Structural Change and Economic Dynamics
16 (2005) 503–521

On the origins of non-proportional economic dynamics: A note on Tugan-Baranowsky's traverse analysis

Giorgio Colacchio *

Department of Juridical Studies, University of Lecce, Complesso Ecotekne – Via per Monteroni, s.n. 73100, Lecce, Italy

Received June 2002; received in revised form September 2004; accepted October 2004
Available online 31 May 2005

Abstract

The article deals with some aspects of Tugan-Baranowsky's contribution. Section 1 presents a brief explanation of Tugan's disproportionality crisis theory. Section 2 gives a brief account of Tugan's business cycle theory, since, as the author maintained, the latter would be organically connected with his disproportionality theory. Section 3 is devoted to Tugan's analysis of an economic system in the presence of different intersectoral growth rates: unbalanced growth is assured by a traverse along which surplus value migrates to the sector which grows faster. In the concluding section, we maintain that Tugan-Baranowsky comes out as a pioneer in the field of non-proportional economic dynamics.

JEL classification: B31; O30; O41

Keywords: Business cycle; Disproportionality theory; Non-proportional dynamics; Traverse analysis

1. Introduction

This paper deals with some crucial aspects of Tugan-Baranowsky's contribution to the theory of capitalist development and to the explanation of economic crises. Section 1

* Tel.: +39 0832 320697; fax: +39 0832 321283.
E-mail address: giorgio.colacchio@ateneo.unile.it.

presents a brief explanation of Tugan's criticism of underconsumption theories, i.e., his "disproportionality crisis theory". Despite the fact that the analysis contained in this section reflects the standard Marxian reproduction schemes—with which many readers may be familiar—it is nevertheless useful to go through it in order to appreciate in full the core of Tugan's theoretical position. In fact, we would argue that this theory contains and implies at the same time a *proportionality* theory, since it focuses on the conditions which would have to be fulfilled in order to assure balanced growth of a multisectoral economic system. In this sense Tugan's emphasis on disproportionality should be regarded as the counterpart to his equilibrium analysis. The meaning of Tugan's position is quite clear: if precise conditions are satisfied, the economic system can expand ad infinitum, without any problem resulting from low consumer goods demand.

In Section 2, we present a brief account of Tugan's business cycle theory, since, as the author himself maintained, it should be considered as organically connected with his disproportionality theory. In particular, we will show that contrary to Tugan's claim, this linkage proves highly problematic.

Section 3 is devoted to one of the most interesting features of Tugan's work, which is the main topic of this article: the attempt—first made by the author through a numerical scheme in 1905—to describe the dynamics of an economic system in the presence of different intersectoral growth rates engendered by the assumption of an *exogenous* negative growth rate of wage. We argue that this attempt represents one of the first numerical formulations of non-proportional economic dynamics to be found in the history of economic analysis: the unbalanced growth is assured by a traverse along which the excesses of surplus value migrate to the sector which must grow at a higher rate in order to assure unbalanced growth. In the above-mentioned section Tugan's numerical exposition is "translated" into a more general analytical form, since we are interested in a rigorous verification of Tugan's statements.

In the concluding section (Section 4), the importance of Tugan-Baranowsky's multisectoral approach is particularly stressed, since only such an approach could have led the author to move from steady state analysis towards non-proportional economic dynamics. We argue that Tugan's traverse analysis too should be regarded as the inevitable consequence of his thorough investigation of equilibrium conditions in non-aggregate models. At the end of this study Tugan-Baranowsky comes out as a pioneer in the field of economic dynamics, deserving as such a much greater consideration.

2. (Dis)proportionality theory

The figure of Tugan-Baranowsky[1] is inevitably associated with his criticism of underconsumption theories which were particularly fashionable between the end of 19th and the beginning of the 20th centuries, especially among Marxist authors. According to these theories, growing accumulation and productive improvements—together with an increas-

[1] For a passionate reconstruction of Tugan-Baranowsky's work and personality, see Kondratiev (1998); for his bibliography and biography see Crisp (1968) and Kowal (1965). An exhaustive bibliography of Tugan's works, and of the works of various authors on his contributions, is given in Amato (1980). We advise the reader that Tugan's name has been transliterated in various ways in the literature: in the references list we have preserved the original spelling.

ing capital/labour ratio and low wage levels—were the root causes of the tendency for the supply of consumer goods to exceed demand, thus explaining the recurring economic crises of capitalist economies.

As Tugan-Baranowsky (1913, pp. 190–191; see also 2002, pp. 3–4) pointed out, in his work on *Industrial Crises in Contemporary England*:[2] "In capitalist economic conditions, the difficulty is not to produce, but to sell, to find outlets. [. . .] Thus the market is the main force regulating the entire capitalist economy, and its inadequacy—that makes itself constantly felt in capitalistic production—is an elastic band hindering capitalist development". So the author asked himself: "But where does this inadequacy of outlets, this difficulty in selling—whereby capitalistic production, constantly higher than demand, always pushes the market—come from? This is exactly what the problem of markets is, an important and difficult problem, and one for which economic science has long been searching for an answer, but in vain" (ibid., pp. 190–191). With his "solution to the problem of markets" Tugan-Baranowsky elaborated a radical critique of underconsumption theories, a critique that became known as the *disproportionality crisis theory*. In short, this theory states that if adequate proportionality among different production branches—i.e. the correct sharing of social labour among productive sectors—is achieved, then the economic system can expand ad infinitum without any problem deriving from the scarcity of effective demand. This would be true *however low social consumption was and even in the presence of its absolute reduction*: "if social production were organised in a planned fashion, [. . .] then no matter how low social consumption may be, the supply of goods could never exceed demand" (Tugan-Baranovsky, 2002, p. 26). Since the absence of economic organisation and planning is a typical feature of capitalism—the choices of capitalists being rather discretionary and the pricing system operating only an *ex-post* form of regulation—exact proportionality is difficult to achieve, thus somewhat unlikely: its absence subsequently leads to economic crises, the composition of aggregate demand being different from that of supply.[3]

To demonstrate that disproportionalities were the only cause of economic crises—consumption level thus playing no significant role in the explanation of capitalist economic dynamics—Tugan-Baranowsky used the Marxian expanded schemes of reproduction (adding a third sector producing *luxury goods* for capitalists) contained in volume

[2] M.I. Tugan-Baranowsky, Promišlennye krizisy v sovremennoj Angli, ich pričiny I vlijanie na narodnyju žizu' (Industrial Crises in Contemporary England: their causes and their influence on National Life), Spb., I. Schorochodov, 1894. A German translation of this work (which was published in four successive Russian editions) was published in 1901 while a French translation (based on the second Russian edition), revised and expanded by the author himself, was published in 1913. For the reader's convenience in the present paper, whenever possible, we will refer to the abridged English translation of the 1901 German edition (Tugan-Baranovsky, 2002); otherwise we will refer to the French translation (the English translations of non-English works are mine).

