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Labour productivity and the law of decreasing labour content

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This paper analyses labour productivity and the law of decreasing labour content (LDLC) originally formulated by Farjoun and Machover. While accepting the validity of that law, it shows that the conventional measures of labour productivity used in the literature may be rather misleading, because they are based on monetary aggregates. Theoretically and empirically sounder measures are provided by standard Marxian real labour values. The notion of labour content and the LDLC are therefore central to understanding capitalist economies. Some rigorous theoretical relations between different forms of profit-driven technical change and productivity are derived in a general input–output framework with fixed capital, and these provide foundations to the LDLC. These theoretical propositions are empirically illustrated using a new dataset of the German economy.

Key words: Labour productivity, Law of falling labour content, Technical change, Labour values, Input–output analysis.
JEL classifications: B51, C67, D57, O33

1. Introduction

Heterodox, and more specifically Marxist, economists have long held the belief that the inherent functioning of the capitalist system—and in particular the forces driving technical change, including class struggle—leads to a tendential decrease in the amount of labour necessary to produce (or, indeed, embodied in) commodities. One of the clearest and most rigorous formulations of this intuition is due to Farjoun and Machover (1983), who derived the celebrated *law of decreasing labour content* (LDLC). In their probabilistic approach: if *C* is a commodity produced over a certain period of time, then 'there is virtual certainty (probability very near 1) that the labour content of one unit of *C* will be lower at the end of the period than it was at the beginning' (ibid., p. 97). Further, more explicitly than other authors, Farjoun and Machover put the LDLC at the centre of the analysis and considered it as one of the key defining features of capitalist economies: it is 'the most basic dynamic law of

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capitalism, archetype of all capitalist development' (ibid., p. 139). Therefore, Farjoun and Machover's (1983) contribution represents a natural starting point for this paper, whose main focus is labour productivity and its relation with technical change in capitalist economies.

Granting that the LDLC characterises capitalist economies, two questions immediately arise. First, why is the LDLC important from a theoretical viewpoint? Second, how can the LDLC be derived, or deduced, from the functioning of capitalist market economies? This paper analyses both questions in a general input–output (IO) model, which is shown to provide a natural framework to formulate and derive the LDLC, and to understand its theoretical relevance.

Section 2 addresses the first question and it shows the salience of the notion of labour content for the understanding of labour productivity. The law is often seen as self-evidently relevant, because it is considered as equivalent to the *law of increasing labour productivity* (see, e.g., ibid., pp. 11, 139 and passim). And labour productivity plays a key role in economic theories of growth and employment, including issues of innovation, structural change, income distribution, etc. Yet the relevance of the LDLC for understanding trends in labour productivity is far from obvious: virtually all of the received productivity measures—as developed for instance in the United Nations' *System of National Accounts* (SNA) (United Nations, 1993; see also OECD, 2001; Bureau of Labor Statistics, 2008)—focus on real GDP per unit of labour, or on some notion of 'real value added' per unit of labour, in order to measure the performance of (different sectors of) the economy. If the conventional SNA measures properly capture labour productivity, then one may argue that the notion of labour content is either misleading or at best redundant.

A thorough critical analysis of the standard SNA measures of sectoral as well as aggregate labour productivity is provided, from an IO perspective. The analysis of the structural features of the economy allowed by the IO framework forcefully shows that the SNA measures are inappropriate to capture production conditions and shifts in efficiency and technology, owing to the central role of relative prices and final demand in their construction. Measures of sectoral and total labour productivity should be based on technological data as much as possible (subject to an unavoidable degree of aggregation) and they should not *definitionally* depend on price variables. The IO employment multipliers—i.e. the labour values of Marxian economic theory[1]—provide (in reciprocal form) theoretically sound measures of sectoral and economy-wide labour productivity, with purely technological foundations—insofar as IO coefficients can be interpreted as pure quantity magnitudes.

Thus, Section 2 proves that the law of increasing labour productivity cannot be properly understood unless the LDLC is formulated. Yet the results also have broader implications for productivity analysis, because they show that the shortcomings of the standard indices are more serious than is acknowledged in the mainstream literature (e.g. Durand, 1994; Cassing, 1996; Schreyer, 2001) and that a proper understanding of labour productivity requires a focus on labour content. IO tables should always be an integral part of the SNA and the point of reference for all productivity measures at the macro- and meso-level of economic activity.[2]

[1] Total labour costs and employment multipliers are identical in Leontief models, but can differ in more general economies. The analysis below can be extended to the general case by using the framework outlined in Flaschel (1983), albeit at the cost of a significant increase in technicalities.

[2] The importance of IO tables in productivity analysis is acknowledged in the mainstream literature (see, e.g., Schreyer, 2001, p. 50).

Critiques of standard SNA productivity measures from an IO perspective and the use of employment multipliers to measure productivity are not novel (see, among others, Gupta and Steedman, 1971; Steedman, 1983; Wolff, 1985, 1994; de Juan and Febrero, 2000; Almon, 2009).[3] This paper presents a new set of arguments that emphasise the central relevance of relative prices and final demand in the definition of the standard measures, but none of the main criticisms crucially depends on *changes* in relative prices over time, so that well-known issues relating to index number construction are not focal. Moreover, a unified theoretical framework for the analysis of productivity measures is provided, which is based on a novel axiomatic method. Rather than comparing different measures in terms of their implications in various scenarios, this paper starts from first principles and formalises some theoretically desirable properties that any measure of labour productivity should satisfy.[4] To be precise, the main axiom focuses on *changes* in productivity and states that labour productivity at t in the production of good i has increased relative to the base period, if a unit increase of the net product of good i demands less labour than in the base period. This is a weak restriction and it incorporates the key intuitions behind the main productivity measures in the literature. Yet it characterises the IO measures, whereas the conventional SNA indices do not satisfy it in general, owing to their inherent dependence on relative prices and final demand.

The second major contribution of this paper, in Section 3, is a rigorous analysis of the conditions under which profitable innovations lower labour values, thereby raising productivity and increasing consumption and investment opportunities. To be precise, in this paper the n-commodity general equilibrium models analysed by Roemer (1977, 1980) are generalised into two main directions. First, following the approach developed by Flaschel (2010), the circulating capital model is extended to the treatment of fixed capital proposed by Bródy (1970) in a seminal contribution. This is important because fixed capital—or, more precisely, capital tied up in production—is a key feature of capitalist economies and is at the centre of innovation processes, but, as various authors have argued, the standard von Neumann framework has serious theoretical and empirical limitations.[5] Second, following one of the key insights of Farjoun and Machover (1983), no condition on uniform profit rates is imposed and the conclusions hold for *any* vector of prices expressed in terms of the wage unit. This extension is both empirically *and* theoretically relevant, because general equilibrium-type constructions, including uniform profit rate models, may be unsatisfactory as representations of allocation in market economies (for a thorough discussion, see Flaschel *et al.*, 2012).

In this general framework, different forms of technical change can be considered and a deterministic theoretical foundation for the LDLC derived. In fact, it can be proved that profitable fixed-capital-using labour-saving innovations lead to productivity increases. Given that capital-using labour-saving technical change has characterised most of the

[3] In Richard Stone's original formulation of the United Nations' SNA, there are definitions of labour productivity that are conceptually analogous to the classical-Marxian measures (e.g. United Nations, 1968, p. 69). This paper suggests that it is unfortunate that this approach has been abandoned. It should be noted that productivity measures based on total labour costs are used both in Marxian theory, and in Sraffian, classical and IO approaches. For this reason, in the rest of the paper, they are sometimes referred to as classical-Marxian indices.

[4] The adoption of an axiomatic approach to analyse classical-Marxian themes is quite novel. Seminal contributions include Yoshihara (2010), Yoshihara and Veneziani (2009), and Veneziani and Yoshihara (2011).

[5] See Bródy (1970) and, more recently, Flaschel *et al.* (2012). For an extension of Roemer's (1977) model to von Neumann economies see Roemer (1979) and Dietzenbacher (1989).

phases in the evolution of capitalism (Marquetti, 2003), this result provides theoretical foundations for the conclusion that labour values tend to fall and labour productivity tends to rise over time in capitalist economies. These results are consistent with the Marxian analysis of technical change and the historical tendencies of capitalism (see Foley, 1986; Duménil and Lévy, 1995, 2003), and identify one of the key dynamic laws of capitalism, describing the link between profitable innovations, the tendential rise in the Marxian technical composition of capital and long-term increases in labour productivity.[6]

The formal analysis also has broader implications concerning the social effects of capitalists' individual decisions. For it can be proved that there is no clear-cut relationship between profitable technical change and social welfare in capitalist economies: capitalists' maximising behaviour is neither necessary nor sufficient for the implementation of productivity-enhancing and welfare-improving innovations.

