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VALUE AND DISTRIBUTION IN THE CLASSICAL ECONOMISTS AND MARX[1]

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I. Introduction

1. THE theory of value and distribution is at present in a situation of unease and uncertainty: we no longer find the same general agreement about its basic elements which obtained until a few decades ago. Two main theoretical developments have undermined the dominant theory which explained distribution and relative prices by means of the "equilibrium" of the two "opposing sets of forces", demand and supply for factors of production.

The first development in order of time has been Keynes's refutation of the doctrine according to which a competitive economic system tends towards the full employment of labour, i.e. towards that equilibrium between "demand and supply" of labour, which was to determine the wage. Keynes' concentration on the short period, and the persistence in the *General Theory* of many traditional premises favoured the successive attempts to reconcile his results with orthodox long-period analysis: but the weakening of the dominant theory which nonetheless resulted from his work can be seen both in the uneasiness which, in ever-changing forms, characterizes the renewed orthodoxy, and in the tendency of Keynes' direct followers towards a more radical departure from traditional theory.

The second development consists in the critique of the notion of capital as a "factor of production" measurable independently of distribution.[2] This critique has shown the invalidity of some propositions of the theory, like the inverse relation between the rate of interest (rate of profit) and the "quantity of capital" per worker, which are basic for the explanation of distribution in terms of demand and supply for "factors of production".

The uncertainty which has resulted from these developments finds its expression in authors who think that new theoretical approaches should be explored. It is also revealed by the nature of some of the work carried out by those who adhere to the traditional approach.[3]

[1] This paper which develops under the impact of Sraffa's *production of commodities by means of commodities* some propositions contained in a Ph.D dissertation of 1955–1958, is based on notes delivered at a conference on "Marx's Transformation of Values into Prices of Production" held in Siena in 1972, and used then for lectures given in Cambridge and elsewhere since 1973–4: in the meantime, references to the ideas contained in them have appeared in other works. I would like to acknowledge the benefit I derived from discussions with Piero Sraffa and from comments from many people and in particular by K. Bharadwaj, A. Campus, B. Cutilli, H. Kurz, and M. Pivetti. Financial assistance by the 'Consiglio Nazionale delle Ricerche' is gratefully acknowledged.

[2] This line of criticism, hints of which may be found in Sraffa's 1951 p. XXII, was first brought to light in print by Robinson 1953. (See also Robinson, 1973, p. 195.)

[3] Thus, the attempt to avoid the difficulties besetting the theory appears to have led to an abandonment of the method based on "long-period positions" of the economic system, characterized by a uniform rate of profit. This notion had been central to the theory of competitive distribution and value since the very inception of systematic economic analysis. (See Garegnani, 1976, pp. 26–29.)

It is perhaps natural that when this kind of uncertainty arises in a scientific field, there should also arise a tendency to go back into the history of the subject, and see when and how theorizing took the turn leading to the present difficulties. When we do so and look back over the two centuries of systematic economic analysis, we find that, at the cost of severe simplification, we can distinguish two successive approaches to the theory of value and distribution. The modern demand-and-supply approach had in fact been preceded by a different approach which had its centre in a notion of "social surplus". This earlier approach found its first systematic expression in Quesnay's *Tableau Economique* of 1758, became dominant with the English Classical economists from Smith to Ricardo, and was then taken over and developed by Marx at a time when the main stream of economic analysis was already moving in a different direction.

2. The purpose of this paper will be to consider this earlier "surplus approach" to value and distribution. Section II will examine the premises which distinguish it from the later demand-and-supply approach. Sections III and IV then set forth the problem of "measuring-value" which arose within it and led to Ricardo's and Marx's explanation of value in terms of the labour necessary to produce the commodities. At the end of Section IV, Marx's error with regard to prices of production will be seen as arising from treating as integral parts of a single method for determining the rate of profit and relative prices, what can be developed as two equivalent but distinct methods for this determination: what we shall call the "Price-equations method" and the "Surplus-equation method". The solution based on the first method will be considered in Section V, where it will be shown to consist of the price equations in Sraffa's *Production of Commodities by Means of Commodities*, 1960. The two solutions obtainable on the basis of the "Surplus-equation method" will then be examined, respectively, in Section VI, dealing with the "Integrated wage-goods sector", and in Section VII dealing with Sraffa's "Standard system".

II. The "core" of the surplus theories

3. The notion of social surplus characteristic of the classical theories can perhaps be seen in its simplest form in Quesnay's *Tableau Economique*, where we find its first systematic expression. Quesnay saw that if the social product—which he considered to consist entirely of agricultural commodities—[4] was to repeat itself year after year without increase or diminution, a part of it had to be put back into production. Besides the necessary replacement of the means of production, this part included the subsistence of the agricultural labourers. What remained of the annual product after deducting this part constituted a "surplus", or "produit net",

[4] As is well known, Quesnay excluded manufactured commodities from the social product on the ground that they were a mere transformation of agricultural products.

of which society could dispose without impairing the conditions of its survival.

The fact that the subsistence of workers was considered necessary for reproduction established a direct link between the analysis of reproduction and that of the distribution of the product among the classes into which society is divided. Thus Quesnay linked the surplus to the landowners' share of the social product. And when Smith extended Quesnay's notion of surplus by showing that surplus originated from production in general and not from agricultural production alone, profits emerged as a second component of the surplus alongside the rent of land, thus providing the basis for the English classical economists' theory of distribution up to Ricardo.

The determination of the size of the social surplus was accordingly the centre around which these theories revolved. In principle this way of determining the non-wage shares is simple. Two magnitudes are assumed to be known prior to the determination of the surplus. They are: (i) the real wage, i.e. the quantities of the several commodities constituting the wage rate,[5] (ii) the social product, i.e. the aggregate of the commodities produced in the year. Since (iii) the technical conditions of production of the various commodities are also known prior to the determination of the surplus, a known social product implies a known number of labourers employed.[6] By multiplying the number of labourers by the known physical wages, we obtain the part of the product that goes to the labourers, which we may call "Necessary consumption", using a phrase by Ricardo (1951–58, VI, p. 108).

The surplus, i.e. the share of the product going to the classes of society other than the labourers, can then be determined by subtracting the "Necessary consumption" from the Social product, taken net of the means of production;[7] that is:

$$\text{Social product} - \text{Necessary consumption} = \text{Shares other than wages (surplus)} \quad (1)$$

[5] We are following the authors here discussed in assuming a single "average" or "natural" wage and thus homogeneous labour. As is well known, the possibility of reducing labour to homogeneity rests on the supposition of a constancy in the ratios between the wages for labour of different qualities: see Ricardo, 1951–1958 I, pp. 20–23 on the constancy of relative wages (see also Smith, 1910, bk. i ch X vol. I, p. 130). These ratios were in fact left to be studied outside what will be indicated below as "core" of these theories. (The difficulty raised by taking these ratios as known in the face of different physical compositions of the wages for different kinds of labour, seems to have been implicitly dealt with by taking the known *real* wage to be that of common unskilled labour and then supposing that the wages of other kinds of labour will tend to remain a constant proportion of it in terms of *value*.)

[6] We are at present assuming that each commodity can be produced by means of one method only. The consideration of alternative methods of production of the same commodity, which provides one of the two bases for the notion of a substitutability between factors of production characteristic of modern theory (cf. par. 7 below), can on the other hand only affect what has been said here by making the employment of labour associated with a given physical social product depend on the wage rate as well: under the hypothesis adopted in this paper the tendency to adopt cheaper methods will bring the economy to a definite technique that giving the highest wage for the given rate of profit. (See Garegnani, 1972, p. 266–7 and 281).

[7] The assumption that the means of production are physically reproduced has the sole purpose of postponing the complications arising out of errors in Smith and Ricardo's notion of capital (par. 12 below).

an equation where "shares other than wages" is the only unknown (see also the diagram Fig. 1 below).

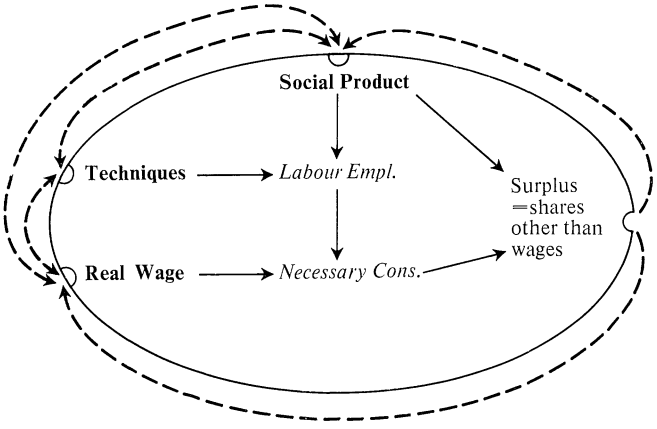

FIG. 1. A diagram of the "core" in the surplus theories. Underlining distinguishes circumstances determined outside the core. Continuous arrows point to dependent magnitudes inside the "core"; discontinuous arrows indicate influences studied outside the "core".

The peculiar feature of these theories—the determination of the shares of the product other than wages as a residuum or "surplus"—thus has its logical basis in the consideration of the real wage and social product as being determinable prior to those shares. It is to the determination of the real wage and to that of the social product that we must therefore turn, however briefly, for an understanding of the view of the economic system which underlies the simple formal structure of the surplus theories.

4. We have seen how Quesnay and the Physiocrats thought that the quantity of the product retained by the agricultural labourers was fixed at the subsistence level (for example Turgot 1786, ch. VI). The same was true for Ricardo who, however, while holding that an increase in wages above the subsistence level would tend to be reversed by the consequent increase in population, also freely considered the possibility that the increase be absorbed into "subsistence" as a result of "improved habits" and thus rendered permanent (Ricardo, 1951–1958, II, p. 115). In assessing Ricardo's and Quesnay's view of the wage, it is in fact important to note that the "subsistence" they referred to was always understood as determined by historical, rather than physiological conditions. Robert Torrens, to whom Ricardo referred as having "most ably illustrated" the subject (1951–1958, I. p. 97) argued that the "habits of the country" act in this respect as a "second nature" and, accordingly, the "natural price of labour" may vary not only from country to country, but also in the same country at the "different stages of national improvement". And Ricardo' for his part,

defined the natural wage as including "those comforts which custom renders absolute necessaries".[8]

Adam Smith's position regarding the "average" or "natural" wage was less clear-cut and, in some respects, more interesting than that of Ricardo. He also saw the wages as tending to an historical subsistence level, but he explained this by the "advantage" which the "masters" have in disputes over wages, rather than by any tendency of the population to grow in excess of the possibility of employment offered by accumulation. Thus, for example, Smith noted how the "masters" could "hold out" much longer than the workers in all wage disputes, since the master's "necessity" for the workman is not so "immediate" as the workman's for his master, and pointed out how "masters are always and everywhere in a sort of tacit but constant and uniform combination, not to raise wages": a "combination" against which, as Smith saw it, the combinations of workers, hindered by law, were of little avail.[9] Marx, for his part, was also far from adhering to a simple theory of wages based on subsistence. He asserted that the "regulating average wage" is given by an historically determined level of subsistence, but the tendency to this "average wage" was the result of a complex interaction between the actual wage and the size of the "industrial reserve army" of unemployed labour: a mechanism which gave considerable flexibility to his position on the probable, long-term, evolution of the "average" wage.[10]

Thus, at a closer inspection, what all these authors had in common was not, as is often held, the idea of a wage determined by subsistence. It was the more general notion of a real wage governed by conditions (often of a conventional or institutional kind) that are *distinct* from those affecting the social product and the other shares in it, and are therefore best studied separately from them. This separation between the determination of the

[8] Cfr. respectively, Torrens, 1815, quoted in Cannan, 1967, pp. 191–193; Ricardo, 1951–1958, 1, p. 94. For the social element in subsistence, see also Adam Smith's definition of "necessaries", 1910, bk. V, ch. II; vol. II, pp. 528–9.