[3] As Tugan (1913, pp. 221–222; see also 2002, p. 26) wrote: "Capitalism possesses no organisation capable of guaranteeing this proportionality. Industrial crises derive from this [. . .] the absence of any organisation designed to permit the proportional sharing-out of production in the capitalist economy has the effect of an elastic band which constantly squeezes production, preventing it from developing all its strengths [. . .] capitalism makes every effort to find an approximation of this proportionality; it gets there through crises and the suppression of firms that have grown excessively" . See also Tugan-Baranovsky (1970, pp. 292–293), and in particular: "True, price is the regulator of capitalist production and in the end establishes a certain rough proportionality in the capitalist economy. But the regulator is a *very imperfect one*, and the restablishment of proportionality is often attained by curtailing production" (ibid., p. 293, italics added).

II of *Capital*.[4] Let us now illustrate the functioning of these schemes, in order to explain the grounding of Tugan's theoretical position, starting from the analysis of expanded reproduction equilibrium conditions in a two-sector model.[5]

The general definitions and assumptions common to both this model and the next one in Section 3 (with the exception of assumptions (b) and (f) that will be relaxed in discussing Tugan's numerical formulation), are as follows:

(a) All magnitudes are expressed in labour hours (labour theory of value) except for l_i and C_i^* (see below).
(b) The economic system consists of two sectors (or departments) producing, respectively, an homogeneous means of production (sector I) and an homogeneous consumption good (sector II). Both sectors employ capital and labour. There are no constraints on the labour market, i.e., labour supply is perfectly elastic at the given wage rate. The length of a productive cycle is equal to "*one year*".
(c) The total value of gross production in each department, called W_i ($i = 1, 2$), subdivides into three parts: C_i = "constant capital", i.e. the total value of fixed and circulating capital involved and used up in the ith department; V_i = "variable capital", i.e. the total value of the wages of workers employed in the ith department; S_i = "surplus value", i.e. the total value produced by the workers in the ith department (numbers of hours worked) minus their wage bills (V_i). Note that all these variables are flows.
(d) C_i^* = total amount of physical capital expressed as the number of units involved and used up in the ith department; l_i = labour force ("labour power", in Marx's words), expressed in units, employed in the ith department; w_i = unit wage expressed in labour hours per year; ρ = capital good's unit value expressed in labour hours; L = yearly total of hours worked by each worker ("length of working year"), assumed to be constant and equal for each sector. Then from (c) it follows that: $C_i = \rho C_i^*$; $V_i = w_i l_i$; $S_i = (L - w_i) l_i$.
(e) The rates of turnover of capital are identical in the two departments and are equal to the length of a productive cycle ("*one year*"). Consequently this assumption implies that, in reality, there is no fixed but only circulating capital.
(f) Workers consume all their earnings while capitalists both consume and save. Savings in each department are fully invested in the same department, according to specified aggregate investment functions.
(g) By the usual definitions (where $i = 1, 2$): $S_i(t)/V_i(t) \equiv \sigma_i$ = "rate of surplus value"; $K_i \equiv C_i(t)/V_i(t)$ = "value composition of capital"; $C_i^*/l_i \equiv Q_i$ = "technical composition of capital"; $p \equiv \sigma_i/(1 + K_i)$ = profit rate.[6]

[4] See Marx (1992, pp. 468–599).

[5] Tugan-Baranowsky's demonstrations (as Marx's) were based on numerical formulations; we will translate them into a more general analytical form for rigorous verification. The elimination of the third sector, as will be clear later, is only designed to simplify calculations and involves no qualitative modifications.

[6] As will be evident in the following, the distinction between technical and value composition of capital is important for the analysis of Tugan's model that we present in Section 3. The technical composition of capital is determined by the ratio between physical capital and labour force, both of them expressed in number of units, while the value composition of capital is nothing but the same ratio expressed in value (see Marx (1990, p. 762)). Clearly, the former ratio may be unambiguously defined only in a one-capital good economy. For an interesting discussion on this matter, see Steedman (1977, pp. 132–136).

The following list of equations and assumptions ($i=1, 2$) describes the framework of the economy at time ("year") t:

$$W_i(t) = C_i(t) + V_i(t) + S_i(t) \tag{1}$$

gross production value produced in the ith department.

$$\frac{S_i(t)}{V_i(t)} = \sigma_i, \quad \sigma_i = \text{constant.} \tag{2}$$

$$K_i = \frac{C_i(t)}{V_i(t)}, \quad K_i = \text{constant.} \tag{3}$$

$$I_i(t) = \alpha_i S_i(t) = C_i(t+1) - C_i(t) + V_i(t+1) - V_i(t) \tag{4}$$

capitalists' investment function ($0<\alpha_i<1$).

Writing Eq. (4) in terms of V_i gives us a first order difference equation whose solution then gives the (monotonically increasing) dynamic behaviour of V_i:

$$V_i(t) = V_i(0)\left[1 + \frac{\alpha_i\sigma_i}{1+K_i}\right]^t. \tag{5}$$

Writing Eq. (1) in terms of V_i and substituting the explicit solution given by Eq. (5) gives:

$$W_i(t) = (K_i + \sigma_i + 1)V_i(0)\left[1 + \frac{\alpha_i\sigma_i}{1+K_i}\right]^t. \tag{6}$$

Eq. (6) describes the dynamic behaviour (monotonically increasing) of W_i. Note that if $\alpha_i=1$, the growth rate of W_i is equal to the profit rate ("*golden age* equilibrium path").

Expanded reproduction equilibrium conditions require that demand for both goods equals supply; then they are given by:

$$\begin{aligned} W_1(t) &= C_1(t+1) + C_2(t+1) \\ W_2(t) &= V_1(t+1) + V_2(t+1) + (1-\alpha_1)S_1(t) + (1-\alpha_2)S_2(t) \end{aligned} \tag{7}$$

Just one of these conditions needs to be considered. Rewriting the first in terms of V_i and defining: $(\alpha_i\sigma_i/1+K_i)\equiv A_i$, gives:

$$V_1(0)\left[\frac{1+K_1+\sigma_1}{1+A_1} - K_1\right](1+A_1)^{t+1} = V_2(0)(1+A_2)^{t+1}K_2. \tag{8}$$

Eq. (8) represents the expanded reproduction equilibrium conditions; it will be satisfied ($\forall t$) if:

$$\begin{aligned} A_1 &= A_2 \\ \frac{V_1(0)}{V_2(0)} &= \frac{K_2(1+A_1)}{1+\sigma_1 - K_1A_1} \end{aligned} \tag{9}$$

If requirements in Eq. (9) are satisfied, then the economic system can expand in *steady state* at the growth rate of $A_1=A_2$.

This initial formulation contains the core of disproportionality crisis theory. The equations in (9) provide sufficient conditions for guaranteed balanced growth at a constant rate, with no problem resulting from the inadequacy of demand for consumer goods: if these conditions are satisfied[7] then the flow of consumer goods—produced within a productive cycle—will be entirely sold. However, there is very little likelihood of these precise requirements being satisfied, since, as Tugan (1913, p. 221) wrote, "capitalism possesses no organisation capable of guaranteeing this proportionality. Industrial crises derive from this", i.e., industrial crises derive from the "*anarchy*" and absence of planning which characterise this mode of production.[8] Disproportionality then represents the *only* explanation, within this expanded reproduction framework, for (recurring) general gluts affecting the capitalist economy.

