Le débat Collard-Bose

David A. Collard
The Production of Commodities
The Economic Journal, Vol. 73, No. 289, March 1963

Arun Bose
The "Labour Approach" and the "Commodity Approach" in Mr. Sraffa's Price Theory
The Economic Journal, Vol. 74, No. 295, September 1964

David A. Collard
The Production of Commodities-A Rejoinder
The Economic Journal, Vol. 74, No. 295, September 1964

Arun Bose
Production of Commodities-A Further Note
The Economic Journal, Vol. 74, No. 295, September 1964

David A. Collard
The Production of Commodities
The Economic Journal, Vol. 73, No. 289, March 1963

IN his review of Mr. Sraffa's book [2] Sir Roy Harrod (ECONOMIC JOURNAL, December 1961) complains of some difficulty in reconciling two approaches to the valuation of commodities. " The reader surely requires a little more help than the author gives him about how (the) conventional labour-cost approach can give the same result as the commodity approach supplied in the earlier chapters." Mr. Sraffa, in his recent Note (ECONOMIC JOURNAL, June 1962), was concerned with other criticisms. The purpose of the present Note is to show that in certain circumstances the two methods give the same result.

I take them in turn, giving first a summary and then a numerical example.

METHOD I: " THE COMMODITY APPROACH "

We write our productive system in the form of a number of equations, one for each industry. For example, for industry A we have [3]

$$[A_a p_a + B_a p_b + C_a p_c \ldots + K_a p_k](1 + r) + L_a w = A p_a \quad . \quad . \quad (1)$$

where L_a is the proportion of the total labour force used in the production of A, r is the general rate of profit, p is price and B, C, etc., are other commodities. Another equation sets the value of the net product of the system equal to unity. If we now put $w = 0$ in order to find the maximum possible rate of profit we shall be in a position to use the property of Mr. Sraffa's " Standard System " that: [4]

$$r = R(1 - w) \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

where R is the ratio of net product to means of production and is quite general in the Standard System. If we now substitute for R the maximum possible rate of profit (which we now know) and decree that (2) shall hold for the actual system, then the wage and prices in the actual system will automatically be measured in terms of the net product of the Standard System. The advantage of such a measure is that it does not vary when the distribution of the surplus between wages and profits changes. By solving the equations we can discover the value of any commodity for each value of w.

[1] The writer is grateful to Mr. Sraffa for having been kind enough to correspond with him on this matter.

[2] *The Production of Commodities by Means of Commodities* (Cambridge University Press, 1960).

[3] Sraffa, *op. cit.*, p. 11.

[4] *Ibid.*, p. 31.

Consider the following numerical example of a system comprising two industries only, each producing a surplus.

$$(1p_a + 1p_b)(1 + r) + \frac{1}{3}w = 3p_a$$

$$(1p_a + 1p_b)(1 + r) + \frac{2}{3}w = 3p_b$$

Putting $w = 0$ and using the extra equation

$$1p_a + 1p_b = 1$$

we obtain 50% as the maximum possible rate of profit. Setting $R = 50\%$, we have

$$r = 50\%(1 - w) \text{ (cf. (2))}$$

Hence once w is given a value, r is also known. Let us put $w = \frac{1}{2}$, in which case $r = 25\%$. Inserting these values in the equations we obtain

$$p_a = 17/36 \text{ and } p_b = 19/36$$

both in terms of the Standard Commodity.

Method II: "The Labour-cost Approach"

We measure wage in the way already described. Then we write down the amount of labour directly used in the production of, say, A, together with the amount of labour used in each preceding period to produce the commodities used up in production.

Suppose that, n years ago, we used L_a labour. Then its present value would be [1]

$$L_{a_n}w(1 + r)^n \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

where r and w are used as previously. If we add up all these values we obtain the labour-cost valuation of the commodity.

As the same standard is being used for each of the valuations we would expect the results to be the same. However, it is not immediately obvious how we are to derive the inputs of previous years until we know what the conditions of production were in the past. I suggest we assume production in the past was carried out in an exactly similar way to production now, *i.e.*, that the same techniques were used and there were constant returns to scale.

Let us take the same case as before and write it in this way:

$$1A + 1B + \frac{1}{3} \rightarrow 3A$$

$$1A + 1B + \frac{2}{3} \rightarrow 3B$$

As we go back into the past we find that the labour ingredient of $3A$ is

$$\frac{1}{3};\ \frac{1}{3};\ \frac{2}{9};\ \frac{4}{27} \text{ etc.}$$

[1] Sraffa, Chap. IV.

