

The Marxian theory of value and heterogeneous labour: a critique and reformulation

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1. Introduction

Central to the Marxist analysis of capitalism has been the observation that the accumulation process tends to obliterate social distinctions among workers. Thus, Marx and Engels write in the *Communist Manifesto* (1972, pp. 336-337):

In the earlier epochs of history we find almost everywhere a complicated arrangement of society into various orders, a manifold gradation of social rank . . . Our epoch, the epoch of the bourgeoisie, possesses, however, this distinctive feature: it has simplified the class antagonisms. Society as a whole is more and more splitting up into two great hostile camps . . . : Bourgeoisie and Proletariat . . . The bourgeoisie, wherever it has got the upper hand, has put an end to all feudal, patriarchal, idyllic relations. It has pitilessly torn assunder the motley feudal ties that bound man to his 'natural superiors', and has left remaining no other nexus between man and man than naked self interest, than callous 'cash payment'.

It is this process which motivates the concept of 'labour in general', which Marx (1968, p. 44) describes in this manner:

The indifference to the particular kind of labour corresponds to a form of society in which individuals pass with ease from one kind of work to another, which makes it immaterial to them what particular kind of work may fall to their share . . . This state of affairs has found its highest development in the most modern of bourgeois societies, the United States. It is only here that the abstraction of the category 'labour', 'labour in general' . . . becomes realized in practice.

This insight is incorporated into standard treatments of the labour theory of value through the (usually implicit) assumption that the sole theoretically important type of labour heterogeneity takes the form of skill differences. This assumption in turn renders possible the reduction of all labour to a common unit by treating skilled labour as a simple 'multiple' of unskilled labour—a procedure followed by value theorists from Marx to the present. Again to quote Marx (1967, p. 44):

Skilled labour counts only as simple labour intensified, or rather, as multiplied simple labour, a given quantity of skilled being considered equal to a greater quantity of simple labour.

Yet the analysis of contemporary capitalism has revealed an opposed tendency: the persistence of divisions within the working class based on race, sex, nationality, ethnicity,

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education and position in the hierarchy of production. Our own study of these divisions, which we call *labour segmentation*, has convinced us, moreover, that they are of no mere secondary importance and cannot be reduced analytically to skill differences (Bowles and Gintis, 1976). While the tendencies in the accumulation process toward the homogenisation of the working class described by Marx and elaborated by Braverman (1974) and others are undoubtedly strong, they are evidently accompanied by powerful opposing tendencies, which reproduce working-class divisions. Specifically, the material basis for labour segmentation is the labour process itself, as well as the uneven development of the capitalist mode of production and the integration of successive waves of workers into wage employment associated with the accumulation process and the undermining of co-existing modes of production. The counterpoint of labour homogenisation and segmentation is thus an integral part of the accumulation process.

It would be difficult to overstate the theoretical rift occasioned in Marxian economics by the appearance of segmentation theories. On the one hand, these theories have by and large eschewed the fundamental categories of value theory, and have failed to ground their analyses in the creation and expropriation of surplus value. On the other hand, value theorists have failed to revise the standard theory to incorporate the conceptual tools needed to encompass the phenomenon of segmentation. Rather than attempt an explanation of the reproduction of internal divisions of the working class *at the point of production*, those writing within the value theory framework have tended to relegate these problems to the sphere of 'culture' operating outside the valuation process, and hence independent of it.† Both views we hold to be deficient. We shall here sketch a theoretical framework which can integrate and enrich the insights of each.

Our subsequent analysis will support three main propositions. First, the reproduction of capitalist social relations and the extraction of surplus value depend on the reproduction of systematic working-class fragmentations based on what we have termed labour segmentation. Second, the method by which Marxists have conventionally integrated heterogeneous labour into value theory is grossly at variance with the nature of the labour process and obscures some central features of the dynamics of capitalism as a system. Third, and conversely, the problem of heterogeneous labour is not, as some have suggested, the Achilles heel of the Marxian economic system. We shall provide an alternative to the standard approach which, while making heterogeneity central to value theory, is also free of the circularity or inconsistency charged by Böhm-Bawerk, Samuelson, Pareto, Morishima and others. We shall then show that the traditional propositions relating surplus value to profits hold in the case of heterogeneous labour.

What we propose, in brief, is that in treating any particular issue the labour force be divided into whatever distinct segments may be dictated by a close historical and class analysis of the social formation in question. These segments will not be reduced to a common unit of account. Rather we propose that the value of a commodity be defined as the direct and indirect labour *of each type* embodied in it. Thus value is no longer a single magnitude, but rather a series of magnitudes (one for each type of labour), or, more technically, a vector. This approach allows for distinct rates of exploitation of

† Marxian value theory thus confronts a problem formally analogous to the dynamic instability of discrimination in the neoclassical competitive model (see Becker, 1964; Arrow, 1972). Arrow (1976), after noting the 'deficiencies' of the Marxian approach, comments: 'Economic arguments of all schools deal with anonymous individuals. The characteristics of sex, race and nation are not incorporated. It may not be totally surprising that economic theory has nothing to say about the causes of sexual, racial or national differences' (p. 236).

different labour segments, as well as obviating the necessity of assuming commensurability among their various labour hours.

We begin with a review of the conventional treatment of heterogeneous labour by Marx and a survey of the major critiques of this method.

2. Abstract labour and heterogeneous labour

Standard approaches to the labour theory of value have followed Marx in regarding difference among workers as a matter of little theoretical importance. To quote Marx (1967, p. 44):

Productive activity, if we leave out of sight its special form, viz. the useful character of the labour, is nothing but the expenditure of human labour-power . . . the value of a commodity represents human labour in the abstract, the expenditure of human labour in general.

Value theory has been content to abstract in this manner from 'all special characteristics which differentiate one kind of labor from another' (Sweezy, 1946, p. 30). Marx clearly thought the matter unimportant (Marx, 1967, p. 198):

We therefore save ourselves a superfluous operation, and simplify our analysis, by the assumption, that the labour of the workman employed by the capitalist is unskilled average labour.

But why is this abstraction appropriate? Because, again to quote Sweezy (1946, p. 31), 'the reduction of all labor to a common denominator . . . is not an arbitrary abstraction . . . It is rather, as Lukacs correctly observes, an abstraction "which belongs to the essence of capitalism".' Yet if the concept of abstract labour derives its validity from the historical reduction of all labour to wage labour, it should be clear that abstract labour is not homogeneous labour. If the use value of the commodity labour power depends on the skills of the worker, the value of a type of labour power itself will depend on the cost of production and reproduction of its skills. Thus abstract labour may be heterogeneous, yet remain commensurable—hence Marx's representation of skilled labour time as a 'multiple' of simple unskilled labour time.† And how is this 'multiple' to be determined?

The value of labour power is determined, as in the case of every other commodity, by the labour time necessary for its production, and consequently also the reproduction of this special article (Marx, 1967, p. 170).

In view of Marx's rather straightforward application of this general principle to the question of skilled labour, the amount of controversy and confusion it has stirred up is rather surprising.‡ A standard critique, which reappears in the writings of Samuelson and other contemporary economists, was supplied by Böhm-Bawerk in his classic *Karl Marx and the Close of His System* (Böhm-Bawerk, 1973). Böhm-Bawerk incorrectly asserts that Marx sought to reduce skilled to simple labour through the exchange process itself, using relative wages as the appropriate weighting scheme.§ He then

† Sweezy comments: 'Under these circumstances, the various specific kinds of labor in existence at any given time and the relative quantities of each have become matters of secondary importance in any general view of the economic system' (Sweezy, 1946, p. 31).