3. Disproportionality crises and the theory of the business cycle

It is necessary to give a brief account[9] of the *core* of Tugan-Baranowsky's business cycle theory—even though this is not the main subject of this article—since, as the author himself maintained, it should be considered as *organically connected* with his disproportionality theory; the latter, in fact, represents the *logical basis* of the former.[10] Particularly, in this section we will show how, contrary to Tugan's claim, the link between his market theory and his cycle theory proves highly problematic.

In the theoretical explanation of the business cycle, presented by Tugan-Baranovsky (2002, pp. 29–44) in the *Industrial Crises in Contemporary England*, a crucial role was played by what the author defined as *free loanable capital* (or, simply, *free capital*). According to Tugan the accumulation of this capital occurs at almost a constant rate, since it originates from the flow of savings of those social groups whose income—in particular rents—is not particularly affected by the economic cycle. On the other hand, the conversion of those savings into fixed capital (i.e. investment) is essentially discontinuous: as we will see, this different dynamic behaviour of supply and demand of loanable funds is the basic cause of economic fluctuations.

Begin with a contraction phase. During recession and stagnation the drop in investments leads free capital, which as we said above accumulates with no rest, to lie idle in banks in

7 Considering the "classic" case of the uniform profit rate, the first equation states that $\alpha_1 = \alpha_2$ while the second imposes restrictions on initial values. As Morishima (1973, pp. 122 ff.) has demonstrated, system instability grows considerably if one supposes a "more reasonable" hypothesis for investment functions, that is letting capitalists invest in both sectors and not only in their own.

8 See above, footnote 3, for a longer Tugan's quotation on this matter.

9 For a more detailed analysis of Tugan's business cycle theory see Barnett (2001), Besomi (2006), Colacchio (1998), Hagemann (1999).

10 See Tugan-Baranowsky (1913, p. 277). It is interesting to point out that in the 1901 German edition of *Industrial Crises* there was no explicit reference to this "organic connection". Tugan felt it necessary to stress it—in the French edition—after having realised, as he himself complained, that while his crises theory had been accepted by many scholars, his theory of markets found only a few supporters. Instead, the former "is organically connected to the theory of markets [. . .] both theories stand and fall together" (Tugan-Baranowsky, 1913, p. 277). Note that when Tugan makes use of the words "crises theory" he actually refers to recurrent crises, i.e. to business cycles theory.

search for a productive use, and this excess supply will cause a continuous fall in the rate of interest.[11] This situation is highly unstable, since "the more such non-functioning capital there is, the stronger must be the drive towards productive investment of the free capital" (Tugan-Baranovsky, 2002, p. 37). Under this pressure, and thanks to the lower interest rates and to the re-establishing of positive profit expectations, the dam will eventually burst and this enormous stock of liquidity will flow to all productive sectors, particularly to those producing capital goods: the economic system is thrown into the expansionary phase of the cycle. The main feature of this phase is represented by an accelerating rate of growth of investments, and then by an accelerating rate of growth of free capital demand which, sooner or later, will become greater than that of supply. The consequent increase in the rate of interest is a testimony to the progressive exhaustion of free capital whose inevitable shortage will manifest itself at first in a fall in share prices—leading to stock market crash[12]—and then in the impossibility of pushing new free capital into fixed capital. This interruption of investments, in a chain reaction, will spread over all departments leading to a fall in the aggregate demand and consequently to general overproduction: it is the downturn and the expansion turns into a recession, which will set the conditions for a new recovery phase.

After having set out the essence of Tugan's business cycle theory, we can go back to our original question: is the cycle depicted above explained by disproportionality crises theory, i.e., are "disproportions" the main cause of recurrent crises? According to Tugan this was the case, since it is the partial overproduction in the departments producing capital goods—given the drop in investments due to the exhaustion of free capital—that sets off the crisis, and this would be nothing but an example of disproportionality crisis: "[. . .] since the producers of the means of production cannot withdraw their capital from their business, and since, furthermore, the size of the capital invested in the form of machines, buildings, etc., requires that production must continue [. . .], overproduction of the means of production ensues. Due to the interdependence of all branches of production, the partial overproduction becomes general overproduction. The prices of all goods fall, and a general business slowdown occurs" (Tugan-Baranovsky, 2002, pp. 41–42). Nevertheless, as we anticipated at the beginning of this section, this reasoning is not convincing, since here Tugan is confusing the *cause* of economic crisis—the exhaustion of free capital—with its *effect*—the disproportionality among the various branches of production. One could argue that, in any case, the ultimate cause of economic crises however lies in the *disproportionality* between supply and demand for loanable capital; but this reasoning seems to run in a circle, since it is just this disproportionality which needs to be explained. Anyway, in a following passage Tugan-Baranovsky (2002, p. 42) gives an explanation of crises which seems to leave out the role of free capital: "But even apart from the effect on the demand for goods of the fall in the number of new businesses being established, social production becomes increasingly disproportional as a result of the upswing, due to the uneven growth of various branches of production. [. . .] The strongest expansion occurs in those industries which constitute the best object of stock market speculation. In this way, by the end of the ascending phase of the industrial cycle the composition of social production lacks all proportionality,

[11] On the interest rate as a monetary variable determined by the relationship between supply and demand of free monetary capital, see also Tugan-Baranovsky (1970, p. 280).

[12] On the distinction between free capital in banks and capital in the stock market see Barnett (2001, pp. 454–455).

which can only be restored by the destruction of some of the capital of those branches of production which have grown excessively". But this is only apparently a disproportionality-based theory of the business cycle, since "disproportions" are again an *effect* of other causes, specifically of speculative stock market manoeuvres. In this case, then, the explanation of economic crises requires one to abandon the field of disproportionality to move towards the analysis of (speculative) money demand. Furthermore it is easy to realise that the stock market manoeuvres can take place—role of credit apart[13]—only thanks to the previous accumulation of free capital, and then we are taken back to our initial considerations. We conclude by saying that the linkage between the theory of disproportionality and that of the business cycle proves to be problematic:[14] if the former may explain the final breakdown of the economic system, the latter requires further investigation—especially with respect to the determinants of investment and money demand—and these unexplained elements can be seen as the legacy of Tugan for the next generation of economists.[15] Besomi (2006, pp. 17–20), on the contrary, has argued that the above mentioned linkage between the theory of markets and that of the business cycle is present, if one evaluates Tugan's statements in the light of the Marxian difference between the *possibility* and the *necessity* of crises:[16] according to this view, Tugan's "theory of markets configures the equilibrium conditions and, by implication, establishes the *possibility* of crises. His theory of credit flows explains how this concrete possibility becomes actual and takes a periodical form" (Besomi, 2006, p. 17). One can agree with this view, as long as it means that disproportionality is a *necessary* condition of economic crises but that it is not sufficient, by itself, to explain the business cycle: only in this sense one can accept Tugan's claim that his theory of markets represents the logical basis of his business cycles theory.