From (3), with a rate of profit of 25%, the value of this labour will be given by $\frac{1}{3}w$ plus a series whose first term is $\frac{1}{3}w$ and whose common ratio is $\left(\frac{2}{3} \cdot \frac{5}{4}\right)$. The total is $\frac{17}{6}w$ or with $w = \frac{1}{2}$, $\frac{17}{12}$. This gives the same result as Method I. Similarly with the labour-cost of *B* in terms of the standard.

The difficulty lies in the interpretation of these rather stringent conditions. If we take them literally, then in any actual economy the labour-cost valuation will always be " wrong," *i.e.*, it will differ from the commodity valuation. An alternative course, that intended by Mr. Sraffa, is not to deal with the real past at all. Previous " years " are then merely successive layers of equations, reached as we convert commodities into dated labour, assuming constant returns as we go. Numerically the process is the same as the one illustrated. It will be noted that one of our conditions is still needed.

Quite apart from its inapplicability to the multi-product industry, it is suggested that because of these difficulties the labour-cost approach is inferior to the commodity approach.

D. A. Collard

University College,
Cardiff.

Arun Bose
The "Labour Approach" and the "Commodity Approach"
in Mr. Sraffa's Price Theory
The Economic Journal, Vol. 74, No. 295, September 1964

BOTH Sir Roy Harrod (ECONOMIC JOURNAL, December 1961) and Mr. Collard (ECONOMIC JOURNAL, March 1963) have suggested that of the two approaches in Mr. Sraffa's book,[2] the so-called "labour–cost approach" is inferior to the "commodity approach." The purpose of this Note is to show that if one uses Mr. Sraffa's device of "sub-systems"[3]—which unfortunately Mr. Sraffa elaborates less fully than the alternative device of "reduction to dated labour"—one gets round the difficulties which allegedly make the "labour–cost approach" inferior.

[1] I am indebted to Mr. Piero Sraffa for going through the first draft of this Note and for clarifying a point about the effect on rates of returns of conversion into sub-systems.

[2] The Production of Commodities by Means of Commodities, 1960.

[3] *Op. cit.*, Appendix A.

By applying successively the " commodity approach " and the " dated labour approach" to his extremely simplified model, Mr. Collard gets the same solutions, viz., $p_a = 17/36$ and $p_b = 19/36$.[1] He argues, however, that one can get identical solutions only by assuming: (*a*) unchanging technique over historic time, and (*b*) constant returns.[2] These assumptions being unrealistic, he concludes that "in any actual economy the labour-cost valuation would always be ' wrong,' *i.e.*, it will differ from the commodity valuation" (p. 146). Mr. Collard could have added that the assumption that r and w in the " reduction equation " have remained unchanged through time is also unrealistic.

However, these objections seem to be beside the point. The use of the slightly confusing phrase " dated labour " notwithstanding, the context makes it clear that by his " Reduction operation " Mr. Sraffa is merely calculating the sum of direct plus potential indirect labour that would be required to produce a commodity, if its commodity inputs were produced under present conditions (*i.e.*, techniques, outputs, the wage–profit division as they are to-day). If one takes the " labour-cost " of producing commodity-inputs in this sense as " replacement-cost " under present conditions, the " difficulties " Mr. Collard speaks of seem to disappear.

The point might be made clear by using the " sub-systems " method as Method III to calculate prices in Mr. Collard's model. Let us first recall Mr. Collard's model.

$$(1p_a + 1p_b)\ (1 + r) + \tfrac{1}{3}w = 3p_a \quad . \quad . \quad . \quad . \quad (1)$$
$$(1p_a + 1p_b)\ (1 + r) + \tfrac{2}{3}w = 3p_b \quad . \quad . \quad . \quad . \quad (2)$$
$$1p_a + 1p_b = 1 \quad . \quad . \quad . \quad . \quad . \quad (3)$$

Since this actual system is also, helpfully, a " standard system " producing a " standard national income," [3] $R = \frac{1}{2}$ is easily obtained. We follow Mr. Collard in putting $w = \frac{1}{2}$, so that we have $r = \frac{1}{4}$.

Now the net product of this system appears in the shape of both commodities A and B. It can therefore be rearranged in the form of two self-replacing systems, with the net product of each appearing in the form of only one commodity. To accomplish this conversion into sub-systems we need two pairs of multipliers, one each for each sub-system, *i.e.*, x applied to Industry A and y applied to Industry B to get sub-system I; x' applied to Industry A and y' applied to Industry B to get sub-system II.

Mr. Sraffa does not discuss the precise method by which these multipliers

[1] If a slight error is corrected. In the Reduction equation appropriate to Mr. Collard's model, the first term in the infinite series is not $\frac{1}{3}\,w$, as wrongly stated in Mr. Collard's article (p. 146), but $\frac{1}{3} \cdot \frac{5}{4} \cdot w$.