‡ Marx's suggestion has only been taken up, however, in recent years. The conceptual framework and an empirical example for an educational system are presented in Bowles (1965), and Chugh (1971). The problem is solved formally in Morishima (1973, pp. 192–193). Rowthorn (1974) offers a detailed solution as well as an insightful survey of the related theoretical issues.

§ The interpretation of Marx's treatment of this question has aroused some controversy. See Bernstein (1899) and Hilferding (1966), as well as the more general discussion in Meek (1973).

introduces Marx's own 'embodied labour time' solution, incorrectly ascribes it to an obscure Russian economist, and objects that as an empirical matter the ratio of embodied labour times of two types of labour is generally not proportional to relative wages. Samuelson (1971) has since developed this point, arguing correctly that relative wages will in general depend on the interest rate. This criticism simply misses the point: Marx never intended that the value of labour power be determined by its wage rate, any more than the value of a commodity be determined by its price.

The more recent critique by Morishima (1973) is more perceptive but, to our mind, ultimately no more persuasive. Morishima begins by showing that when skill differences among workers are allowed, a uniform rate of exploitation of labour across skill levels is achieved only if relative wages are equal to the relative labour times embodied in the production of these skills. The problem of heterogeneous labour, Morishima then observes, confronts Marxian economists with a dilemma:

as soon as the heterogeneity of labour is allowed for, the theory of value is seen to conflict with Marx's law of the equalization of the rate of exploitation through society, unless the different sorts of labour are reduced to homogeneous abstract human labour in proportion to their wage rates (Morishima, 1973, p. 180).

The latter method, he correctly points out, contradicts Marx's 'intention of obtaining an intrinsic value system completely independent of markets'. On the other hand, reduction to simple embodied labour implies different rates of exploitation for the various labour categories. This, Morishima comments, is obviously 'not compatible with Marx's view of the polarization of society into two classes . . . There is no easy way out of this predicament'. 'Our solution', he concludes, is 'a Marxian economics without the labour theory of value' (p. 181).

There is indeed a way out—a way in fact which we believe considerably strengthens and enriches our understanding of the capitalist mode of production and its dynamics. First, as we have seen, the concept of abstract labour does not require the homogenisation of labour as an historical tendency. Second, the reduction of labour to a common unit of measurement is not required to prove the most basic propositions of the labour theory of value. We shall demonstrate this point in section 4 and the Appendix. Third, the assumption of equal rates of exploitation is in no way required by historical materialism and is inconsistent with a critical Marxian concept: uneven development.

For these reasons we find Morishima's critique of the Marxian treatment of heterogeneous labour inadequate. This does not mean, however, that we accept the standard reduction to simple labour proposed by Marx. Indeed, we shall presently show that the procedure of reducing differences among workers to differential labour embodied in the production of skills to be seriously flawed.

3. Labour segmentation and the extraction of surplus value

The traditional Marxian treatment of heterogeneous labour is based on the assertion that competition among capitals will reduce to economic insignificance all differences among workers not rooted in differences in embodied labour. We shall suggest, on the contrary, that forms of labour segmentation unrelated to skill differences are reproduced through the process of extraction of surplus value at the point of production. Moreover, labour segmentation influences the determinants of prices and profits through its effect on the fragmentation of the working class, both at the point of production and in the

social formation as a whole. Thus a value theory which abstracts from labour heterogeneity or reduces it to a matter of skills is an inauspicious foundation for understanding surplus value itself.

We shall begin by sketching a Marxian theory of the firm (Gintis, 1976; Bowles and Gintis, 1976). Marx notes (1967, p. 170):

the peculiar nature of labour-power as a commodity is, that its use value does not, on the conclusion of the contract between the buyer and the seller, immediately pass into the hands of the former.

This 'peculiarity' is so critical that it renders labour power conceptually distinct from any other commodity—to the point where it is quite misleading to identify labour with the use value of labour power to the capitalist. The enjoyment of the use value of any other commodity is non-problematic: the bread does not resist being eaten. The use value of labour power, however, depends not only on the technical attributes of the worker, but also on the ability of the capitalist to induce the worker to perform. The essence of exchange is a legally enforceable *quid pro quo*. This is not the case in the labour power–wage transaction, where in return for a wage the worker agrees only to submit to the political authority of the firm for a given period of time. What the worker must do in order to generate profits for the capitalist goes far beyond the terms of legal contract, which will in general specify the hours of work, the wage rate, health and safety conditions, pensions and the like, but not the amount of labour services to be performed. Authority at the point of production must be used to *evince* worker behaviour not guaranteed by the wage labour contract.

This observation directly implies (Gintis, 1976) that the use value of a worker is not limited to his or her 'technical attributes', such as skills. The surplus extracted in the work process depends, in addition, on states of consciousness, degrees of solidarity with other workers, the size of reserve armies, and the social organisation of the work process. In short, it follows from the 'peculiar nature of labour-power as a commodity' that the organisation of production must reflect essential elements of class struggle. Not only must such traditional issues as the length of the working day and the division of revenue between capital and labour be understood in terms of the extraction of surplus value, but also the structure of hierarchical authority, job fragmentation, racism and sexism as basic aspects of the capitalist firm.

We shall develop this point by investigating the way in which differences among workers bear upon the determination of the profit rate and the process of surplus value extraction. The approach via the labour theory of value expresses the profit rate in terms of a relationship between the rate of exploitation and a weighted average of the organic compositions of capital in the various sectors of the economy. We shall deal substantively only with the role of heterogeneous labour in influencing the rate of exploitation; arguments with respect to the organic composition of capital are parallel.†

The rate of exploitation may be defined as 'unpaid' labour time divided by 'paid' labour time, or $e = (T - v)/v$, where T represents the number of hours of wage labour performed per day by the members of a household, and v is the value of the household's wage bundle, the direct and indirect abstract labour time embodied in their total daily consumption of purchased commodities. If b and A are vectors representing the wage

† These input–output relationships are regulated by the available technologies, the prices of capital goods and labour, and the profit rate. All are influenced by the evolution of class conflict which, as we shall argue, depends critically upon forms of heterogeneity of the work force. On the relation between technology and class conflict, see Marglin (1974), Stone (1974) and Marx (1967, ch. 15).

bundle and the unit value of each item in it, then $v = Ab$, and we can rewrite the rate of exploitation as $e = (T - Ab)/Ab$. It is clear that e may be increased by increasing T , the number of hours worked for wages by each household, by reducing some elements of b , the wage bundle, or by reducing the labour time embodied in some of its components. With a given set of input-output relations the latter can be accomplished only by increasing the intensity of labour.[†]

Now it is evident that T , A and b are all affected by the forces and social relations of production, as both are manifested in and transformed by past and present class conflict. Both T and b are more or less directly affected by class conflict as well. The relative power of the contending parties is based on the specific material conditions of the social formation. It is here that differences among workers have a direct bearing, particularly via their effects on the degree of unity of the working class, its capacity for collective economic and political action, and its ability to create coalitions with other classes or strata in the social formation.

