A note on the formal treatment of exploitation in a model with heterogenous labor

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This note is devoted to presenting the formal generalization of Marx's analysis of value, exploitation, and prices to a capitalist commodity production system in which various categories of labor are distinguished according to their unequal capability to create exchange value. Wages are not necessarily proportional to the value productivity of workers. Thus, the various categories of concrete labor may not be exploited to the same degree.[1] In the determination of the rate of surplus-value, we use both the Price of Net Product-Unallocated Purchasing Power (PNP-UPP) interpretation of Marx's theory of value, often referred to as the "New Interpretation"[2], and the traditional framework. The problem is the same in the two frameworks. Only quantitative results differ since definitions are distinct, notably the definition of the rate of exploitation.

We do not address here the problem of measuring "abstract" (or "universal") labor in real capitalist commodity production systems, and take the relative value productivity of workers, from more simple to more complex categories of labor, as given in the derivation. In order to recover a measure of this value productivity from real-world price and wage data by sector, some additional assumption about relative rates of exploitation (which Marx often explicitly assumes to be equal) is required.

[1]See Dong-Min Rieu's paper in the present issue of the journal, "Estimating Sectoral Rates of Surplus-Value: Methodological Issues". Among earlier studies devoted to heterogenous labor, one can mention: Bowles S., Gintis H., 1977, "The Marxian Theory of Value and Heterogeneous Labour: A Critique and Reformulation", *Cambridge Journal of Economics*, Vol. 1(2), pp. 173-192. Morishima M., 1978, "S. Bowles and H. Gintis on the Marxian theory of value and heterogeneous labour", *Cambridge Journal of Economics*, Vol. 2, pp. 305-309. Steedman I, 1980, "Heterogeneous labour and 'classical' theory", *Metroeconomica*, Vol. 32(1), pp. 39-50. Krause U., 1981, "Heterogeneous Labour and the Fundamental Marxian Theorem", *The Review of Economic Studies*, Vol. 48(1), pp. 173-178. Cayatte J.L., 1984, "Travail simple et travail complexe chez Marx", *Revue Economique*, Vol. 35(2), pp. 221-245.

[2]Duménil G., 1980, *De la valeur aux prix de production*, Économica, Paris. Foley D., 1982, "Value of Money, the Value of Labor Power and the Marxian Transformation Problem", *Review of Radical Political Economics*, Vol. 14, pp. 37-47. Duménil G., Foley D., 2008, "The Marxian Transformation Problem", in Durlauf S.N., Blume L.B. (ed.), *The New Palgrave: A Dictionary of Economics*, The Macmillan Press, London, Basingstoke.

A preliminary issue is the definition of the unit of labor (section 2). While Marx refers to the reduction of "complex labor" to "simple labor", we adopt an alternative notion of an "average complexity" of labor.[3] There is no theoretical implication to this option, and the alternative formalism can be easily derived from that presented in the paper. The values and prices of individual commodities are, then, determined, given the relative value productivity of different types of concrete labor and their relative wages (section 3). Our approach includes the distinction between productive and unproductive labor, as unproductive labor is a category of labor that does not create value at all (section 4). In the two following sections, the framework is that of the PNP-UPP interpretation. Section 5 defines the monetary expressions of value and labor time. Section 6 determines the rates of exploitation of the various categories of labor, in the various industries, and in the total economy. Exploitation is also addressed in the traditional interpretation of transformation (section 7).

A last section is devoted to a reduced form of the model in which only one category of labor is considered in each industry.

(1) **Notation and basic aggregates.**

The basic framework is as follows: (1) n commodities are produced within n industries, with given single-production techniques; (2) m categories of labor exist. The subscript i and the superscript j respectively denote commodities and labors.

Technology is defined by the two matrices of material and labor inputs:

$$a = \begin{pmatrix} a_{11} & \dots & a_{1n} \\ \vdots & & \vdots \\ a_{n1} & \dots & a_{nn} \end{pmatrix} \quad \text{and} \quad l = \begin{pmatrix} l_1^1 & \dots & l_1^m \\ \vdots & & \vdots \\ l_n^1 & \dots & l_n^m \end{pmatrix}$$

Vectors l_i and l^j denote respectively the lines and columns of matrix l:

$$l_i = (l_i^1, \dots, l_i^m) \quad \text{and} \quad l^j = \begin{pmatrix} l_1^j \\ \vdots \\ l_n^j \end{pmatrix}$$

[3]In this definition, labor of average complexity is not chronologically and spacially invariant, a property it shares Marx's original notion of "simple average labor", which "varies in character in different countries and at different cultural epochs", although "in a particular society it is given" (Marx, K. *Capital, Vol. I* Vintage books, New York, 1876, p. 135).