[2] Mr. Collard did mention Mr. Sraffa's own interpretation of " reduction to dated labour " which eliminates the unreal assumption (*a*) but remains, in Mr. Collard's view, tied to the restrictive assumption (*b*).

[3] This is most unrealistic. But it is shown in the Appendix that the argument of this Note holds even if we use a more realistic actual system as a basis.

are obtained. Nor, so far as the writer is aware, has anyone else set out the method in print. So a brief explanation of the method used here may not be out of place.

We first construct two equations expressing conditions that must be fulfilled if each sub-system is to eliminate one of the commodities from its net product, viz.,

$$\tfrac{5}{4}x + \tfrac{5}{4}y = 3y\text{—which reduces to } x = \tfrac{7}{5}y$$
$$\tfrac{5}{4}x' + \tfrac{5}{4}y' = 3x'\text{—which reduces to } x' = \tfrac{5}{7}y'$$

We now add two additional equations which impose the condition that the rearranged system has the same set of aggregates as the original system, viz.,

$$x + x' = 1$$
$$y + y' = 1$$

Solving these equations, we get the results:

$$x = \tfrac{7}{12};\ y = \tfrac{5}{12};\ x' = \tfrac{5}{12};\ y' = \tfrac{7}{12}$$

Putting $r = \frac{1}{4}$, $w = \frac{1}{2}$, and applying the multiplier $x = \frac{7}{12}$ to equation (1), and $y = \frac{5}{12}$ to equation (2), we get Sub-system I, whose net product is $\frac{1}{2}p_a$.

$$\begin{array}{l}
\tfrac{35}{48}p_a + \tfrac{35}{48}p_b + \tfrac{7}{72} = \tfrac{84}{48}p_a \\
\tfrac{25}{48}p_a + \tfrac{25}{48}p_b + \tfrac{10}{72} = \tfrac{60}{48}p_b \\
\hline
\tfrac{60}{48}p_a + \tfrac{60}{48}p_b + \tfrac{17}{72} \quad \tfrac{84}{48}p_a + \tfrac{60}{48}p_b
\end{array}$$

In the same way, applying multipliers $x' = \frac{5}{12}$ and $y' = \frac{7}{12}$, we get Sub-system II, whose net product is $\frac{1}{2}p_b$.

$$\begin{array}{l}
\tfrac{35}{48}p_a + \tfrac{25}{48}p_b + \tfrac{5}{72} = \tfrac{60}{48}p_a \\
\tfrac{35}{48}p_a + \tfrac{35}{48}p_b + \tfrac{14}{72} = \tfrac{84}{48}p_b \\
\hline
\tfrac{60}{48}p_a + \tfrac{60}{48}p_b + \tfrac{19}{72} \quad \tfrac{60}{48}p_a + \tfrac{84}{48}p_b
\end{array}$$

From Sub-system I we know that the value of $\frac{1}{2}p_a$ in terms of the standard is $\frac{17}{72}$, so that $1\,p_a = \frac{17}{36}$. Similarly, from Sub-system II, we get $1\,p_b = \frac{19}{36}$ in terms of the standard.

In this way, by using the Sub-systems method, we solve the price problem by using a variant of the so-called "labour-cost approach,"[1] without having to make special assumptions about past techniques, wage-profit division,

[1] The phrase "labour-cost approach" is inexact. Mr. Sraffa makes it quite clear that only "when the wage absorbs the whole net product (is) the commodity equal in value to the labour that directly or indirectly has been required to produce it." In all other situations "the commodity forming the net product of a sub-system is equal in value to the wages of labour employed plus the profits on the means of production" (*op. cit.*, Appendix A). Incidentally, the "commodity approach" too, in all cases of production with a surplus, takes into account not only commodity inputs but also quantities of direct labour required.

constant returns,[1] etc. It is surprising that Mr. Collard overlooked this method.

Sir Roy Harrod, in his review, has said:

> "In the case of joint production the reduction to 'dated labour' is said to be impossible, and as joint production is omnipresent . . . it follows that we must take the theory of price formation through the production of commodities by commodities to be the over-riding one, and the more familiar approach by 'dated labour' to be of not much help" (p. 786).

Price theory based on the "commodity approach" would indeed be the only one available once we brought joint production and fixed capital into the picture, if the "dated labour method" were the only alternative available. Actually, the sub-systems method—which is in many ways a simpler variant of the "labour approach" than the "dated labour method"—is especially useful in taking care of joint production and fixed capital, as Mr. Sraffa shows in some detail in Chapters 9 and 10 of his book.