The analysis in Section 3 is related to the classical literature on technical change, distribution and the evolution of capitalism (for recent contributions, see Duménil and Lévy, 2003; Foley, 2003; Petith, 2008). Yet unlike in the latter contributions, an explicit microeconomic perspective is adopted, which emphasises capitalists' profit-maximising behaviour in highly disaggregated economies. Moreover, although the paper sheds some light on the influence of distributive conflict on technical change, the focus is not on the general relation between technical change and distribution or on the much debated effect of technical progress on profitability.[7] Instead, the effect of individually optimal capitalist decisions on productivity and social welfare is thoroughly explored. Finally, although the process generating innovations is not explicitly formalised, the analysis can be supplemented with the classical-Marxian evolutionary model of technical change developed by Duménil and Lévy (1995, 2003).

The analysis is not purely theoretical, though. In Section 4 an empirical appraisal of the main theoretical conclusions is provided, based on the new IO dataset of the German economy constructed by Kalmbach *et al.* (2005). The empirical evidence confirms the main conclusions: first, SNA measures of labour productivity can be rather misleading and quite different from the theoretically sound IO indices. Second, the LDLC holds for the German economy, a fact that is not easily visible by just looking at the IO tables. It should be noted, however, that the main aim of this paper is not to provide a fully rigorous econometric analysis of productivity measures or of long-term trends of technical change in capitalist economies. The focus is primarily theoretical and methodological: the paper provides a general analysis of the relationships between prices, technical change and labour productivity. From this viewpoint, the discussion of the German economy (1991–2000) does not aim to be exhaustive: it only illustrates the main theoretical points and the empirical results should be taken as a first step towards a more detailed analysis.

2. Labour content and labour productivity

The point of departure of the analysis is the standard IO table shown in Table 1, which illustrates economic activity in a particular year in the n sectors of the economy. The

[6] Given the focus of the paper, the LDLC is not analysed in the context of the broader set of dynamic laws of capitalism. However, some possible links and avenues for further research are briefly discussed below and in the concluding section.

[7] See, for example, Himmelweit (1974), Michl (1994) and the literature therein. Some implications of the analysis for these classical debates are briefly discussed in Section 5 below.

Table 1. *The standard form of an input–output table*

Delivery from ↓ to →	Sector 1 ... sector n	Final demand	Row sum
Sector 1	$x_{11}(t)p_1(t) \ldots x_{1n}(t)p_1(t)$	$f_1(t)p_1(t)$	$x_1(t)p_1(t)$
.	.	.	.
.	.	.	.
.	.	.	.
Sector n	$x_{n1}(t)p_n(t) \ldots x_{nn}(t)p_n(t)$	$f_n(t)p_n(t)$	$x_n(t)p_n(t)$
Value added	$Y_1(t) \ldots Y_n(t)$	–	$Y(t)$
Column sum	$x_1(t)p_1(t) \ldots x_n(t)p_n(t)$	$F(t)$	

notation is standard: $p(t) = [p_1(t),\ldots,p_n(t)]$ is the $1 \times n$ vector of prices of the n commodities at time t; $x_{ij}(t)$ is the amount of good i used as intermediate input in the production of good j; $x_i(t)$ is the gross output of good i; $f_i(t)$ is the final demand of good i.

At the most general level, labour productivity can be defined as a ratio between an index of output and an index of labour input. One possibility is to use *gross output* as a measure of real product and to define labour productivity as gross output per unit of direct labour. As is well known, however, this measure is appropriate only in the rather special case of technical progress affecting all factors proportionally. Further, gross-output-based indices of productivity are sensitive to the degree of vertical integration: *ceteris paribus*, gross-output-based productivity rises as a consequence of outsourcing, even if there are no changes in technology and production conditions. Therefore most of the literature focuses on value added.[8] Two methods are used to obtain *real* output measures starting from value-added data. The *single-deflation* method requires deflating all entries (both outputs and inputs) in the nominal Table 1 by a common price deflator, say P. Single-deflated value added in sector i is then $Y_i^s(t) = Y_i(t)/P$, and at the aggregate level $Y^s(t) = \sum_{i=1}^{n} Y_i^s(t)$. In contrast, the *double-deflation* method attempts to measure everything in constant prices, i.e. with regard to Table 1 it attempts to replace current prices $p(t)$ with the prices $p(0)$ of a base year $t = 0$. This method, however, cannot be directly applied to the 'value added' row in Table 1, whose entries are pure value magnitudes, and the double-deflated sectoral values added $Y_i^d(t)$ are obtained indirectly by applying the accounting consistency requirement of the nominal Table 1 to its analogue in constant prices. This means that $Y_i^d(t)$ is the value added that would have resulted in sector i, had the prices in Table 1 remained constant after the base year.

Value added in base year prices remains a value magnitude and not a quantity independent of relative prices, and therefore both single- and double-deflated value added are problematic notions in productivity analysis. 'Value added is … not an immediately plausible measure of output: contrary to gross output, there is no physical quantity that corresponds to a volume measure of value-added' (Schreyer, 2001, p. 41). Rather than measures of sectoral real output, *single-deflated* values, $Y_i^s(t)$, should be interpreted as indices of sectoral real incomes, with only a distant relation with technological conditions. Any such measure as $Y_i^s(t)/L_i(t)$—where $L_i(t)$ denotes the work hours employed in sector i—represents at best real purchasing power per unit of labour,

[8] For an approach focusing on gross output, see Hart (1996) and Stiroh (2002).

rather than sectoral labour productivity. In contrast, the economic meaning of sectoral *double-deflated* value added is rather unclear: since $Y_i^d(t)$ in general differs from $Y_i^s(t)$, for any i, then $Y_i^d(t)$ does not measure output correctly and in addition it has nothing to do with real purchasing power. It is a purely fictitious quantity representing the income per worker that would have emerged *if* prices had remained constant at the level of the base year.

These well-known conceptual problems, though, are usually considered as minor and in virtually all of the literature on labour productivity, value-added measures of real output, and in particular the double-deflated values $Y_i^d(t)$ and $Y^d(t)$, are used. Sectoral and macroeconomic labour productivity are defined, respectively, as $\pi_i^c(t) = Y_i^d(t)/L_i(t)$, and $\pi^c(t) = Y^d(t)/L(t)$, where $L(t) = \sum_{i=1}^{n} L_i(t)$, and

$$\pi^c(t) = \sum_i \left(\frac{L_i(t)}{L(t)}\right) \cdot \left(\frac{Y_i^d(t)}{L_i(t)}\right) = \sum_i \left(\frac{L_i(t)}{L(t)}\right) \cdot \pi_i^c(t). \tag{1}$$

Value-added-based indices are considered theoretically and empirically meaningful. Indices based on single-deflated value added are deemed appropriate to analyse issues relating to economic welfare, whereas 'for the purposes of measuring efficiency and productivity [double-deflated measures are] to be preferred' (Stoneman and Francis, 1994, p. 425; see also Cassing, 1996). Several doubts can be raised on both claims and in general on the standard approach to productivity analysis.

For any vector $z \in \mathcal{R}^n$, let z' denote its transpose[9] and let $\hat{z}$ denote the diagonal matrix with z as its main diagonal. In IO analysis it is common to choose the units of the n commodities so that, in the base period, $p(0) = e' \equiv (1,\ldots,1)$. The double or row-wise 'price deflated' Table 1 can then be expressed in matrix notation as in Table 2. Following common practice in IO analysis, the matrix of intermediate inputs X can be transformed into the matrix of input coefficients $A = X\hat{x}^{-1}$ and the $1 \times n$ vector of direct labour inputs $\ell = (\ell_1,\ldots,\ell_n)$ can be similarly transformed into a vector of labour coefficients $l = \ell\hat{x}^{-1}$.[10] Then, the macro-identity $Y^d = p(0)f = F^d$ behind Table 2 can be expressed in matrix notation as follows:

$$Y^d = p(0)y^d = p(0)\,(I-A)x = p(0)f = F^d.$$

In contrast, the labour time spent, directly and indirectly, in the production of the n goods is given by $v = (v_1,\ldots,v_n) = l(I-A)^{-1}$ and the IO, or classical-Marxian measures of sectoral labour productivity are defined as $\pi_i^m = 1/v_i$.[11] In the rest of this section, a general framework is provided to compare productivity measures. In order to avoid problems of interpretation, the structural coefficients (A, l) are considered as the parameters of a linear technology, as in standard IO practice.

