MARXISM 21
Feature

Reflections on the Concept of Exploitation

Takao Fujimoto*

This paper describes the author's three reflections on the concept of exploitation. The first one is concerned with the Fundamental Marxian Theorem. It is shown that exploitation as defined in a usual way exists irrespective of whether profits are positive or negative, so long as workers do not save and an extended productiveness condition is met. The second reflection deals with the concept of exploitation in models with heterogeneous labour. We may consider the rates of exploitation separately for individual type of labour, or the common rate of exploitation based upon abstract labour, fusing heterogeneity into one abstract category. Some features of these two concepts are explained. In the third one, I argue that the unemployed should be counted as exploited, and thus present some reasons why the unemployed are exploited, and a simple way to define the rate of exploitation for a capitalist system as a whole.

Keywords: abstract labour, exploitation, Fundamental Marxian Theorem, heterogeneous labour, joint production, unemployed workers.

---

* Department of Economics, Fukuoka University, Fukuoka, Japan.
takao@econ.fukuoka-u.ac.jp.
Thanks are due to two referees whose critical comments and suggestions have been helpful to improve this article.

1. Introduction

In this article, I present three reflections on the concept of exploitation, reviewing mainly my own recent papers. The first theme is about the Fundamental Marxian Theorem with exploitation in the usual sense. Basically this theorem is believed to state that positive profits imply exploitation. We can indeed prove exploitation exists regardless of whether profits are positive or not in each sector. All we have to assume is that workers receive no-saving wage rate: wages are not so high as to allow them to save up. Our proposition can be generalized to models with heterogeneous labour as well as joint production.

The second reflection is concerned with the concept of exploitation in models with heterogeneous labour. Either we deal with various types of labour services separately, or devise abstract labour to realize the uniform rate of exploitation among all types of labour. These two methods may yield considerable discrepancies among the magnitudes of exploitation rates. We explain a reason for this phenomenon.

The third one is to argue that the unemployed workers are to be counted as exploited. That is, there should be two rates of exploitation, the workplace rate and the system rate. The former is the usual one, while the latter takes into consideration the unemployed. The third reflection has not been published, and seems to be a new contribution.

The final section gives conclusions.

2. Fundamental Marxian Theorem

Morishima and Seton (1961: 209) got a formula which relates the rate of exploitation with the uniform rate of profit, telling that the latter is positive, if and only if the former is positive. Then, Okishio (1963) proved that if each industry earns positive profits, then exploitation exists. The reader should note that in Okishio (1963) the profit rates need not be uniform. It is also important to understand that these authors never denied the existence of exploitation under disequilibrium while negative profits are observed in some set of sectors. Thus, Fujimoto (1978) showed that exploitation exists when a sort of average rate of profit is positive.

Let us review what Okishio (1963) proved as the Fundamental Marxian Theorem, so named later by Morishima (1973: 53). Okishio (1963) used a Leontief model of circulating capital with homogeneous labour, and assumed the following:

(A1) There exists a positive price vector such that $p > pA + wL$.

(A2) The money wage rate w satisfies $w = pC$.[1)]

Given these two assumptions, we can show, using the Perron-Frobenius theorem, that $\Lambda C < 1$, i.e., exploitation exists.[2)]

The above two assumptions imply $p > p(A + CL)$, which in turn guarantees the existence of a positive column n-vector z such that $z > (A + CL)z$. Then, it is easy to note the existence of a positive column n-vector x and a positive scalar y such that

$$\begin{cases} x > Ax + Cy, \text{ and} \\ y > Lx. \end{cases}$$

1) Here, A is the given $n \times n$ material input coefficient matrix with a process as a column, L the given row n-vector of labour input coefficients, C the given column n-vector of commodity basket consumed by workers, p a row n-vector of market prices, and w the wage rate. In vector comparison, $x > y$ means $x_i > y_i$ for all i, and $x \geqq y$ means $x_i \geqq y_i$ for all i.

2) We denote by Λ the row n-vector of labour values defined by $\Lambda = \Lambda A + L$.