Three aspects of the influence of labour heterogeneity in the determination of the rate of exploitation appear to be particularly important.

Divisions within the working class allow the capitalist to bargain separately with each group of workers. Any individual capitalist, or the entire class, may combine with one group of workers against another. The resulting disunity of the workforce may allow the capitalist to impose lower wages or more unfavourable working conditions upon the workforce as a whole, while surrendering only part of the increased profits to the colluding workers. To this end the capitalist will, should the occasion warrant, draw on whatever racist, sexist, nationalist, credentialist or other divisive sentiments are found, or can be created, in working-class culture. The outcome of a successful manoeuvre of this type is a fragmented work force, higher profits and unequal rates of exploitation among labour segments.

Labour heterogeneity influences the rate of exploitation indirectly as well, through its relationship to the size and composition of the reserve army of labour. The reserve army affects the rate of exploitation through its ability to curb wage demands and increase the intensity of labour. The reserve army is generated by the accumulation process itself. The problem for the capitalists is to maintain near capacity utilisation of capital goods and to contain the explosive political potential of unemployment. Labour segmentation is critical to this objective. Different probabilities of unemployment among labour segments militate against unity between the employed and unemployed, while concentrating the most severe burden of job insecurity on those groups least able to impose political or economic costs on the capitalists.

Last, heterogeneous labour directly affects the rate of exploitation via the intensification of labour. Until recently, Marxist treatments of this issue have not gone beyond Marx's discussion in *Capital*, where the question of heterogeneity is barely discussed.[‡] Yet modern conditions require a more extensive analysis. Whereas in the 19th-century

[†] Each individual capitalist can increase profits by increasing the intensity of labour above that of his competitors. A uniform increase in the intensity of labour throughout a competitive industry, however, merely reduces the value of the product. In the case of a luxury good, the profit rate will not be affected. In the case of a wage good, the reduction in value increases the economy-wide rate of exploitation, and hence the profit rate. In the case of a capital good, which enters, directly or indirectly, into the production of wage goods, the intensification of labour also lowers the organic composition of capital, thereby again raising the economy-wide profit rate.

[‡] Braverman's (1974) important contribution in this area likewise abstracts from the role of heterogeneous labour.

factory the capitalist directly oversaw the behaviour of a few technicians, supervisors and the mass of production workers, the modern corporate enterprise exhibits a complex social division of labour, in which the capitalist's direct control over the production process derives from his position at the apex of a hierarchically ordered system of authority relations. Here the labour force cannot be directly overseen; nor, often, as in many of the fast growing service sectors, can the worker's product be readily measured. In this new framework, the issue of heterogeneous labour becomes of paramount importance.

To see this, we must ask: what are the resources the capitalist has at hand to evoke 'proper behaviour' on the part of the worker, in order to draw labour out of labour power? Given the legal structure of capitalism, the answer is simply the ability to dismiss, promote or change the wage of the worker. The prudent capitalist will seek to undermine any forms of social relations among workers which curb the potency of these sanctions and rewards. Maintaining and fostering fragmentations is a major strategy toward this end.

The most obvious threat to the capitalist's power is collusion among workers. Individuals can be dismissed for insubordination, but the cost of dismissing entire blocs of workers is prohibitive. Thus it is essential to stratify the workforce in order to minimise worker solidarity. Turning the division of labour into a hierarchy of diverse and antagonistic fragments is thus an effective means toward the intensification of labour.

It follows that the capitalist, operating independently of any class collusion, will in general reproduce in the enterprise the hierarchy of statuses reflecting the historical development of the working class. This will be directly evident in the day-to-day staffing of jobs—e.g. white over black, male over female, schooled over less schooled. This process, moreover, is substantially independent of the skill levels of the labour segments in question. The different economic positions of these various labour segments will tend to coincide with and thus serve to perpetuate their status positions in the larger society, barring the intervention of counteracting forces.

To the extent that labour segmentation contributes to a higher rate of exploitation, and thus indirectly to a more rapid rate of accumulation, it will in general accelerate the development of the forces of production. But there are important counter-effects. Labour segmentation will represent a barrier to the expansion of the productive forces to the extent that it restricts the supply of labour, limits the development of labour power through unequal access to education and training, and places inefficient restrictions on job assignment and the organisation of job structures. This retardation of the forces of production is entirely consistent with maximum long-run profits, given the problematic nature of the extraction of labour from labour power.

The conflict between the efficient development and deployment of labour power and the reproduction of capitalist social relations is a manifestation of the ubiquitous contradiction between the forces and relations of production.† By retarding the productive forces, and thus raising the amount of labour embodied in a unit of wage goods (i.e. by raising some elements in A), working-class fragmentation produces negative as well as positive pressures on the rate of exploitation. The negative effects may also operate through downward pressure on T , the number of hours worked by the average household. This is most clearly seen, perhaps, in the case of the sexual division of labour and its ideological manifestations which, by limiting the paid labour force participation of

† Bowles (1977) analyses schooling in the underdeveloped economies from this perspective.

women, lowers the total hours of wage labour supplied by the average family. The heterogeneity of the working class can thus represent an internal barrier to the accumulation process.

Since labour segmentation is but one among many influences on the accumulation process, the theoretical arguments here developed do not inform us of its specific importance in any particular capitalist social formation at a given historical moment. Yet numerous empirical and historical studies, to which we can here give but brief mention, do attest to the significant impact of labour segmentation in the US. Thus there is considerable historical evidence that employers have used ethnic and racial divisions among workers to maintain control over the production process and to raise the rate of exploitation (Edwards, 1977; Edwards, Reich and Gordon, 1973; Stone, 1974). Racial antagonisms have been shown to be a major determinant of the extent of unionisation and trade union weakness (Reich, 1973; Brody, 1966; Gutman, 1968; Spero and Harris, 1968). The existence of segmented labour markets with quite distinct reward and mobility structures has been documented in a number of empirical studies (Gordon, 1971; Carnoy and Rumberger, 1975; Edwards, Reich and Gordon, 1973; Buchele, 1976). Moreover, a wide variety of statistical evidence suggests that skills, at least as conventionally measured by training and cognitive achievement scores, are a weak determinant of occupational position, job performance and income. The importance of age, race and sex differences independent of skill differences has been quite widely demonstrated (Edwards, 1976; Bluestone, 1971). We have shown that even the higher income and privileged job assignments enjoyed by more schooled workers, though much celebrated by the human capital school, cannot be explained by the cognitive skills or on-the-job-training associated with higher levels of education and longer job experience (Bowles and Gintis, 1975, 1976; Ryan, 1977; Rainwater, 1974; Griliches and Mason, 1972).

In addition, the role of racial divisions in weakening labour's bargaining position is supported by the fact that the degree of racial discrimination in US urban labour markets is strongly associated with the share of income received by the top 1% of income recipients, and likewise is negatively correlated with the income share of lower and middle income whites (Reich, 1973).

Finally, numerous studies indicate that neither the capitalist organisation of work nor the assignment of individuals to positions in the firm hierarchy can be accounted for in terms of production efficiency. Worker controlled production in various countries has been associated with higher productivity and worker satisfaction (Blumberg, 1969; Riskin, 1974; Zimbalist, 1975). Even moderate movements in this direction in capitalist countries have shown similar effects, although they are sometimes discontinued due to the threat they pose to the authority relations of the enterprise (Blumberg, 1969; Whyte, 1955; Kaus, 1973).