One of the key shortcomings of the SNA measures is their sensitivity to changes in relative prices that do not reflect any shift in production conditions. Consider,

[9] In this paper, vectors are always column vectors, unless otherwise stated.

[10] For the sake of notational simplicity, in the rest of the paper, the timing of vectors will be omitted, whenever this is clear from the context.

[11] v can be derived even if the assumptions of this paper are relaxed: see Gupta and Steedman (1971) for the treatment of fixed capital and imports, and Flaschel (1983) on joint production.

Table 2. *Elementary input–output table in matrix notation*

\	1…n		
1 . . . n	X	f	x
	$y^{d\prime}$	–	Y^d
	x'	F^d	–

for example, a simple economy with one capital good and one pure consumption good, such that in period t the technical coefficients, a_{ij}, are $0 < a_{11} < 1$, $a_{12} > 0$ and $a_{21} = a_{22} = 0$. If a single price deflator P is used, which includes prices of all sectors, as in standard index number theory, then quite puzzlingly real value added in sector 1 may be affected by changes occurring in sector 2, even if good 2 does not enter the production of good 1, either directly or indirectly. In general, when output prices change relative to input prices, the single deflation method will detect variations in productivity even if production conditions are unchanged.[12]

Productivity indices based on double-deflated value added fare no better. Consider the IO matrix $\tilde{A}$ in constant prices where the standard normalisation $p(0) = e'$ is not adopted, so that $\tilde{a}_{ij} = p_i(0)a_{ij}/p_j(0)$, for all i, j. Similarly, $\tilde{l}_j = l_j/p_j(0)$ and thus the same relationship holds for labour values: $\tilde{v}_j = v_j/p_j(0)$. Because the investment good sector is homogeneous with respect to inputs and outputs:

$$\pi_1^c = \frac{1 - p_1(0)a_{11}/p_1(0)}{l_1/p_1(0)} = \frac{1 - a_{11}}{l_1/p_1(0)} = \frac{p_1(0)}{v_1},$$

so that relative prices do not distort π_1^c, which coincides with the IO measure. For the consumption good sector, however, a different conclusion holds:

$$\pi_2^c = \frac{1 - p_1(0)a_{12}/p_2(0)}{l_2/p_2(0)} = \frac{p_2(0) - p_1(0)a_{12}}{l_2} \neq$$

$$\frac{1}{v_2/p_2(0)} = \frac{1}{(v_1/p_1(0))p_1(0)a_{12}/p_2(0) + l_2/p_2(0)} = \frac{1}{(v_1a_{12} + l_2)/p_2(0)}.$$

The numerator of π_2^c depends on relative prices, and thus on their structure and on the base period used: different vectors $p(0)$ lead to different values of π_2^c regardless of production conditions. Certainly, labour values are also measured relative to output value, but this only means that each time series of labour values is divided by the constant price of the corresponding good, which does not distort the internal structure of the

[12] For related analyses of the sensitivity of the SNA measures to changes in relative prices see Durand (1994), Hart (1996) and Almon (2009).

time series itself: for any given j, $1/v_j$ is only rescaled and its growth rate is independent of prices. In general, whereas the indices π^c_j depend on the conceptually dubious double-deflated values added, the vector v is derived from the meaningful, volume-oriented double-deflated entries of the IO table $\tilde{A}$.

The previous conclusions can be generalised and made more rigorous by analysing alternative approaches in a unified framework, in which some desirable properties of productivity measures are defined *ex ante*. Let $e_i = (0,\ldots,1,\ldots,0)'$ be the i-th unity base vector. Definition 1 formalises the notion of *increases* in labour productivity.[13]

Definition 1

1. Labour productivity at t has increased *with regard to commodity* i, relative to the base period, if and only if an increase of the net product f by one unit of commodity i demands less labour than in the base period. Formally, let $x_i(t) = (I - A(t))^{-1} e_i$ and let $\ell_i(t) = l(t)x_i(t)$: labour productivity has increased if and only if $\ell_i(t) < \ell_i(0)$.
2. If $\ell(t) \leq \ell(0)$ then labour productivity at t has increased *in the whole economy*, with respect to the base period.

Definition 1 does not aim to capture all aspects of labour productivity and it only constrains *changes* in productivity. From an epistemological viewpoint, it can be seen as an axiom: whatever else a measure of productivity may do, it should satisfy Definition 1, which sets some *minimal* restrictions on productivity measures. From this perspective, Definition 1 has a number of attractive features. First, it has a firm technological foundation that captures only shifts in productive conditions and efficiency: purely monetary magnitudes are irrelevant and final demand plays only an auxiliary role.[14] This is certainly a desirable property of labour productivity measures, as many authors have argued (e.g. OECD, 2001). Second, by focusing on goods, rather than sectors, Definition 1(1) incorporates the interdependencies between sectors and it allows one to capture the relation between technical change and social welfare. This may seem more controversial, but a similar concern for the role of intermediate inputs and vertical integration actually motivates the use of value-added-based indices—as opposed to gross-output-based indices—in the mainstream literature (e.g. Schreyer, 2001, p. 41 ff.): they are preferred because they capture interindustry transactions and 'provide an indication of the importance of the productivity measurement for the economy as a whole. They indicate how much extra delivery to final demand per unit of primary inputs an industry generates' (Schreyer, 2001, p. 42). Third, Definition 1(2) may be deemed rather stringent, especially if n is large, as it requires (weakly) monotonic increases for all goods. From an axiomatic perspective, however, it sets a very weak and intuitive restriction on any productivity measure. This is even more evident if a (neoclassical) notion of productivity as measuring economic welfare is adopted, for in this case Definition 1(2) is analogous to a paretian condition capturing vectorwise improvements in consumption and investment opportunities.

Definition 1 is by no means trivial, however. For example, in Definition 1 all labour is implicitly treated as productive. This may be deemed objectionable from a Marxist

[13] The following notation holds for vector inequalities: for all $x, y \in \mathcal{R}^n$, $x \geqq y$ if and only if $x_i \geqq y_i$, all i; $x \geq y$ if and only if $x_i \geqq y_i$ and $x \neq y$; and $x > y$ if and only if $x_i > y_i$, all i.

[14] The original net product f is irrelevant in Definition 1, thanks to the linearity of the technology.

viewpoint: some labour might be considered unproductive and therefore not count.[15] This forcefully shows that Definition 1 is not vacuous and it does incorporate theoretically substantive assumptions. It is not clear theoretically how to incorporate a productive–unproductive labour distinction into a micro-based IO framework, because what is productive from the perspective of an individual capital might be unproductive from the perspective of total social capital, and from this perspective Definition 1(2) does not hold. Thus, the IO approach requires the maintenance of vertical additivity in an IO table and the productive–unproductive labour distinction effectively contests it. But it is worth remarking (i) that the distinction between productive and unproductive labour is far from being widely accepted among Marxist economists,[16] and (ii) that the issue is irrelevant in a comparison with standard SNA productivity measures, because the latter treat all labour as productive anyway.

The next result states that Definition 1 characterises the classical-Marxian measures of labour productivity.

Proposition 1

For a given commodity i, $\ell_i(t) < \ell_i(0)$ if and only if $\pi_i^m(t) > \pi_i^m(0)$.

Furthermore, if the whole economy is considered $\ell(t) \leq \ell(0)$ if and only if $\pi_i^m(t) \geq \pi_i^m(0)$, for all $i = 1,\ldots,n$, with strict inequality for some i.

Proof

By the definition of v, for any final demand f, $L = \ell e = lx = l(I-A)^{-1} f = vf$. The latter expression implies $\ell_i(t) = v(t)e_i = v_i(t)$ and the desired result follows. QED.

