

# Ladislaus Von Bortkiewicz

**“Wertrechnung und Preisrechnung im Marxschen System”**  
*Archiv für Sozialwissenschaft und Sozialpolitik*, vol. XXV, 1907

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*International Economic Papers*, no 2, MacMillan, 1952



# VALUE AND PRICE IN THE MARXIAN SYSTEM

By LADISLAUS VON BORTKIEWICZ

Translated from German\* by J. Kahane

The quantitative incongruity of value and price (more precisely: price of production) forms a specific characteristic of Marx's theory of the capitalist economy<sup>1</sup>.

In this context, value can have no other meaning than that of a magnitude which indicates how many units of the good serving as a measure of value are obtained in exchange for a commodity or for a unit of this commodity. In this sense, value is merely the index of an exchange relationship and must not be confused with the so-called "absolute value" of a commodity, which is identical with the quantity of labour employed in its production<sup>2</sup>.

Whilst, however, "value" itself (for the sake of brevity, I do not say either relative value" or "exchange value") and "absolute value" mean two quite different things, a firm quantitative relationship nevertheless prevails between them: the values of different goods bear the same proportion to each other as their absolute values, and this proportionality, which constitutes the substance of the Marxian Law of Value, holds for any measure of value.

Labour, or more exactly, wage-labour, can be used as such a measure of value<sup>3</sup>. The value of a good A could then be expressed as a certain number of time units of labour, e.g. as 12 days of labour. This would mean that good A or its equivalent, could pay the wages for 12 days of labour. If the value of

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\* "Wertrechnung und Preisrechnung im Marxschen System", *Archiv für Sozialwissenschaft und Sozialpolitik*, vol. XXV, 1907, pp. 10-51. 445-488. This is the second and third of a series of 3 articles published under this heading in 1906/7. The first article summarises previous discussion of Marx's theory of prices and profit by such writers as Lexis, Böhm-Bawerk, Hilferding and Tugan-Baranowski.

<sup>1</sup> *Translator's Note*: The author quotes from the Third German edition of *Das Kapital* and from *Theorien über den Mehrwert*, edited by Karl Kautsky. Stuttgart, 1905. In some cases, page references to English language editions have here been added, in square brackets and italics. Unless, otherwise stated, these refer to *Capital*, translated from the First German Edition by Ernest Untermann, edited by Frederick Engels, Chicago, 1909, and to *Theories of Surplus Value*, translated by G. A. Banner and Emile Burns, London, 1951.

<sup>2</sup> Marx himself avoids the term "absolute value" and instead occasionally uses either "real value" (e.g. in *Theorien über den Mehrwert*, Vol. II, Part I, p. 150, footnote), or "immanent value" (*Das Kapital* III<sub>1</sub>, p. 147). As a rule however, Marx uses the word "value" by itself even when he has absolute value in mind, (e.g. in *Das Kapital*, I, pp. 6-7). This does not give rise to misunderstandings, since the context always reveals clearly which value is meant. Critics have repeatedly dealt with the problem of absolute value in Marx. Thus A. v. Wenckstern (Marx, 1896, pp. 17-21) blames Marx strongly for operating with this concept, whilst, on the contrary, S. Frank (*Die Wertheorie von Marx irhe Bedeutung*, in Russian, St. Petersburg, 1900, p. 182) regards it as a great merit of Marx to have kept the two concepts of relative and absolute value strictly separate. This whole problem is in reality of no practical importance; the question is simply whether it serves any useful purpose to call the quantity of labour needed to produce a commodity "value", whilst at the same time talking of "value" when dealing with the index of an exchange-relationship. There is no point in either attacking or defending the concept of absolute value, since firstly, this concept does not, in itself, involve the notion that goods are exchanged in proportion to the quantities of labour they contain, or, alternatively, in proportion to their absolute values, and, secondly, the problem of an absolute value is by no means the same as the problem of an unchangeable (and in this sense "absolute") measure of value. Ricardo did not always pay due regard to these two points and thereby caused much trouble with his "real value". namely by rising sterile discussions. This concept of "real value" to express the quantity of labour required to produce a commodity was better developed in McCulloch, *Principles of Political Economy*, London, 1870 (reprint of the first edition of 1825), pp. 116-118. Cf. Frank, *loc. cit.*, p.175.