[9] In keeping with this view, pointing to the relative bargaining power of the two classes, Smith saw that in an "advancing state of society", "masters voluntarily break trough [their] natural combination ..... not to raise wages", whereas the contrary would be true in a "declining state of society" (Smith 1910, bk. I, ch. VIII; vol. I, pp. 56, 64). On the idea in Hicks–Hollander 1977, in Samuelson, 1978, and in Casarosa, 1978 according to which Smith and/or Ricardo would have determined the "equilibrium" real wage as that balancing the growth of the supply of labour with that of its demand, resulting from accumulation, cfr Garegnani [1983], p. 311.

[10] Thus, for example, in *Capital*, vol. I, 1969a, ch. XXV p. 580. Marx admits that the real wage could rise in the long run to the extent in which the corresponding "diminution of the unpaid labour ... would [not] threaten the [capitalist] system itself". This position has important implications. In particular the fact that, under sufficiently general hypotheses, technical change cannot lower the rate of profit corresponding to a given real wage (cf. n. 6 above), entails the possibility of a long-term rise in the real wage which does not "threaten" the system: Marx's erroneous notion as to the possible effects of technical change on profits led him to discount this possibility. (Samuelson seems thus to move on questionable grounds when in 1971, p. 422 he reduces the question of the validity of Marx's approach to distribution to the question of "whether real wages rise or stagnate over a century". For further evidence against the idea that Marx held any simple subsistence theory of wages, cf Baumol, 1983).

wage and that of the social product is evident when, as in Quesnay or Ricardo, the wage is explained in terms of a customary subsistence, but the same separation between the two problems emerges in Marx and Smith, who admitted a greater influence of current economic conditions on the real wage. It is this separate determination of the real wage that is expressed in its treatment as a magnitude which is known when the determination of the other shares of the product is approached.[11]

As for the physical social product the circumstances that were seen to determine it, that is, basically, the accumulation of capital and the technical conditions of production,[12] were such that it was natural to suppose it was known prior to its division among the classes.

5. It is important to stress here that this *separate* determination of real wage and social product entails a structuring of the analysis which is radically different from that of the theories which were to become dominant later. The surplus theories have, so to speak, a *core* which is isolated from the rest of the analysis because the wage, the social product and the technical conditions of production appear there as already determined. It is in this "core" that we find the determination of the shares other than wages as a residual: a determination which, as we shall see in the next section, will also entail the determination of the relative prices of commodities. Further, as a natural extension of this, we shall find in the "core" an analysis of the relations between, on the one hand, the real wage, the social product and the technical conditions of production (the independent variables) and, on the other hand, the shares other than wages constituting the surplus, and the relative prices (the dependent variables).

An important point to notice is that the treatment of the real wage, the social product and the technical conditions of production as independent variables in the "core" in no way entailed denying the existence of influences of any single one of these three sets of variables over the remaining two. The interaction between these circumstances was in fact freely admitted by the classical economists and by Marx. An example is Marx's discussion of the "realization" of surplus value, in which the real wage played a key role in the determination of the size of the social product (cf. e.g. Marx 1969a, III, pp. 492–49). Another example is the reverse influence which the speed of growth of the social product was generally recognized to have for a shorter or longer period on the real wage.

[11] As Marx observed "The foundation of modern political economy . . . . is the conception of the value of labour power as something fixed, as a given magnitude" (Marx, 1969, vol. I, p. 45).

[12] In fact if we attempt to reduce analyses as different as those of Quesnay, Smith, Malthus or Ricardo to their common basic elements, what we find is the view that the *volume* of the social product depends on: (i) the stage reached by accumulation, which governs the number of "productive" labourers employed; (ii) the technical conditions of production which regulate the physical product per labourer and depend in turn on the stage reached by accumulation. (See Smith, 1910, vol. I, pp. 1–2). The *commodity composition* of the social product, on the other hand, was studied from the angle of the needs of reproduction (cfr. for example Quesnay's *Tableau Economique* or Marx's reproduction schemes in chapters XX–XXI of vol. II in *Capital*), or else was left to be studied case by case as the need arose.

Moreover, the fact that those three sets of circumstances appeared as independent variables in the determination of the surplus did not prevent the classical economists from freely admitting influences of the surplus upon them: e.g. the classical economists generally admitted the influences which the level of profit could have on the real wage, via the speed of accumulation, and Marx went further by considering how a fall of the rate of profit, consequent upon a rise of the wage rate, may reverse that very rise by checking accumulation and causing technical changes, thus re-creating a sufficient level of labour unemployment (cf. par. 5 above).

What the structure of classical analysis did imply was that these interactions and reverse influences, like the influences of the other factors determining wages, social product and available techniques, were left to be studied outside the 'core'. The multiplicity of these influences and their variability according to circumstances was in fact understood to make it impossible to reconduct them to *necessary* quantitative relations like those, studied in the "core", between distributive variables and relative prices and between outputs or techniques and the dependent distributive variables and prices.

6. Now this separation of the analysis into distinct logical stages contrasts sharply with what we find in the later marginalist theories. In these, the determination of the wage is in fact inseparable from, and symmetrical to, that of the other shares of the product. Moreover, the demand-and-supply mechanism used in that determination implies that real wages and the other distributive variables (and hence relative values) can only be determined *simultaneously*, and simultaneously with the volume and composition of the product. Indeed, in later theory, distribution, outputs, and relative values of commodities are all determined simultaneously taking as data the tastes of consumers, the endowments of "factors of production" and the technical conditions of production. The determination of these three sets of data is then seen as falling largely outside the domain of economics. As a result, in these theories the determination of revenues other than wages and of relative prices comes to include most of economics. Instead of constituting a limited "core" of economic analysis—dealing with the necessary quantitative relationships among distributive variables and among them and prices—it becomes almost co-extensive with economics itself. The more limited scope which the theory of value has in the surplus approach[13] may however give it the greater flexibility which is required by a subject as complex as economics.[14,15]

[13] The "core" might in fact be described as constituting the "theory of value" such as we find in the surplus theories.

[14] An important example of the greater flexibility of the "surplus theories" seems provided by the attitude to possible deficiencies of aggregate demand: while Ricardo held that no "general gluts of commodities" were possible, a different view of the problem was taken by Malthus or Marx, (who also worked within the same "surplus" approach to value and distribution, and were no less consistent with it on the necessity of "short chains of reasoning" in economics cf. Marshall, 1961, I, p. 773.) (*footnote 15 on p. 298*)

The remainder of this paper will be exclusively concerned with what has here been described as the 'core' of the surplus theories. More particularly we shall be concerned with the part of this "core" which consists of treating the real wages, as distinct from outputs or techniques, as the independent variable.

7. An effect of the above contrast in the structuring of the two theories may be seen in how modern authors, used to focusing attention on "consumers' choice" in the determination of outputs, seem often surprised by the fact that in classical theories the changes in real wages are considered separately from changes in outputs, even in outputs of wage goods. The modern focus on consumers' choice and the corresponding simultaneous determination of prices and outputs is however an integral part of the demand-and-supply mechanism for determining distribution just mentioned. Consumers' tastes for goods requiring different proportions of factors of production are in fact supposed to determine, together with the choice between alternative methods of production, the relative "scarcity" of these factors. When the explanation of distribution is different—as we saw to be the case for the classical economist—the need to study the effects of changes in wages on prices *simultaneously* with their effects on outputs is no longer evident (a similar need to study the effects of a change in the technical conditions of production simultaneously with its effects on the tastes of consumers—is for example denied in the marginalist theories using those tastes as data.) It might however be insisted that if non-constant returns prevail, the output changes resulting from changes in real wages (and the consequent changes in prices and techniques adopted) will in turn affect prices and the distributive variables other than wages, and hence modify the relationships studied under the assumption of given outputs. A simultaneous determination of prices and quantities would then seem required in order to study those relationships. This line of argument presumes however the possibility of expressing the dependence of outputs upon changes in distribution by means of functional relations of the same nature as those postulated in modern theory, endowed, that is, with known properties of

[15] The classical surplus theories are characterized by some authors as being concentrated on reproducible commodities, and hence "production", as opposed to the concentration on commodities of the scarcity type and hence on "exchange" which would be the hallmark of the dominant marginalist theories. Accordingly the two kinds of theory would deal with two distinct series of problems, with an opposite practical relevance in relation to time, the classical theory becoming relevant just when (in the long run) the marginalist theory becomes irrelevant (cfr. e.g. Pasinetti, 1977, pp. 6, 31–33). Whereas it aptly describes some differences between the two approaches, this distinction seems not to go to the roots of the difference, which lies in the way in which *both* "production" and "exchange" are treated in each approach. The determination of the wage on the basis of the forces mentioned at par. 5 above—entailing the determination of profits as a surplus and not by the "scarcity of capital" relative to labour—also implies that the problems of exchange themselves cannot be viewed as problems of "scarcity". This remains true whether we consider a short period, or a long period, in which plant in the several industries can adapt to outputs. (For a critical view of this criterion of distinction see also Roncaglia, 1979 pp. 145–6.)

sufficient generality and with persistence over time. If this were in fact possible it would be natural to consider the changes in the quantities produced simultaneously with the change in distribution from which they originate. But this view was not that which the Classical economists took of the matter and, it seems, we today have even better reasons than they had for not taking it. Thus for example important changes in the real wage may have a multiplicity of effects on aggregate demand, the intensity of which will depend on the particular circumstances in which they occur and cannot, in our present state of knowledge, be reconducted to known functional relations of sufficient generality and persistence. If this is admitted, it will appear that a general determination of outputs simultaneously with relative prices is impossible, and that the basic procedure can only be that followed by the classical authors. They analysed changes in prices and outputs by what we may describe as two distinct logical stages. In the first, the effect of the change in real wage was examined while taking the outputs as given. In the second stage, the possible effect on *outputs* of the initial change was analysed in accordance with the circumstances of the case under consideration, jointly with its possible secondary effects on prices and distribution, in the case of non constant returns to scale (cf. Garegnani, 1983 pp 311–312).