These results cast serious doubt on the usual treatment of heterogeneous labour in Marxian value theory. Our discussion, though brief, supports two theoretical guidelines in revising this treatment.

First, even abstracting from the role of the state apparatus, the essential relations of capitalism are not captured by technical relations at the point of production and market relations among economic actors. Yet this is precisely the attempt of the conventional treatment of heterogeneous labour in the labour theory of value. Such an approach necessarily abstracts from the critical structures and processes upon which the expropriation of surplus value and the reproduction of the social relations of production depend—

the phenomenon of labour segmentation being the case in point. It follows immediately that economically relevant differences among workers—those relating to the internal structure of the working class and its relationship to capital—can no more be understood by reference to technology, skills or embodied labour time alone, than by reference to biology. Indeed, differences in these productivity-related attributes of workers are less the cause than the outcome of labour segmentation, and such differences certainly cannot explain the reproduction of such segmentation over time.

Second, rates of exploitation for different types of wage labour and in different sectors of the economy need not tend toward equalisation. The tendencies towards inequality operating at the point of production, augmented by uneven development within the capitalist mode of production, among modes of production, and even among social formations, are an integral part of the accumulation process. There is no basis in theory for their relegation to the secondary role of 'counter tendencies'. Unequal exploitation is a continuous and continually reproduced manifestation of the dynamics of the capitalist mode of production.

4. The labour theory of value with heterogeneous labour

The raw materials of our model will be the specification of relevant labour segments, the wage bundles of the various types of wage labour, the matrix of input-output relationships among goods in the economy, and the matrix of direct labour inputs involved in their production. Though we have stressed that labour segmentation is reproduced in the economic sphere, the determination of the segments and their boundaries appropriate to any particular investigation will in general depend on whatever political and ideological, as well as economic, aspects are relevant to class relations and class conflict. We shall take the wage bundles as given outside our model. These could be treated as formally endogenous, using the traditional Marxist observation that the value of labour power is simply the 'cost of reproduction of the commodity labour power'. But this approach is at best tautological, for these costs of reproduction depend on the total configuration of forces and social relations of production as reflected in the accumulation process, technical change, the class struggle and the size of reserve armies, all of which go beyond the limits of our model. We shall also take the input-output matrix and the direct labour input coefficients as given. These do not represent technical 'facts', but rather are products of the interaction of the forces and social relations of production, as reflected in the technologies in use and the state of class relations. The labour input coefficients represent a particular—but, as we have stressed, by no means technically given—outcome of the capitalist's attempt to extract labour from labour power.

For a mathematical description of this model the reader is referred to the Appendix, where we show that our proposed treatment of heterogeneous labour preserves the formal structure of value relations and the transformation of values into prices and profits. Specifically we show that: (a) the rate of profit can be expressed in terms of the vectors of rates of exploitation and organic compositions of capital; (b) there is normally a strictly inverse relationship between wages and the profit rate; (c) the profit rate is positive if and only if all the rates of exploitation, as defined below, are positive, assuming labour values are positive; (d) while negative rates of exploitation in 'common labour hours', as also defined below, are possible in our formulation, the existence of profits requires at least one positive rate of exploitation. Unpaid labour time is thus a necessary condition of profits, reaffirming Marx's central assertion concerning the nature of

exploitation for the case of heterogeneous labour, and providing an analogue to the well known Seton–Okishio–Morishima result for homogeneous labour (Morishima and Seton, 1961; Okishio, 1963); (e) in an economy undergoing simple expanded reproduction, the profit rate equals the ratio of surplus to necessary labour time, labour time being considered a vector.

To illustrate our method, consider the following example. We assume there are just three types of workers: supervisory personnel, blue-collar workers in the primary segment of the labour force, and secondary, or marginal, workers.† Four commodities are produced: a wage and capital good, ‘food’; a pure capital good, ‘steel’; a pure wage good, ‘housing’; and finally a pure luxury good, ‘Mercedes’. The production relationships for the four goods and the wage bundles of the three types of labour are illustrated in Table 1.‡

Table 1. *Production relations and wage bundles*

A. Production relations: inputs required per unit of output

Inputs	Outputs			
	Food	Steel	Housing	Mercedes
Food	0.10	0.20	0.30	0.20
Steel	0.30	0.10	0.20	0.30
Housing	—	—	—	—
Mercedes	—	—	—	—
Supervisory labour	0.07	0.03	0.10	1.00
Primary labour	0.42	0.09	0.40	0.70
Secondary labour	0.21	0.18	0.70	0.40

B. Wage bundles: amount of goods per hour of labour

Type of labour	Commodities			
	Food	Steel	Housing	Mercedes
Supervisory labour	0.25	—	1.00	—
Primary labour	0.10	—	0.50	—
Secondary labour	0.00	—	0.25	—

What can we say about this economy? The basic results, solving equations (2) and (6) in the Appendix, are presented in Table 2. By inverting the matrix of input–output coefficients of the production equations and multiplying the result by the matrix of labour inputs, we arrive in the usual manner at the value of each commodity. The value of a unit of food, for example, is a vector (the first column of Table 2A) representing 0.10 hours of supervisory labour, 0.54 hours of primary labour, and 0.32 hours of

† For simplicity we assume that there are no skill or training differences among these three types of workers. To construct an explicit submodel of training and education would be a useful complement to the method, but would merely complicate our exposition. Similarly, we assume for simplicity that there are no such differences among members of each labour segment. Note that when labour time embodied in worker training is included, their magnitudes will be multidimensional. Hence the ‘simple’ labour time of every type of labour will itself be multidimensional!

‡ The first column of Table 1A, for example, indicates that the production of a unit of food requires 0.21 hours of secondary labour, 0.42 hours of primary labour, 0.07 hours of supervisory labour, 0.1 units of food, and 0.3 units of steel. Similarly, Table 1B indicates that the wage bundle of supervisory labour consists of 0.25 units of food and one unit of housing.

Table 2. *Values, prices, money wages and profits*
A. *Values and prices*

	Food	Steel	Commodities Housing	Mercedes
1. Values in terms of:				
Supervisory labour	0.10	0.05	0.14	1.04
Primary labour	0.54	0.22	0.61	0.87
Secondary labour	0.32	0.27	0.85	0.55
2. Prices per unit	\$2.77	\$1.53	\$4.00	\$9.57

B. *Money wages and profits*

1. Money wages per hour of	
Supervisory labour	\$4.69
Primary labour	\$2.28
Secondary labour	\$1.00
2. Rate of profit	24.26%

secondary labour. The profit rate, related in the Appendix to the vector of rates of exploitation and the vector of organic compositions of capital, is applied to the prices of production equations in the usual manner to yield the relative prices of the goods and the various money wage rates. Arbitrarily setting the wage of secondary labour at \$1.00, we calculate the price of food as \$2.77, and similarly for other prices and wage rates.

This model exhibits some novel properties. First, it is clear that we cannot ask how well values approximate to relative prices, for values are three-dimensional vectors. Thus while the relative price of steel *vis-à-vis* food is 0.55, the relative value is the vector (0.50, 0.41, 0.84). This is no problem, however, for as Medio (1973) and Morishima (1973) have shown, the labour theory of value attempts first to express the profit rate in terms of labour time, and then determines prices from costs of production. The historically renowned approximation between relative value and relative price, a hold-over from Ricardian economics, may indeed obtain at times as an empirical result, but it is a theoretical casualty of modern treatments of the traditional 'transformation problem'.