<sup>3</sup> According to Marx one should talk here not of labour, but of *labour power*. This point is developed further on.

another good B is 6 days of labour, and if therefore 2 B are given in exchange for A, then we must deduce, according to the Law of Value, that A requires for its production twice the labour required for the production of B, or, in other words, that the absolute value of A is double the absolute value of B. These absolute values would, however, be expressed not by 12 and 6 days of labour respectively, but, if we assume the rate of surplus value to be 50%, for instance, by 8 and 4 days of labour respectively. If wages are 4 Mark a day, the value of A would be 48 Mark, but the production of A would have required the capitalist to spend only 32 Mark on wages.

This much needed to be said in order to avoid misunderstandings due to the multiple significance of the concept of value, in what follows, value will always be taken to mean the index of an exchange-relationship, unless the contrary is explicitly stated. It is of the essence of that concept of value that its magnitude be determined according to the (Marxian) Law of Value.

This in fact constitutes the difference between value and the price of production<sup>4</sup> (for which we shall briefly say "price"), since the latter is formed not according to the Law of Value, but according to the Law of the Equal Rate of Profit. Price is also, however, like value, the index (or exponent<sup>5</sup>) of an exchange-relationship, and, again, just like value, represents a purely theoretical structure, although price, *i.e.* the price of production, which is essentially the same as the "natural price" of the classical economists, represents a higher degree of approximation to reality than does value<sup>6</sup>. Value-calculation means to determine the exchange-relationships of goods according to the Law of Value. Price-calculation means to determine the same exchange-relationships according to the Law of the Equal Rate of Profit<sup>7</sup>.

Marx uses the following model to elucidate the relationship between value-calculation and price-calculation<sup>8</sup>.

He distinguishes between several spheres of production, which differ with respect to the organic composition of the capital sums invested in them. In each sphere of production, let  $c$  be the value of the constant capital,  $v$  that of the variable capital,  $m$  that of the surplus value produced,  $a$  the fraction of the constant capital entering into the value of the product, and  $W$  the value of the (yearly) output. On these assumptions:

$$(1) \quad W = ac + v + m$$

The rate of surplus value  $\frac{m}{v}$  is assumed to be the same in all spheres of production. It follows that the rate of profit  $\frac{m}{c+v}$  varies in the different spheres

<sup>4</sup> The distinction made by Marx between "price of production" and "real price of production" (*Das Kapital*.III<sub>1</sub>, p. 274) need not be considered here. This distinction is connected with the peculiar part assigned by Marx to commercial (in contrast to industrial) capital. This is discussed further on.

<sup>5</sup> *Das Kapital*, I, p. 72 Cf. *Das Kapital*.III<sub>1</sub>, p. 339.

<sup>6</sup> Cf. *Das Kapital*.III<sub>1</sub>, pp. 1-2

<sup>7</sup> Marx himself talks occasionally of a "capitalistic method of calculation, which is *prima facie* fatuous and which appears to contradict the laws of formation of value". *Das Kapital*, I, p; 395, footnote 110.

<sup>8</sup> *Das Kapital* III<sub>1</sub>, pp. 132-151

of production: it is higher or lower according to whether constant capital is more weakly or more strongly represented in the sphere of production concerned. This is a consequence of the principle of value-calculation.