Prices, wages, and outputs (levels of activity) are:

$$p = \begin{pmatrix} p_1 \\ \vdots \\ p_n \end{pmatrix}, \quad w = \begin{pmatrix} w^1 \\ \vdots \\ w^m \end{pmatrix}, \quad \text{and} \quad y = (y_1, \ldots, y_n)$$

The auxiliary notation u (a column vector of 1) and I (a matrix $n \times n$, whose diagonal is made of 1) are used:

$$u = \begin{pmatrix} 1 \\ \vdots \\ 1 \end{pmatrix} \quad \text{and} \quad I = \begin{pmatrix} 1 & & 0 \\ & \ddots & \\ 0 & & 1 \end{pmatrix}$$

With this notation, it is possible to determine employment and wages within each industry, for each category of labor, and within the total economy, as in table 1.

Table - Employment and wages

	Industry	Categories of labor	Total
Employment	$L_i = y_i l_i u$	$L^j = y\, l^j$	$L = y\, l u$
Wages	$W_i = y_i l_i w$	$W^j = y\, l^j w^j$	$W = y\, l w$

The average wage, $\overline{w}$, in the total economy and the ratio, ν^j, of the wage of each category of labor to the average wage are:

$$\overline{w} = \frac{W}{L} \quad \text{and} \quad \nu^j = \frac{w^j}{\overline{w}}, \quad \text{with} \quad \sum_{j=1}^{m} \frac{L^j}{L} \nu^j = 1$$

The values created by one hour of labor of category j and the individual value of commodities i are:

$$\mu = \begin{pmatrix} \mu^1 \\ \vdots \\ \mu^m \end{pmatrix} \quad \text{and} \quad \lambda = \begin{pmatrix} \lambda_1 \\ \vdots \\ \lambda_n \end{pmatrix}$$

(2) **The unit of value.**

Marx's framework of analysis suggests the choice of the category of labor that produces less value as "simple labor", and the expression of other categories of labor as quantities of this standard. Using the above notation this is equivalent to setting $\mu^1 = 1$, if the simplest category of labor corresponds to $j = 1$.

The alternative option used in this note is to refer to an *average* capability to create value in the economy (an "average complexity" instead of a minimum), writing that the total value created during the period is equal to the total number of hours L. Thus, the value created by labors of all categories is equal to the value of the net product, and their common value is L:

$$y\,l\mu = y(I-a)\lambda = L$$

With this normalization, the total value created during one year is set equal to the sum of productive labor, independently of the comparative capabilities of various categories of labor to create value.

The equation $y\,l\mu = L$ "normalizes" the set of coefficients μ^j. Since $L = y\,lu$ (table 1), one has: $y\,l\mu = y\,lu$. Some among the μ^j are larger than 1, others are smaller. The average of the coefficients μ^j, weighted by the shares, $\frac{y\,l^j}{y\,lu} = \frac{L^j}{L}$, of each category j of labor within total labor, is equal to 1: $\sum_j \frac{L^j}{L}\mu^j = 1$. The following line can be added to table 1:

	Industry	Categories of labor	Total
Value created	$\Lambda_i = y_i l_i \mu$	$\Lambda^j = L^j \mu^j$	$\Lambda = L$

(3) **Values and prices of production.**

The values of individual commodities can now be determined:

$$\lambda = a\lambda + l\mu \quad \text{or} \quad \lambda = (I-a)^{-1} l\mu$$

The following equation defines prices of production, with a uniform profit rate r:

$$p = (1+r)(ap + lw)$$

That is, since $w = \nu\overline{w}$:

$$p = (1+r)(I-(1+r)a)^{-1} l\nu\,\overline{w}$$

(4) **Productive and unproductive labor.**

In this framework, it is easy to distinguish between productive and unproductive labor. For any category, k, of unproductive labor (with a wage

w^k), one has: $\mu^k = 0$. Such categories of labor disappear in the equation of values above, but are considered in the calculation of prices of production.

(5) **The monetary expressions of value and labor time.**

In the framework of the *Price of Net Product-Unallocated Purchasing Power* (PNP-UPP) interpretation (known as the "new interpretation") of the so-called "transformation problem", a first principle establishes that the price of the net product (PNP) of the economy is the price form of the total value-creating labor expanded during the period. Thus, a coefficient m can be defined, as the Monetary Expression of Value (MEV), by:

$$m \underbrace{L}_{\text{Value created}} = \underbrace{y(I-a)p}_{\text{Price of the net product}}$$

It is an average for the total economy, attached to Marx's price form: the ratio between the price and the value of the net product.