In other words, labour productivity with regard to good i increases if and only if the amount of labour directly and indirectly embodied in good i decreases. Further, any index of aggregate labour productivity satisfies Definition 1(2) if and only if it is monotonic in the vector of labour values. Proposition 1 provides theoretical foundations to the classical-Marxian indices as the appropriate indicators of labour productivity. Certainly, one may argue that the indices π_j^m have the disadvantage that they cannot be deduced only from data that characterise sector j, and it is this property that drives Proposition 1. Yet the standard value-added-based measures cannot be defined based only on data from sector j either, even though the dependence on the other sectors is less evident than in π_j^m. It is in fact *impossible* to formulate and interpret nominal value added Y_j—as well as 'real' value added Y_j^s or Y_j^d—without reference to a price system (even if prices may not appear explicitly, owing to the normalisation $p(0) = e'$). SNA measures do depend on the data of the other sectors via the price vector, but—unlike for π_j^m—the sectoral influences are unexplained and depend on the contingent institutional and market conditions of the base year. The rigorous technological foundation that characterises the classical-Marxian indices is lost. Therefore, it should not be surprising that the standard SNA measures cannot correctly capture labour productivity either at the sectoral or at the aggregate level. This is proved in the following propositions.

Proposition 2 states that the SNA and the classical-Marxian indices of sectoral labour productivity coincide only in a very special case.

[15] This would also force a distinction between labour embodied and Marxian value. We are grateful to an anonymous referee for bringing these issues to our attention.

[16] For a thorough discussion, see Mohun (1996) and the subsequent debate.

Proposition 2

Let $p(0) = e'$. The equality $\pi^c_j = \pi^m_j = 1/v_j$, for all $j = 1,\ldots,n$, holds if and only if $\pi^c_j = \pi^c$, for all $j = 1,\ldots,n$.

Proof

($\Leftarrow$) Suppose that $\pi^c_j = \pi^c$, for all $j = 1,\ldots,n$. Then $e' - e'A = \pi^c l$, or equivalently $(1/\pi^c)e' = l(I - A)^{-1} = v$.

($\Rightarrow$) Suppose that $\pi^c_j = 1/v_j$, for all $j = 1,\ldots,n$. Let $\hat{\pi}^c$ denote the diagonal matrix with π^c_j, $j = 1,\ldots,n$, on the main diagonal. Since $\pi^c_j = 1/v_j$, for all $j = 1,\ldots,n$, then $v\hat{\pi}^c = e'$, or equivalently, $v = e'\left(\hat{\pi}^c\right)^{-1}$. By definition, $v = l(I - A)^{-1}$ and thus, by the latter equation: (i) $e' = l\left(I - A\right)^{-1}\hat{\pi}^c$. Further, by definition $e'\left(I - A\right) = l\hat{\pi}^c$, or: (ii) $e' = l\hat{\pi}^c(I - A)^{-1}$. Then it is immediate to show that (i) and (ii) have a meaningful solution only if $\hat{\pi}^c = \pi^c I$, for some positive π^c. QED.

By Proposition 2, any differences in the two sectoral indices must be examined in relation to sectoral productivity differences. The next result instead shows that the SNA measure of aggregate productivity satisfies Definition 1(2), *if final demand is constant*.

Proposition 3

Suppose that $f(t) = f(0) = f > 0$. If $v(t) \leq v(0)$ then $\pi^c(t) > \pi^c(0)$.

Furthermore, $\pi^c(t) > \pi^c(0)$ if and only if $v(t)f < v(0)f$.

Proof

The result follows noting that $\pi^c(t) = p(0)f/L(t)$ and that the equality $L(t) = v(t)f$ holds, as shown in Proposition 1. QED.

In other words, technical change yielding increases in productivity according to Definition 1(2) implies a corresponding change in the SNA macroeconomic measure of labour productivity. Further, the change in technology decreases the expenditure of human labour for the production of a given vector of final demand f. Thus, Proposition 3 suggests that movements in the SNA aggregate measure map changes in the IO indicators, *if final demand is constant*. Yet Proposition 3 does not necessarily hold if final demand varies, nor does it hold at the sectoral level.

Consider the two-sector economy described in Table 3, where process 1 is subject to technical change between $t = 0$ and $t = 1$. Let $p(0) = e'$ and assume $w = 1$. First, technical change in sector 1 is *capital using* and *labour saving* in the sense that it increases the value of intermediate inputs but lowers labour costs, at current prices. Second, technical change is *profitable*, because unit costs in sector 1 decrease from 0.9 to 0.86. Third, the SNA sectoral productivity measure *increases* in sector 1 and remains *constant* in sector 2:

$$\pi^c_1(1) \approx 1.44 > \pi^c_1(0) = 1.25, \qquad \pi^c_2(1) = \pi^c_2(0) = 8.$$

In contrast, fourth, the classical-Marxian measures π^m_1, π^m_2 *decrease*:

$$\pi^m_1(1) \approx 1.58 < \pi^m_1(0) \approx 1.70, \qquad \text{and}$$

$$\pi^m_2(1) \approx 2.92 < \pi^m_2(0) \approx 3.09.$$

The technical change described in Table 3 leads to a sharp divergence in the standard indices, π_i^c, and the IO indices, π_i^m, which can move in opposite directions. Therefore, by Proposition 1, the example in Table 3 proves that the SNA sectoral measures, π_i^c, do not satisfy Definition 1. Noting that these conclusions can be generalised to n-good economies, they can be summarised in the next proposition.[17]

Proposition 4

Suppose that $f(t) = f(0) = f > 0$. For any good i, if $\ell_i(t) < \ell_i(0)$ then $\pi_i^c(t)$ may increase, decrease or remain constant relative to $\pi_i^c(0)$. Furthermore, it is possible to have $\ell(t) \leq \ell(0)$, but $\pi_i^c(t) \leqq \pi_i^c(0)$, for all i, with strict inequality for at least some i.

In other words, the standard sectoral productivity indices do not satisfy the minimal requirements set out in Definition 1, even under the restrictive assumption of a constant final demand. The shortcomings of the SNA measures π_i^c derive primarily from the fact that they crucially rely on price information and do not properly reflect changes in technology. As a result, they can show increases in productivity in every sector even if the net production possibilities of the economy are deteriorating. Actually, by Proposition 3, the SNA aggregate index π^c correctly reflects changes in the whole economy whenever final demand is constant, but Table 3 shows that π^c and the sectoral measures π_j^c can move in opposite directions (in the example, π^c decreases) if the sectoral allocation of labour changes appropriately (see equation (1)). Hence, the SNA sectoral measures do not provide useful information concerning the sectors leading to movements in aggregate labour productivity.

It is worth stressing that the proof of Proposition 4 is completely general. In Table 3, only *profitable* technical change is considered, but this is unnecessary to establish the proposition. It is, however, theoretically relevant because it shows that the result is not driven by some peculiar or economically meaningless combination of parameters. Further, none of the conclusions depends on the assumption of capital-using, labour-saving technical change and it is easy to construct similar examples with other types of innovations.

Although the previous analysis has focused on sectoral productivity measures, the standard approach to aggregate productivity is also unsatisfactory and the SNA measure π^c does not satisfy Definition 1(2) in general. To see this, consider again a two-good economy with technical change between $t = 0$ and $t = 1$. At any t, let $L(t) = l(t)x(t)$, so that, by the definition of labour values, $L(t) = v(t)f(t) = v_1(t)f_1(t) + v_2(t)f_2(t)$. Then,

Table 3. *A two-sector economy with profitable capital-using and labour-saving technical change (at constant prices $p(0) = e'$, $w = 1$)*

Structure\period	$t = 0$	$t = 1$
Matrix of intermediate inputs A	$\begin{matrix} 0.1 & 0.3 \\ 0.4 & 0.3 \end{matrix}$	$\begin{matrix} 0.44 & 0.3 \\ 0.1 & 0.3 \end{matrix}$
Labour inputs l	0.4 0.05	0.32 0.05

[17] In Table 3 the reciprocal of the direct labour time per unit of output, $1/l_i(t)$, also increases in sector 1 and remains constant in sector 2. Therefore Proposition 4 can be extended to the indices $\pi_i^l(t) = 1/l_i(t)$, which are also sometimes used to measure productivity.

dropping time subscripts for the sake of notational simplicity, for a given technology (A, l), the *net product transformation line* is given by:

$$f_2 = (L - v_1 f_1)/v_2 = L - \pi_2^m f_1/\pi_1^m, \quad \text{with} \quad \pi_1^m = 1/v_1, \pi_2^m = 1/v_2.$$

Figure 1 shows that if $\pi_1^m/\pi_2^m \neq p_2(0)/p_1(0)$ there can be a change in final demand from f^0 to f^1 and a simultaneous change in technology (A, l), such that $v(t) \leq v(0)$ and the net product transformation line shifts out, but $\pi^c(0) = p(0)f^0 > \pi^c(1) = p(0)f^1$. Noting that this argument can be easily generalised to n-good economies, it can be summarised in the next proposition.

