values of the technical coefficients, the relative Sraffian prices and the relative equilibrium prices of the system (4)–(7) are equal to $p_a/p_b = 1.0686$, and the corresponding rate of profit to $r = 1.2139$. If we now let all the technical coefficients vary for three consecutive periods and compare, for each period, the prices and the rates of profit obtained by our discrete system (p_{a_t}, p_{b_t}, r_t) with those that result from the static Sraffian system. The symbols $\Delta\%p$ and $\Delta\%r$ indicate, for each period, the percentage deviations in the discrete prices and rate of profit with respect to the static prices and rate of profit.

1.

$$A_b = 8\;;\;\; B_a = 4\;;\;\; B_{\ell_a} = 7\;;\;\; B_{\ell_b} = 7\;;\;\; \ell = 16\,.$$

$p_{a_1}/p_{b_1} = 1.0089$ $\qquad\qquad$ $p_a/p_b = 1.0427$

$r_1 = 1.4721$ $\qquad\qquad$ $r = 1.6082$

$$\Delta\%p = -3.242\%$$

$$\Delta\%r = -8.463\%$$

2.

$$A_b = 6\;;\;\; B_a = 3.5\;;\;\; B_{\ell_a} = 7\;;\;\; B_{\ell_b} = 5\;;\;\; \ell = 12\,.$$

$p_{a_2}/p_{b_2} = 1.2670$ $\qquad\qquad$ $p_a/p_b = 1.1667$

$r_2 = 2.2216$ $\qquad\qquad$ $r = 2.3341$

$$\Delta\%p = +8.597\%$$

$$\Delta\%r = -4.820\%$$

3.

$$A_b = 5\ ;\ B_a = 3.5\ ;\ B_{\ell_a} = 5\ ;\ B_{\ell_b} = 4\ ;\ \ell = 10\ .$$

$p_{a_3}/p_{b_3} = 1.0320$ $\qquad\qquad$ $p_a/p_b = 1.1149$

$r_3 = 2.9903$ $\qquad\qquad$ $r = 3.4326$

$$\Delta\%p = -7.436\%$$

$$\Delta\%r = -12.885\%$$

7. Conclusions

This article has demonstrated how Marx's theory of the prices of production, better known as the "transformation of values into prices," not only presents no logical deficiency, but also is dynamically superior to Sraffa's widely circulated and accepted formulation when expressed in its corresponding mathematical terms, which *cannot* be those customarily used by economists of the static-linear tradition. In the context of a discrete dynamic system, which is essential if we are to consider the part played by circulation alongside production, the quantities of labor performed in the various production sectors (and thus the production of value) are absolutely essential to define the general rate of profit. This shows that the prices of production are nothing more than values redistributed over the various invested capitals. The fact that the equilibrium solutions of a dynamical discrete system and the solutions of a static system are identical when the technical coefficients are constant is of very little practical importance and only shows that the Sraffian system of equations amounts to no more than the static case of Marx's theory of the prices of production.[9]

The discrete system defined here, however, is insufficient and may still lead to major misunderstandings if its intrinsic limits are not grasped. The system lacks a description of the mechanism that continuously leads to the formation of a uniform rate of profit—which is a mere assumption here—from the different sectoral rates. As a result, it is not possible to study thoroughly the effects of technological progress and increments in productivity achieved by changes in the means of production and in the composition of the capital within the individual branches of the social production. Future work will extend the model presented here to the mechanism of formation of the uniform rate of profit and to the variation of labor productivity in the various sectors through the introduction of new and more advanced means of production.