4. Non-proportional economic dynamics

Tugan-Baranowsky's point—as mentioned before[17]—was not only to prove that "disproportions" were the *only* cause for economic crisis, but also to point out *the irrelevance* of

[13] According to Tugan, credit amplifies economic fluctuations—especially in the ascending phase of the cycle—being an almost independent purchasing power: "The capitalist economy creates a new medium of circulation—credit. Credit does not eliminate the dependency of the prices of goods on supply, but in the credit economy this dependency becomes extremely complex. [. . .] In the credit economy, the market's purchasing power is a complex, elastic, immaterial, but at the same time fragile, structure resting on a real money base; the market's purchasing power can rise or fall in line with the greater or lesser inclination of buyers to make use of credit, without any change whatever in the real conditions of supply of goods and money" (Tugan-Baranovsky, 2002, pp. 23–24). See also Tugan-Baranovsky (1987, p. 113). For further considerations on Tugan's monetary theory, see Hagemann (1999, pp. 92–93) and Koropeckyj (1991, pp. 63–67).

[14] For a similar statement, see Barnett (2001, p. 455): "In one sense there is a contradiction between Tugan's empirical account of the progress of crises and his theoretical explanation(s) of them. The latter involved quasi-Marxist notions such as "disproportionality" and "maldistribution" that were not fully integrated into empirical description of actual crises, which did employ the concept of "free loanable capital".

[15] It is interesting to recall Keynes' "sympathy" for Tugan's business cycle theory (Keynes, 1930, II, pp. 100–101). For an exhaustive account of the influence of *The Industrial Crises* on later economists, see Barnett (2001, pp. 458–464) and Reijnders (1998, pp. 226–234).

[16] See Marx (1969, pp. 513–517).

[17] See above, Section 1.

consumption level to the expanded reproduction process. The demonstration of this proposition, offered in *Industrial Crises in Contemporary England*, was based on the comparison between a "simple reproduction" scheme[18] and an expanded one. Total production value being the same in both schemes, in the second scheme (expanded reproduction) consumption level is reduced by the share of surplus value invested in the means of production, and is thus lower (compared to the simple reproduction case) over a few periods ("years"). So Tugan-Baranovsky (2002, p. 20) could maintain that "comparison of the simple reproduction of social capital with its reproduction on an expanded scale enables us to draw the highly important conclusion that in the capitalist economy, demand for goods is, in a certain sense, independent of the total volume of social consumption: the total volume of social consumption can fall at the same time as total social demand for goods is rising, absurd as this may seem from the standpoint of "common sense". [. . .] Thus total social production of goods in schema no. II [. . .] is considerably greater than that in schema no. I, but production of consumption goods is less, without this disturbing the equilibrium between supply and demand".

On the basis of this analysis Tugan-Baranowsky went further and claimed that accumulation could also proceed—without gluts—*in the presence of zero, or even negative, growth rate in supply of consumer goods*, provided that certain precise conditions were satisfied, i.e., provided that the absolute reduction in consumer goods were counterbalanced by increasing investment.[19]

The above explanation was rightly criticised by Kautsky (1901, p. 116): "Tugan's schemes furnish just one case inside where reduction in consumption does not lead to crisis: in the transition from simple to expanded reproduction. This one case in Tugan is raised to the status of a characteristic of capitalism; moreover, this is a case that never actually arises in such a system".[20]

To answer this, it would have to be shown how social consumption could diminish (either absolutely or relatively) and—at the same time—the economic system expand with no gluts or crises, *starting from an expanded reproduction scheme*. Tugan-Baranowsky set himself this task in his succeeding work, *Theoretical Foundations of Marxism*,[21] where—in

[18] One obtains the simple reproduction case by putting $\alpha_i = 0$ ($i = 1, 2$) in the previous model in Section 1.

[19] According to Tugan-Baranovsky (2002, pp. 21–22) this is what really happens in the dynamics of the capitalist economy where "technical progress expresses itself in a constant increase in the importance of the means of production, the machine, relative to living labour, the worker himself. [. . .] Workers take second place relative to machines, and at the same time, demand arising from workers' consumption takes second place relative to demand arising from the productive consumption of the means of production. All the interlocking gears of the capitalist economy assume the character of a mechanism existing, as it were, in its own right, in which human consumption appears as merely one element in the production of the reproduction and circulation of capital". Mainwaring (1995), following Tugan's suggestions, has analysed the dynamical proprieties of a two-sector model characterised by constant capitalists' consumption (and constant technical coefficients). In this model capitalists' desire for accumulation in itself—i.e., their being engaged in the production of machines for the sake of producing machines—is regarded as the birth (and the expansion) of a bubble (a "T-bubble", in Mainwaring's words) which will eventually burst, leading to a crash and then to economic crisis. The reader is referred to Mainwaring's article for a fuller account of the model (especially for the explanation of how repeated bubbles may generate quasi-periodic crises).

[20] R. Luxemburg too emphasised the importance of Kautsky's criticism (see Luxemburg, 1951, pp. 318–320).

[21] Tugan (1905). Note that Kautsky's criticism was addressed to the first German translation of *Promišlennye krizisy v sovremennoj Anglii*, published, as we said (see above, footnote 1), in 1901, and thus before *Theoretishe Grundlagen*.

reply to Kautsky's criticism—he developed an expanded reproduction scheme under the hypothesis that real wages decrease at a constant rate, with capitalists' consumption taken to be constant. We now report Tugan's (1905, pp. 224-227) numerical exposition,[22] bearing in mind that the Russian economist worked on three-sector reproduction schemes, the first sector producing capital goods, the second one consumer goods for workers (wage goods) and the third one consumer goods for capitalists ("luxuries").

First year	Second year	Third year
I 1632**C** + 544**V** + 544**S** = 2720	I 1987.4**C** + 496.8**V** + 828.1**S** = 3312.3	I 2585.4**C** + 484.6**V** + 1239**S** = 4309
II 408**C** + 136**V** + 136**S** = 680	II 372.6**C** + 93.2**V** + 155.2**S** = 621	II 366.9**C** + 68.9**V** + 175.5**S** = 611.3
III 360**C** + 120**V** + 120**S** = 600	III 360**C** + 90**V** + 150**S** = 600	III 360**C** + 67.5**V** + 172.5**S** = 600

Tugan used these schemes to demonstrate how "the expansion of social production keeps step with the reduction in social consumption; the demand for and supply of goods, however, remain in perfect equilibrium" (Tugan-Baranowsky, 1905, p. 226). A few years later he summarised the main conclusions of his analysis as follows: "In my own Theoretical Foundations of Marxism, I have presented a scheme of capital accumulation under the assumption of an absolute reduction in social consumption. There is no production excess, for the simple reason that the reduction in *demand for consumer goods has been balanced by an increase in demand for means of production*. One could ask how the means of production are to be employed, if demand for consumer goods is falling. The answer is easy. The means of production will at this stage be increasingly employed for the production of new means of production. All workmen, bar one, will be replaced by machinery. This one remaining worker will then have the job of putting this enormous mass of machines into motion, for the production of new machines and consumer goods for the capitalist class. [. . .] It is still possible that capitalists, driven by their passion for accumulation, decide to reduce their consumption. [. . .] In this case production will be entirely geared towards capital accumulation. [. . .] It is obvious that in truth I do not mean this arbitrary hypothesis whereby the replacement of workmen with machinery would lead to the virtual abolition of workers as such (I have used this hypothesis *so as to explain how my theory remains valid when taken to the very extreme*), but rather the hypothesis suggesting that with the proportional re-partition of social production, no reduction in social consumption will lead to the formation of a production excess" (Tugan-Baranowsky, 1913, pp. 216–217, italics added).[23]

It may be useful to underline the most important features of Tugan's scheme:

(a) Real wages decrease uniformly at a rate of 25% ("per year") in each sector.
(b) Capitalists' total consumption is constant, being equal to 75% of starting year surplus value: so the third sector growth rate is equal to zero.