Proposition 5

Suppose that $f(t) \neq f(0)$. If $\ell(t) \leq \ell(0)$, then $\pi^c(t)$ may increase, decrease or remain constant relative to $\pi^c(0)$.

Proposition 5 concludes the theoretical analysis of labour productivity measures. The previous results prove that the SNA sectoral measures do not meet the requirement set out in Definition 1(1). By Proposition 5, the SNA aggregate measure π^c does not satisfy the very weak condition in Definition 1(2) either: it can detect a decline in

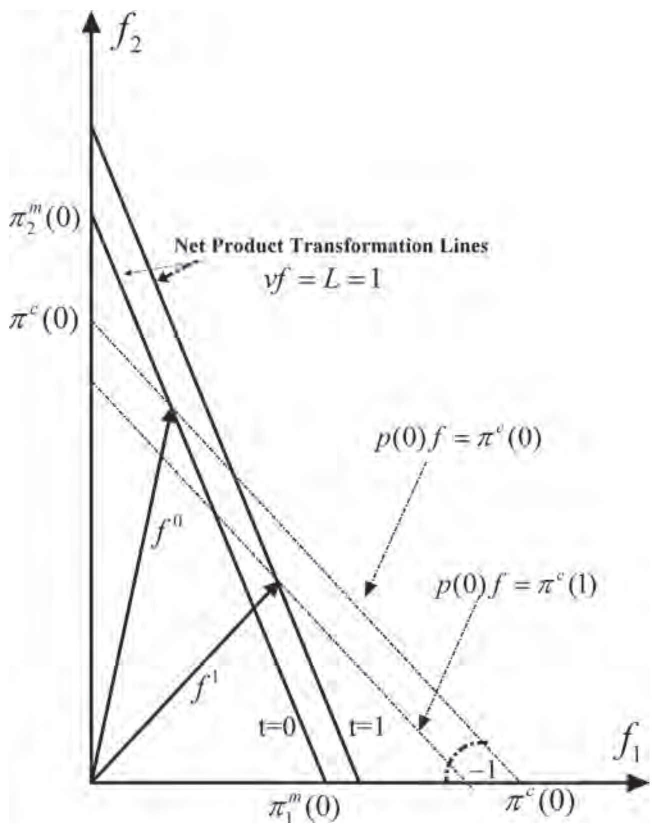

Fig. 1. *An increase in net production possibilities and a decrease in the conventional measure of aggregate labour productivity ($p(0) = e'$).*

productivity in the economy even if the net production possibilities unambiguously expand. Neither the sectoral nor the aggregate SNA productivity measures are adequate to capture shifts in technology and efficiency. Besides, Propositions 4 and 5 imply that, contrary to the received view, value-added-based measures are also inadequate to capture economic welfare, for an expansion of the net production possibilities increases social welfare.[18] Again, the problem with standard measures is that they crucially depend on relative prices and final demand, in a manner that is independent from technical conditions.

It is important to clarify the scope and generality of these conclusions. As noted above, in the mainstream literature, changes in relative prices over time have long been known to cause significant problems in index number construction and especially in analyses of movements in SNA productivity measures based on fixed-base Paasche or Laspeyres indices. This has motivated a move towards the adoption of the chained Fisher index of real value added, whereby the square root of the product of Paasche and Laspeyres indices is taken for pairs of adjacent years, which are then chained together. It is still debated whether this effectively removes the problems associated with relative prices changing over time, especially given that it comes at the cost of losing additivity of the components and that the Fisher index is undefined when either the Laspeyres or the Paasche index is negative—a not unlikely occurrence (see Schreyer, 2004; Meade, 2010). But the key point here is that although the main shortcoming of standard measures is shown to be their reliance on relative prices (and final demand), nothing in the above analysis hinges upon *changes* in relative prices over time. The issue of the choice of the appropriate index number is therefore secondary for the key arguments, and the adoption of chained Fisher indices does not solve any of the problems highlighted above.

First, the propositions hold for any two periods and therefore *a fortiori* for any two adjacent periods. Hence, *per se* chaining is not relevant for the present analysis, for chained indices coincide with the standard, fixed-base indices when two adjacent periods are considered. Second, and perhaps more importantly, none of the above results depends on relative prices changing between periods 0 and t, and therefore they hold even if relative prices remain constant over time and equal to the base period prices. In this case, however, there is no issue concerning the choice of the appropriate index number capturing changes in the standard SNA productivity measure, as they all coincide. Thus, the adoption of a chained Fisher index makes no difference for the key conclusions of this paper.

Certainly, if technological conditions change, relative prices are likely to vary and therefore it may be unrealistic to assume them to remain constant over time. Yet, from a theoretical perspective, this is an appropriate assumption that allows us to identify a number of key limitations of standard productivity measures *in addition* to the well-known problems caused by changes in relative prices. The limitations of the productivity indices based on some notion of real value added are deep and suggest that the notion of labour content is essential to capture labour productivity. For the purposes of this paper, they also imply that the law of increasing labour productivity cannot be properly understood unless the LDLC is formulated.

[18] Proposition 5 also applies to measures based on single deflated aggregate value added.

3. Technical change and the LDLC

Section 2 proves that the classical-Marxian indices $\pi_j^m = 1/v_j$ represent the only theoretically sound measures of labour productivity, which capture both its technological and its welfare aspects, and thus the LDLC is crucial in order to understand the dynamics of a capitalist economy. In this section some propositions are derived on the relationship between prices and productivity, by analysing the conditions under which profitable innovations lower labour values.

Technologies are now more generally described by a 3-tuple (K, A, l), where K is a stock matrix whose generic entry K_{ij} denotes the amount of commodity i that is tied up in the production of commodity j.[19] Everything is expressed again per unit of commodity output. For the sake of simplicity, it is assumed that the output matrix is equal to the identity matrix, I, but all results can be extended to technologies with multiple activities as well as joint production, provided the framework outlined in Flaschel (1983) to define labour content is adopted.

In order to avoid a number of uninteresting technicalities, and with no loss of generality, the following standard assumption is made on technology.

Assumption 1

For any technology (K, A, l), A is productive and indecomposable, and $l > 0$.

Assumption 1 has two main implications. First, in this paper, technical changes in the various sectors of the economy are considered separately and are assumed to occur in individual sectors.[20] Yet Assumption 1 implies that the effects of sectoral innovations extend throughout the economy. Second, let $p_{wj} = p_j/w$ be the price of good j in terms of the wage unit, so that $p_w = p/w$ is the vector of wage prices. In what follows, it is not assumed that p_w represents long-term production prices: it may well be a vector of (normalised) market prices. By Assumption 1, the Leontief inverse exists and is strictly positive, and so Lemma 1 immediately follows, which extends a well-known property of prices of production with uniform profit rates to *any* vector of wage prices which allows for positive profits.

Lemma 1
Under Assumption 1, for any p_w such that $p_w > p_w A + l$ it follows that $p_w > v = l(I - A)^{-1} > 0$.

Thus, labour-commanded prices are a useful upper estimate for embodied labour costs even if no restrictive assumption on uniform profit rates is made.

Let r_j be the profit rate on capital advanced in sector j. Definition 2 distinguishes various forms of technical change, depending on their effect on unit costs and on labour values, and on whether they tend to substitute labour for capital or vice versa.

[19] For a detailed explanation of the treatment of fixed capital see Bródy (1970) and Flaschel *et al.* (2012). In this section, it is still assumed that the matrix of depreciation of fixed capital is equal to zero, i.e. $A^\delta = 0$, but all results can be extended to the matrix $\bar{A} = A + A^\delta$ and the corresponding labour values.