The capitalist economy, however, cannot tolerate this consequence and removes it in the following manner: the total surplus value—which we shall call  $M$ —created in all spheres of production, is distributed among the separate spheres of production in proportion to the total capital ( $c + v$ ) invested in each. That part of the total surplus value which is so allotted to a given sphere of production, Marx calls profit. Let profit be  $m'$ , the sum of the values of all constant capital  $C$ , and that of all variable capital  $V$ ; it follows that:

$$(2) \quad m' = \frac{c + v}{C + V} M$$

$W$  is now replaced by the (production) price  $P$ , to which the formula

$$P = ac + v + m$$

applies. Marx employs the expression “cost price” for the sum  $ac + v$ . The quotient

$$(3) \quad \frac{M}{C + V}$$

which we shall describe by  $\rho$ . Marx calls average rate of profit. The latter prevails—according to the principles of price-calculation—not only in all the spheres of production taken together, but in each separate sphere of production, for:

$$(4) \quad P = ac + v + \rho(c + v)$$

Let  $r$  be the rate of surplus value  $\left(\frac{m}{v} \text{ and } \frac{M}{V}\right)$ ,  $q$  the share of constant capital in

the total capital of the sphere of production concerned  $\left(\frac{c}{c + v}\right)$ , and  $q_0$ , the same relationship for the total capital of all spheres of production taken together  $\left(\frac{C}{C + V}\right)$ , then:

$$(5) \quad \rho = (1 + q_0)r$$

and on the basis of (1) and (4)

$$(6) \quad P = W + (c + v)(q - q_0)r$$

This formula, which is not to be found in Marx, shows clearly that price, owing to its construction, will be higher or lower than value, according to whether  $q$  is larger or smaller than  $q_0$ .

Marx illustrates his model with a numerical example, which is reproduced here. In view of our subsequent discussion, it will, however, be convenient

slightly to modify Marx's assumed figures, by taking 50 and 52—instead of 51 twice—for  $ac$  in the spheres of production II and III. This is quite permissible, since Marx's numerical assumptions are completely arbitrary. We then get the following two tables:

Table 1: Value-calculation

Sphere of Production	Constant Capital ( $c$ )	Variable Capital ( $v$ )	Constant Capital used up ( $ac$ )	Surplus Value ( $m$ )	Value ( $W$ )	Rate of Profit $\left(\frac{m}{c+v}\right)$
I	80	20	50	20	90	20 %
II	70	30	50	30	110	30 %
III	60	40	52	40	132	40 %
IV	85	15	40	15	70	15 %
V	95	5	10	5	20	5 %
I-V	390	110	202	110	422	22 %

Table 11: Price-calculation

Sphere of Production	Constant Capital ( $c$ )	Variable Capital ( $v$ )	Constant Capital used up ( $ac$ )	Cost Price ( $ac+v$ )	Profit ( $m'$ )	Price ( $P$ )	Divergence Of Price From Value ( $P-W$ )	Rate of Profit $\left(\frac{m'}{c+v}\right)$
I	80	20	50	80	22	92	+ 2	22 %
II	70	30	50	80	22	102	- 8	22 %
III	60	40	52	92	22	114	- 18	22 %
IV	85	15	40	55	22	77	+ 7	22 %
V	95	5	10	15	22	37	+ 17	22 %
I-V	390	110	202	312	110	422	0	22 %

Marx thought that a comparison between these tables revealed the quantitative relationships expressed in them to be identical so long as they comprised all spheres of production, or all kinds of goods. The equalization of the rates of profit (20%, 30%, *etc.*) brought about by competition, or, in Marx's terms, the reduction of the different rates of profit in the various spheres of production to a common average rate of profit (22%), Marx believed to result merely in a different distribution of the total surplus value (110) among the separate spheres of production or groups of capitalists. The total price (422) coincided with the total value. The positive divergences of prices from values (2+7+17 = 26) balanced the negative divergences (8+18 = 26).<sup>9</sup>

It is easy to show that the procedure employed by Marx for the transformation of values into prices is erroneous, since it fails to keep separate rigorously enough the two principles of value- and price-calculation.