Nor can we in general ask whether the 'profit rate in value terms', or the ratio of surplus to necessary labour time, approximates the actual rate of profit. For in our model with heterogeneous labour there are three such profit rates in value terms—one for each type of labour. In our model, the actual rate of profit is 24.26%, while the profit rate in value terms is the vector (44.69%, 20.49%, 15.00%). A formal transformation of these rates into the actual rate of profit is presented in the Appendix. As an approximation, we may weight each profit rate in value terms by the proportion of the total labour force in the corresponding labour segment, which, not surprisingly, gives us the rather inaccurate figure of 20.69%. If, following Bortkiewicz and later treatments, we exclude the luxury good sector from this calculation (since it does not affect the actual rate of profit), this weighted profit rate in value terms becomes 24.45%—a good approximation indeed.† In short, this example illustrates what is shown in the Appendix; that the transformation of value and surplus value into price and profit is not rendered unfeasible by the assumption of heterogeneous labour.

† The first approximation is suggested by Marx's own incorrect solution to the transformation problem where each sector is weighted by its contribution to total value. Marx considered this solution to be exact, but it is not in fact. The second approximation has been suggested by Bortkiewicz and others. Why this approximation can be expected to be in general adequate is discussed in the Appendix.

Can we say more? Does our approach suggest any important empirical applications? We think it does, and will provide here but one illustration by way of a numerical example addressed to the politically important question of the structure of exploitation in the working class. Since Lenin popularised the notion of the 'aristocracy of labour', Marxists have grappled with the question: does a part of the working class 'exploit' the rest, including workers in the peripheral countries of the capitalist world system (Lenin, 1939; Emmanuel, 1972)? The practical implications of this question require close attention to political and ideological considerations: value theory, by itself, cannot settle questions of political strategy. Nonetheless, such fundamental issues as the identification of the boundaries of the working class, and the depth of the contradictions among various segments of the working class, cannot be successfully addressed without an answer to this question. And answers, unfortunately, have ranged from the unlikely assertion that, in the advanced capitalist countries, the transfer of value through unequal international exchange renders all workers on balance not exploited but rather exploiters, to the bizarre contention that *all* employees, by dint of their wage labour status, are to be considered the proletariat.

This paper contributes to the debate by showing that the question can be formulated in terms compatible with an extended Marxian value theory, and showing that labour segmentation can be understood in terms of the fundamental dynamics of surplus value extraction. In the framework of our model (all of whose numerical magnitudes are of course hypothetical) we can also provide quantitative answers to the question: who is consuming the fruits of whose labour, and in what quantities? This of course does not decide the issue, but it does, we believe, provide a necessary part of the analysis. From the wage bundles (Table 1B) and the commodity values (Table 2A) we calculate by matrix multiplication the value of labour power for each type of labour (Table 3).

Table 3. *The value of labour power and the structure of exploitation*

	Type of labour			
	Supervisory labour (1)	Primary labour (2)	Secondary labour (3)	Total labour hours (4)
<i>Hours of labour embodied in the hourly wage bundle of</i>				
Supervisory labour	0.16	0.74	0.93	1.83
Primary labour	0.08	0.36	0.46	0.90
Secondary labour	0.03	0.15	0.21	0.39
<i>Rate of exploitation in common labour hours</i>				
	-46%	12%	150%	

Notes: column (4) is the sum of columns (1), (2) and (3). The rate of exploitation in common labour hours is unity minus column (4) divided by column (4).

For instance, we see that for one hour of work the supervisor receives consumption goods embodying 0.16 hours of his own type of labour, 0.74 hours of primary labour, and 0.93 hours of secondary labour. If we ignore for the moment the segmentation of the labour force, and measure the wage bundle in 'common labour hours', the total labour hours in the hourly wage bundle of the three types of labour are, by addition, 1.83 hours for supervisory personnel, 0.90 hours for primary labour, and 0.39 hours for secondary

labour. The fact that on balance supervisory personnel consume a wage bundle embodying more common labour hours than they work motivates us to introduce a concept which, while formally distinct, is in the spirit of Marx's rate of exploitation. We define the 'rate of exploitation in common labour hours' as the quantity one *minus* total labour hours embodied in the wage bundle *divided by* total labour hours embodied in the wage bundle. This appears in the last row of Table 3. We arrive at the interesting result that supervisors enjoy a negative 'rate of exploitation in common labour hours'.

As a final example of how this framework may be used to discuss the distribution of labour values, we suppose that capitalists spend 40% of their income on luxuries, and divide the remainder equally for investment in food and steel (the numbers are by no means critical). Then by solving the production equations presented in the Appendix, and arbitrarily specifying the scale of the economy, we arrive at Table 4, which shows the distribution of the aggregate labour times among types of labour and between labour and capital.†

Table 4. *Distribution of labour time among labour and capital*

	Number of hours of			Total common labour hours received
	Supervisory labour	Primary labour	Secondary labour	
Received by:				
Supervisory labour	1895	8580	10,798	21,275 (1)
Primary labour	3224	14,486	18,693	36,304 (2)
Secondary labour	1671	7248	10,185	19,104 (3)
Capitalist	4787	10,263	8264	23,315 (4)
Total labour hours expended	11,579	40,578	47,841	100,000 (5)

Note: numbers do not sum precisely, due to rounding.

The fact that all workers, including supervisors, surrender their distinct types of surplus value to the capitalist (Table 4, row 4) allows us to give a consistent economic interpretation to the contradictory class location of the supervisory employees.‡ While all types of labour are exploited *vis-à-vis* the capitalist, some workers are in effect 'exploiters' *vis-à-vis* others. The fact that supervisory labour is a net recipient of surplus labour time (measured in common labour hours) dramatises the discrepancy between delimitations of the boundaries of the working class based on wage labour status and those based on an accounting of surplus labour time.

Appendix

In this Appendix we shall show that many of the results of Marxian value theory can be extended to the case of heterogeneous labour. Our general result will be that if proper weights are given to the various industries in the value calculations, the actual rate of profit is equal to (a) the ratio of surplus to necessary labour time, and (b) the ratio of the rate of exploitation to the organic composition of capital plus unity. As in the case of homogeneous labour, the proper industry weights will be their relative weights in contributing to uniform expanded reproduction (Morishima, 1973; Medio, 1973). The novelty in our extension is that the value magnitudes are multi-dimensional, so it is not obvious that the ratios in question exist.

† Because of the linearity of the model, the results in Table 4 are independent of the scale of operation, except for a factor of proportionality.

‡ See Wright's (1976) admirable discussion of contradictory class locations.

In addition, assuming the extended input-output matrix is irreducible except for the presence of luxury goods, we shall prove the following results. First, we shall exhibit the strictly inverse relationship between wages and the profit rate. Second, we shall show that the economy can support a positive profit rate if and only if all the rates of exploitation (one for each type of labour) are positive, assuming that some of each type of labour is used directly or indirectly in the production of all non-luxury goods. Third, we shall demonstrate the heterogeneous analogue to the so-called Seton-Morishima-Okishio theorem: the profit rate must be less than the largest of the various rates of exploitation, and hence is limited by them. Lastly, we shall show that the profit rate can be positive only if at least one of the rates of exploitation in terms of common labour hours is positive.