[22] In the following we use our symbols that differs from the Tugan's ones.

[23] Here Tugan (from "*All workmen* . . ." to the end) was quoting in its entirety a very famous passage from his book published in 1905 (see Tugan, 1905, pp. 230–231).

(c) The growth rate of *value composition* of capital reflects almost exclusively the fall in real wages. So the *technical composition* of capital is implicitly assumed to be constant and takes the same value in all departments.[24]

Given the above assumptions, sector I exhibits an accelerating growth rate. In Tugan's opinion this is not only due to the increasing rate of surplus value within this sector (which at the same time implies a continuous increase in the share of surplus value invested there), *but also to the transfer*—in order to guarantee smooth expanded reproduction—*of surplus value excesses spilling out from sectors II and III to the first sector*: "Falling wages and stationary capitalists' consumption led to a reduction in invested capital in the last two production sectors and to its transfer to the first sector, which experienced significant expansion" (Tugan-Baranowsky, 1905, p. 226).

It may be interesting to carry out a deeper analysis of this tableau in order to discover the algorithm that Tugan-Baranowsky used in constructing the tableau itself and, more generally, in order to rigorously verify Tugan's statements. Symbols have the same meaning of previous sections, even if now we work, consistent with Tugan's exposition, on a tri-sectoral model (remember that the third sector produces consumer goods for capitalists). In particular, we will assume that $\rho_i = 1$, that the technical composition of capital is constant and uniform for all sectors, i.e. $C_i/l_i = Q^* =$ constant, $i = 1, 2, 3$, and that workers saving is zero.

The general formulation of (equilibrium) expanded reproduction conditions for Tugan's scheme is:

$$\begin{aligned} W_1(t) &= C_1(t) + V_1(t) + S_1(t) = C_1(t+1) + C_2(t+1) + \bar{C}_3 \\ W_2(t) &= C_2(t) + V_2(t) + S_2(t) = V_1(t+1) + V_2(t+1) + V_3(t+1) \\ W_3(t) &= \bar{C}_3(t) + V_3(t) + S_3(t) = \bar{S}_1 + \bar{S}_2 + \bar{S}_3 \end{aligned} \quad (1')$$

while the *aggregate* investment function is given by:

$$\sum_{i=1}^{3}[S_i(t) - \bar{S}_i] = \sum_{i=1}^{3}[\Delta C_i(t) + \Delta V_i(t)] \quad (2')$$

where $\bar{S}_i < S_i$ is the constant amount of surplus value consumed by capitalist of ith sector, and where we have used the usual notation: $\Delta x(t) \equiv x(t+1) - x(t)$. Note furthermore that in Eq. (1′), we have already incorporated the hypothesis that $C_3 (= \bar{C}_3)$ and W_3 are constants, as emerges from Tugan's numerical tableau.

Two particular features of this model can already be noted.

Firstly, sector III plays no relevant role in expanded reproduction: in fact since *by hypothesis* W_3 and C_3 are constants, the third Eq. in (1′) is independent from the other two and will be satisfied if $l_3(t) = l^* =$ constant, $\forall t$, where l_3, as before, is the third sector labour

[24] This assertion can be easily demonstrated. Taking $\rho = 1$, the value composition of capital can be expressed as: $K_i = C_i^*/l_i w_i$, where $C_i^*/l_i \equiv Q_i$ is the technical composition of capital. If Q_i is constant and $g(w_i) = -25\%$, (where $g(x) \equiv [x(t+1) - x(t)]/x(t)$ we get: $g(K_i) = [g(Q_i) - g(w_i)]/[1 + g(w_i)] = 25\%/75\% = 33.3\%$, which, except for rounding off, is the same value as in Tugan's tableau.

force and where[25]

$$l^* = \left(\sum_{i=1}^{3} \bar{S}_i - \bar{C}_3\right)\frac{1}{L}. \tag{3'}$$

Secondly, Eq. (2′) represents the aggregate equilibrium condition: saving = investment; it may be discarded being implied by Eq. (1′).

Following Tugan-Baranowsky we assume a uniform wage w which decreases at the constant rate of $1-\gamma$, $0<\gamma<1$:

$$w(t) = w(0)\gamma^t \quad (\gamma = 0.75 \text{ in Tugan's tableau}) \tag{4'}$$

From Eq. (1′) after some manipulations we obtain the following equilibrium condition:

$$\frac{l_1(t)}{Q^*} = \frac{l_2(t)}{w(t+1)} \tag{5'}$$

which may be rewritten as follows:

$$\gamma V_1(t) = C_2(t) \tag{6'}$$

Eq. (6′), clearly, imposes a fundamental restriction on initial values,[26] implying that $\gamma V_1(0) = C_2(0)$.

Note that in the starting year (the "first year") Tugan freely assumes:[27]

$$K_i(0) = \frac{C_i(0)}{V_i(0)} = 3; \quad \sigma_i(0) = \frac{S_i(0)}{V_i(0)} = 1; \quad W_3 = 600; \quad \bar{C}_3 = 360\,(i = 1, 2, 3). \tag{7'}$$

Then, choosing the arbitrary value $V_1(0) = 544$ (or $C_2(0) = 408$), Eq. (6′) and assumptions (7′) determine all the other first year numerical values.

To obtain the numerical values of the second year we need to specify the value of L, the length of the "working year". This choice is completely arbitrary since we have only to respect the positivity constraint: $L > 0$. For the sake of simplicity we put:[28]

$$L = 1 \tag{8'}$$

---

[25] Obtaining Eq. (3) is straightforward if you write the third Eq. in (1) as: $\bar{C}_3 + wl_3 + (L-w)l_3 = \sum_{i=1}^{3} \bar{S}_i$.

[26] We advise the reader that in the following the "first year" will have time-index "0", the second year "1" etc.

[27] Obviously in fixing W_3 value one has to respect the constraint represented by the expanded reproduction condition: $\sum_{i=1}^{3} S_i(0) > W_3(0)$, i.e., W_3 value should be less then the aggregate surplus value of the starting year (in Tugan's formulation 800 > 600).

[28] Note that (8) implies (from $\sigma_i(0)$, $V_i(0)$ and $K_i(0)$ values) that: $w(0) = 0.5$; $l_1(0) = 1088$; $l_2(0) = 272$; $Q^* = 1.5$.