We define two nonnegative $(n+1)\times(n+1)$ matrices B and A as follows:[3]

$$B = \begin{pmatrix} I_n & 0 \\ 0 & 1 \end{pmatrix} \text{ and } A = \begin{pmatrix} A & C \\ L & 0 \end{pmatrix}, \qquad (1)$$

where I_n is the $n \times n$ identity matrix. These two matrices may be called the complete matrix of output coefficients and that of input coefficients, both with each production process and a household activity as a column, respectively. The two assumptions of Okishio then imply the existence of positive column (n+1)-vector x such that $Bx > Ax$. Let us rewrite this condition as

(A1*) There exists a nonnegative column (n+1)-vector x such that

$$Bx > Ax.$$

Now, Fujimoto and Opocher (2010: 448) demonstrated that if the assumption (A1*) is given, then $\Lambda C < 1$. Therefore, when the assumption (A2) is in addition assumed, exploitation exists. The reader should remember that our assumption (A1*) has nothing to do with prices, and it is an extended condition of productiveness, concerned with the quantity side of the economy. Thus, exploitation can exist irrespective of profit rates of industries, under whatever prices so far as (A2) is satisfied.[4] This proposition is called "Fundamental Theorem for Capitalist Reproduction Systems" in Fujimoto and Ranade (2010).

Certainly, the assumption (A1*) implies Okishio's (A1), and hence there may seem to be nothing new. This is not so. In Okishio, the vector p is common in (A1) and (A2), while p in our (A2) need not satisfy (A1). Besides, (A1*) does not in general imply (A1) beyond simple Leontief models. The same restrictive requirements are made also in Morishima (1973: 53-4).

---

3) In Bródy (1970: 23), these are called complete matrices.

4) It is easy, however, to prove that given the assumptions (A1*) and (A2), there can be no price vector under which all the industries make losses.

A simple numerical example may be useful. Let the data are given as:

$$A = \begin{pmatrix} 0.3, & 0.6 \\ 0.5, & 0.2 \end{pmatrix},\ L = \begin{pmatrix} 0.1, & 0.1 \end{pmatrix},\ \text{and}\ C = \begin{pmatrix} 1 \\ 0 \end{pmatrix}.$$

Okishio's assumptions are satisfied by a price vector $p = (1,\quad 1)$ and the money wage rate $w = 1$.

(A1) The costs of two sectors are $pA + wL = (0.9,\quad 0.9)$, thus both sectors make positive profits.

(A2) $w = 1 = pC$.

On the other hand, our (A1*) is verified by $x' = (9,\quad 6,\quad 2)$, while our (A2) can be satisfied by $w = 1$ and $p = (1,\quad 0.5)$. The costs of two sectors are $pA + wL = (0.65,\quad 0.8)$, and so the second industry incurs loss. And yet, there is exploitation as was proved by Okishio (1963), or we can see it directly because $\Lambda = L \cdot (I - A)^{-1} = (0.5,\quad 0.5)$ which leads to $\Lambda C = 0.5 < 1$.

Then, Fujimoto and Opocher (2010) allow for heterogeneous labour, joint production, durable consumption goods, and joint production of labour services by households. In short, the model defined by (1) becomes

$$B = \begin{pmatrix} B & D \\ F & J \end{pmatrix} \text{ and } A = \begin{pmatrix} A & C \\ L & L_h \end{pmatrix}, \tag{2}$$

where B and A are now $m \times n$ rectangular matrices as in von Neumann models, D and C $m \times h$ matrices, F and L $k \times n$ matrices, and J and L_h $k \times h$ matrices. There are m goods and services, k types of labour services, n normal production processes, and h household activities. The symbol D stands for old durable consumption goods left over from C, L_h means the direct labour services required in household activities, J the matrix of labour outputs through household activities allowing for joint production of labour of various types, and the matrix F shows the outputs of labour services of special types created in normal production processes.[5] When the reader wishes to insist that F should be a zero matrix, regarding those special services as normal *commodities*, then let $F = 0$.

In such a model, Fujimoto and Opocher (2010) define commodity content of various commodities with a particular commodity or a particular type of labour service as the standard of value, say i-th labour service, and proved that the own content of the standard commodity is less than unity (Fujimoto and Opocher, 2010: 448), given an assumption a little weaker than (A1*), i.e.,

(A1-FO) There exists a nonnegative column $(n+h)$-vector x such that

$$Bx \geqq \boldsymbol{Ax} + \boldsymbol{e}_i,$$

where $\boldsymbol{e}_i$ is the column $(n+h)$-vector whose i-th element is unity with the remaining entries being all zero. This proposition should not, however, be misunderstood to imply the exploitation of normal commodities other than labour services as is argued in Fujimoto and Fujita (2008).