ARUN BOSE

Kirori Mal College,
Delhi University.

APPENDIX

1. Mr. Collard's model of an actual system is excessively simplified, as already noted above. Its advantage is that it allows us to estimate R (and also the standard national income) almost at a glance. The gain in analytical convenience is offset, however, by the fact that Collard's actual system is a most unlikely one. No actual system is ever likely to be a ready-made standard system. Would the argument of this Note hold if we operate with a more realistic actual system? In principle, there is no reason why it should not. This is confirmed by what follows below.

2. Consider a system possessing the following "technological skeleton" (*i.e.*, technological conditions of production):

$$2A + 2B + \tfrac{3}{4}L \longrightarrow 8A \qquad (1)$$
$$2A + 5B + \tfrac{1}{4}L \longrightarrow 8B \qquad (2)$$

Each line here represents quantities of both commodities and the fraction of the total annual labour of society required to produce either A or B.

3. By applying appropriate multipliers, we derive from the above a standard technological skeleton corresponding to it, viz.,

$$\tfrac{8}{5}A + \tfrac{8}{5}B + \tfrac{3}{5}L \longrightarrow \tfrac{32}{5}A \qquad (3)$$
$$\tfrac{16}{5}A + 8B + \tfrac{2}{5}L \longrightarrow \tfrac{64}{5}B \qquad (4)$$
$$\overline{\tfrac{24}{5}A + \tfrac{48}{5}B + 1L} \qquad \overline{\tfrac{32}{5}A + \tfrac{64}{5}B} \qquad (5)$$

[1] There is no difference between outputs of, and factor proportions in, industries A and B when we compare Collard's model in its original form with the same model in its converted form. The question of variation or constancy of returns would therefore simply not arise.

4. From (5) we get:

$$\text{The Standard National Income} = \tfrac{8}{5}A + \tfrac{16}{5}B$$
$$R^{1} = \tfrac{1}{3}$$

Let $w = \frac{2}{3}$ and $r = \frac{1}{9}$.
Our actual system then is:

$$(2p_a + 2p_b)\left(\tfrac{10}{9}\right) + \tfrac{3}{4}\cdot\tfrac{2}{3} = 8p_a \quad . \quad . \quad . \quad . \quad . \quad . \quad (6)$$
$$(2p_a + 5p_b)\left(\tfrac{10}{9}\right) + \tfrac{1}{4}\cdot\tfrac{2}{3} = 8p_b \quad . \quad . \quad . \quad . \quad . \quad . \quad (7)$$

5. Using the "commodity approach," we solve simultaneous equations (6) and (7) to get

$$1p_a = \tfrac{43}{248};\ 1p_b = \tfrac{14}{62}.$$

6. In order to be able to use the sub-systems variant of the "labour approach" for price determination, we first use the method indicated above to obtain two sets of sub-systems multipliers, which are:

Sub-system I multipliers $\frac{88}{93}$ for Industry A, $\frac{80}{93}$ for Industry B; Sub-system II multipliers $\frac{5}{93}$ for Industry A, $\frac{13}{93}$ for Industry B.

7. Using these multipliers, we get:

from Sub-system I, "labour cost" of $\frac{992}{279}p_a = \frac{172}{279}$,
so that $1p_a = \frac{43}{248}$
from Sub-system II, "labour cost" of $\frac{62}{279}p_b = \frac{28}{558}$,
so that $1p_b = \frac{14}{62}$.

David A. Collard
The Production of Commodities-A Rejoinder
The Economic Journal, Vol. 74, No. 295, September 1964

WE must be grateful to Mr. Bose for setting out a numerical illustration, for single-product industries, of Mr. Sraffa's method of sub-systems. If I understand him correctly, he has two major criticisms to make of my note (ECONOMIC JOURNAL, March 1963): that my conclusions are misleading and that I somewhat surprisingly neglected a third method, that of sub-systems. First there are two minor points to clear up. The terminology "commodity" and "labour" approaches is perhaps a little misleading, but it is that in which the issue was first raised. I should have preferred the labels "neo-Walrasian" and "Classical." The other point is that I must apologise for having allowed the fraction $\frac{5}{4}$ to slip out of my note at some stage in preparation.