III. Ricardo's measurement of the value of aggregates by means of embodied labour

8. We have seen the rationale behind taking the Social product and the Necessary consumption as given physical aggregates when determining the shares other than wages in the surplus theories. But how are we to measure those two magnitudes in equation (1)? As we shall presently see, the theory cannot stop at conceiving them as physical aggregates and must proceed to their measurement in value; will these values also be given when the physical aggregates are given? It is in connection with this problem of measurement that the surplus theories of distribution meet the question of value and with it, their chief analytical difficulty. The remainder of this paper will turn on this question which will be taken up in the comparatively advanced form it assumed in the theory of profits of Ricardo's *Principles*.[16]

Since we are concerned with aspects of the Classical problem of value which are independent of the rent of land, we shall assume that fertile land abounds and that rent can accordingly be ignored.[17] Thus, on the right hand

[16] We are thus not concerned here with the physical measurement underlying the reasoning in both Ricardo's earlier *Essay on the Influence of a Low Price of Corn on the Rate of Profit*, and Quesnay's *Tableau Economique* (on these points cf. respectively, Sraffa, 1951, pp. xxxi–xxxii. n. 4, and Sraffa, 1960, p. 93).

[17] See above par. 6. We may however notice how the characteristic separation between determining the quantities produced and determining the shares of the product other than wages (par. 6 above) allowed Ricardo to isolate the share of land rent. This separation (and the implicit assumption that each worker is assisted by the same capital, whatever the land cultivated, or the intensity of its cultivation) allowed Ricardo to take as given the productivity of labour on the no-rent land or, even, that of the last labourer empoyed on land already under cultivation (on the latter point, cf. Ricardo 1951–1958, vol. I, p. 71).

side of equation (1) we shall find aggregate profits. Two observations are however necessary in order to proceed to the *rate* of profit, on which Ricardo's interest was in fact focused. The first is that in Ricardo, as in the other Classical economists, a yearly production cycle is implicitly assumed;[18] the wages are supposed to be advanced at the beginning of the cycle and are therefore a part of capital. The second observation concerns the fact that in determining the rate of profit Ricardo operated *as if* capital consisted entirely of the wages advanced for the year. In fact he saw the division of the product between wages and profits as the only factor capable of influencing the rate of profit, thus ignoring the independent influence exerted by the proportion between labour and means of production.[19] In order to give an account of his theory of profit which is both consistent and sufficiently faithful, we shall assume that production requires only very simple means of production, which can be ignored.

Under the above assumptions, the annual rate of profit will be expressed by the ratio between the social surplus and the annual wages or "Necessary consumption", i.e.

$$\text{Rate of profit} = \frac{\text{Social product} - \text{Necessary consumption}}{\text{Necessary consumption}} \qquad (2)$$

9. Suppose, now, that the "Necessary consumption" advanced to workers is reproduced in kind during the year and the yearly aggregate profits can accordingly be reckoned in physical terms, that is, as a *surplus product.* Even under this most favourable hypothesis it would be impossible to stop at a

[18] "A year is assumed in political economy as the period which includes a revolving cycle of production and consumption" (James Mill, 1821, p. 185).

[19] Cf. for example: "whether these increased productions, and the consequent demand they occasion, shall or shall not lower profits depends *solely* on the rise of wages" (Ricardo, 1951–1958, I, pag. 298, our italics; for a similar statement cf. also *ibid* pp. 289–292). More generally, Ricardo failed to show any awareness that the rate of profit can change for causes other than changes in the proportion between wages and profits in the (net) social product. The origin of this error, which Ricardo shared with his contemporaries, can be traced back to Adam Smith. In a well-known passage of *Wealth of Nations* he had argued that, although it may be thought that besides wages, profits and rents, the price of a commodity also includes all that is necessary for replacing the means of production, yet since the prices of the latter are in turn made of those parts, the entire price "resolves itself either immediately or ultimately into the same three parts of rent, labour and profit" (Smith, 1910, I, pp. 44–45). From this view of prices, in which we find in germ the correct idea of reducing capital to wages (and rents) advanced for *various periods of time*, Smith often slipped into the altogether different idea that capital can be reduced to the wages advanced for the current year.

Marx indicated the above deficiency of Ricardo's analysis as an erroneous identification between rate of profit and rate of surplus value (cf. for example, 1969, II, p. 463). Marx's criticism of Ricardo seems to have been misapprehended as an accusation that Ricardo "ignores non-wage capital, at least when referring to the economy as a whole" (Steedman, 1982, p. 126). However Marx did not deny that Smith and Ricardo saw the existence of means of production (cf. for example the carefully worded passage in 1969a, III, p. 841 and n. 51): what he said was that Ricardo ignored the effect of changes in the proportion of labour to means of production on the rate of profit (Marx, 1969, II, p. 373; on this point cf. also G. De Vivo 1982, pp. 91–92).

physical notion of the two magnitudes involved, because the surplus product and the necessary consumption would generally consist of commodities which are different, or are taken in different proportions. The ratio between the two aggregates would give the quantity of "surplus product" (a composite commodity) per unit of "necessary consumption" (a different composite commodity), but not the *rate of profit*, i.e. the ratio between the *values* of the two magnitudes.

In the *Principles* Ricardo is accordingly faced with the need to measure the Social product and the Necessary consumption in value terms and, hence, with the problem mentioned above: will these given physical aggregates also be given when expressed as value magnitudes? If the value expression of either aggregate were to depend on the rate of profit, the determination of profits as a surplus in accordance with equation (1) or (2) above is threatened by circular reasoning.

Ricardo's starting point in dealing with value was Smith's theory of the "natural price". Smith had defined the natural price as the sum of the wages and profits (we are ignoring rents) which must be paid in order to produce the commodity, reckoned at their "natural" or "average" rates (Smith, 1910, bk. I, ch. VII, pp. 48–51). As for the unit in which these natural prices should be expressed, Smith had suggested a "real" or "invariable" measure of value consisting of the quantity of labour which a commodity can "command", that is, in modern terms, the wage unit (*ibid*, bk. I, ch. V, vol. I, p. 28). If, however, we use Smith's measure in equations (1) or (2) we are faced with exactly the difficulty we mentioned above: the *value* of the *physically given* social product will *not* be known before the rate of profit is known. Take, for example, an economy employing 3 million workers. The Necessary consumption will "command" 3 million labour-years and be a known magnitude. But the same will not be true for the Social product: with, for example, the capital consisting only of the wages advanced yearly we assumed above, the Social product will command $3(1+r)$, million labour-years, where r is the rate of profit, namely 3.3 m. if $r = 10\%$, but 6.6 m. if $r = 120\%$. We may accordingly seem to be reasoning in a circle when we follow the surplus approach and attempt to determine profits by *difference* in accordance with equation (2): in order to do that we need to know the size of the Social product, which is not known until we know the very rate of profit which is to be determined.[20] This dependence of the value of the product upon distribution means that, when we look at the Social product and the Necessary consumption in value terms, the constraint by which one class cannot have more without the other class having less—so evident if we could look at the product in physical terms—is no longer

[20] The difficulty, however, is ultimately that of expressing the capital required in production independently of distribution, (cf. Garegnani, 1960, pp. 18–19). Thus, we shall see in Section VI below how measurements in terms of commanded labour are in fact compatible with the determination of the rate of profits in accordance with equation (2) above.

apparent: might not the real wage rise without affecting the rate of profit, or vice-versa? Indeed Smith himself often lost sight of the constraint and envisaged the rate of profit and the wage as determined *independently of each other.* He wrote that "the natural price varies with the natural rate of each of its component parts" (Smith, 1910, bk. I, ch. VI, vol. I, p. 56) giving rise to what has been described as Smith's "adding up theory of prices" (Sraffa, 1951, p. xxxv).[21] And, after Smith Malthus could argue that a tariff on corn would raise *both* the rent of land and the rate of profits, without apparently seeing the consequences these rises would be bound to have upon the real wage (e.g. Malthus 1951, p. 398, and passim).

10. Ricardo's great merit was in fact that he saw through these "appearances" (cf. par. 11 below) and brought consistency back into economic theory. This achievement of Ricardo's *Principles* was rendered possible by relating the exchange value of the commodities to the quantity of labour necessary to produce them.

Let us in fact suppose that the proportion between wages and profits in Smith's natural prices are the same for all commodities—as they would be under our assumption at par. 8 of a capital consisting only of the advanced wages. All commodities would then exchange according to the quantity of labour required for their production. The ratio between the values of any two aggregates of commodities—or between sums or differences of such values—would accordingly be equal to the ratios between the respective quantities of labour embodied.[22] The value of Social product and of the Necessary consumption in equation (2) could then be "measured" in terms of the quantities of labour embodied, here indicated by P and N, and we

[21] Adam Smith's "adding-up theory of prices" seems often to have been construed as anticipating the demand-and-supply explanation of value and distribution of the later marginalist theories, in contrast with the explanation of the same phenomena later provided by Ricardo. Thus, in 1954, p. 189, Schumpeter writes of Smith's conception of the "natural price" as "the rudimentary equilibrium theory of chapter 7 [of the Wealth of Nations which]... points towards Say and, through the latter's work, to Walras", and originates a "Smith, Mill, Marshall line" which was rival to that of Ricardo (*ibid.* 530, cf. also *ibid* pp. 557–558; 567–568, 599). A similar interpretation is adopted by Maurice Dobb who, in 1973, refers to Smith's "determination of the general level [of wages and profits] by conditions of supply and demand for labour and capital respectively" (p. 50, cf. also p. 112, and passim). This view of Smith's approach to value and distribution seems to overlook that, behind Smith's vague references to the rate of profit as determined by the "competition" of capitalists, there lay, as we saw, the inconsistency of rates of wages, profits and rent determined independently of one another. And it was just this inconsistency that Ricardo and Marx criticised Smith for. The demand and supply forces of the modern theories, founded as they are, on the "substitutability" between "factors of production", (cf. above p. 76) are as absent in Smith as they are in Ricardo.

[22] As we saw in par. 9 the rate of profit can be directly envisaged as a relative value, namely the value of the Surplus product (a composite commodity) in terms of the Necessary consumption (a second composite commodity): if all individual commodities exchange according to the quantity of labour embodied, the same will be true of these two composite commodities: When the economy is not replacing its wage capital, equation (3), written as $r = (P/N) - 1$, shows how the rate of profit can still be derived from a relative value: that of the Social product, relative to the Necessary consumption.

would obtain the rate of profit r as

$$r = \frac{P - N}{N} \tag{3}$$

where P and N are now *known* magnitudes. Thus, if we return to our example of an economy with 3 m workers, a doubling of the real wage and, hence, of the labour required for the production of its constituents from, for example, 1/3 to 2/3 of a labour-year, would make the rate of profit fall from $(3-1)/1 = 200\%$ to $(3-2)/2 = 50\%$. The constraint binding changes in wages and changes in the rate of profit becomes self evident and no space is left for the illusion, generated by the appearance of price as a sum of wages and profits, that the rivalry between capital and labour "tends to increase the value of the product to such an extent that each receives a larger piece" (Marx, 1969a, III, p. 503). In yet another quotation from Marx, relating particularly to Ricardo:

> "It is the great merit of classical economy to have destroyed this false appearance and illustion, this mutual independence ... of the various social elements of wealth" (Marx, 1969b, III, p. 830).