Taking the expression "Monetary Expression of Labor Time" (MELT) at face value, one can also define, $\overline{m}^k$, the monetary expression of concrete labor time for each category of labor. MEV and MELT are related: $\overline{m}^k = \mu^k m$.

The two notions, the "Monetary Expression of (concrete) Labor Time" (MELT) and the "Monetary Expression of Value" (MEV), which can also be called the "Monetary Expression of *value-creating* (abstract) Labor Time" are distinct.

(6) **Rates of exploitation in the PNP-UPP interpretation.**

The second basic principle underlying the PNP-UPP interpretation is that exploitation is assessed by the fraction of the value of the net product that workers create, in comparison to the purchasing power over a fraction of the total net product embodied in their wages. Thus, exploitation does not depend on the specific allocation of this purchasing power (UPP) according to the needs or preferences of workers. (Workers receiving the same wage for the same value-creating labor are equally exploited.)

It is possible to determine rates of exploitation (rates of surplus-value), respectively, for each category of labor, within each industry, and for the total economy.

For each category of labor j:

$$e^j = \frac{m\mu^j - w^j}{w^j} \quad \text{or, with} \quad w^j = \nu^j \overline{w}, \quad e^j = \frac{m}{\overline{w}}\frac{\mu^j}{\nu^j} - 1$$

For industry i:

$$e_i = \frac{ml_i\mu - l_i w}{l_i w} \quad \text{or} \quad e_i = \frac{m}{\overline{w}}\frac{l_i\mu}{l_i\nu} - 1$$

For the total economy:

$$e = \frac{mL - W}{W} \quad \text{or} \quad e = \frac{m}{\overline{w}} - 1$$

The distinction between these three rates of exploitation follows from the assumption that wages are not necessarily proportional to the capacity of each category of labor to create value. If proportionality is assumed ($\mu = \nu$), as Marx typically does (a uniform rate of surplus-value), one recovers: $e^j = e_i = e$. In addition, the prices of production of individual commodities are equal to their values for $r = 0$.

(7) **Exploitation in the traditional interpretation.**

The same categories of issues can be addressed in the traditional interpretation, in which the vector of consumption goods bought by workers is considered, instead of an unallocated purchasing power. Thus, exploitation simultaneously depends on the capacity of each category of labor to create value, their wage and the composition of the bundle of goods they buy.

The consumption of each category of workers, j, is assumed to be proportional to a given bundle of commodities $d^j = (d_1^j, \ldots, d_n^j)$ that workers buy with the wage received for one hour of labor. One can define the rectangular matrix of consumptions:

$$d = \begin{pmatrix} d_1 \\ \vdots \\ d_m \end{pmatrix}$$

Exploitation can be defined for each category of labor j:

$$e'^j = \frac{\mu^j - d^j\lambda}{d^j\lambda}$$

For industry i:

$$e'_i = \frac{l_i\mu - l_i d\lambda}{l_i d\lambda}$$

For the total economy:

$$e' = \frac{L - y\,l d\lambda}{y\,l d\lambda}$$

In the particular case in which wages are proportional to the value created and all workers (in spite of the difference in purchasing power) consume goods according to the same proportions, $\overline{d}$ (with $d^j = \mu^j \overline{d}$), one recovers:

$$e'^j = e'_i = e' = \frac{1 - \overline{d}\lambda}{\overline{d}\lambda}$$

(8) **A reduced form of the model.**

Only one category of labor is considered in each industry.[4] The various categories of labors, as in the present paper, can be aggregated in each industry. This is formally equivalent to summing all labors in each industry to determine a vector $\tilde{l}$:

$$\tilde{l} = \begin{pmatrix} \tilde{l}_1 \\ \vdots \\ \tilde{l}_n \end{pmatrix} \quad \text{with} \quad \tilde{l} = lu$$

The average wage rate in industry i is:

$$w_i = \frac{W_i}{L_i} = \frac{l_i w}{l_i u}$$

The average value created by one hour of labor time is:

$$\mu_i = \frac{l_i \mu}{l_i u}$$

In this framework it is possible to define a MELT (at face value) in each industry: $\overline{m}_i = \mu_i m$, with μ_i defined as above.

[4] As in Dong-Min Rieu's paper.