We assume an n -good economy in which the period of turnover of capital is one. In this economy, the production of one unit of good j uses up c_{ij} units of good i and l_{rj} units of labour of type r (we shall call this ' r -labour'), for $r = 1, \dots, m$. We assume that the hourly wage bundle of each type of worker is given, and b_{ir} is the amount of good i in the wage bundle of r -labour. Finally, let $C = (c_{ij})$, $L = (l_{rj})$, and $B = (b_{ir})$.

If $x = (x_1, \dots, x_n)$ is the gross output of the economy and F is a vector representing the amount of each good acquired by the capitalists (the surplus product), the production conditions consistent with full employment of non-labour resources are given by

$$x = (C + BL)x + F, \quad x \geq 0, F \geq 0. \quad (1)$$

For simplicity, (1) abstracts from the sector of the economy in which labour is embodied in workers through education and training. These production conditions allow us to derive commodity values as follows. Let the value of good i be given by $\lambda_i = (\lambda_{1i}, \dots, \lambda_{mi})$, a vector of the direct and indirect labour hours of each of our m types embodied in a unit of good i . If we let $A = (\lambda_{ri})$, then we have $A = AC + L$, so

$$A = L(I - C)^{-1}. \quad (2)$$

We call λ_{ri} the ' r -value of good i '.

The value of labour power in this model is clearly a matrix, since there are m wage bundles and the value of each is an m -vector. If we define v_{rs} as the r -value of the wage bundle of s -labour, and define $V = (v_{rs})$, it is clear that $V = AB$.

We now proceed to construct expressions for necessary and surplus labour time, the rate of exploitation and the organic composition of capital for the economy.† Let $y = (y_1, \dots, y_n)$ be any set of non-negative outputs for the n industries. We shall call the economy operating at level y the ' y -economy', and extend our previous definitions to aggregates in the y -economy. Thus the value of output in the y -economy is $\lambda(y) = Ay$; constant capital in the y -economy is $k(y) = ACy$; variable capital, finally, is given by $v(y) = ABLy = VLy$. Thus we arrive immediately at the following expressions, natural extensions of the usual Marxian concepts:

$$\text{necessary labour time} \quad n(y) = k(y) + v(y) = A(C + BL)y \quad (3)$$

$$\text{surplus labour time} \quad s(y) = \lambda(y) - n(y) = (I - AB)Ly. \quad (4)$$

If we define the quotient of two equi-dimensional vectors as the quotient of their respective terms, we may represent the 'profit rate in value terms' as $\pi(y) = s(y)/n(y)$. This is a natural extension of the corresponding concept in the case of homogeneous labour. Of course there is no reason to suppose in general that the various terms $\pi_r(y)$, where $\pi(y) = (\pi_1(y), \dots, \pi_m(y)) = s(y)/n(y)$ will be equal, much less that they equal the economy-wide actual rate of profit. Neither is $s(y)/n(y)$ equal to the profit rate in the case of homogeneous labour.

To define natural extensions of the concepts of rate of exploitation and organic composition of capital, we note that since Ly is the total direct labour time and $v(y)$ the total paid labour in the y -economy, unpaid labour time is just $(L - VL)y$. Thus the rate of exploitation, being the ratio of unpaid to paid labour time, can be expressed as $\sigma(y) = (I - V)Ly/VLy$. Formally, $\sigma(y)$ has its traditional meaning when applied to the labour force as a whole; for we have, corresponding to

† The procedure followed here is an extension of the method and simplification of the proofs of Medio (1973). Medio suggests an extension to differentially exploited homogeneous labour and Roemer (1975) analyses this case in depth. Our treatment differs in that labour is heterogeneous as well as differentially exploited.

the traditional equation 'total value = constant capital + variable capital \times (1 + rate of exploitation)', the following:

$$\lambda(y) = k(y) + v(y)(1^* + \sigma(y)), \quad (5)$$

where 1^* is the m -vector with entries unity.

Substantively, however, $\sigma_r(y)$ does not represent the 'own rate of exploitation' of r -labour (or in everyday language, 'how exploited' r -labour is), since r -labour contains in its wage bundle other types of labour, which are not expressed in any way in $v_r(y)$, and hence neither in $\sigma_r(y)$. Conversely, r -labour embodied in the goods consumed by the working class need not be consumed by r -labour itself. In short, $\sigma_r(y)$ does not represent the relationship between hours worked and hours embodied in the wage bundle of r -labour. We shall term this latter, conceptually distinct, concept the ' r -rate of exploitation in common labour hours', and write $e_r(y) = (1 - \tilde{v}_r)/\tilde{v}_r$, where

$$\tilde{v}_r = \sum_{i,s} \lambda_{si} b_{ir}$$

(see Table 3 for an illustration). Keeping this important distinction in mind, we may now represent the organic composition of capital as the m -vector $\omega(y) = k(y)/v(y)$. This leads to the expression $\sigma(y)/(\omega(y) + 1^*)$ for the profit rate in value terms for the y -economy, where 1^* is defined as in (5). The reader can easily show that $\pi(y) = \sigma(y)/(\omega(y) + 1^*)$, corresponding to the familiar assertion that the 'rate of profit in value terms equals the rate of exploitation divided by the organic composition of capital plus unity'.

How do these m profit rates in value terms relate to the rate of profit? We shall find an expression for the profit rate which will be independent of the actual level of outputs in the economy, provided $C + BL$ is irreducible except for the presence of luxury goods. Let $p = (p_1, \dots, p_n)$ and π represent the price structure and the profit rate for such a situation. Then the price equations are given by $(1 + \pi)p(C + BL) = p$, so we have

$$p \left[\frac{1}{1 + \pi} I - (C + BL) \right] = 0. \quad (6)$$

That is, $1/(1 + \pi)$ is a left-eigenvalue of the matrix $(C + BL)$, with left-eigenvector p . By a theorem of Frobenius (Gantmacher, 1959), if $(C + BL)$ is irreducible, this matrix has a positive eigenvector, and this corresponds to a maximal, positive eigenvalue. In fact, $C + BL$ is not irreducible because of the existence of luxury goods, but it is trivial to extend the theorem to this case.

Since the profit rate is then uniquely determined by the extended input-output matrix $(C + BL)$, it is clear that the profit rate is determined independently of the actual levels of output in the economy.[†] But since the various profit rates in value terms are defined only with respect to a particular output structure y , it is clear that we can specify an equation relating these rates to the actual rate of profit only by specifying a particular—and hopefully reasonable—output structure y^* . We shall show that such a y^* exists providing the profit rate is positive, and corresponds to the sectoral proportions in the economy in the situation where it follows the path of maximal balanced growth—i.e. where luxuries are not consumed and investment funds are allocated to achieve market clearance.

In our formulation the heterogeneous labour inputs are represented by their wage bundles added together with all other inputs. The labour categories are thus suppressed, rendering our solution compatible with any number of labour segments. Equally important, because the economy follows balanced growth, input and output vectors will be proportional, so we may compare their magnitudes without reference to prices. Thus our y^* -economy may be treated as though it were composed of a single commodity. As in the case of Ricardo's corn model, we may determine the profit rate for the entire economy in terms of the surplus of outputs over inputs. This is done as follows.