Mr. Bose's opening remarks are less than fair. I can claim not to have fallen into the trap laid for the unwary by the terminology of the chapter on

[1] Another way of calculating R, suggested by Sraffa (*op. cit.*, p. 31) is, of course, to convert (1) and (2) from expressions denoting technological requirements into proper production equations. We then take $w = 0$, $r = R$, p_a (or p_b) $= 1$, and solve for R. That would give us R without the need to construct a standard system; but it would not give us the standard national income.

reduction to dated labour.[1] In fact, I offered two interpretations, only the first of which he quoted. I also wrote " . . . an alternative course, and that intended by Mr. Sraffa, is not to deal with the real past at all. Previous ' years ' are then merely successive layers of equations, reached as we convert commodities into dated labour, assuming constant returns as we go."

This whole question gives rise to a major problem of interpretation. We can agree that there is more than one way of solving a set of simultaneous equations. We can also agree that any viable method will do and that the argument as to which method is " best " simply concerns its relative convenience. Complications arise when some of these methods have analogies in economic theory. It is sometimes possible to solve a set of equations by a " reduction " approach, and it will not at all be a matter for surprise if the results are the same as those obtained by other methods. One would not normally use such a method—its interest lies solely in that it has a labour-cost analogy in economic theory. In fact, the reduction method of solution provides two possible analogies, the one needing more assumptions than the other. Sometimes the method will not work and the analogy has to be abandoned. For example, in the case of joint-products we would have " negative quantities of labour, for which no reasonable economic interpretation could be suggested " (Sraffa, p. 58).

The method of sub-systems, as illustrated by Mr. Bose for the single-product case, is capable, like the reduction approach, of two interpretations, a mathematical one (in which case it is otiose) or an economic one (in which case its assumptions should be made more explicit). In the latter sense each sub-system reflects in miniature the technical conditions of the whole, so that constant returns must be taken to prevail if the analogy is to hold.

In the joint-product case Mr. Sraffa has suggested that we derive " labour contained " by an extension of the approach by sub-systems. Rather in the early Leontief manner, the net output of one good is increased and the value of the extra labour required by the system at zero profit is calculated. This will give the same result as setting $w = 1$ and solving the neo-Walrasian equations, but its interest lies in the fact that once again there is an economic analogy and that once again constant returns is the assumption needed in order to make the analogy hold.

To sum up. If, by " different " methods, we merely indicate different ways of solving a set of simultaneous equations the identity of results obtained is neither interesting nor surprising. Their economic analogies, on the other hand, *are* interesting but involve important assumptions, notably constant returns.

David Collard

University College,
Cardiff.

[1] Mr. Sraffa's comparison with " old wine " and " oak chests " is not helpful to those with Ricardian deviations.

Arun Bose
Production of Commodities-A Further Note
The Economic Journal, Vol. 74, No. 295, September 1964

I AGREE that the three alternative methods are simply different ways of solving simultaneous equations, and are of no interest from the mathematical point of view. From the point of view of economic theory, what is of interest is that the neo-Walrasian " commodity approach " can be compared with the post-classical " labour approach " with a view to finding out which is the theory with a more general application.

Mr. Collard's statement in his original comment (ECONOMIC JOURNAL, March 1963, p. 146) that " in any actual economy the labour-cost valuation would always be ' wrong ', *i.e.*, it will differ from the commodity valuation " can only mean, so far as I can see, that the former in his view is " inferior " to the latter because it is tied to the constant returns assumption, while the latter is not. The purpose of my Note was simply to contest this view.

It seems to me that in his Rejoinder Mr. Collard continues to adhere to this view. Not impressed by my footnote, he detects an implicit constant returns assumption when the sub-systems method is applied to the single-product case because " each sub-system reflects in miniature the technical conditions of the whole so that constant returns must be taken to prevail . . . "

I am afraid I cannot agree that each sub-system reflects in miniature the technical conditions of the whole. In my illustration, derived from his own two-commodity-two-industry model, it is easy to see that outputs of *A* and *B* and input-proportions employed in the production of *A* and *B*—are both unchanged when we construct sub-systems. In fact, the construction of sub-systems is a purely mental manipulation leaving technical conditions unaltered. Consequently, no question of variation or constancy of returns arises. I am sure Mr. Collard will not retort: what is the use of constructing such imaginary sub-systems with no counterpart in the real world? As he himself clarifies very neatly in his Rejoinder, both the sub-systems and the commodity method are different ways of solving simultaneous equations. To solve such equations, you have to manipulate on paper the original equations to eliminate some of the variables. When you do that you don't produce anything differently in the real world, nor do you require any additional information not given to you in the original production equations. When evaluating the " commodity method " Mr. Collard apparently treats such manipulations as purely mental, so that the results are not vitiated and restricted by any constant returns assumption. But when it comes to the sub-systems method, the multiplication of the original industry equations by fractions are taken to create " miniature industries " (even though the fractions always add up to 1). I fail to see why.

ARUN BOSE

Kirori Mal College,
Delhi University.