[20] The reader is referred to Bródy (1970) for a discussion of the prerequisites for an analysis of technical change in a Leontief IO system.

Definition 2

1. Technical change $(K_j, A_j, l_j) \mapsto (K_j^*, A_j^*, l_j^*)$ is *profitable* if and only if, at initially given prices p_w such that $p_{wj} = r_j p_w K_j + p_w A_j + l_j$ and $r_j > 0$:

$$r_j p_w K_j + p_w A_j + l_j > r_j p_w K_j^* + p_w A_j^* + l_j^*.$$

2. Technical change $(K_j, A_j, l_j) \mapsto (K_j^*, A_j^*, l_j^*)$ is *progressive* if and only if:

$$v = vA + l > v^* A^* + l^* = v^*.$$

   Similarly, technical change is *regressive* if and only if $v < v^*$.

3. Technical change $(K_j, A_j, l_j) \mapsto (K_j^*, A_j^*, l_j^*)$ is: *capital using* (KU) if and only if $p_w K_j < p_w K_j^*$; *capital saving* (KS) if and only if $p_w K_j > p_w K_j^*$; *labour using* (LU) if and only if $l_j < l_j^*$; and *labour saving* (LS) if and only if $l_j > l_j^*$.

Definition 2 generalises the definitions in Roemer (1977) to economies with capital tied up in production *and* to any vector of wage prices p_w: profits are treated as a mere residual and no assumptions are made on the uniformity of profit rates or on the determination of p_w.[21] It is important to note that in Definition 2(3), innovations are defined in monetary terms and thus are significantly more general than in Roemer (1977), in that they allow for non-monotonic changes in capital requirements. Finally, it is worth noting that in Definition 2(2) it is not restrictive to focus on technical changes where all labour values change in the same direction. If technical change occurs in one sector at a time, this will not produce value changes in opposite directions in different sectors (see Roemer, 1977, p. 410).

Next, define the following auxiliary intermediate input matrix: $A^{*+} = \max\{A^*, A\} \geqq A^*$.

The auxiliary matrix A^{*+} is a mathematical construct that will be useful to derive the formal theorems below. In particular, if j is the sector subject to technical change, then $(K_j, A_j, l_j) \mapsto (K_j^*, A_j^{*+}, l_j^*)$ might be loosely interpreted as technical change using the most circulating capital intensive technique. Note also that $A_j^{*+} \geq A_j$ if and only if $A_{ij}^* > A_{ij}$, for at least some i. Based on A^{*+}, a specific class of innovations is considered below and the following assumption is made.

Assumption 2

For any profitable KU-LS technical change $(K_j, A_j, l_j) \mapsto (K_j^*, A_j^*, l_j^*)$, the following inequality holds: $p_w A_j + l_j > p_w A_j^{*+} + l_j^*$.

Assumption 2 states that the main part of the cost-reduction process occurs via changes in the capital that is tied up in production, which allows for significant reductions in labour costs, whereas changes in intermediate inputs are unsystematic and secondary, and therefore profitable even if the auxiliary matrix A^{*+} is considered. Assumption 2 rules out only secondary profitable technical changes and yields no major loss of generality in the analysis of LS innovations. Formally, Assumption 2 provides a link between the effect of technical progress on fixed capital and changes in the use of

[21] In Roemer (1977), cost-reducing innovations are called *viable*, but the notion of *profitability* more explicitly conveys the idea of monetary, rather than physical, magnitudes.

intermediate inputs. Then, the first key result on technical change in general economies with fixed capital can be derived.[22]

Theorem 1
Assume Assumption 1. Let $p_w > p_w A + l$.

(i) Under Assumption 2, all KU-LS profitable technical changes are progressive.
(ii) However, there are KU-LS progressive technical changes that are not profitable.

Theorem 1 is quite general and by no means obvious. For it proves that cost-reducing innovations that substitute fixed capital for labour are progressive, even if no stringent assumption is made concerning the effect of technical change on intermediate inputs. Therefore, in general, LS innovations will reduce the labour content of goods and increase net production possibilities. Yet profitable KU-LS innovations do not fully exploit the potential of technical progress to increase labour productivity. For there exist feasible technologies that will not be adopted by capitalists that would yield social welfare improvements by increasing net production possibilities.

The proof that profitable KU-LS innovations increase consumption and investment opportunities has relevant implications for the LDLC and the understanding of capitalist economies. For it derives a systematic relationship between certain forms of technical change, profit-maximising behaviour and labour values. Empirically, one may conjecture that distributive conflict and increasing wages have introduced a bias in the direction of technical change towards KU-LS changes that may partly explain the secular increase in labour productivity observed in capitalist economies. Theoretically, although class conflict is not analysed in this paper, one may construct a plausible scenario in which wage increases induce KU-LS technical change and so a decrease in labour content. This argument may provide microfoundations to the LDLC, which need not be based on—but, of course, can be supplemented by—probabilistic considerations. The price implications of technical changes may indeed be chaotic, as Farjoun and Machover (1983) argued, but the quantity implications investigated in this paper are independent of such chaotic behaviour.

The result in Theorem 1, however, cannot be extended to other types of innovations. Theorem 2 proves that there may be profitable KS-LU innovations that reduce the economy's net production possibilities and thus social welfare.

Theorem 2
Assume Assumptions 1–2. Let $p_w > p_w A + l$.

(i) All KS-LU progressive technical changes are weakly profitable.
(ii) However, there are KS-LU profitable technical changes that are not progressive. More precisely, technical change is progressive if and only if $v_j > vA_j^* + l_j^*$.

Together with Theorem 1, Theorem 2 provides a full description of technical change in a capitalist economy with capital tied up in production. Theorem 2 characterises the conditions under which KS-LU progressive technical change occurs: KS-LU innovations are progressive and thus increase social welfare if and only if they reduce the labour content of a commodity in terms of the *old* labour values. Thus, Theorem 2 implies that the problematic situation with respect to technological regress is, generally speaking, the

[22] The proofs of Theorem 1 and of Theorem 2 below are in the Appendix.

labour-using case. To be specific, labour productivity falls if the following inequalities hold simultaneously: $r_j p_w K_j + p_w A_j + l_j > r_j p_w K_j^* + p_w A_j^* + l_j^*$, $l_j < l_j^*$, $v_j < vA_j^* + l_j^*$. In Theorem 2, labour values all move in the same direction, i.e. if labour productivity falls in some sectors, then it falls in all of them. Therefore, it is unambiguously clear whether the set of net production possibilities expands or contracts. In the KS-LU case with $v_j < vA_j^* + l_j^*$ it contracts, as the labour contents of all commodities rise. Hence, capitalist choices leading to KS-LU technical change may have adverse effects on economic development, since they may undermine the LDLC and thus decrease consumption and investment opportunities, and periods characterised by KS-LU technical change may be plagued by productivity slowdowns.

Theorems 1 and 2 generalise Roemer's (1977) results in economies with circulating capital and they identify some systematic connections 'between the visible and the invisible—between price and labour-content' (Farjoun and Machover, 1983, p. 84). As noted above, given the KU-LS nature of technical progress in actual capitalist economies, Theorem 1 sheds some light on the LDLC, by identifying a link between profit-driven individual actions and the behaviour of labour content. In contrast, Theorem 2 can be interpreted as identifying another (potential) failure of the invisible hand. The case $v_j = vA_j^* + l_j^*$ is the dividing line that separates strictly falling from strictly rising labour contents. This dividing line is expressed in terms of labour values and thus it is not visible to agents in the economy, who take their profit-maximising decisions based on price magnitudes. As a result, individually rational decisions may lead to socially suboptimal outcomes.

4. Productivity measures and the LDLC: empirical results

This section provides an empirical illustration of the main concepts and propositions discussed above. For this purpose the IO dataset constructed by Kalmbach *et al.* (2005) in their study of the German economy (1991–2000) is considered. This dataset provides one of the most detailed and rigorous IO time series available, including data on fixed capital and, crucially for the computation of labour values, capital depreciation matrices.