Let a^* and y^* be the maximal right-eigenvalue and corresponding right-eigenvector of $(C + BL)$. Then since the maximal left- and right-eigenvalues must be equal (Franklin, 1968), we have $a^* = 1/(1 + \pi)$. By Frobenius's theorem, we also know that y^* is positive, except that luxury goods clearly have weight zero. By the definition of the eigenvalue, we have $(C + BL)y^* = a^*y^*$. That is, the outputs y^* are produced by inputs a^*y^* , so the y^* -economy undergoes balanced expanded

[†] Of course the input-output matrix itself is affected by, among other things, the past history of profit rates, but this point is irrelevant to our conclusion.

reproduction at the rate $(y^* - a^*y^*)/y^* = (1 - a^*)/a^* = \pi$, where π is the profit rate in the original *x*-economy.

We may now show that if the various sectors in the *x*-economy are weighted by the proportions y^* (where y^* is scaled down so that the sum of its entries is unity), then the m profit rates in value terms are all equal, and equal the money rate of profit. We have $k(y^*) + v(y^*) = ACy^* + VLy^* = A(C + BL)y^* = \Lambda a^*y^* = a^*\lambda(y^*)$. By equation (5), we then have $\sigma(y^*) = (1 - a^*)\lambda(y^*)/v(y^*)$. But also, $\omega(y^*) = k(y^*)/v(y^*) = a^*\lambda(y^*)/v(y^*) - 1$. Thus we have

$$\pi(y^*) = \frac{\sigma(y^*)}{\omega(y^*) + 1} = \frac{(1 - a^*)\lambda(y^*)/v(y^*)}{a^*\lambda(y^*)/v(y^*)}.$$

But this last expression is just the m -vector all of whose entries are $(1 - a^*)/a^*$, which is simply π . This proves the assertion.

This assertion also has the following obvious corollary. If we weight luxury goods industries in the *x*-economy with weights zero, and weight other industries according to the production levels they would exhibit were investment geared towards balanced growth, then, given the existing consumption patterns of workers the profit rates in value terms will equal the money rate of profit. This explains the ability of the profit rates in value terms in our example of section 4 to predict the profit rate so closely.

We shall now demonstrate some of the traditional theorems relating profit rates to surplus value and the rate of exploitation, extended to the case of heterogeneous labour. These will depend on the properties of what we shall call quasi-irreducible matrices. We call a matrix M *quasi-irreducible* if there is a non-negative $n_1 \times n_1$ irreducible matrix A_1 and a non-negative $n_1 \times (n - n_1)$ matrix A_{11} such that, with a proper re-ordering of the columns of M , we have

$$M = \begin{pmatrix} A_1 & A_{11} \\ 0 & 0 \end{pmatrix}.$$

Finally, every column of M is assumed to have a positive entry. We begin by proving some preliminary lemmas concerning quasi-irreducible matrices which will be used in the sequel.

Lemma 1. Let M be quasi-irreducible. Then M has an eigenvector $q = (q_1, 0)$, where q_1 is a positive n_1 -vector. The vector q is unique up to a factor of proportionality, and its corresponding eigenvalue is positive and of maximum absolute value. We call this eigenvector the Frobenius vector of M , and the corresponding eigenvalue the Frobenius root of M . We denote the Frobenius root by $F(M)$.

Proof: let A_1 and A_{11} be defined as in the definition of quasi-irreducibility. Then by Frobenius' Theorem, A_1 has a positive eigenvector u and a corresponding positive eigenvalue $u' = F(A_1)$. It is then clear that $q = (u, 0)$ represents the eigenvector of M in question, and corresponds to the Frobenius root $u' = F(M)$.

Lemma 2. If M is quasi-irreducible, its Frobenius root is a strictly increasing and continuous function of all m_{ij} , where $i, j \leq n_1$.

Proof: let $A_1 = (a_{ij})$ and $u' = F(A_1)$. Then u' is a strictly increasing (Gantmacher, 1959) and continuous (Franklin, 1968) function of each a_{ij} . But $F(M) = F(A_1) = u'$, and $m_{ij} = a_{ij}$ for $i, j \leq n_1$.

Lemma 3. Suppose M is quasi-irreducible, $F(M) < 1$, and let $N = (n_{ij}) = (I - M)^{-1}$. Then N exists, is non-negative, and if $i \leq n_1$, we have $n_{ij} > 0$.

Proof: given the assumptions of the lemma, there is, by Lemma 1, an n_1 -vector, u , such that $u > 0$ and $(u, 0) > (u, 0)M$. Then $u > uA_1$, so $F(A_1) < 1$. Thus $(I - A_1)^{-1} > 0$ (Morishima, 1973). But

$$N = (I - M)^{-1} = \begin{pmatrix} (I - A_1)^{-1} & (I - A_1)^{-1}A_{11} \\ 0 & I \end{pmatrix},$$

which clearly satisfies the lemma. For A_{11} has a positive entry in every column, so $(I - A_1)^{-1}A_{11}$ is strictly positive.

We now apply these lemmas to our economic model, as defined in equations (1) through (6). We begin with Lemma 4.

Lemma 4. The profit rate is a strictly decreasing function of all b_{ij} , as well as of all c_{ij} and l_{ij} , where $j \leq n_1$. That is, the profit rate varies strictly inversely with all non-luxury entries in the matrices B , C and L .

Proof: since $(1 + \pi)^{-1} = F(C + BL)$, the lemma follows immediately from the previous lemmas.

Lemma 5. Suppose some labour is used to produce each non-luxury good. Then all labour values are non-decreasing functions of the input-output coefficients c_{ij} , and strictly increasing functions of l_{ij} where i and j represent non-luxury goods.

Proof: by assumption C is quasi-irreducible and $(C + BL)$ is productive. Thus $F(C) < F(C + BL) < 1$, so $F(A_1) = F(C) < 1$, where A_1 is the upper right-hand corner of C . Now write $A = (A_1, A_{11})$ and $L = (L_1, L_{11})$ in the obvious way so that

$$(A_1, A_{11}) = (L_1, L_{11}) \begin{pmatrix} (I - A_1)^{-1} & (I - A_1)^{-1} A_{11} \\ 0 & I \end{pmatrix}. \quad (7)$$

Thus we have $A_1 = L_1(I - A_1)^{-1}$ and $A_{11} = A_1 A_{11} + L_{11}$. Suppose ΔA_1 represents a suitably small increase in the non-luxury inputs to non-luxury goods, and let ΔA represent the corresponding change in A . Then clearly $(A_1 + \Delta A_1)(I - A_1 - \Delta A_1) = L_1$, and multiplying both sides by $(I - A_1)^{-1}$, we get $(A_1 + \Delta A_1)(I - \Delta A_1(I - A_1)^{-1}) = A_1$. Hence $\Delta A_1 = (A_1 + \Delta A_1)\Delta A_1(I - A_1)^{-1}$. Now A_1 and $(I - A_1)^{-1}$ are strictly positive by Lemma 3, and since ΔA_1 has at least one non-negative entry, it is easy to see that the right-hand side of the last equation is strictly positive. Thus $\Delta A_1 > 0$. Since $A_{11} = A_1 A_{11} + L_{11}$, and every column of A_{11} has a positive entry, we must have $\Delta A_{11} > 0$. Thus $\Delta A > 0$. The rest of the proof, involving changes in A_{11} , is now trivial.