Hence, if there are some household activities which produce i-th labour service as a single product, denote by $S(i)$ the index set of such activities, and if the wage rate of one unit of type j labour service is w_j, the inequality $w_i \leqq pC^a - pD^a + wL_h^a$ for $a \in S(i)$ implies the exploitation of i-th type labour service involved in those activities in $S(i)$.[6] We may also conceive of the exploitation of 'households' which produces various labour services jointly.

5) Okishio's method in Okishio (1963) cannot be used in Morishima-von Neumann models where alternative, both efficient and inefficient, processes are listed. On the other hand, Morishima' approach in Morishima (1974) involves too much sophisticated concepts such as the "warranted rate of profit"and the "capacity rate of growth".

6) The superscript a indicates the corresponding column in $S(i)$ of the respective matrices, and $w \equiv (w_1, w_2, \ldots, w_k)$. Besides, the relevant entries in J in B are supposed to be normalized to unity.

Some words are in order here because some people regard the assumption (A1*) or (A1-FO) as restrictive, e.g., net outputs of older types of CPU's are negative. First of all, the vector x in these assumptions is not an actual activity vector, but an imaginary one. Thus, we need not use all the processes in actual use to verify a mathematical condition: we can avoid the use of those processes involving, say, 80386 chips. Moreover, we do not care at all about imaginary production scales or about imaginary levels of employment required by the vector x. Second, durable capital goods may be classified not by specific lot numbers or CPU generation identities, but kinds of services they supply. Thus, the chip 80386 can enter, with less efficiency, the same commodity class as the most recent ones. A method for constructing abstract labour can be utilized to form this sort of composite commodity. Finally, the requirement in (A1-FO) can be weakened to

$$Bx \geqq (1-\varepsilon)Ax + \boldsymbol{e}_i,$$

with a small positive scalar ε, and we make minor modifications in the arguments when necessary. Many durable capital goods often keep their original 'efficiency' almost intact through many years of production, and they are gradually or from time to time suddenly discarded simply because more efficient ones are introduced.

3. Heterogeneous Labour and Abstract Labour

In this section we explain a way of dealing with the heterogeneity of labour in a little more detail, by use of Fujimoto (2009) and Fujimoto and Opocher (2010). We assume that workers of any type receive a no-saving wage rate as in the previous section. More precisely prices and wages rates are such that workers of any type cannot save after carrying out a set of household activities determined by an optimal programming problem.

Potron (1913), Bowles and Gintis (1977), and Krause (1981) all treated a Leontief model with heterogeneous labour. Their model is described by

$$\mathrm{B}=\begin{pmatrix} I_n & 0 \\ 0 & I_k \end{pmatrix} \text{ and } \mathrm{A}=\begin{pmatrix} A & C \\ L & 0 \end{pmatrix}, \tag{1a}$$

which model is just between the models (1) and (2) in terms of generality. These authors made a misinterpretation of the matrix $L\cdot(I-A)^{-1}\cdot C$, and yet Potron (1913) and Krause (1981) proposed an interesting method to define abstract labour.7)

Fujimoto and Opocher (2010) considered the model (2) above in section 2, and defined the values in terms of labour service *i* by solving

(Problem (H)): Find out $\Lambda^{[i]} \geqq 0$ such that $\lambda_i^{[i]}$ should be maximized

$$\text{subject to } \Lambda^{[i]}\cdot \mathrm{B} \leqq \Lambda_{[i]}^{[i]}\cdot \mathrm{A}.$$

Here

$$\Lambda^{[i]} \equiv (\lambda_1^{[i]}, \lambda_2^{[i]}, \ldots, \lambda_{i-1}^{[i]}, \lambda_i^{[i]}, \lambda_{i+1}^{[i]}, \ldots, \lambda_{m+k}^{[i]}), \text{ and}$$

$$\Lambda_{[i]}^{[i]} \equiv (\lambda_1^{[i]}, \lambda_2^{[i]}, \ldots, \lambda_{i-1}^{[i]}, 1, \lambda_{i+1}^{[i]}, \ldots, \lambda_{m+k}^{[i]}).$$

That is, $\Lambda^{[i]}$ is the vector of values with *i*-th labour service being the standard of value, and the entry $\lambda_j^{[i]}$ stands for the value of commodity *j* with *i*-th labour as the standard of value. On the other hand, $\Lambda_{[i]}^{[i]}$ is $\Lambda^{[i]}$ with its *i*-th entry replaced by unity.