Writing the system (1′) in the equivalent form:[29]

$$\begin{aligned} W_1(t) - \bar{C}_3 &= Q^*[l_1(t+1) + l_2(t+1)] \\ W_2(t) - V_3(t+1) &= w(t+1)[l_1(t+1) + l_2(t+1)] \end{aligned} \tag{9'}$$

we need to consider only one of the two equations. Dividing the first equation for $l_2(t+1)$ we have:

$$\frac{W_1(t) - \bar{C}_3}{l_2(t+1)} = Q^* \left[\frac{l_1(t+1)}{l_2(t+1)} + 1 \right] \tag{10'}$$

which—considering Eq. (5′) and using the fact that by definition $W_1(t) = (L+Q^*)l_1(t)$—leads to the difference equation:

$$l_1(t+1) = \frac{L+Q^*}{\Psi(t)} l_1(t) - \frac{\bar{C}_3}{\Psi(t)} \quad \text{where } \Psi(t) \equiv Q^* + \gamma^2 w(0) \gamma^t \tag{11'}$$

Note that Eqs. (5′) and (11′) (or Eq. (10′)) are the fundamental equations of Tugan's algorithm.

Substituting numerical values in Eq. (11′) we get the l_2 value for the second year[30] and then we can obtain the numerical values of all the other second year variables which, except for rounding off, are the same as in Tugan's tableau.[31] In the same manner one can proceed for the following year (the "third year").

Notice that since by hypothesis $(L+Q^*)l(0) - \bar{C}_3 > 0$, $l_1(t)$ is a strictly increasing function (see Eq. (11′)), and from Eq. (5′) it is easy to see that the growth rate of sector II depends *crucially upon the initial conditions and parameter values*. In Tugan's numerical formulation, in fact, $g(l_2)$ becomes *positive* in the fourth year—we use the usual notation: $g(x) \equiv [x(t+1) - x(t)]/x(t)$—and in the fifth year l_2 is greater than its starting value $l_2(0)$: then sector II undergoes an *overshooting effect* as w begins to decrease.

For a deeper insight into the dynamic behaviour of the system, write the solution of Eq. (11′), obtained through successive substitutions, as:

$$\begin{aligned} l_1(t) = \frac{1}{Q^*} \left(\frac{L+Q^*}{Q^*} \right)^{t-1} \Bigg\{ \left[\prod_{i=0}^{t-1} \frac{1}{T(i)} \right] \left[(L+Q^*) l_1(0) - \bar{C}_3 \right] \\ - \sum_{r=1}^{t-1} \left(\frac{L+Q^*}{Q^*} \right)^{-r} \left[\prod_{i=r}^{t-1} \frac{1}{T(i)} \right] \bar{C}_3 \Bigg\} \end{aligned} \tag{12'}$$

where $T(i) \equiv \Psi(i)/Q^*$.

[29] We have used the definitions: $Q^* \equiv C_i/l_i$, $V_i \equiv wl_i$. As we have already said, in the following we will not consider the third equation of (1) whose solution $l_3(t) = l^*$ is independent from the first two. Remember, furthermore, that we know the $V_3(t)$ value $\forall t$ (since l_3 is constant and w exogenously determined).

[30] Remember that the time index for the second year is 1.

[31] From: $2360/l_2(1) = 1.5(1 + 5.\bar{3})$ (the value in parenthesis descends from Eq. (5)) we get: $l_2(1) \approx 248.421$, and then: $V_2(1) = w_2(1) l_2(1) \approx 93.16$; $l_1(1) \approx 1324.9122$; $V_1(1) \approx 496.84$; $C_1 \approx 1987.3$; $C_2 \approx 372.63$, etc.

From Eq. (12′) it is easy to see[32] the relevance of the values of L, Q^* and γ in determining $g(l_2)$: as $t \to \infty$, $T(t) \to 1$, then $g(l_1)$ approaches the value L/Q^* and, from (5), $g(l_2)$ approaches the value $(1-\gamma)L/Q^* - \gamma$, implying that as $t \to \infty$ the growth rate of each sector (recall that $g(W_i) = g(l_i)$) tends to different finite values.[33] Note, furthermore, that the overshooting of W_2 is by no means a necessary event: for large values of γ, in fact, $g(l_2)$ will be positive and increasing $\forall t$.

The analysis of surplus value transfer from sector II to the first is not quite straightforward especially at the beginning of the adjustment process. The problem is caused by the fact that we do not know exactly the sectoral investment functions because we do not know the individual values of each $\bar{S}_i$, having only the aggregate investment function described by Eq. (2′): as we will see, what happens, especially in the first periods, depends crucially upon the values of the $\bar{S}_i$ (besides the other parameter values).

To begin with, consider the third sector: as we have already said, it does not play any relevant role in the system dynamics, its equation being satisfied independently from those of the first two sectors. Moreover we can be sure that S_3 will be entirely spent in luxuries goods, that is, inside its own sector. This point may not be evident because of our (arbitrary) assumption: $S_3 - \bar{S}_3 > 0$, but it becomes straightforward if one thinks about the fact that the difference $S_3 - \bar{S}_3$ has to be consumed by someone else (that is, by capitalists of the other two sectors): consequently as S_3 increases there will be an *outflow of surplus value excesses from sector III into the first two exactly counterbalanced by an identical inflow from these latter into the former*, so as to keep $\bar{S}_1 + \bar{S}_2$ constant. Then, contrary to Tugan's claim, *there is no net investment* from sector III towards the first two sectors.[34]

In the light of these considerations we can abandon sector III and concentrate on one of the remaining sectors. Consider sector I, whose net investment function is given by:

$$I_1(t) = \Delta C_1 + \Delta V_1 = Q^*[l_1(t+1) - l_1(t)] + w(t)[\gamma l_1(t+1) - l_1(t)] \quad (13')$$

while disposable surplus value is given by:

$$S_1 - \bar{S}_1 = (L - w(t))l_1(t) - \bar{S}_1 \quad (14')$$

Substituting from Eq. (11′) for $l_1(t+1)$, after simple manipulations we obtain the following condition:

$$I_1(t) > S_1 - \bar{S}_1 \quad \text{if}: \quad \bar{S}_1(t) > \frac{\bar{C}_3[Q^* + \gamma w(t)]}{\Psi(t)} - \frac{(L+Q^*)\gamma(1-\gamma)w(t)}{\Psi(t)} l_1(t) \quad (15')$$

[32] Studying the behaviour of Eq. (12) through the limit values of its coefficients as $t \to \infty$ may be seen as an application of the Poincaré method for approximating difference equations with non-constant coefficients. For a deep insight into this method see Elaydi (1996, pp. 303 and ff.).

[33] In Tugan's numerical formulation, as $t \to \infty$ $g(l_1) = g(W_1) \to 0.\bar{6}$, $g(l_2) = g(W_2) \to 0.25$ (and obviously $g(W_3) = 0\,\forall t$). It may be interesting to point out that if $\gamma < Q^*/(Q^* + L) = 0.6$, sector II, after the overshooting phase, will tend to a negative asymptotic growth rate, i.e. the increase in l_1 will not sufficient to compensate w diminishing (see Eq. (5)) $\forall t$.

[34] Note, furthermore, that the assumption $S_3(t) = \bar{S}_3(t)$ is also consistent with the model, leading however to an implausible consumption reduction of capitalists of the first two sectors (in this case as $t \to \infty$ $\bar{S}_1(t) + \bar{S}_2(t) \to \bar{C}_3$).