11. It may here be appropriate to notice how the nature of Ricardo's contribution in overcoming the error implicit in Smith's adding-up theory of prices may help us to comprehend Marx's often misunderstood position with respect to what he called "vulgar political economy". Ricardo had begun to overcome the difficulties which had prevented his predecessors from seeing the constraint binding wages to profits and rents: however his very success in bringing this to light had the result of exposing the class antagonisms which underlie the division of the product. In this situation, the attempt to preserve a harmonious view of society took—Marx thought—the form of turning a blind eye upon the analytical advances of Ricardo and keeping closer to the "appearances" by which the price of the product, seen as the *sum* of profits, wages and rents, may *seem* capable of accommodating the rise in one of these elements without a decrease in the others. In Marx's ironic words, already referred to above, these economists held that:

> "Even if this occasionally brings them to blows, nevertheless the outcome of this competition between land, capital and labour finally shows that, although they quarrel with one another over the division, their rivalry tends to increase the value of the product to such an extent that each receives a larger piece, so that their competition, which spurs them on, is merely the expression of their harmony" Marx, 1969a, III, p. 503.

The fact that these views were the result of adhering, as popular thought often does, to "appearances" explains the specific adjective "vulgar", i.e. popular, which Marx applied to these economists. Accordingly, Marx defines as "vulgar economy" that "economy" "which deals with appearances only", in contrast with "classical political economy which has investigated the real relations of production in bourgeois society" (Marx, 1969b, p. 85, n. 1). In

this "vulgar economy" he includes, if not the work of Say himself, that of his followers in France and Germany (Marx, 1969a, III, p. 500), and that of Senior and his followers (Marx, 1969b, pp. 596–7). Marx's frequent reference to the existence of "vulgar representations" in Smith, an author for whose scientific achievement he had the highest respect, seems on the other hand apt to bring out the specific, and not merely derogatory, meaning which he attributed to the expression "vulgar economy".

The distinction by Marx between 'vulgar' and 'classical' political economy turned thus on a second, even more basic distinction between two kinds of representation of the economic relations of bourgeois society. There are, on the one hand, the "apparent relations" or "connections", which are those perceived by the unsystematic observer and which are represented in Adam Smith's "adding up" theory of prices, when "instead of resolving exchange-value into wages, profit and rent [he] constructs the exchange—value of the commodity from the value of wages, profit and rent, which are determined independently and separately" (Marx, 1969a, II, p. 217). In such an inconsistent representation of the economic system "[the] contradictory character [of capital] is totally concealed and effaced . . . no contradiction to labour [is evident]" (Marx, 1969a, III, p. 467). There are, on the other hand, the "real relations" constituting the "intrinsic", or "inner connections" of the bourgeois system. These are the relations brought to light by systematic scientific analysis. They centre on the constraint that binds changes in wages to changes in profits and rents and reveal the economic antagonism between classes (for example, Marx, 1969a, II, p. 166). Now, for Marx, these "inner connections" required, in order to be revealed, that the product be measured *independently* of its division between the three classes. Hence the role of Ricardo's measurement of values in terms of labour embodied in which, in Marx's own words, the value of the commodity "*does not depend upon its division into wages, profits and rents*" and constitutes instead "the limit . . . for the dividends which the labourers, capitalist and landlord will be able to draw from this value in the form of revenue, wages, profits and rents"; Marx, 1969b, III, p. 854; cf. also p. 274 and 1969a, II, p. 219.[23]

[23] When faced with passages like the ones above, it seems surprising that various authors following what looks like an established tradition, should attribute to Marx's theory of value some "qualitative" role different from that which the theory had in Ricardo, i.e. that of measuring the product independently of its division (cfr. P. Sweezy, 1946, p. 33; the idea finds an early expression in Hilferding 1949, e.g. pp. 130–132. A recent clear-cut expression of this position may be found when it is claimed that "the idea that the theory of value developed in vol. I of CAPITAL is a (bad) theory of relative prices is . . . untenable" (Medio, 1977, p. 382). Medio (a participant in the Siena conference mentioned above) appears however to contradict this claim of his, when in the very next line, he admits that "the cost of production theory of price seems to be unsatisfactory [because] it contained an apparent element of circularity . . . the calculation of the rate of profit requires valuing [product, wage goods and means of production] at their equilibrium prices. The latter, however, can not be calculated without knowing the rate of profit". Here indeed we have a very good reason for Marx's "bad" theory of relative prices: for what means were available to Marx for breaking that "apparent circularity" if not the labour theory of value developed by Ricardo for that very purpose?

IV. Marx's "prices of production"

12. And for this analysis of the inner organic connection" binding wages and profits, Marx took the road which Ricardo's theory of value had opened up for him.

As we saw (par. 8 and n. 20) when determining the rate of profit Ricardo had operated *as if* capital could be resolved entirely into the wages advanced for the year. Marx started by clearing up this confusion which Ricardo had inherited from Smith. He showed that the proportion which the means of production bear to the labour (and hence, to the wages advanced) constitutes a factor which can influence the rate of profit independently of the proportion in which the product is divided between wages and profits. Accordingly, he distinguished capital into two parts: the wages advanced or "variable capital" and the means of production or "constant capital". To simplify our exposition, we shall assume that constant capital is entirely consumed during the yearly production cycle (i.e. consists of circulating capital) and shall retain the assumption of free land. If, then, commodities exchanged according to the quantities of labour embodied, the rate of profit would be determined as

$$r = \frac{s}{c+v} \tag{4}$$

where c and v are respectively the "constant" and "variable" capitals, measured in terms of the labour necessary to produce them, whereas s, the social "surplus value", is Ricardo's $(P-N)$ and is measured by the quantity of "surplus labour", the labour exerted in the year over and above that necessary for reproducing the wages.[24]

Commodities however do not exchange according to the quantities of labour embodied. If we look at "natural" prices as resolving themselves into wages and profits, as Ricardo did, we find that these wages and profits are present in different proportions in the prices of different commodities and the latter do not, therefore, exchange in proportion to the labour necessary for their production. Ricardo had admitted this in terms of "modifications" to the rule that commodities exchange according to the labour embodied: his argument concerning profits continued however to rest ultimately on that rule.[25]

[24] Equation (3), by which we expressed Ricardo's determination becomes, in the new symbols, $r = s/v$ and its difference from equation (4), due to Marx's consideration of the organic composition of capital, may become clearer if we rewrite equation (4) as

$$r = \frac{s/v}{(c/v)+1}$$

[25] Ricardo struggled with that problem until the end of his life. Cfr. the paper *On Absolute and Exchangeable Value*, Ricardo 1951–58 V, which was in fact written in the summer of 1823, just before his death. A misunderstanding of the position of Ricardo and of Marx's criticism of it seems to occur when in Steedman 1982, it is asserted that Marx was "quite wrong . . . to say that Ricardo's approach was inherently incapable of providing a theory of the rate of profit",

(*continued overleaf*)

Marx faces the question in the manuscripts posthumously published by Engels as vol. III of Capital and, in the sketch of a solution he provides there, he comes within one step of a correct general solution. He even indicates the step which is yet to be taken—though, as we shall see, he failed to realize its consequences.[26]

13. There, Marx starts by asking himself why commodities do not generally exchange according to the quantities of labour necessary to produce them. The answer he gives is simple: if commodities exchanged according to that principle, those produced with capital of a higher "organic composition" (i.e. a higher proportion of "constant" to "variable" capital) would give a lower rate of profit. Thus, suppose that only corn and steel are

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where, as the context shows, "Ricardo's approach", as distinct from Marx's, would be founded on the idea that "the general rate of profit and the prices of production must be determined simultaneously within the theory" (*ibid.* p. 124). In fact the basis of Ricardo's argument in the *Principles*, is the assumption of the *constancy* in the value of commodities "in the production of which no additional quantity of labour is required" (e.g. Ricardo, 1951–58 I, pp. 110–111). Ricardo's actual procedure seems therefore no different from Marx's in admitting that commodities do not exchange according to the quantity of labour embodied, but in determining the rate of profit as if they did so exchange—by looking, that is, for a commodity which would constitute a "medium", such that "Those on the side of this medium would rise in comparative value with a rise in the price of labour . . . and those on the other side might fall from the same cause" (Ricardo, 1951–58, for Marx cf. p. 34 below). Moreover Ricardo, like Marx, uses the rate of profit so determined to ascertain how far the relative prices of commodities deviate from, or change independently of, the relative quantities of labour embodied.

The main difference between Ricardo and Marx lies therefore in whether this logical order is explicitly followed and justified, as it is by Marx, or has instead to be extracted from an argument which, if taken literally, far from being a "simultaneous" determination of prices and the rate of profit, would be contradictory, since it would rest on ignoring that very dependence of relative prices on distribution which is admitted elsewhere in the argument. Now, this criticism of Ricardo is precisely the one expressed by Marx when for example he charges Ricardo with a "erroneous confusion of cost prices and values" (Marx, 1969, II, p. 199). It would on the other hand contradict the facts to view Ricardo's procedure as a better preparation than Marx's for the later simultaneous determination of prices and profits: when the latter came, with Bortkiewicz, Seton and Sraffa (cfr. n 32 below) it grew out of Marx's equations for prices of production, rather than directly out of Ricardo's less definite procedure.

[26] We may here notice the curious misunderstanding according to which the labour theory of value is held to be incompatible with the existence of alternative techniques (cf. Morishima, 1973, p. 189, Steedman, 1982, p. 65). The dependence of labour values on the technique adopted would not in fact prevent their use in determining the rate of profit, any more than the similar dependence of physical inputs prevents them from playing the same role in price equations (8) below: labour values would allow determining the rate of profit corresponding to each technique, and hence that which could be shown to rule under free competition at the given wage (cf. p. 3 n. 6, above).