Lemma 6. Suppose all money wages are positive and the profit rate is zero. Then all the rates of exploitation are zero.

Proof: let p and w represent prices and money wages, so we have $p = pC + wL$ and $w = pB$. From $A = AC + L$ we derive $wA = wAC + wL$. Thus $(p - wA) = (p - wA)C$. Since BL is positive by assumption, we have $F(C) < F(C + BL) = 1$, as the profit rate is assumed zero. Thus C cannot have an eigenvalue of unity, by Lemma 1. Hence we must have $p = wA$. Now if L_r and A_r are the r th rows of the matrices L and A , respectively, we have $\sigma_r(x) = (L_r x - A_r B L x) / A_r B L x$. Suppose $\sigma_r(x) \neq 0$ for some r , so $L_r x \neq A_r B L x$. Then since w is strictly positive, $w L_r x \neq w A_r B L x$. Thus we have $w L x \neq p B L x = w B L x = w L x$, which is a contradiction. Hence all the rates of exploitation are zero.

We may now exhibit some basic properties of the extension of the labour theory of value to heterogeneous labour. From Lemma 4 we have Theorem 1.

Theorem 1. The profit rate is strictly inversely related to the size of the wage bundle of any labour segment.

We may also demonstrate Theorem 2.

Theorem 2. The profit rate is positive if and only if all rates of exploitation are positive, assuming some of each type of labour appears directly or indirectly in each non-luxury good.

Proof: this fundamental theorem is rather simple if we assume $F \geq 0$ in equation (1); i.e. if we assume the capitalists replace all depreciated capital goods before investing and consuming. But this assumption is unduly restrictive. Instead, we define a series of new economies, called C^* -economies, which have the same wage bundles B and labour inputs L as our original, but new input-output structures C^* . We denote prices, profits, values and money wages in the C^* -economy by $p^* = p^*(C^*)$, $\pi^* = \pi^*(C^*)$, $A^* = A^*(C^*)$, and $w^* = w^*(C^*)$, respectively. By Lemma 4, π^* is a strictly decreasing and continuous function of the non-luxury entries of C^* ; and by Lemma 5, A^* is a strictly increasing function of the non-luxury entries in C^* if the profit rate is positive.

We first assume that all the rates of exploitation are positive, and prove the profit rate is positive. We begin by showing that $\pi^*(0) > 0$. For assume the contrary. Then $p^* \leq p^* B L$, and $w^* = p^* B$. But since the rates of exploitation are positive, we have $L_r x > A_r B L x \geq L_r B L x$. Hence $w^* L_r x > w^* L_r B L x$, from which we derive $w^* L x > w^* L B L x = p^* B L x = w^* L x$, which is a contradiction. Hence $\pi^*(0) > 0$. Now suppose $\pi = \pi(C) < 0$, and let $f(a) = \pi^*(aC)$, where $0 < a \leq 1$. Clearly aC is quasi-irreducible, so $f(a)$ is a continuous and strictly increasing function of a , by Lemma 4. Moreover $f(1) < 0$ by assumption, and $f(0) = \pi^*(0) > 0$, as we have shown above. Hence there

is some number $0 < b < 1$ such that $f(b) = 0$. If we set $C^* = bC$ it follows that $\pi^*(C^*) = 0$ and $C^* \leq C$. Clearly C^* is irreducible and from Lemma 5 it is obvious that $A^* < A$. Thus $L_{rx} > A_r BLx \geq A_r^* BLx$, which implies that the rates of exploitation for the C^* -economy are positive. But by Lemma 6, the profit rate for the C^* -economy cannot be zero. Hence we have a contradiction. Thus $\pi > 0$.

Now assume $\pi > 0$, and we show the rates of exploitation are all positive. It is clear that there is some number $b > 1$ such that if we set $C^* = bC$, then $\pi^*(C^*) = 0$. Clearly C^* is irreducible and $C^* > C$, and from Lemma 5 it is obvious that $A^* > A$. But by Lemma 6, we have $L_{rx} = A_r^* BLx$ for all r , so $L_{rx} = A_r^* BLx > A_r BLx$. Hence all rates of exploitation are positive, and the theorem is proved.

The final general theorem we shall prove is an extension of the Seton-Okishio-Morishima theorem cited above. This theorem states that the profit rate is always less than the rate of exploitation, and therefore limited by it. In the case of heterogeneous labour, we have Theorem 3.

Theorem 3. The profit rate is less than or equal to at least one of the rates of exploitation.

Proof: suppose $\sigma_r(x) = (L_{rx} - A_r BLx)/A_r BLx < \pi$ for all r . Then $L_{rx} < (1 + \pi)A_r BLx$, and we have $Lx < (1 + \pi)ABLx$. Let p and w be money prices and wages, so $p = (1 + \pi)(pC + wL)$ and $w = pB$. Thus we have $p(I - (1 + \pi)C) = p((I - C) - \pi C) = (1 + \pi)wL$, so it follows that $p(I - \pi C)(I - C)^{-1} = (1 + \pi)wL(I - C)^{-1} = (1 + \pi)wL(I - C)^{-1}$. But $(I - C)^{-1}$ is strictly positive by Lemma 3. Hence $p > (1 + \pi)wA$. Thus we have $wLx < (1 + \pi)wABLx < pBLx = wLx$, a contradiction. Hence $\sigma_r(x) \geq \pi$ for some r , and the theorem is proved.

It should be noted that all of the above propositions have been demonstrated without having to reduce labour to a common unit. A casualty of the process, however, is the attractive connotation of the homogeneous rate of exploitation as the ratio of unpaid to paid labour for an individual worker or group of workers. By contrast, our formulation of the rate of exploitation emphasises the fact that expropriation of surplus value is a relationship between capital and the working class as a whole, rather than, as it might appear, a relationship between capital and each labour segment taken separately. To rehabilitate the former notion of exploitation, we have defined in the text the rate of exploitation in common labour hours, for r -labour by $e_r = (1 - \tilde{v}_r)/\tilde{v}_r$, where

$$\tilde{v}_r = \sum_s v_{rs}.$$

This concept will have substantive import only in those situations in which it is reasonable to attribute to all labour hours an 'equal worth'.†

We can state the following theorem in this case.

Theorem 4. The profit rate is positive only if at least one rate of exploitation in common labour hours is positive.

Proof: assume all the rates of exploitation are non-positive. Then we must have $\tilde{v}_r \geq 1$ for all r . Since $\sigma_r(x) = (L_{rx} - V_r)x/V_r x$, we have $L_{rx} > V_r x = A_r BLx$. This can be written as

$$\sum_j L_{rj} x_j > \sum_{i,s,j} \lambda_{ri} b_{is} l_{sj} x_j.$$

Thus

$$\begin{aligned} \sum_{r,j} L_{rj} x_j &> \sum_{i,s,r,j} \lambda_{ri} b_{is} l_{sj} x_j = \sum_{r,s,j} v_{rs} l_{sj} x_j \\ &= \sum_{s,j} \tilde{v}_s l_{sj} x_j < \sum_{s,j} l_{sj} x_j. \end{aligned}$$

This is a contradiction. Hence $e_r > 0$ for some r .

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† Note that the rate of exploitation is as presented in equation (5), and does not represent the vector of rates of exploitation in common labour hours as defined by $e_r = (1 - \tilde{v}_r)/\tilde{v}_r$.

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