From Eq. (15′), we can see the relevance of parameter values, particularly of $\bar{S}_1$, in determining the flow of surplus value between the two sectors. Given the values of Q^*, L and γ, *at some point* the condition expressed by Eq. (15′) will be necessarily satisfied (since $l_1(t)$ is a strictly increasing function), implying from then onwards a continuous (and increasing) transfer of surplus value from the second sector into the first one. A "small" $\bar{S}_1$, however, may require at the beginning of the adjustment process *an opposite transfer* of surplus value from the first into the second sector (the duration of this phase also depending upon $\bar{S}_1$ value). In Tugan's tableau it is easy to see that only if $\bar{S}_1 > 235.79$, will the condition expressed by Eq. (15′) be satisfied from the first year, while if $\bar{S}_1 < 235.79$, contrary to Tugan's claim, it is the first sector that transfers surplus value (to sector II) over a number of periods.[35] Then, from Eq. (2′) and from what we have said about the way S_3 is used, the intersectoral flow of surplus value between the two departments can be represented by:

$$I_1(t) - (S_1 - \bar{S}_1) = -[I_2(t) - (S_2 - \bar{S}_2)] \tag{16'}$$

where all the values in Eq. (16′) are given by Eqs. (4′), (5′) and (12′).

The economic meaning of the model is straightforward. The assumption of a wage which decreases at a constant rate $(1-\gamma)$ has the same effect as assuming a sharp increase in the growth rate of sector II. Now in fact the same amount of food wage "can command" a greater number of workers: with a constant capital/labour ratio this will require an acceleration in the rate of growth of sector I, and this acceleration will be possible only through the continuous transfer of part of surplus value of sector II to the first one. Depending on the value of γ, at the beginning of the adjustment process there may occur—though not necessarily—an overshooting effect on W_2: after this phase, however, sector II will converge to its asymptotic growth rate that for "small" γ can also be negative.[36] Furthermore, as we have seen, the beginning of the adjustment process may be much more complicated: depending on other parameter values, in fact, it may be necessary over a number of periods to have an opposite transfer of surplus value from the first sector to the second in order to guarantee unbalanced equilibrium growth. The following figure sums up the main features of Tugan's model[37] (Fig. 1).

The importance of this model, and then of Tugan's explanation, is clear, standing as it does as a remarkable exercise in non-proportional economic dynamics (one of the first numerical formulations of this kind in the history of economic thought). Strictly speaking this contribution should be regarded as a forerunner of Hicks' (1985, pp. 131–143) *traverse analysis*. In fact Tugan investigates the existence of a dynamic path—for an economic system subject to a "perturbation"—along which the necessary transfers of surplus value

[35] If we assume that the previous "story" of the economy is represented by the unperturbated form of system (1) (i.e., the case for which $w(t) = w \forall t$), it is easy to see, from the equilibrium conditions of this system, that $\bar{S}_1 > 235.79$ is the case consistent with the model (precisely in the unperturbated system we have that $\bar{S}_2 > 0$ if $\bar{S}_1 > 309$). Obtaining the equilibrium conditions of the unperturbated system is straightforward: you have only to follow the same procedure we used in Section 1.

[36] See above, footnote 33.

[37] In Fig. 1 we have assumed that the growth rate of the unperturbated system (see above, footnote 35) is equal to 0.20 (note that the asymptotic growth rate of the unperturbated system is equal to profit rate, i.e., is equal to 0.25).

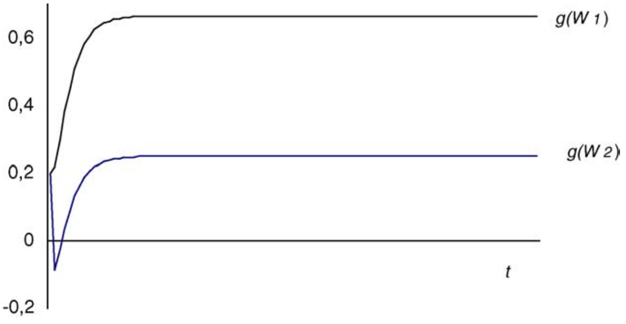

Fig. 1. The overshooting effect in Tugan's model.

between sectors take place in order to guarantee economic equilibrium: in the model we have set out, the existence of a traverse is defined by Eq. (16′).

5. Conclusions

As Tugan-Baranovsky (2002, p. 16) wrote in the introduction to his analysis of crisis theory, "prior to Marx, the greatest error made in analysing the process of the social reproduction of capital was precisely to disregard the significance of the means of production as a necessary component of the social product. [. . .] So in analysing the social reproduction of capital it is imperative to take account of the fact that social capital is used to manufacture means of production as well as consumer goods". Tugan's position is quite clear here: in the analysis of the capitalist reproduction process one should necessarily start from the basic premise that at least two different kinds of good exist: consumption goods and capital goods (the means of production), which should for no reason be confused or superimposed, the one over the other, by theoretical investigation. Thus, it is clear that a low or even negative rate of growth of consumer goods, if matched by a high rate of capital accumulation, will not necessarily disturb the smooth expansion of the economic system. In such a case: "machinery has taken the place of workers, the means of production have replaced consumer goods in the market, and consequently total social demand has not changed" (Tugan-Baranowsky, 1913, p. 201).[38] This correct definition of national product, the direct result of a multisectoral approach to economic analysis, lies at the very centre of Tugan's criticism of underconsumption theories. In fact, these theories consider all goods, ultimately, as consumer goods, hence, the disappearance of capital goods (means of

[38] This is a clear criticism which anticipates Keynes (1936, p. 370) judgement of Hobson's underconsumption theory: "Mr. Hobson laid too much emphasis (especially in his later books) on under-consumption leading to over-investment, in the sense of unprofitable investment, instead of explaining that a relatively weak propensity to consume helps to cause unemployment by requiring and not receiving the accompaniment of a compensating volume of investment which, even if it may sometimes occur temporarily through errors of optimism, is in general prevented from happening at all by the prospective profit falling below the standard set by the rate of interest".

production); in other words, social production is simply reduced to social consumption.[39] Moreover, as has already been mentioned, only his multisectoral approach could have led Tugan-Baranowsky to analyse quantitative–qualitative transformations generated by different intersectoral growth rates, and thus to identify, in the existence of a possible traverse, the conditions for smooth economic expansion. In other words, only such an approach could have led Tugan-Baranowsky to move from *steady-state analysis* towards non-proportional economic dynamics.[40]

However, these considerations do not also imply acceptance of a *disproportionality crisis theory*. In truth, by considering expanded reproduction schemes Tugan, like Marx, was correctly investigating the *logical* equilibrium conditions of a multisectoral economic system (i.e. long-run analysis); but the extension of this analysis to the explanation of economic *crises* would inevitably prove problematic, since these schemes are not suited to this purpose. Indeed, as we have seen in Section 2, when Tugan integrates his crises analysis with that of the business cycle, he is forced to depart from his "theory of markets" moving towards an explanation of economic fluctuations in terms of the discrepancy between *investments and loanable funds*: disproportionalities become the *effect* of capitalists' investment decisions, and their determinants need to be investigated. At least, as we have said, disproportionality may be regarded as the necessary condition of economic crises—i.e., every crisis manifests itself in the disporportionality among economic sectors—but it is not sufficient, by itself, to explain the *causes* of both economic crises and business cycles.