The values are calculated in a systematic way first by solving the following linear programming problem (LP):

$$\text{(LP) } \max q_i \text{ subject to } q\cdot B \leqq q\cdot A + b^{(i)} \text{ and } q \in R_+^{m+k},$$

where *q* is a row $(m+k)$-vector, and $b^{(i)}$ is the *i*-th row of *B*. Once an opti-

7) See Fujimoto and Ekuni (2011).

mal vector q^* is obtained, the values are computed by

$$\lambda_i^{[i]} = \frac{q_i^* - 1}{q_i^*} \text{ and } \lambda_j^{[i]} = \frac{q_j^*}{q_i^*} \text{ for } j = 1, \ldots, m+k,\ j \neq i.$$ [8]

It is important to note that our labour values thus defined include those in Leontief models and in Morishima-von Neumann models as special cases.

Given $\lambda_i^{[i]}$, we can discuss about the existence of exploitation when actual prices and the wage rate of labour type *i* are observed. If the wage rate is so low that a worker of type *i* can carry out only those household activities which bind their own labour content in the above programming problem (LP), workers of type *i* are exploited.[9]

In the framework of Fujimoto and Opocher (2010), there can be some more concepts of exploitation: the reader is referred to Fujimoto (2009).

Now we shift to abstract labour. Before entering the explanation of our method, we touch upon the contributions by others. The method of handling heterogeneous labour by Okishio (1965, 1977), which is adopted also by Morishima (1973), requires the life-long data on how various types of labour are created and/or maintained, and each type of labour should be formed in a unique way. The necessity of life-long data is in contradiction to the snap-shot nature of production coefficients. After all, the Okishio's method can work only in models without joint production. Fujimori's approach (1982) to the abstract labour is somewhat similar to Okishio's (1965). He uses, however, the actual employment data of various labour types, thus the method can be affected by those data which are neither technological nor biological.[10] A de-

8) For derivation of these equations, see Fujimoto and Opocher (2010: 445-7) or Fujimoto (2009: 60-3). A solution q^* may not be unique, then adopt one of them that has the maximum number of positive entries.

9) Note that the greater is q_i^*, the greater becomes $\lambda_i^{[i]}$.

10) See also Hollander (1978).

fect in Okishio's method is that while the normal production coefficients keep a short-run property, life-long data are required to calculate conversion rates. In our method, those life-long data are thought of as reflected in respective consumption baskets.

Our method of contriving abstract labour is presented in Fujimoto and Ekuni (2011). Here we describe its key-steps. Let us consider the model described by (2) in section 2. Take up an efficiency conversion row *k*-vector α in the $(k-1)$ dimensional simplex S^{k-1}, and new symbols are defined as follows:

$$\Lambda_C \equiv (\lambda_1, \lambda_2, \ldots, \lambda_m), \text{ and}$$
$$\Lambda_L \equiv (\lambda_{m+1}, \lambda_{m+2}, \ldots, \lambda_{m+k}).$$

We have left out the superscript [*i*] because no particular labour service is the standard. Our linear programming problem becomes:

(Problem (A)) max p subject to

$$(\Lambda_C, p) \cdot \begin{pmatrix} B & D \\ \alpha F & \alpha J \end{pmatrix} \leqq (\Lambda_C, 1) \cdot \begin{pmatrix} A & C \\ \alpha L & \alpha L_h \end{pmatrix} \text{ for } \Lambda_C \in R_+^m \text{ and } p \geqq 0.$$

Note that in this problem, *p* is a scalar, α is a parameter vector, and the relation $\Lambda_L = p \cdot \alpha$ is implicitly involved. That is, the resulting values of various labour types should be proportional to efficiency conversion rates. The above problem can be solved as we have done through the auxiliary problem (LP). Since an optimal solution p^* depends on a parameter vector α, we write $p^*(\alpha)$. The last problem to solve is

$$\max\ p^*(\alpha) \text{ for } \alpha \in S^{k-1}.$$ [11)]