Here we may also notice the misconception by which Samuelson in 1974, p. 292, writes that "in a regime of values . . . the technique that minimizes values at $r = 0$ will minimize them for all r's, a shortcoming of the values model". A confusion occurs here between the labour theory of value, by which the several *commodities* exchange according to embodied labour—and the different conditions under which costs for the *same* commodity, produced with alternative techniques, should equal the ratio between the respective quantities of labour embodied, when the technique maximizing the net product per worker would in fact dominate at all levels of the real wage. Equal organic composition *as between commodities* does not conceptually entail equal organic composition *as between alternative techniques*, as shown for example by Samuelson's "surrogate production function", 1962, which rests entirely on labour value conditions, but clearly does not entail domination of one technique over all others.

produced in the economy and, the "rate of surplus value" s/v being for example 1, the organic composition c/v is 1 for corn and 3 for steel: steel production would give a rate of profit of 25% as against the 50% of corn prodution. The competitive tendency to a uniform rate of profit will then make steel exchange for more than the quantity of corn embodying the same amount of labour, so as to raise the surplus value (the difference between value of output and value of capital) in steel production and lower it in corn production until the two rates of profit become the same. It may then appear that the divergence of the price of steel in terms of corn from the ratio of the respective quantities of labour embodied, has the meaning of *redistributing* surplus value away from the surplus industries to the "deficit" industries (from the corn industry to the steel industry in our example). Marx in fact arrived at this conclusion and argued that when the redistribution has been completed, we shall find in each branch of production the same rate of profit we obtain in equation (4), by distributing *total* surplus value over *total* capital:[27] just as when 5 sacks of corn are re-distributed proportionally among 10 people, each will end up with $\frac{5}{10}$ of a sack, irrespective of the initial distribution. The prices of production that ensure this result will be those obtained by applying the rate of profit calculated in equation (4) to the capital used in each branch. In our steel and corn economy:

$$r = \frac{(s_s A_s + s_c A_c)}{(c_s A_s + c_c A_c) + (v_s A_s + v_c A_c)} \tag{5}$$

$$\left.\begin{aligned} p_s &= (c_s + v_s)(1+r) \\ p_c &= (c_c + v_c)(1+r) \end{aligned}\right\} \tag{6}$$

in which the prices of production p_s of steel and p_c of corn are referred to physical units of the two commodities requiring a unit of labour for their production (so that $c_s + v_s + s = c_c + v_c + s_c = 1$), and A_s, A_c indicate the quantities produced of the two commodities. The way in which r is calculated implies on the other hand that the prices p_s and p_c are expressed in terms of the composite commodity constituting the social product, taken in the quantity requiring a unit of labour for its production.[28]

This reasoning (according to which the rate of profit—the key variable of the system—is determined by equation (5) *as if* commodities exchanged in proportion to the quantities of labour necessary for their production, may go a long way towards explaining why Marx thought it possible to conduct his argument in vol. I and II of *Capital* on the basis of just that assumption, postponing to vol. III the determination of the prices of production.)

14. If this is what we find in the posthumously published Chapter IX of Book III of *Capital*, Marx did not fail to notice a shortcoming in this

[27] Cfr. for example the illustration of the cotton mill in Marx 1969a, III, p. 155.

[28] Marx's condition that total prices should equal total values is in fact equivalent to taking as numeraire this particular composite commodity, which is also that produced with the "average" organic composition.

intended solution. Competition will distribute profits in proportion to the "prices" of the constant and variable capitals, and not in proportion to their "values" (i.e. the quantities of labour embodied). He wrote accordingly "there is always the possibility of an error if the cost price of a commodity in any particular sphere is identified with the value of the means of production [and variable capital] consumed by it".[29]

Marx seems to have left the question there. Had he attempted to correct that "error", he would soon have found that the price equations modified by estimating the capitals at their prices must determine not only the prices of production but also the rate of profit: equation (5) would therefore have stood out as incorrect. Let us in fact modify equations (6) by expressing both the constant capital, assumed here to consist of steel, and the variable capital, consisting of "corn", in terms of prices. We obtain

$$\left.\begin{aligned} p_c &= (c_c p_s + v_c p_c)(1+r) \\ p_s &= (c_s p_s + v_s p_c)(1+r) \end{aligned}\right\} \tag{7}$$

It is sufficient to divide both equations by one of the two prices, say p_c to realize that Marx's two independent price equations contain in fact only one unknown, the *relative* price p_s/p_c, and are therefore contradictory if r is to be determined by equation (5): in the price equations the uniform rate of profit can only be determined *simultaneously* with the relative price of the two commodities.

It is not difficult to see where lay the fault in the notion of a redistribution of surplus value which, as we saw, led Marx to equation (5). Unlike the 5 sacks of corn which do not change in size *relative* to the 10 people in the course of the redistribution, the size of the social surplus value does so change *relative* to capital. This surplus value is in fact the price of production of the surplus product, and cannot but *change* relative to that of social capital when, with the redistribution of surplus value, relative "prices" in general come to diverge from relative "values".[30] As we saw in par. 9 the profit rate is but the relative value of those two composite commodities and it will not be equal to the ratio between the quantities of labour embodied in them any more than the relative price of any two commodities.

15. There is, however, a sense in which Marx's error was suggestive. The error can be envisaged by us as the result of treating as integral parts of a single method (for the determination of the rate of profit) what are in fact when consistently developed, two equivalent methods, each of which is

[29] Marx, 1969a, III, p. 165. As is well known Marx indicates by 'cost-price' the value (price) of the capital (constant *and* variable) used up in the production of the commodity in question. Marx continues the passage quoted above with the words "our present analysis does not necessitate a closer analysis of this point".

[30] It is curious to note how Hilferding, in his answer to Böhm Bawerk's critique of Marx, neatly misses this point when he writes: "It is obvious that the change in distribution makes no difference in the total amount . . . of surplus value undergoing distribution". Hilferding 1949, p. 160.

sufficient to determine that rate. The first, which we may call the *price-equations method*, is exemplified by equations (7) and determines the rate of profits—or, more generally, the relation between the wage and the profit rate—*simultaneously* with relative prices. However, the basic idea of profits (the non-wage share) as a *surplus product*, which they can be seen to be whenever the economy is in a self-replacing state, inevitably leads one on to attempt some simpler method. The latter method, which we may here call the *Surplus-equation method*, is exemplified by equation (4) or (3) for the case in which commodities exchange according to the labour embodied. Essentially, it depends on the possibility of expressing both the surplus and the capital that appear in the equation in terms which are proportionate to their values but do not contain the unknown prices, so that the profit rate is the only unknown.

As sections VI and VII below indicate, this second method is also available for sufficiently general hypotheses and it appears to exhibit some advantages of simplicity and transparency over the Price-equations method.

V. The "Price-Equations Method" of determining profits

16. The "Price-equations method" consists of the generalization of equations (6). In these equations we assumed that constant capital consists of one commodity only. When that assumption is abandoned, the constant capital of each industry has to be distinguished into as many quantities of embodied labour as there are kinds of means of production. To each of those kinds a different price of production applies: the additional unknown price thus introduced will entail an additional price equation.[31] Matters are even simpler for the variable capital: the assumption of a uniform real wage ensures that in all industries variable capital consists of the same composite "wage commodity": we may therefore apply to it the single price obtainable from the prices of its constituent commodities.

We may now write the price equations obtained by generalizing equations (6) for the case of any number k of commodities $a, b, \ldots k$.

[31] A determination of the rate of profit based on Marx's price equations was proposed as early as 1907 in L. von Bortkiewicz, 1949. In that article Bortkiewicz acknowledged a debt to Tugan Baranowsky, who had used a similar method to show that Marx was mistaken in his determination of the rate of profit. It is from the latter author that Bortkiewicz apparently derived the grouping into three sectors (means of production, subsistence goods and luxury goods) of the various industries to which Marx had separately referred when dealing with prices of production. The aggregation of the means of production into a single sector is however in contrast with the need to disaggregate constant capital, and Bortkiewicz had to refer to a single price of production, thus treating the price system as if only one capital good existed in the economy. Bortkiewicz's aggregation of constant capital also helped to hide the fact that, as we shall see below, the measurement in terms of labour embodied of the elements of capital can be replaced by a physical measurement. However, a second solution to Marx's problem of the determination of the rate of profit which did not suffer from the deficiency of the aggregation of capital goods and was based on work by Dmitriev was advanced in Bortkiewicz, 1954 (cf, below p. 23, n. 39).

Let:

a be produced during the year by L_a labourers assisted by the constant capitals $A_a, B_a, \ldots K_a$, (some of which may be zero) consisting of commodities $a, b \ldots k$; thus requiring a total quantity $A = L_a + A_a + B_a + \ldots + K_a$ of direct and indirect labour;

$L_b, A_b, B_b, \ldots K_b, B; \ldots; L_k, A_k, B_k, \ldots K_k, K$, be the analogous quantities in the production of commodities $b \ldots k$; w be the quantity of labour necessary to produce the given real wage;

$\lambda_a, \lambda_b, \ldots \lambda_g$ (such that $\lambda_a + \lambda_b \ldots + \lambda_g = 1$) be the quantities of labour embodied in the wage goods, g in number, constituting a unit of the "wage commodity" λ, which we then choose as the numeraire.

We shall have:

$$\begin{aligned}
[(A_a p_a + B_a p_b + \ldots K_a p_k) + L_a w](1+r) &= A p_a \\
[(A_b p_a + B_b p_b + \ldots K_b p_k) + L_b w](1+r) &= B p_b \\
&\ldots\ldots\ldots \\
&\ldots\ldots\ldots \\
[(A_k p_a + B_k p_b + \ldots K_k p_k) + L_k w](1+r) &= K p_k \\
\lambda_a p_a + \lambda_b p_b + \ldots \lambda_g p_g &= 1
\end{aligned} \tag{8}$$

Equations (8) are $(k+1)$ in number and contain the same number of unknowns: the rate of profit r and the k prices of production $p_a, p_b \ldots p_k$.

It can now easily be seen that the need to distinguish the constant capital of an industry, say a, into the quantities $A_a, B_a, \ldots K_a$ also makes it inessential to measure them in terms of labour embodied: the prices of production $p_a, p_b, \ldots p_k$ can be directly applied to the physical inputs of $a, b, \ldots k$, and the same applies to the variable capitals $(L_a w)$, $(L_b w)$, etc., consisting of the composite "wage commodity" (which, being our numeraire, has a unit price).[32] These physical measurements are clearly preferable, because they only depend on the method of production of the commodity concerned and not, in addition, on the methods of its direct and indirect means of production, like the corresponding quantities of labour embodied. Equation (8) can therefore also be read with the quantities w, A_a, B_a, etc., taken as physical quantities.[33] Henceforth we shall adopt this alternative reading of equations (8).

Equations (8) are however those we find in Sraffa's *Production of Commodities by Means of Commodities*, Chapter II.[34] Sraffa's own symbols were

[32] The possibility of physical measurements in Marx's equations of the prices of production, when modified by applying prices to the capitals, appears to have been first noted in print in Seton, 1957, p. 151, n. 3.

[33] Even the numerical values of the coefficients would remain unchanged if we choose for the unit of each commodity $a, b, \ldots k$, the quantity of it requiring a unit of labour for its direct and indirect production.

[34] Cf. the equations in Sraffa 1960 p. 6, which differ from equations (8) above, because in Sraffa the wages are included, commodity by commodity, in the quantities of the means of production.

chosen for equations (8) so as to bring out the fact that his equations are the same as Marx's equations (5) and (6) once these are modified by applying the prices of production to variable and constant capital. The fact that such a modification had been suggested by Marx himself has not prevented it from changing his equations (5) and (6) beyond easy recognition. The essential point however is that equations (8) provide a general solution for precisely the same problem which Ricardo and Marx had faced by means of the labour theory of value. The characteristic premises of the surplus approach, for which the real wage and the social product are given when determining the rate of profit and relative prices (par. 4 above) have remained unaffected and, therefore, the notion of profits as a residual and the associated view of the forces determining distribution have also remained unaffected.