Kalmbach *et al.* (2005) group the 71 original sectors into seven macrosectors. They divide the industrial sector into agriculture, manufacturing and construction. Within manufacturing itself, they further distinguish more traditional industries from the so-called '*export core*' (a crucial subsector in an export-oriented country such as Germany), which comprises the four single production sectors with the highest exports: chemical, pharmaceutical, machinery and motor vehicle. They also distinguish between three main types of services: business-related services, consumer services and social services. For their aggregation, Kalmbach *et al.* (2005) adopt a broad definition of business-related services by including wholesale trade, communications, finance, leasing, computer and related services, and research and development services, in addition to business-related services *in a narrow sense*. Consumer services instead include retail trade, repair, transport, insurance, real-estate services and personal services. Table 4 summarises the seven (macro)sectors thus obtained and the sectoral output shares (in percentages, for the year 2000).[23]

[23] As noted by an anonymous referee, the empirical analysis in this section focuses on production sectors, rather than produced commodities, in contrast with Definition 1 above. Albeit theoretically relevant, this distinction is practically secondary given the high level of aggregation and the specific choice of the seven macrosectors by Kalmbach *et al.* (2005).

The technological coefficients of the seven-sectoral aggregation are reported in Table 5, which shows the intermediate IO matrix A of the German economy for the year 1995 per 10^6 euro of output value. The double-deflated coefficients $\tilde{a}_{ij}$ are used to characterise the entries of A. There are also (not shown) a depreciation matrix, A^δ, a fixed capital matrix, K, and a vector of labour coefficients, l.

In order to calculate the labour values of the seven sectors, the formula $v = l(I - A - A^\delta)^{-1}$ is used in each of the 10 years under consideration. The classical-Marxian measures, π_j^m, are then derived as the reciprocal of the entries of v. Instead, dividing each of the 70 real value-added items (per 10^6 euro output value) by the corresponding labour coefficient (per 10^6 euro output value), one obtains the conventional measures of labour productivity, π_j^c. The time series of the two productivity measures for six of the seven sectors are shown in Figure 2.[24]

The empirical evidence confirms the main conclusions of the paper. Concerning the measurement of labour productivity, the data shows that the two series π_j^m, π_j^c are very different, as expected from the analysis in Section 2. First of all, apart from the remarkable exception of sector 3, the *levels* of the two measures are sharply different in all sectors and in virtually every year of the sample, with no recognisable overall pattern (in some sectors the standard measures are higher than the IO indices, but the opposite happens in other sectors) and with differences even in the relative ranking of sectors in terms of their labour productivity. By Proposition 2 this is to be expected, given the wide sectoral differences in productivity. Second, even the *qualitative behaviour* of the

Table 4. *The seven-sectoral structure of the economy*

1	Agriculture	1.33
2	Manufacturing, the export core	12.37
3	Other manufacturing	22.55
4	Construction	6.29
5	Business-related services	21.36
6	Consumer services	23.35
7	Social services	12.75

Table 5. *Technological coefficients of the seven-sectoral aggregation (Germany, 1995)*

	1	2	3	4	5	6	7
1	0.030	0.000	0.047	0.000	0.000	0.002	0.002
2	0.081	0.241	0.050	0.021	0.003	0.008	0.014
3	0.159	0.226	0.338	0.286	0.030	0.060	0.065
4	0.010	0.005	0.009	0.020	0.007	0.034	0.020
5	0.137	0.107	0.126	0.088	0.291	0.118	0.080
6	0.032	0.044	0.045	0.100	0.071	0.139	0.044
7	0.034	0.008	0.013	0.007	0.009	0.014	0.025

[24] Social services are omitted because they are subject to processes that in general are not determined by profit-maximising firms. Details of the computations of the time series of the two indices are available from the authors upon request.

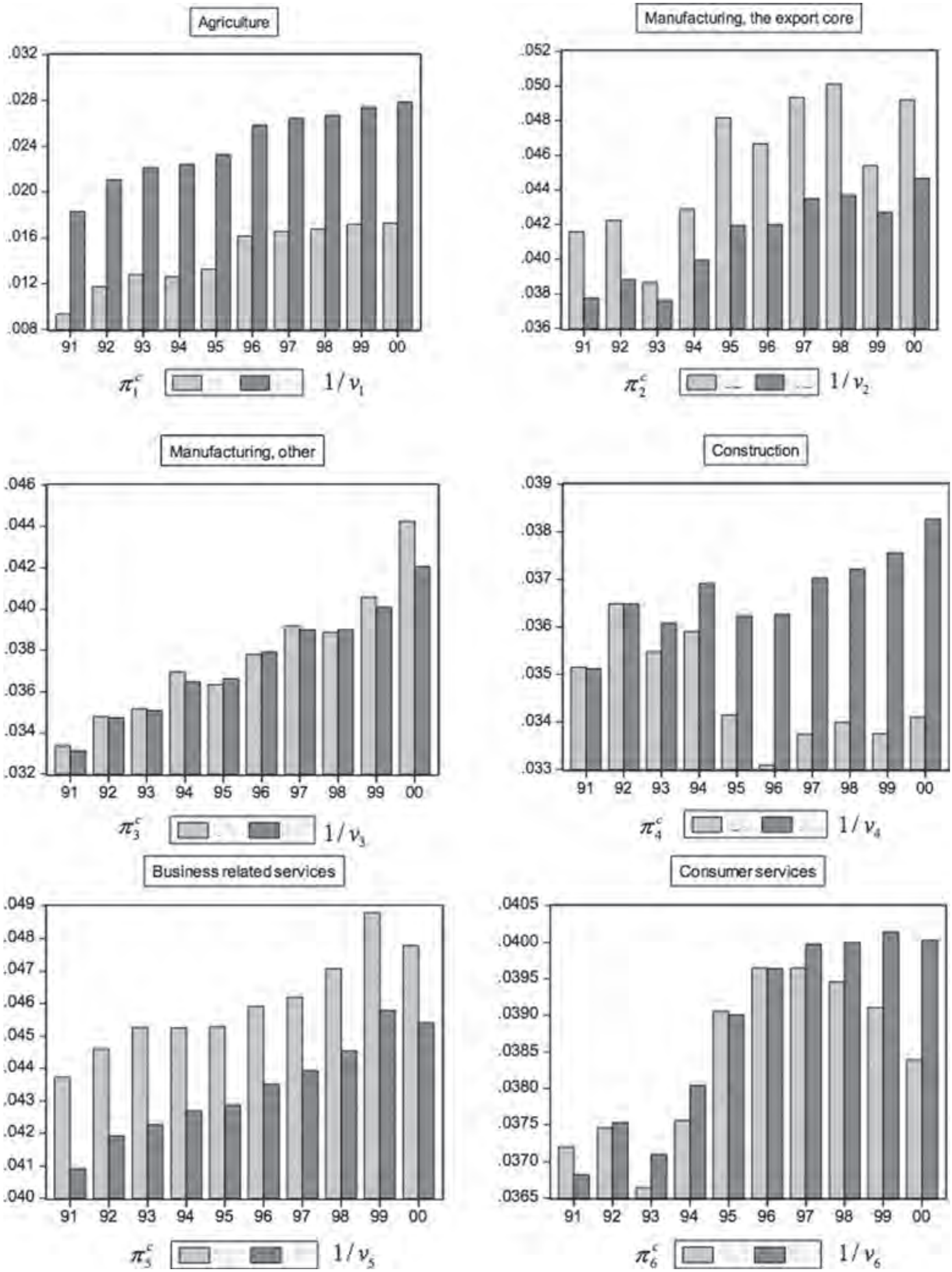

Fig. 2. *Comparing conventional and Marxian labour productivity indices:* π^c_j, $1/v_j$.

two indices over time is very different, as expected from Proposition 4. In sector 4, both the *trend* and the year-on-year behaviour of the two variables are markedly different. The Marxian measure of productivity has risen over time, while the conventional

SNA measure shows a sharp increase immediately after the German reunification, but a significant decline thereafter. Even setting aside the construction sector (where measurement problems may play a role), in various instances the two indices provide opposite verdicts concerning the *direction of change* of labour productivity over time. Particularly striking examples are sector 2: 1995–96 (and to a lesser extent 1997–98); sector 3: 1994–95 (and to a lesser extent 1997–98); sector 5: 1993–95; and last but not least, sector 6: 1997–2000, which is characterised by a similar, if less pronounced, overall pattern as sector 4.[25]

In summary, the *theoretical* differences between the two measures do give rise to significant *empirical* discrepancies. The standard SNA indices π_j^c lack theoretical foundations, as argued in Section 2 above, and they can also be very misleading in empirical analysis, as the evidence in Figure 2 forcefully shows.