In considering first the value-calculation (Table I), we may assume that

<sup>9</sup> It must not be overlooked that the values and prices in Marx's models refer, not to quantitative units of the goods concerned, but to their total quantities.

spheres of production I and V serve the production of subsistence goods for the workers, since the value of these goods (90 + 20) amounts to exactly as much as the wages received by the workers (110). We may further assume that the spheres of production III and IV manufacture means of production, since the value of the goods concerned (132 + 70) coincides with the value of the constant capital used up (202) in all spheres of production. The goods produced in sphere II, finally, would represent the consumption goods of the capitalist class, as the value of these goods (110) is equal to the total surplus value. "Simple reproduction" is assumed throughout.

What happens now, when price-calculation (Table II) replaces value-calculation? Production spheres I and V will still be making consumption goods for the workers, sphere II consumption goods for the capitalists, and spheres III and IV means of production. The sum of wages has not altered. Table II, too, gives 110 for the variable capital in all the spheres of production taken together. The workers should therefore be able to acquire for this sum the goods produced in I and V, neither more nor less. These goods, however, now have a price of 92 + 37, *i.e.* a total of 129. The workers thus must go short, or, put in another way, some of the goods made in I and V find no outlet. In this regard, therefore, the price model breaks down; nor is it correct with regard to the capitalists' consumption goods and to producer goods. We find a total profit of 110, as against the figure of 102 for the price of the goods in sphere II; whilst, for the producer goods, we get the figures 202 by taking the total constant capital used up in all the spheres of production, and 191 by taking the price of the goods made in III and IV.

We have thus proved that we would involve ourselves in internal contradictions by deducing prices from values in the way in which this is done by Marx. We made the mistake of carrying over certain magnitudes without alteration from the table of values into that of prices. In transforming values into prices, it is inadmissible to exclude from the recalculation the constant and variable capital invested in the various spheres of production.

Up to a certain point, Marx himself foresaw this objection. He says:<sup>10</sup> "Aside from the fact that the price of a certain product, for instance the product of capital B, differs from its value, because the surplus value realised in B may be greater or smaller than the profit of others contained in the product of B, the same fact applies also to those commodities which form the constant part of its capital, and which indirectly, as necessities of life for the labourers, form its variable part. So far as the constant part is concerned, it is itself equal to the cost price plus surplus value, which now means cost price plus profit, and this profit may again be greater or smaller than the surplus value in whose place it stands. And so far as the variable capital is concerned, it is true that the average daily wage is equal to the values produced by the labourers in the time which they must work in order to produce their necessities of life. But this time is in its turn modified by the deviation of the prices of production of the necessities of life from their values. However, this always amounts in the end to saying that one commodity receives too little of the surplus value while

<sup>10</sup> *Das Kapital*. III, 1, pp. 139—140 [pp. 189—190].

another receives too much, so that the deviations from the value shown by the prices of production mutually compensate one another. In short, under capitalist production, the general law of value enforces itself merely as the prevailing tendency, in a very complicated and approximate manner, as a never ascertainable average of ceaseless fluctuations.”

Thus, in the first half of the quotation, Marx points out that the results he has obtained by recalculating values into prices, appear to require a modification of the numerical bases on which he constructed his table of prices, and which he simply took over from his table of values. However, instead of drawing the only appropriate conclusion, namely that the whole construction of prices is useless, Marx tries, in the second half of the above quotation, to rescue the sense and meaning of this construction by two arguments: firstly, that the divergences of prices is compensate each other; and secondly, that the capitalist economy is a field in which strict laws never have an undisputed validity.

The following considerations should be borne in mind against the first of these arguments. The fact that the positive divergences of prices from values match the negative ones, or, in other words, that total value equals total price, is merely the consequence of Marx's having equated certain prices—namely those relating to constant and variable capital and to total profit—to the corresponding values. Marx himself admits, however, that this equation represents an inaccuracy—at least with regard to constant and variable capital—and there would seem to be no reason why this inaccuracy should fall to have an effect on the reliability of the numerical expression of the total price.