17. Equations (8) are however less transparent than Ricardo's and Marx's equations (3) and (4) were about the forces governing the rate of profit. The basic asymmetry between a wage independently determined and profits resulting as a residual is obscured: in equations (8), to envisage profits as a difference between the value of the product and that of the wages and means of production makes no more sense than the reverse procedure of obtaining the value of the product by adding the profits, the wages and the value of the means of production together. More fundamentally the question is that these "appearances"—the very ones which had misled Smith and later authors into thinking that profits and wages could be determined independently of one another—make it more difficult to grasp the properties of the system. These "appearances", which Ricardo had overcome by measuring value in terms of embodied labour, are in fact engendered by the difficulty of viewing the effects of the interdependence of prices, and are here dispelled by the mathematical consideration of the *system* of price equations.[35] Thus, from equations (8) it results that once the real wage is given, the rate of profit is determined, so that the two cannot change independently of one another. Moreover, system (8) reveals that, under sufficiently general conditions, there exists an inverse relation between the real wage and the rate of profit (below par. 20). But a reasoning that relies on theorems which abstract from the *content* of the problems analysed cannot fully overcome the difficulty of grasping the effects of the interdependence of $k+1$ unknowns, and cannot therefore have the transparency of surplus equations like (3) or (5).[36] That transparency was in fact ultimately due to the

[35] The individual price equation may indeed misleadingly suggest, with its seeming symmetry between profits and wages, and, above all, its seeming representation of price as a sum, the idea that the two rates can be determined "independently and separately" from one another (cf. par. 9 above). Not surprisingly it is only later in the development of theory that the price equations reveal their implications as to the constraint linking the real wage and the rate of profit.

[36] Thus, the nature of profits as a residual can be clarified by Sraffa in the first chapter of 1960 only by showing how prices just sufficient to repay the wages and replace the means of production become contradictory when a surplus product comes into existence. The fact that this indirect route had to be followed to exhibit what would have been so evident in equations

(*continued overleaf*)

"picture" which corresponded to those equations: that of a known product to be divided between wages and profits; with the rate of profit originating from the distribution in proportion to the amount of capital of the surplus which this product shows above the known amount of wages. This "picture" allowed a concrete mental representation of a highly abstract analysis: in the 'picture' the dependence of the rate of profit on real wages was *seen*, and the properties of the economic system associated with this key relation were under a correspondingly easier grip.[37]

Naturally, reality need not be simple, and need not allow for the existence of a surplus equation like (3) or (5), beyond the hypotheses necessary to validate the labour theory of value. But the point we wish to make is that the recovery, if at all possible, of that "picture" under the present more general assumptions, would constitute an important scientific advance. It would do so because it would entail a better grip over the economic system's known properties: and it would make it correspondingly easier to gain greater knowledge by asking further questions.[38]

In the next section we shall indicate how, under our present assumptions, a determination of the rate of profit along the lines of the "Surplus equation method" becomes possible, provided we focus our attention on the sector of the economy where the general rate of profit is in fact determined. We shall then see, in Section VII, that a similar, even simpler "picture" of distribution becomes available in the shape of Sraffa's "standard system" when we envisage the rate of profit, and not the wage, as the variable which can be determined outside the "core" of the theory.

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(3) or (5) provides an example of the loss of transparency referred to. A further and perhaps more striking example of this lack of transparency is the sense of novelty which, as late as 1961, under the name of "non-substitution theorem", greeted the proposition that, in an economy like that of equations (8), the real wage is given irrespective of the "demand of consumers", when the rate of profit (rate of interest) is given (cf. Samuelson, (1966) p. 528)—a proposition which would have been obvious from equations (3) and (5), or from (9) and (10) below.

[37] Of course Social product, Necessary consumption and Social capital, are, in themselves, highly abstract notions. The mind can however fit them into a "picture" and proceed to operate with them (just as the mind of a child can work with an abacus), *as if* they were concrete objects connected by the simple relations of part and whole. This mental procedure drastically differs from that which we have when entirely abstract mathematical notions must be resorted to in order to deal with variables like the prices and quantities of commodities, which, though more concrete (i.e. more directly experienced) are both numerous and interdependent.

[38] The loss of the "picture" which was associated with equations (3) or (5) should in fact not be confused with the loss of a mere didactic device, which could always be made good by using appropriate, simplifying assumptions. The nature of this loss may perhaps be best illustrated by referring to the uneasiness which some physicists feel at the loss of the "pictures" of physical reality which were provided by the notions of "waves" and "particles". This uneasiness induces one physicist to hold that "as a system of calculation which gives the right answers [quantum theory] is overwhelmingly successful. But whether it tells us anything about what matter is actually like is another question". And the comment of a science historian to this was "if we were forbidden to talk in terms of models at all, we should have no expectations at all, and we should then be *imprisoned ... inside the range of our existing experiments*", where "model" appears here to mean something closer to what we have called "picture", than to what economists are at present in the habit of calling "model" (Pippard, 1962, pp. 33 and Hesse, 1962, pp. 56–7, our italics).

VI. The surplus-equation method and the wage-goods sector

18. Let us single out the price equations of the commodities a, b, ...h, consisting of the wage goods a, b, ... g, constituting the "wage commodity" (par. 16 above) and of their direct and indirect means of production $g+1$, ..., h. The definition of these commodities, h in number (with $h \leqslant k$), implies that in their h price-equations we find as unknowns only their h prices plus the rate of profit r. It follows that these h equations, together with the last equation in system (8), defining the wage commodity as the numeraire, will be sufficient to determine the rate of profit and the h prices independently of the remaining $(k-h)$ price equations.

This means that, once the real wage is given, the general rate of profit will depend exclusively upon the technical conditions of production of the wage goods and their direct and indirect means of production. The technical conditions of production of the others commodities—i.e. the luxury goods and the means of prodution specific to them—will only be relevant in order to determine, by means of the remaining $(k-h)$ equations of system (8), such prices for these commodities as will yield the rate of profit determined by the first h equations.[39]

19. Let us now look more closely at the part of the productive system which directly or indirectly reproduces the aggregate wages advanced to the workers for the year.[40] This part of the economy constitutes what we may call the "vertically integrated sector of the wage-goods", or *integrated wage-goods sector* for short. Let us then express both the net yearly product of this sector and the wages paid in it, in terms of Smith's "labour-commanded" standard of value, i.e. in terms of the quantity of labour which those aggregates of commodities can buy. Both these two quantities will be known *before* the rate of profit and prices are known. The net product, being the yearly wages of L labourers, will evidently "command" L labour years,

[39] The principle according to which the rate of profit is determined by the conditions of production of wage goods, and of their direct and indirect means of production alone, is a generalization of that which Ricardo appears to have used in his *Essay on Profit* (1815), when he concluded that "it is the profits of the farmer that regulate the profits of all other trades" (1951–1958, IV, p. 23; VI, p. 104). If we in fact assume that wages consist entirely of "corn", and that this is produced by itself and labour, as is required in order to validate Ricardo's argument, then the h commodities entering directly or indirectly the wage narrow down to the single commodity 'corn'. The role of wage goods in determining the rate of profit was first pointed out, it appears, by Dmitrieff, 1974, p. 59 ff. It then emerged again in the solutions of the so-called problem of "transforming" Marx's "values" into "prices of production", advanced in Bortkiewicz, 1952, where the author recognised his debt to Dmitrieff, and also in Bortkiewicz, 1949.

[40] For the unique determination of such a sector cf. Garegnani, 1972, p. 264, and, more generally the notion of a "sub-system" in Sraffa, 1960, p. 69. We may note that the determining role of this "sector" of the economy in no way depends on actually being able to isolate it within the real economy: the rate of profit would be determined in the same way, if the wage-goods advanced to the workers were not being reproduced. The "sector" is therefore best considered as a purely logical construction, which, like Sraffa's "standard product" to be considered in the next section, has the purpose of giving transparency to the forces regulating the rate of profit.

where L is the known number of labourers employed in the economy.[41] The wages paid in the sector, on the other hand, will be those of the L_v labourers required for the direct and indirect production of the "necessary consumption", a known magnitude, since the real, wage and the technical conditions of the direct and indirect production of its constituents are known. These wages will therefore "command" L_v labour-years.

It follows that in the integrated wage-goods sector, the amount of profits in terms of "commanded labour" constitutes a "surplus value" $(L - L_v)$ which is known before the rate of profit and the relative prices are determined. This surplus value coincides numerically with Marx's own social surplus value s (since $L = v + s$ and $L_v = v$), though it differs from the latter in conception because: (i) it is the surplus value of the wage-goods sector, and not that of the entire economy; (ii) it is expressed in terms of labour commanded and not in terms of labour embodied.[42]

When we proceed, as we must, from the amount of profits (surplus value) to the rate of profit, the obstacle we meet is that, unlike the value of the product and of the wages, the value in "commanded labour" of the means of production of the integrated wage-goods sector is not known independently of the prices. We seem to be unable to proceed with a "surplus equation" where the rate of profit is the only unknown (above par. 15), and to be forced to return to the price equations. This obstacle is not insuperable however. Under our present hypotheses[43] it can be overcome by the device of "reducing to labour" the means of production: as will be presently seen this device allows us to express the value in commanded labour of the means of production as a function where the rate of profit is indeed the only unknown.

The capital required for the integrated production of any commodity can in fact be regarded in either of two alternative but equivalent ways. The first is that, confined to a single yearly production cycle—between, say, moments

[41] The fact that the value of this net product, expressed in labour commanded, is independent of the rate of profit does not contradict the fact, stated in par. 9 above, that the value in labour commanded of *any* commodity is not independent of that rate. The constancy of the *value* of net output of the wage goods sector is in fact due to the changes in *physical size* which this net product undergoes as the real wage changes with the rate of profit. It may help the reader to note that the use made here of the labour commanded standard has no connection with that suggested by Sraffa, in *op. cit.* par. 41–42, that is of using as numeraire a quantity of labour commanded which would vary with the rate of profit in such a way as to remain equivalent to the "Standard product".

[42] It should be noted that the net output of the integrated wage-goods sector is *physically homogeneous* with the wages paid there, since both consist of the same composite wage-commodity. The amount of profits in the sector could then be determined as the difference between two physical quantities of the wage commodity, and the rate of profit could be seen to arise from the distribution of this surplus product over the capital of the wage-good sector, reduced to wages in the manner we shall consider in the next paragraph. The commanded labour standard used in the text has however the advantage that in terms of this, the product of the integrated wage-goods sector is constant as the real wage changes either in level or in composition.

[43] Fixed capital of constant efficiency can however be easily shown to leave the present argument unaffected, (cf. p. 53 n. 46 below).