As has been said, "Tugan's interest in economics was wide-ranging" (Koropeckyj, 1991, p. 61) and in recent years there has been renewed interest in certain aspects of his theoretical contribution, especially in his pioneering research on monetary economics.[41] We believe that those features of Tugan's work we have outlined here also make him a pioneer in the history of economic dynamics, and his work deserves greater consideration for this reason.

[39] The famous model presented in Sweezy (1942, pp. 186–189) (one of the severest critics of Tugan's work) in which the author provided a formal demonstration of the unavoidable tendency towards underconsumption in "mature" capitalist economies, is a clear example of this. What should be noticed is that Sweezy drew his conclusions from an *aggregate* model, because only in a one-sector economy is it possible to omit the dynamics of the department producing means of production: so, in his model, while investments immediately turned into new consumption goods, the accumulation of new capital goods played no relevant role.

[40] In his 1913 work, Otto Bauer (1986) presented a two-sector model—similar to Tugan's 1905 scheme—characterised by different intersectoral growth rates due to the assumption of a continuous increase in the value composition of capital (in the absence of technological advance). Under this assumption department I must grow faster then department II and thus, in order to guarantee smooth economic expansion, a continuous transfer of part of surplus value from the second department to the first will be needed (together with a progressive decline in capitalists' consumption in department II). Sooner or later, contrary to Bauer's own opinion, the economic system will break down as a result of the shortage of surplus value, the growth rate of variable capital (equal to the growth rate of surplus value) being lower than the growth rate of constant capital. For a formal analysis of Bauer's model see Orzech and Groll (1983). However, the fact that Bauer's scheme does not exhibit sustained growth should not be ascribed, as Orzech and Groll seem to suggest, to its unbalanced "nature" (this, indeed, is a very interesting feature of this model), but rather to the misleading assumption that an increase in the value (and technical) composition of capital would involve no productive improvements. In fact, the conclusions may be very different if one assumes that growing mechanisation similarly implies an increase in relative surplus value.

[41] See Tugan-Baranovsky (1987), the Italian translation of *Ḃumažnyja den′ gi metall* (Paper Money and Metal), and Koropeckyj (1991).

Acknowledgements

The present paper was developed as part of the National Research Programme (PRIN-MIUR) "Dynamic capabilities, firms' organisation and local systems of production", co-ordinated by the University of Bologna. The author also wishes to thank the post-graduate Institute ISUFI of the University of Lecce for sponsoring part of this research. I am grateful to Daniele Besomi and to three anonymous referees for helpful comments and suggestions. The remaining errors are my sole responsibility.

References

Amato, S., 1980. Ricerca bibliografica di S.Amato: M.I. Tugan-Baranowsky (1865–1919). Argomenti Storici, Quaderno III–IV. Università di Firenze, Facoltà di Magistero, Florence.

Barnett, V., 2001. Tugan-Baranowsky as a pioneer of trade cycle analysis. Journal of the History of Economic Thought 23 (4), 443–466.

Besomi, D., 2006. "Marxism gone mad": Tugan-Baranowsky on crises, their possibility and periodicity. Mimeo (forthcoming in Review of Political Economy).

Bauer, O., 1986. The accumulation of capital. History of Political Economy 18 (1), 87–110 (or. ed. 1913).

Colacchio, G., 1998. Dal sottoconsumo alle sproporzioni: il caso Tugan-Baranovskij. Storia del Pensiero Economico 36, 3–43.

Crisp, O., 1968. Tugan-Baranovskii, Mikhail I. In: International Encyclopaedia of The Social Sciences, vol. 16. The Macmillan Co., New York, pp. 164–166.

Elaydi, S.N., 1996. An Introduction to Difference Equations. Springer-Verlag, New York.

Hagemann, H., 1999. The development of business-cycle theory in the German language area 1900–1930. Storia del Pensiero Economico 37, 87–122.

Hicks, J.R., 1985. Methods of Dynamic Economics. Oxford Clarendon Press, Oxford.

Kautsky, K., 1901. Krisentheorien. Die Neue Zeit XX, 2.

Keynes, J.M., 1930. A Treatise on Money. Macmillan and Co., London.

Keynes, J.M., 1936. The General Theory of Employment Interest and Money. Macmillan and Co., London.

Kondratiev, N.D., 1998. M.I. Tugan-Baranovsky (basic features of his scientific world-view). In: Makasheva, N., Samuels, W.J. (Eds.), The Works of Nikolai D Kondratiev, vol. 4. Pickering and Chatto, London, pp. 317–338 (or. ed. 1923).

Koropeckyj, I.S., 1991. The contribution of Mykhailo Tugan-Baranovsky to monetary economics. History of Political Economy 23 (1), 61–78.

Kowal, L.M., 1965. Economic Doctrines of M.I. Tugan-Baranovsky, Dissertation. University of Illinois, Urbana.

Luxemburg, R., 1951. The Accumulation of Capital. Routledge and Kegan Paul LTD., London (or. ed. 1913).

Mainwaring, L., 1995. Tugan's "bubble": underconsumption and crises in a Marxian model. Cambridge Journal of Economics 19, 305–321.

Marx, K., 1969. Theories of Surplus Value, vol. II. Lawrence and Wishart, London.

Marx, K., 1990. Capital. A Critique of Political Economy, vol. I. Penguin Books LTD. in association with New Left Review, London.

Marx, K., 1992. Capital. A Critique of Political Economy, vol. II. Penguin Books LTD. in association with New Left Review, London.

Morishima, M., 1973. Marx's Economics. Cambridge at the University Press, Cambridge.

Orzech, Z.B., Groll, S., 1983. Otto Bauer's scheme of expanded reproduction: an early Harrodian growth model. History of Political Economy 15 (4), 529–548.

Reijnders, J., 1998. Tugan-Baranowsky's breakthrough in business cycle theory. In: Glombowski, J., Gronert, A., Plasmeijer, H.W. (Eds.), History of Continental Economic Thought. Metropolis, Marburg, pp. 211–238.

Steedman, I., 1977. Marx After Sraffa. New Left Books, London.

Sweezy, P.M., 1942. The Theory of Capitalist Development. Monthly Review Press, New York.

Tugan-Baranovsky, M.I., 2002. Studies in the theory and history of commercial crises in England. In: Hagemann, H. (Ed.), Business Cycle Theory, Selected Texts 1860–1939, vol. 2. Pickering and Chatto, London, pp. 3–44 (or. ed. 1901).
Tugan-Baranowsky, M.I., 1905. Theoretische Grundlagen des Marxismus. Duncker & Humblot, Leipzig.
Tugan-Baranowsky, M.I., 1913. Les Crises Industrielles en Angleterre, Traduit sur la 2e édition Russe et Augmentée par l'Auteur par Joseph Schapiro. M. Giard & E. Brere, Paris.
Tugan-Baranovsky, M.I., 1970. The Russian factory in the 19th Century. Richard D. Irwin Inc., Homewood, Illinois (or. ed. 1907).
Tugan-Baranovsky, M.I., 1987. Cartamoneta e Metallo (Paper Money and Metal). Edizioni Scientifiche Italiane, Naples, Rome (or. ed. 1919).