Nor is this all: without entering into a discussion of the details of the transformation of values into prices, it is possible to bring positive proof that the theory of the equality of total value and total price—a theorem to which Marx and the Marxists<sup>12</sup> attach so great an importance—is generally wrong.

Let G be the good which serves as measure of value and of price. The figures 90 and 92, which indicate the value and the price of the total output of sphere 1, would accordingly signify that this total output is exchanged for 90 units of G according to the principles of the value-calculation, and for 92 units according to those of the price-calculation. Such differences between price and value are due to differences in the organic composition of the capital invested in the various spheres of production. These differences obviously also depend, with respect to their signs and their magnitude, on the organic composition of the capital invested in the production of G.

Let us now assume that this capital has the lowest organic composition of all, *i.e.* that in this capital, constant capital constitutes relatively a smaller part than it does elsewhere. On this assumption, the transition from value-calculation to price-calculation should result in all goods being exchanged for more units of G than formerly, in other words, all prices should be higher than their corresponding values. The total price would consequently be greater than the total value.

<sup>12</sup> Cf. *e.g.*, P. Fireman in *Conrads Jahrbücher*, 3rd series, III (1892), p. 808, or K. Kautsky, *Karl Marx's ökonomische Lehren*, 8th ed., Stuttgart, 1903, pp. 99—100.

In the opposite case, where the capital employed to produce G has the highest organic composition, the total price would prove to be a lower figure than that expressing total value.

This situation is in no way altered by the fact that Marx thought of values and prices in terms of money<sup>13</sup>. To him, monetary terms were, *e.g.* in the case of a gold standard, nothing other than certain quantities of gold<sup>14</sup>, and he always regarded the proportion in which gold, whether coined or in bullion, was exchanged against goods, or rather against other goods, as being subject to the general laws of value and of price.

From this standpoint, it would also be quite wrong to link the equality of total value and total price, insofar as both are expressed in money and in the same monetary unit, with the idea of an unchangeable “value of money”. What Marx means by unchangeable or “constant value of money”, is that, in the case of the gold standard, the same amount of labour is always required to produce a given quantity of gold<sup>15</sup>. In other words, “constant value of money” is equivalent to saying that the absolute value of the good serving as money remains constant. It is, however, obvious that the recalculation of values into prices presupposes the constancy of the absolute values of all goods and therefore also of that good which fulfils the function of money. Thus, when we demonstrated above that total price could equally well be larger or smaller than total value, this was done precisely on the assumption of a “constant value of money” in the Marxian sense.

Indeed, recourse would have to be had to the so-called Quantity Theory of Money if, starting from a discussion of the “value of money”, one were to construct an accordance between total price and total value. This solution is, however, barred in this case—if for no other reason than because Marx was the sworn enemy of the Quantity Theory, which he used to call alternatively an “illusion” and “a fatuous hypothesis”<sup>16</sup>.

The above naturally does not preclude the possibility that total price could coincide with total value. This would occur if the organic composition of the capital employed in the production of the monetary good, *e.g.* gold, bore a certain relation—which need not be discussed here—to the organic composition of all other capital. Nowhere in Marx, however, is there any mention of such a qualification. Without paying the slightest regard to the conditions of production of the good serving to measure values and prices, Marx simply asserts in general terms that total price equals total value. This assertion is not only unproven, it is false.

Marx’s error is due to the illogical method lie used in deriving prices from values; it is not caused primarily by any confusion between the concept of value as the index of an exchange-relationship and the concept of absolute value. Such a confusion might at most have been an accessory, for when Marx’s calculation led him to the result that total price = total value, it is possible that he should have seen in that a confirmation of the view that the value of all

<sup>13</sup> *Das Kapital*, III, p. 138.

<sup>14</sup> *Das Kapital*, I, p. 67.

<sup>15</sup> *Das Kapital*, I, p. 69.