(−1) and (0)—which we adopted for prices equations (8): capital there includes the means of production, besides the wages, both kinds of capital being advanced at moment (−1): the value of the capital so conceived involves the unknown prices of the means of production. The second way of looking at capital, however, proceeds to consider the *production* of these means of production and considers it as the result of previous *stages* in the production of the final commodity: the stage between moments (−2) and (−1), in which the means of production of the commodity have been produced; the stage between (−3) and (−2) in which the means of production of those means of production have been produced, and so forth. The result of this procedure is that capital is reduced to wages with, generally (in the case of circular production, when the commodity, or one of its means of production, requires itself directly or indirectly in order to be produced) a residual of means of production which may be rendered small at will by carrying the process on through a sufficient number of stages. These wages are however conceived as having been advanced for varying periods of time; not only at moment (−1) but also at (−2), (−3), etc. The advantage of this device is that, when measured in "labour commanded", these advanced wages will be given by the quantities of "dated labour" they are the wages of. Unlike the prices of the means of production[44] in the other view of capital, they will therefore be *known* quantities since the methods of production of both the commodity and its means of production are known.

A simple example will show how this second view of capital can be applied to the means of production of the integrated wage-goods sector to obtain a "Surplus equation" determining the rate of profit. Consider an economy where (besides the general assumptions already made in par. 12) we suppose that wages consist only of "corn". "Corn" is produced with one "plough" per worker: the "plough", entirely consumed during the year, is in turn produced by one unassisted worker. The L_0 labourers employed in the integrated wage-goods sector will therefore be distributed half in (directly) producing the "corn", and half in reproducing the "ploughs": the capital advanced for, the year (reckoned, therefore, according to the first of the two views above) will consist of the wages *and* the plough. If we take now the

[44] We are in fact expressing the prices of the means of production by means of these "advanced wages" and the profits on them for the relevant periods of time. This is made clear by the procedure for this "reduction of the commodity to dated labour", which consists of taking the price equation of the commodity and replacing the prices of means of production with the expressions given by the respective price equations. The opeation is then repeated with the prices of the second layer of means of prodution appearing in the equation thus modified, and so forth for the further layers, which appear in succession after each round of substitutions (see for example Sraffa, 1960 pp. 34–35). When looked at from this "dated labour" angle the other view of capital appears to be the result of "collapsing" all the successive stages of production into a single year—a "collapsing" which is made possible by the availability at the beginning of the year of the products of all the intermediate stages, that is the means of production. It should however be remembered that the reduction to dated labour constitutes a more logical device, as is shown by the fact that, when the production of a commodity is "circular", it cannot be conceived as starting with unassisted labour.

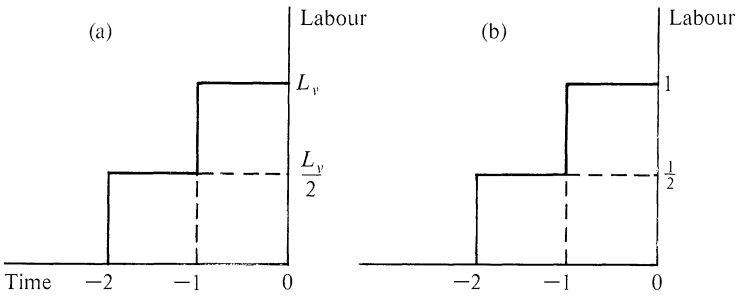

FIG. 2. The "dated labour" curve for the aggregate of wage goods paid as yearly wages (curve *a*) and the proportional distribution of labour over time in the production of the wage commodity (curve *b*).

second, "dated-labour" view of capital, the latter emerges instead as the wages of the two quantities of "dated labour" shown in Fig. 2a: $L_v/2$ labour years applied at moment (−2) for producing the "ploughs", which are then used in the successive "stage" of corn production—together with a further $L_v/2$ labour years applied at (−1). Expressed in "commanded labour" the wages, to which the entire capital of the integrated wage-goods sector has thus been reduced, will equal the corresponding quantities of dated labour i.e. $L_v/2$ advanced at (−2) and $L_v/2$ advanced at (−1). The rate of profit can then be seen to emerge from the distribution of the surplus value $L - L_v$, in proportion *both* to the wages advanced, and to the time by which they have been so advanced, account being taken of compound profits, that is

$$L - L_v = r\frac{L_v}{2} + 2r\frac{L_v}{2} + r^2\frac{L_v}{2} \tag{9}$$

where the term $rL_v/2$ indicates the share of surplus value allotted to the capitalists advancing the wages paid at time (−1); and the term $2rL_v/2$, together with the compound profit term $r^2L_v/2$, indicates the share allotted to those paying the wages at time (−2). The rate of profit is the only unknown in (9).

20. It is now convenient to divide both sides of equation (9) by L_v obtaining

$$\frac{L - L_v}{L_v} = r\frac{1}{2} + 2r\frac{1}{2} + r^2\frac{1}{2} \tag{9a}$$

and proceed to some properties of our "surplus equation" (9a) which can be easily shown to hold beyond our present simple example.

On the left of the equality sign we find, expressed in commanded labour, the amount of surplus value per worker in the integrated wage-goods sector. This amount is identical to Marx's rate of surplus value s/v and depends purely on the level of the real wage and on the labour required for the direct

and indirect production of its constituents. On the right of the equality sign we find instead a function expressing the amount of profits per worker, also expressed in commanded labour, which would be necessary in the sector in order to pay a rate of profit r. This function, which we may call "profit function" for short, depends purely on the *proportional* time distribution of the labour necessary to produce the wage commodity (see Fig. 2b): it does not therefore depend on the *level* of the real wage, but only on its *composition* and on the methods for the direct and indirect production of the wage-goods. The "profit function" has an important property which can easily be seen to hold with any kind of circulating capital, and with fixed capital of constant efficiency: it is zero when $r = 0$ and then it rises monotonically with r.[45]

The solution of the "surplus equation" (9a) can now be represented in the diagram of Fig. 3a where r is measured horizontally and the rate of surplus value is measured vertically. We have there the curve Os representing the "profit function", which rises monotonically from the origin (In the case of our example it will rise indefinitely as r rises indefinitely as in Fig. 3a; in the case of "circular production" it rises indefinitely as the rate of profit approaches a "maximum rate of profit" R as in Fig. 3c). The rate of surplus value, $(L - L_v)/L_v$, can on the other hand be represented by a horizontal line.

The level of the rate of profit which solves the equation will be that for which the "profits curve" Os cuts the surplus-value line. The fact that the "profits curve" is monotonically rising ensures that the solution will be unique and positive for any positive rate of surplus value—i.e. for any level of the real wage less than the product per head in the integrated wage goods sector). Figure 3a or 3c make it clear that this single positive rate of profit, depends exclusively on two circumstances: (i) the rate of surplus value; (ii) the proportional time-distribution of the labour necessary to produce the wage commodity, which determines the shape of the "profit function".

The fact that the "profit function" is an increasing functon also makes clear a second set of properties of the system, pertaining to the relation between the wage and the rate of profit. A rise in the wage, leaving its

[45] When we admit fixed capital of constant efficiency the validity of the property of the "profit function" mentioned in the text can easily be seen if we start from the function $g(r)$ giving the profits obtaining on a "pool" of n machines of kind K, one of age O, one of age 1 and so on up to the last, of age $(n-1)$, where n years is the life of machines K. For $r = 0$ these profits will be zero, and for $r > 0$ they will be given by

$$g(r) = p_k\left[\frac{nr(1+r)^n}{(1+r)^n - 1} - 1\right]$$

where p_k is the price in labour commanded of a new machine K. Now, the function in square brackets is a monotonically increasing function of r. It follows that the "profits function", in which functions of the above form will be multiplied by, or added to, those of form $[(1+r)^n - 1]$, will also be zero for $r = 0$ and will then monotonically rise with r. It should also be noticed that the integrated wage-goods sector will now have to be defined so as to include among its fixed means of production only sets of machines of uniform age distribution, with the corresponding constant yearly replacement appearing in the gross output of the sector.

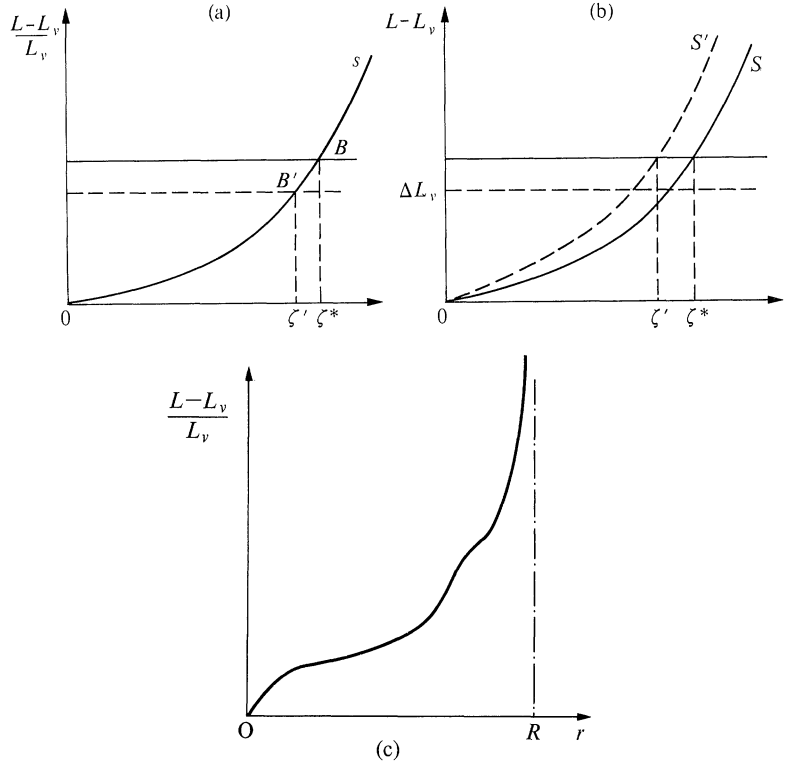

FIG. 3. Determination of the rate of profit with reference to (a) the rate of "surplus value" (b) the "surplus value" expressed in labour commanded of the integrated wage-goods sector. (c) indicates the shape of the "profits function" when production is "circular".

commodity composition unchanged will decrease the rate of profit. This will be so because the rate of surplus value $(L/L_r)-1$ will decrease with the rise of L_v, and will therefore intersect the unchanged "profits function" for a lower rate of profit: cf. the shift from B to B' in Fig. 3a.

The same conclusion can be seen to apply if the real wage changes in composition, but in such a way that it increases in one or more components with no decrease in any of the others. In order to see this, it is however convenient to shift our attention from Fig. 3a which relates to rates of surplus value $(L-L_v)/L_v$ to Fig. 3b which refers instead to the aggregate surplus value $(L-L_v)$ in the integrated wage-goods sector as a whole. With the assumed increment in some components of the real wage, the amount of surplus value will decrease by the amount ΔL_v of the labour required in order to produce that increment for the aggregate wages. On the other hand, for $r>0$, the curve OS of total profits in the wage-goods sector will "rotate" upwards from OS to OS' because of the profits accruing on the

wages paid for the additional labour ΔL_v. Both changes will have the effect of decreasing the rate of profit.[46]

However, when the composition of the real wage changes in such a way that while the quantity of some components rises, that of others falls, and it becomes accordingly impossible to refer unambiguously to a rise or fall in the real wage—the profit curves, whether that of profits per worker (Fig. 3a), or that of total profits (Fig. 3b), can change in any way whatever. The rate of profit may then change in a direction opposite to that in which the rate of surplus value has changed—in line with what Marx saw as possible when the organic composition of social capital changed together with the real wage (cf. e.g. Marx, 1969b, III, p. 869).