When we denote by α^* an optimal solution vector of this last problem, the exploitation rate for type *i* labour is calculated as

11) This maximization problem reminds us of the Perron-Frobenius theorem on the greatest eigenvalue of nonnegative square matrices.

$$e_i = \frac{\alpha_i^* - \lambda_i}{\lambda_i} = \frac{\alpha_i^* - p^*(\alpha^*)\cdot\alpha_i^*}{p^*(\alpha^*)\cdot\alpha_i^*} = \frac{1-p^*(\alpha^*)}{p^*(\alpha^*)} = e_a,$$

showing the common rate, e_a, among all the types. We may say that abstract labour can be constructed by use of the conversion rate vector α^*.

We can prove that the common rate of exploitation e_a is not greater than all the individual rates of exploitation for labour type *i*.[12] This can be seen intuitively from the two problems above, (H) for heterogeneous labour, and (A) for abstract labour. In (H), the standard labour service has a magnitude unity for its direct input, while others assume $\lambda_j^{[i]}$ on both sides of inequalities. We know $\lambda_i^{[i]} < 1$. Therefore, if there is more than one type of labour, the vector Λ_L in a solution to (H) cannot form an efficiency conversion vector after normalization. To realize the proportionality, we multiply $\lambda_j^{[i]}$ on the right-hand side by a factor $1/\lambda_i^{[i]}$. Then, we may be able to increase $\lambda_i^{[i]}$ on the left-hand side, destroying proportionality again. Readjusting in successive iterations can reach a solution of (A). It is certain that in the limit $\lambda_i^{[i]}$ on the left-hand side is greater than or equal to the original magnitude, implying the common rate is not greater than the rate of exploitation for labour type *i*, $(1-\lambda_i^{[i]})/\lambda_i^{[i]}$.

A still more intuitive explanation is that when calculating the exploitation rate of labour type *i*, workers of type *i* are dictatorial, and those workers of other types may be regarded as horses or slaves. On the other hand, when we consider abstract labour, all the types of workers enter the value calculation on equal terms, thus increasing the amount of necessary abstract labour, pressing down the common rate of exploitation.

12) A numerical example of large discrepancies among these rates is presented in Fujimoto and Ekuni (2011).

It is proved in Fujimoto and Ekuni (2011) that our definition of abstract labour contains those proposed by Krause (1980, 1981) as a special case.

4. Unemployed Workers

In this section, we argue in a brief way that there should be another concept of exploitation, in which the unemployed are counted as exploited. Most economists, based upon Marx's definition, define the exploited labour as surplus labour. Hence, unless workers are employed, there can be no exploitation of the fruits of their labour. This may be all right so long as individual workers are concerned. When the unemployed workers form a massive reserve army, they influence the level of wage rates, giving a constant pressure downward, while their very existence cut down the production of goods and services. These show that the existence of unemployment decreases what workers as a whole could get with full employment.

Thus, we may define the system rate of exploitation as

$$e_s \equiv \frac{\text{total possible labour} - \text{necessary labour}}{\text{necessary labour}},$$

where total possible labour is the sum of actual labour and lost labour by unemployment. When calculating the latter, the average annual working hours can be used depending on the types of labour. On the other hand, necessary labour should include those labour time required to produce the consumption covered by unemployment pay and other supplementary benefits. More complications will surely turn up when we calculate the system rate in the real world.

If we neglect the consumption made by the unemployed using unemployment pay, the above formula can be rewritten as

$$e_s = \frac{u}{\lambda \cdot (1-u)} + e = \frac{u}{\lambda \cdot (1-u)} + \frac{1-\lambda}{\lambda},$$

where u is the rate of unemployment, e the normal rate of exploitation, and λ the value of (abstract) labour force.[13]

It should be stressed again that all we have to have to compute the traditional rate of exploitation are technological data and prices as well as wage rates: actual production or employment levels are unnecessary. On the other hand, our definition of system exploitation rate requires in addition the information about unemployment.

5. Conclusions

We have shown that the Fundamental Marxian Theorem as advocated has too narrow a scope. In order to assert exploitation, it is unnecessary to examine the existence of a price vector under which each sector realizes positive profits. Exploitation exists when a sort of productiveness condition on the quantity side is satisfied including household activities, and the wage rate is not so high as to make it possible for workers to save. In sum, the existence of exploitation hinges upon the level of wage rate, and has little to do with the positivity of profits in each production process. This seems natural enough.