<sup>16</sup> *Das Kapital*, I, p. 96, and footnotes 79 and 80.

goods taken together had a significance which could not be modified by the “capitalistic method of calculation” (*i.e.* by applying the principle of price calculation). Since, however, this view can only be put forward on condition that the value of all goods is taken to mean their absolute value, this might, in fact, mean a confusion of the two concepts of value on the part of Marx<sup>17</sup>.

Critics have argued against Marx that, regardless of the truth or falseness of this thesis that total price equals total value, there is no point in it<sup>18</sup>. In a certain sense this is quite true: total price cannot in fact teach us anything about the exchange-relationships of goods. The critics overlook, however, the special point of view—characteristic of Marx—which he tried to express in this thesis. It was essential for Marx’s argument to show that price and profit could be constructed without having to bring into account the successive “mark-ups” arising in the course of the circulation of goods<sup>19</sup>. And it must be admitted that, by proving the equality of total price with total value, he would have refuted the “Theory of Mark-ups”, *i.e.* the theory that profit arises from the mark-ups. It is equally true, however, that there is no need at all of this proof in order to refute that theory. The fact that total price can, as we have shown, be *both greater and smaller* than total value, according to the conditions of production of the good serving to measure values and prices, is in itself sufficient to remove the ground from under the theory of mark-ups.

One final point may be permitted with regard to Marx’s assertion that total price is identical with total value. So long as one is comparing, not certain magnitudes of value and price, but certain relationships of magnitude within the system of value-calculation with analogous relationships of magnitude within the system of price-calculation, one is not at all tied to the condition that the unit of price should be the same as the unit of value. If the latter is represented by 1 ounce of gold, the former may be represented by 3/4 or 1 1/2 ounces of gold. In these circumstances, one can always, with any given model of values (such as Table 1) select such a unit of price as will make one particular element of the price-model (*e.g.* the price of the total output of I, or the variable capital invested in III, *etc.*) equal the corresponding element in the table of values. Similarly, there is nothing to prevent one making a sum of certain elements in the table of prices coincide with the sum of the analogous elements of the table of values, and thus, for instance, equating total price with total value. Such a method of determination can, however, obviously be applied only to one single magnitude among those listed in the table of prices, or to one single function of these magnitudes. It would thus not be permissible to equate total price with total value whilst simultaneously equating total profit with total surplus value. In Marx’s exposition, however, the identity total price = total value appears not as a permissible, though arbitrary, assumption, but as the consequence of a series of mutually incompatible identifications of certain magnitudes of price with the corresponding magnitudes of value. The

<sup>17</sup> “This confusion appears more clearly in Hilferding’s *Marx-Studien*. I. p. 32.

<sup>18</sup> “Böhm-Bawerk is wrong in doubting the justification for working with total value and total price. Value is not an exchange-relationship, but the index of an exchange-relationship, and one may very well sum up a series of magnitudes of value. The same holds true of price.

<sup>19</sup> Cf. Hilferding, *loc. cit.*, p. 31.

incompatibility of these identifications can be seen from the mere fact that they lead to the conclusion that total price = total value, which, when the measure of prices is identical with that of values—as is the case with Marx—, is notoriously wrong, or can lie right only by accident.

This concludes our discussion of the first argument put forward by Marx to justify his disregard for the inaccuracies which, self -admittedly, are involved in his method of deducing prices from values.

His second argument (see above p. 10) is equally unconvincing, but all the more characteristic of the author of *Das Kapital*. As so often elsewhere, here too, he holds the nature of the object to which his theoretical construction refers, responsible for the inner contradictions afflicting this construction. The laws of economics, including the law of the equal rate of profit, do not, indeed, ever find a pure concrete expression. In actual fact, divergences from the norm occur under the influence of various factors which, in formulating these laws, theory must needs disregard. In this particular instance, however, we find divergences which are inherent to the theoretical model itself, and which have thus nothing to do with any disturbing factors.