21. These properties, made easily visible by surplus-equations (9) and (9a) are thus closer than we might perhaps have expected to Marx's conclusions on the matter. In particular, it is confirmed that the rate of profit depends on two factors only, the rate of surplus value s/v, and the proportions between means of prodution and labour. However, the correction of the error implicit in equation (5) modifies Marx's specification of the second circumstance, in terms of the "organic composition" c/v of social capital, in two important respect.

In the first place the proportion of labour to means of production on which the rate of profit depends is that of the integrated wage-goods sector, and not that of the economy as a whole as Marx thought. This in turn implies that he was mistaken in believing that changes in the relative outputs of commodities could affect the rate of profit, through variations in the proportion of labour to the means of production in the community.[47] It also implies that Marx was equally wrong when he thought that, through the same variations, changes in the technical conditions of production of "luxuries", or of their specific means of production, could affect the rate of profit.

In the second place, the proportion of labour to means of production cannot be expressed by the ratio c/v, and must instead be expressed by the proportionate distribution over time of the labour necessary to produce the wage commodity, or by the quantities of the several means of production, in the price equations. This is a consequence of the fact that it is impossible to measure capital by a single magnitude independent of distribution. This fact, which deeply affects the validity of the marginalist theories (above par. 1),

[46] The monotonically increasing character of the profits function also implies that in equation (9a) a single value of L_v will correspond to any positive level of r or, in the case of circular production, to any value of r where $0 = r < R$. Equation (9a) thus shows that as the rate of interest rises, the wage-rate must fall when measured in terms of *any* commodity (which, then chosen as a measure of the wage, will play the role of the "wage commodity" in that equation).

[47] Cf. for example Marx, 1969a, III, p. 162,: "There will naturally be a very great difference [in the general rate of profit], depending on whether a greater or smaller part of total capital produces a higher or lower rate of profit [i.e. whether capital has, respectively, a lower or higher organic composition]." (The deficiency of this thesis of Marx is ignored, e.g., in Mandel, 1975, p. , , , , and *passim* and Rowthorn, 1976, pp. 62–63 and *passim*).

has some important consequences also here: thus if Marx had been correct the relative price of the products of two distinct processes of production would always change monotonically with r, whereas in fact such relative prices may invert their *direction* of change as r rises (falls) monotonically.[48]

VII. The "surplus-equation method" and Sraffa's standard system

22. The basis of our discussion of the surplus theories has so far been the Classical economists' premise according to which, when approaching the determination of profits and prices, the real wage should be treated as an independent variable. A different view of the wage is however suggested in Sraffa, 1960. Sraffa begins by observing (p. 9) that the wage, besides a minimum consisting of the necessary subsistence, may include a share of the surplus. He proceeds to argue (*ibid.*, p. 33) that under these conditions the wage would have to be taken as given "in terms of a more of less abstract standard" and, accordingly,

> "it would not acquire a definite meaning until the prices of commodities are determined".

Then, he continues,

> "the position is reversed [and] the rate of profits, as a ratio, has a significance which is independent of any prices, and can well be "given" before the prices are fixed".

A discussion of the view of the operation of the economic system which seems to underlie Sraffa's suggestion would bring us beyond that "core" of the theory with which we are here exclusively concerned (cf. above par. 7). Our present interest—confined to examining Sraffa's use of the "Surplus equation method"—requires us only to consider how far the suggested change in the independent distributive variable is compatible with the surplus approach to value and distribution which is the subject of this article.

23. When within this approach to distribution we envisage changes in the rate of real wages over time, we may attribute these changes to either of two circumstances: a long-term evolution of the social conditions determining the level of subsistence, *or* the kind of economic circumstances which authors like Smith or Marx thought might keep the wage above the level of subsistence even for long periods of time (par. 4 above).[49] In the first case,

[48] There is a further aspect of Marx's views which, when appropriately modified finds confirmation in the surplus equation of the wage-goods sector. It regards the role of prices in "re-distributing" total surplus value in proportion to the capital employed in the individual industries. This role can be detected when we look at the integrated wage-goods sector and at the prices appearing there. The same cannot however be said when we look at the economy as a whole, since in that case the amount of surplus value is not given before the prices and rate of profit are given.

[49] The view that the wage can exceed the level of subsistence for long periods of time seems indeed implied also in the very idea of a rising subsistence level. This rise can result only from wages remaining above the previous subsistence level for a period of time which is long enough to engender those 'habits' which may then become a 'second nature' in Torrens's phrase later adopted by Ricardo (par. 4 above) and by Marx in 1969b, III, p. 859).

the real wage will evidently have to be taken as a given magnitude in the "core" of the theory (par. 5). The same will be true in the second case only if the share of the surplus taken up by the wage depends on circumstances acting through the wages. The real wage will then appear in the "core" as the magnitude which has been determined in both level and composition by the circumstances in question: profits will continue to be determined as a pure residual, though now they will not constitute the *entire* surplus.

The share of wages in the surplus might however be alternatively determined by circumstances acting through profits, like those which Sraffa envisages when, after the passage already quoted, he writes of the rate of profit as a magnitude

> "susceptible of being determined .... by the level of the money rates of interest" (Sraffa 1960, p. 33.)

Then, the rate of profit will become the independent variable within the "core" and the wage, an unknown, can be treated as an "abstract" value magnitude which can be measured in terms of any standard.[50,51]

Thus, Sraffa's replacement of the wage by the rate of profit as the independent distributive variable can be seen to result from his choice of one of the two views that can be taken of the circumstances determining the division of the surplus between wages and profits. As such it appears to be no less compatible with the surplus approach to distribution than the other view, adopted by Smith and Marx, when they saw those circumstances as acting through the wage. In either case we find the basic distinction betwen subsistence and surplus, and we also find the determination of the division of the surplus by means of social and economic circumstances which, whether they act through wages or through profits, are best studied outside the "core" where the unknown distributive variable is determined together with relative prices. In formal terms, what we have if we follow Sraffa is a modification of the analysis conducted in that "core": the modification consists of the addition, so to speak, of a second layer to the determination of the surplus by the difference between social product and "necessary consumption" (par. 6 above). In this second layer we have the sharing of the surplus between wages and profits, and the rate of profit appears there as a "datum" or independent variable, with the surplus wage as the unknown. The diagram (Fig. 1) may accordingly be modified as in Fig. 4 below.

[50] The treatment of the wage as a residual need not however entail the treatment of the wage as an abstract value quantity. Commodities which play a primary role in workers' consumption may provide a more significant measure of the wage than is provided by other commodities. The concept of the wage as an abstract value magnitude does not, on the other hand, prevent the use of the "integrated wage-goods sector", with respect to any of the commodities in terms of which we may wish to measure the wage. With r as the independent variable in equation (9a), the rate of surplus value would be the unknown, giving the unique wage corresponding to the given level of r (cf. above n. 46).

[51] E. Burmeister fails to realize this implication of Sraffa's suggestion about an exogenous determination of the rate of profit when in 1977, p. 68 n., he writes "Sraffa's measure of the real wage is flawed ... [the] weights are not related in any way to human needs".

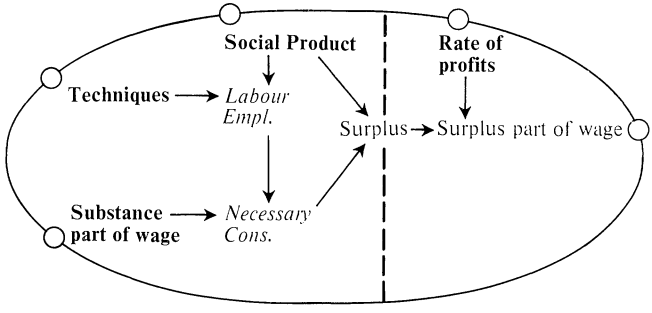

FIG. 4. A diagram of the "core" of the surplus theories when we admit that the wage may share in the surplus because of causes acting through the rate of profit (compare with Fig. 1 above).

24. The treatment of the wage as an abstract value quantity makes a very simple Surplus equation possible for the study of the relation between the wage rate—or preferably its surplus component—and the rate of profit (Sraffa, 1960, pp. 9–10). As is well-known, under the hypothesis of single-product industries a unique set of positive proportions always exists such that the net product consists of a composite commodity which requires itself as its only means of production. The unique composite commodity thus defined, which depends exclusively on the technical conditions of production, is what Sraffa calls "Standard commodity". A replica of the real economy, employing the same number of workers, but producing exclusively the Standard commodity constitutes them what Sraffa calls the "Standard system". The consideration of the wage as an abstract value quantity can then be used in order to measure the wage in terms of the Standard commodity. In the "Standard system" we shall accordingly have a physical homogeneity between all *three* magnitudes on which the relation between the wage and the rate of profit depends, namely the net product, the wages and the means of production.

Let us then take the standard net product as our unit of the standard commodity. Let us also choose as a unit of labour, the labour employed in the real economy, and hence in the Standard system, so that the wage rate w, paid *post factum*, coincides with the total wages in either. Looking at the standard system we may write

$$w = 1 - rM \tag{10}$$

where M is the amount of the standard commodity used as means of production in the standard system. If we then refer to Sraffa's "Standard ratio" R, between the standard net product and M we have $R = 1/M$ or $M = 1/R$ which when substituted in equation (10) gives

$$w = 1 - \frac{r}{R} \tag{10a}$$

The linear relation (10) or (10a) between r and w also applies to the real economy as soon as wages are measured in terms of the standard commodity. The price equations of the standard system are then identical to the equations of the real economy, except for the multipliers applied to the latter in order to take the industries in the proportions of the standard system.

What the standard system does, is only to provide a 'surplus equation method' of arriving at that relation, with the corresponding 'picture' of the relations of distribution (par. 17 above); in Sraffa's own words, the purpose is "to give transparency to a system and render visible what was hidden" (1960, p. 23).[52]

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[52] Sraffa's reference here is to the transparency aquired by properties of the system like those pertaining to the inverse relations between w and r, or to switches in methods of production, etc., To refer to a 'direct link' between the use of the standard commodity and the use of the labour theory of value by Marx, "to expose the nature of (the) exploitation [of labour]" (cf. for example, Eatwell, 1975, pp. 543–4) would therefore seem misleading. The basic analytical role of the labour theory of value which we saw in Sections III and IV leaves no room for its use in expressing exploitation, (which, it seems, would only consist of showing what is already clear, that is that workers do not get the entire product: cf. p. 13, n. 23 above). Moreover, Sraffa's use of the standard product, though strictly analytical, is "auxiliary" (Sraffa, 1960, p. 31) and is therefore unlike that of Ricardo and Marx, who depended on the labour theory of value for their conclusions regarding the rate of profit and relative prices.

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