As a second reflection, a snapshot approach to heterogeneous labour has been explained. Using the complete matrices, i.e., dealing with normal production processes and household activities in a symmetrical way, we can conceive labour values and exploitation even for models with joint production. Abstract labour can also be constructed by solving some pro-

13) This formula is suggested by one of the referees.

gramming problems. Salient differences between two rates of exploitation, one based upon heterogeneity as it is and the other upon abstract labour, are clarified.

Finally we have proposed another concept of exploitation, taking into consideration unemployed workers, and defined the system rate of exploitation. This rate may be able to measure more accurately the degree of suffering among workers in an economic system, and the system rate may also show different trends from the ordinary rate. When we say systems, we have in mind capitalist systems, in which production coefficients are well averaged out through mass reproduction and rapid diffusion of new technologies, and those coefficients show a degree of stability, even though we analyze the existence of exploitation in a snapshot of the system.

(received 2010-12-14. revised 2011-01-09. accepted 2011-01-09)

□ References

Bowles, Samuel and Herbert Gintis. 1977. "The Marxian Theory of Value and Heterogeneous Labour: A Critique and Reformulation." *Cambridge Journal of Economics*, 2: 173-92.

Bródy, András. 1970. *Proportions, Prices and Planning: A Mathematical Restatement of the Labor Theory of Value*. Budapest: Akadémiai Kiadó.

Fujimori, Yoriaki. 1982. *Modern Analysis of Value Theory*. Berlin: Springer-Verlag.

Fujimoto, Takao and Arrigo Opocher. 2010. "Commodity Content in a General Input-Output Model." *Metroeconomica*, 61, 3: 442-53.

Fujimoto, Takao and Fumiko Ekuni. 2011. "Abstract Labour in a Model of Joint Production." to appear in *Política y Cultura*, 35 (http://polcul.xoc.uam.mx/).

Fujimoto, Takao and Ravindra R. Ranade. 2010. "Fundamental Theorems for Capitalist Reproduction Systems with Heterogeneous Labour." *Fukuoka University Review of Economics*, 55, 2: 165-88.

Fujimoto, Takao and Yukihiko Fujita. 2008. "A Refutation of Commodity Exploitation Theorem." *Metroeconomica*, 59, 3: 530-40.

Fujimoto, Takao. 1978. "Exploitation, Profits and Growth: A Disequilibrium Analysis." *Economic Studies Quarterly*, 29: 268-75.

_____. 2009. "The Concept of Exploitation in a General Linear Model with Heterogeneous Labour." *Investigación Económica* (UNAM), 68 (267): 51-82. http://redalyc.uaemex.mx/redalyc/src/inicio/HomRevRed.jsp?iCveEntRev=601).

Hollander, H. 1978. "A Note on Heterogeneous Labour and Exploitation." *Discussionsbeiträge zur Politischen Ökonomie*, 14.

Krause, Ulrich. 1980. "Abstract Labour in General Joint Systems." *Metroeconomica*, 32: 115-35.

_____. 1981. "Heterogeneous Labour and the Fundamental Marxian Theorem." *Review of Economic Studies*, 48: 173-8.

Morishima, Michio and Francis Seton. 1961. "Aggregation in Leontief Matrices and the Labour Theory of Value." *Econometrica*, 29, 2: 203-20.

Morishima, Michio. 1973. *Marx's Economics: A Dual Theory of Value and Growth*. Cambridge University Press.

_____. 1974. "Marx in the Light of Modern Economics." *Econometrica*, 42, 4: 611-32.

Okishio, Nobuo. 1963. "Mathematical Note on Marxian Theorems." *Weltwirtschaftlisches Archiv*, 91: 287-99.

_____. 1965. *The Fundamental Theory of Capitalist Economy* (in Japanese). Tokyo: Soubunsha.

_____. 1977. *Marx's Economics: Values and Prices* (in Japanese). Tokyo: Chikuma-Shobou.

Potron, Maurice. 1913. "Quelques Propriétés des Substitutions Linéaires à Coefficients $\geq$ 0 et Leur Application aux Problèmes de la Production et des Salaires." *Annales Scientifiques de l'É.N.S.*, 3e série, 30: 53-76.