Notes

1. See Sraffa (1960). Here we are abstracting completely from the logical criticisms directed against the Sraffa theory by Marxist economists, although this criticism has demolished the myth of that theory's coherence. See, for example, Savran (1979, 1980) and Stamatis (1990).
2. See Dumenil (1983), Foley (1982), and Glick-Ehrbar (1987).
3. Marx himself implicitly recognizes the need for a time spread in the formulation of the process of formation of the prices of production on the basis of values when, in chapter 9 of volume 3 of *Capital*, he mentions a "quantitative error *in the past*" with respect to the possible variance between the value of the constant and variable capital used in production and its price of production. See Marx (1977c), p. 252.
4. Recent statements of this argument are to be found in Freeman (1984) and in Carchedi (1991). While Kliman and McGlove (1989), although adhering to a viewpoint rather similar to that set forth in the present work on the problem of the transformation of values into prices, do not explain why a time lag should be placed between the prices of the inputs and the prices of the outputs. These authors invoke the "dialectic" and the "dialectical method" of Marx, as opposed to the standard method of the bourgeois economists, but this is like the story of Don Quixote and Sancho Panza, who saw the same phenomena appear in very different forms depending on the different attitudes with which they approached the adventure.
5. The importance of what we have called here direct prices follows directly from the theory of rent. Independently of differences in the natural fertility of the various types of land and the different amounts of capital invested, the prices of products subject to ground rent will tend toward the direct prices of equations (6), since private property hinders the free circulation of

money capitals among the sectors with respect to land, which is the practical mechanism that leads to the equalization of the various rates of profit or the redistribution of the social surplus value to the different branches of capitalist production. The fact that agricultural and mining products are sold at direct prices of equations (6) generates *absolute* rent, which is equal to the difference between the direct price of the commodity yielding a rent and the price of production at which this would be sold were there no barrier to the circulation of capital among the production sectors. Disregard and ignorance of the fact that the magnitudes of value are determining factors of prices have led the neo-Ricardians and Sraffians to reject absolute rent, due to the impossibility of formulating a theory for it, and have led many economists of the "Third-Worldist" tradition (Amin, Dos Santos) utterly to confuse absolute rent with monopoly rent.

6. Some readers may doubt that the numerator of equation (7) constitutes a sum of homogeneous terms, but they would be wrong. In the entire system (4)–(7), quantities provided with index t are either unknown magnitudes (functions) or known magnitudes (functions). In systems of this kind, the unknown magnitudes (functions), by the nature of things, assume the quality (time, work, weight, mass, etc.) of the known magnitudes (functions), which in our case are the quantities of labor time l_a and l_b expended in the two sectors of production.

7. Upon request the author will provide the complete mathematical treatment of the discrete dynamical system presented here, and its relations with the static system of the Sraffian type.

8. The essential difference between our discrete system and the static Sraffian system emerges with great clarity in the case of production without the use of labor (fully automated production), which corresponds to zero value of the quantities l_a and l_b. For l_a, the Sraffian system (9) yields positive solutions for the prices of production, as for any other value of l_a and l_b, while the difference system (4)-(7) has no solutions. In the absence of human labor expended in production, the two systems (with constant technical coefficients) no longer converge to the same equilibrium solution. This circumstance corresponds to the fundamental principle of the Marxian theory, according to which the calculation of values is indissolubly linked to the existence of the commodity economy, which is incompatible with a fully automated production. The discrete system developed here, however, should not be confused with a simple iterative calculation of the prices of production from values. It is a rather well known fact that the Sraffian prices of production can eventually be obtained by using the same technical coefficients, starting with the quantities of value of the commodities (or other positive quantities), and repeating the calculation (see Shaikh, 1977). However, the iterative calculation from values is only a numerical calculation procedure in which the intermediate values of the prices (before the final equilibrium value) do not count, since they are purely imaginary quantities; in our system (also assuming fixed technical coefficients and amount of labor), every intermediate (not equilibrium) value counts as an effective price, since the equations are built on the assumption of the circulation of capital and the necessary succession of the three circuits of capital

(productive, commodity, and money) (see Marx, 1977b, pp. 26–123), an assumption that, as can be seen, exists neither in the Sraffian theory nor in the various static Marxist derivations of the of production prices. In this work we use the term circuit of capital (productive, commodity, or money) instead of the term *cycle* of capital used by Marx in the first chapters of volume 2 of *Capital* to indicate the various metamorphoses of capital, in order to avoid any possible confusion with the economic cycle, that is, the cyclical oscillations of capitalist production.

9. It is perhaps for this reason that, whatever anyone said (e.g., Joan Robinson), Piero Sraffa declared: "The labor theory of value is absolutely correct."

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