Concerning the relation between prices, profits and labour values, all the tables in Figure 2 show that the LDLC holds for the German economy (1991–2000). The classical-Marxian indices of labour productivity show a clear upward trend in all sectors. This result seems robust and is consistent with the findings of previous studies (e.g. Gupta and Steedman, 1971; Wolff, 1985; de Juan and Febrero, 2000), although only few contributions explicitly focus on sectoral productivities.

5. Conclusions

This paper analyses the LDLC originally formulated by Farjoun and Machover (1983). First, the relevance of the LDLC is shown. It is argued that the IO indices based on the Marxian labour values are theoretically sound measures of labour productivity, whereas conventional indices based on real value added per worker are theoretically questionable and less reliable empirically. The notion of labour content is necessary to analyse labour productivity and the LDLC is central in order to understand the dynamics of capitalist economies.

Second, the dynamics of labour productivity in capitalist economies is analysed in a general linear model with fixed capital. It is proved that capitalists' maximising behaviour is neither necessary nor sufficient for the implementation of productivity-enhancing and welfare-improving innovations. Further, it is shown that the type of capital-using labour-saving profitable innovations that have characterised capitalist economies tend to lower labour values, which provides a deterministic foundation for the LDLC. Some empirical evidence is also provided, which shows that the LDLC holds in the German economy after the reunification.

The analysis in this paper can be extended in various directions. From the empirical viewpoint, the discussion in Section 4 is preliminary and only a first step towards a comprehensive investigation of alternative productivity measures. Further, a systematic econometric investigation of the theoretical relations between technical change and productivity explored in Section 3 would be interesting.

From the theoretical viewpoint, this paper can be interpreted as showing that profitable innovations that increase the Marxian technical composition of capital, raise labour productivity. This is a strong result in itself, but it immediately raises two questions. First, why does this sort of technical change occur? And, second, what are its

[25] It is worth noting in passing that sector 7, social services (not shown in Figure 2), has a similar pattern as sector 6.

implications in terms, for example, of class incomes and distribution? As noted in the Introduction, both questions lie beyond the scope of this paper, but they are important issues in political economy and suggest important avenues for further research. A framework that may be worth considering in order to analyse both questions, which is consistent with the approach adopted in this paper, has been recently proposed by Flaschel *et al.* (2012). In the latter paper, a system of prices of production with the usual properties is derived that allows for non-uniform profit rates and wage rates, provided intersectoral wage and profit differentials are assumed to be fixed. In particular, a generalised wage–profit curve can be derived (ibid., Theorem 2.2), which may allow one to extend Himmelweit's (1974) argument. Further, within this framework, it should be possible to prove a generalisation of the Okishio theorem, whereby any profitable technical changes lead to an increase in *all* of the sectoral profit rates, provided the real wage and the structure of profit and wage rate differentials remain constant. Yet, a thorough exploration of these issues must be left here for future research.

Appendix

Proof of Theorem 1

Part (i). Suppose first that $A_j^* \leqq A_j$, and thus $A^* \leqq A$. Since $l \geq l^*$, then by Assumption 1 it immediately follows that $v^* < v$. Suppose next that $A_{ij}^* > A_{ij}$, for some i. Consider the auxiliary matrix A^{*+} and define the vector of auxiliary labour values $v^{*+} = v^{*+}A^{*+} + l^*$. By Assumption 2, $p_w A_j + l_j > p_w A_j^{*+} + l_j^*$, or, equivalently, $p_w(A^{*+} - A) - (l - l^*) \leq 0$, with both terms in brackets being semi-positive by assumption. By Lemma 1, $0 < v < p_w$, and so the latter inequality implies:

$$v(A^{*+} - A) - (l - l^*) \leq 0,$$

and thus:

$$vA^{*+} + l^* \leq vA + l = v.$$

By recursive application of the latter inequality, we get:

$$v(t+1) = v(t)A^{*+} + l^* \leq v(t), \ t = 0,1,2,\ldots, \text{with } v(0) = v.$$

This sequence is bounded below and monotonically decreasing and thus converges to:

$$v(\infty)A^{*+} + l^* = v(\infty) = v^{*+}.$$

Therefore, by Assumption 1 it follows that $v^{*+} < v$, so that $\left(A_j^{*+}, l_j^*\right)$ is progressive with respect to $\left(A_j, l_j\right)$. Finally, note that by definition $A_j^{*+} \geqq A_j^*$, and therefore $v^* = v^*A^* + l^* \leqq v^{*+} = v^{*+}A^{*+} + l^*$, which implies $v > v^{*+} \geqq v^*$.

Part (ii). The desired result follows noting that there exist technical changes with $v^* < v$, such that $p_w A_j + l_j \leqq p_w A_j^* + l_j^*$ at the initial price vector $p_w > p_w A + l$, because the latter is not proportional to v in general, and noting that for KU-LS technical changes this implies $r_j p_w K_j + p_w A_j + l_j < r_j p_w K_j^* + p_w A_j^* + l_j^*$. QED.

Remark. The recursive argument used in Part (i) can be modified to provide an alternative demonstration of Proposition 8 in Roemer (1977).

Proof of Theorem 2

Part (i). If $p_w A_j + l_j \geqq p_w A_j^* + l_j^*$, then the desired result immediately follows from Definition 2(3). Therefore suppose $p_w A_j + l_j < p_w A_j^* + l_j^*$. Since technical change is progressive, then by Lemma 1, $p_w > v > v^*$. The latter inequalities imply that $p_w > p_w A^* + l^*$. Suppose, by way of contradiction, that $p_{wj} = r_j p_w K_j + p_w A_j + l_j < r_j p_w K_j^* + p_w A_j^* + l_j^*$. Since $p_{wj} > p_w A_j^* + l_j^*$, this implies that there is a $r_j' \in (0, r_j)$, such that $p_{wj} = r_j' \, p_w K_j^* + p_w A_j^* + l_j^*$ and since by assumption $p_w K_j > p_w K_j^*$, it follows that $r_j' \, p_w K_j + p_w A_j + l_j < r_j' \, p_w K_j^* + p_w A_j^* + l_j^* = p_{wj}$. The latter inequality implies that the KU-LS technical change $(K_j^*, A_j^*, l_j^*) \to (K_j, A_j, l_j)$ is profitable and therefore, since the premises of Theorem 1 are satisfied, it is progressive so that $v^* > v$, a contradiction. Therefore, we have $p_{wj} = r_j p_w K_j + p_w A_j + l_j \geqq r_j p_w K_j^* + p_w A_j^* + l_j^*$.

Part (ii). First of all, note that if KS-LU technical change $(K_j, A_j, l_j) \to (K_j^*, A_j^*, l_j^*)$ is profitable, this has no implication on the inequality $v_j \gtreqless v A_j^* + l_j^*$. Then, we prove that technical change is progressive if and only if $v_j > v A_j^* + l_j^*$.

First, note that $v_j > v A_j^* + l_j^*$ implies $vA^* + l^* \leq vA + l = v$, and therefore it is possible to construct an infinite sequence:

$$v(t+1) = v(t)A^* + l^* \leq v(t), \; t = 0,1,2,\ldots, \text{ with } v(0) = v,$$

which is monotonically decreasing, and bounded below, and thus converges to $v(\infty)A^* + l^* = v(\infty) = v^*$, $v^* > 0$. By Assumption 1 it follows that $v > v^*$.

Next, note that if $v_j = v A_j^* + l_j^*$, then $v = v^*$. Finally, suppose $v_j < v A_j^* + l_j^*$. Then $v \leq vA^* + l^*$ and we can consider the following monotonically increasing sequence:

$$v(t) \leq v(t)A^* + l^* = v(t+1), \; t = 0,1,2,3,\ldots, \text{ with } v(0) = v.$$

By Lemma 1, $v < p_w$ and by profitability it follows that $p_w A^* + l^* \leq p_w$. Therefore:

$$v(t) \leq v(t)A^* + l^* = v(t+1) < p_w A^* + l^* \leq p_w, \; t = 0,1,2,\ldots$$

Hence the sequence is bounded above by p_w, and so it converges to:

$$v(\infty) = v(\infty)A^* + l^* = v^*, \;\; v^* > 0.$$

By Assumption 1 $v < v^*$ must hold. QED.

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