We are thus driven to reject Marx's derivation of price and profit from value and surplus value. This derivation has indeed one advantage: that of simplicity, which is why it could seem to lie "self-evident" even to one who only partly supported Marx. This advantage is, however, counterbalanced by a not inessential fault: namely, that the derivation in question is erroneous.

Although Marx's attempt to recalculate values into prices must be regarded as a failure, yet the idea of such a double calculation should not be dismissed off-hand. A correct solution of the theoretical problem which Marx had set himself, is very apt to strengthen an insight into important economic relationships. To reach such a solution, it is advisable to reduce to wage outlays all of the outlays of all capitalists who took part in the production of a commodity. We shall present algebraically, from this point of view, first values and then prices.

Let  $w$  be the value of a quantity of any commodity, and  $A$  the number of units of labour, *e.g.* labour-days, embodied in it. Let  $l$  be the wages, *e.g.* per working day, and  $r$ —as formerly—the rate of surplus value, then it follows that:

$$(7) \quad w = Al + rAl$$

The correctness of this formula is obvious so long as it is assumed that the production of the commodity concerned does not cause the capitalist to incur any outlays other than wage-payments, or, in other words, that only variable capital is engaged in this production. It can, however, easily be shown that formula (7) does not lose its validity through the addition of constant capital.

Should this constant capital, namely, be itself created without the aid of another constant capital, then its value could, without any further ado, be expressed by a formula of exactly the same structure as formula (7).  $A$  would then indicate how many working days are embodied in the constant capital concerned. Constant capital enters into the value of the product to the extent

of the whole or of part of its own value. Formula (7) will consequently still be valid for the value of the product, with  $A$  representing the whole amount of labour employed in the production of the commodity concerned, both directly and indirectly (*i.e.* through the intermediary of the constant capital).

Should, however, another constant capital have participated in the production of the constant capital concerned, then the analysis of the value of the product would have to be carried further to the point where a constant capital is found which is wholly the result of direct labour. The general validity of formula (7) would then become apparent.

Equation (7) brings to light the manner in which the value of the product is composed of wages ( $Al$ ) and of the capitalist's profit or surplus value ( $rAl$ ).

The same equation, written in the form

$$(8) \quad w = (1 + r)lA$$

indicates that value ( $w$ ) is proportional to the amount of labour used ( $A$ ).  $(1 + r)l$  is the same for all products or goods and appears, for that very reason, as a factor of proportionality. In order to determine the value of a certain quantity of a commodity, or, briefly, the value of a commodity, one must therefore know: firstly, the magnitude of  $A$ , *i.e.* of the quantity of labour embodied in one unit of the commodity concerned, and secondly, the size of the proportionality factor, which depends on the rate of surplus value ( $r$ ) and on wages ( $l$ ).

For this reason one would be quite wrong in thinking that formula (8)—taken by itself—can afford an answer to the problem of the determination of value as conceived by Marx. Whoever attempts to solve this theoretical problem, is not entitled to treat the rate of surplus value and wages as given quantities. They must, on the contrary, be regarded as unknowns.

Let  $n$  be the number of all goods bought and sold in the market. In algebraic terms, the problem is to determine the values ( $w_1, w_2, w_3, \dots, w_n$ ) of these goods. The given factors are  $A_1, A_2, A_3, \dots, A_n$ , each of which represents the amount of labour embodied in one unit of the product concerned. On the basis of formula (8), one can set up the set of equations

$$(9) \quad \left\{ \begin{array}{l} w_1 = (1 + r)lA_1 \\ w_2 = (1 + r)lA_2 \\ \dots\dots\dots \\ \dots\dots\dots \\ w_n = (1 + r)lA_n \end{array} \right.$$

If this is to be soluble, then the set must be completed by two further equations, as otherwise the number of unknowns ( $w_1, w_2, w_3, \dots, w_n, r$  and  $l$ ) would surpass the number of equations by 2.

One of these missing equations can be discovered by considering that amongst the  $n$  goods, there is also that which serves as measure of value, or as money. Let  $y$  be the ordinal number of this good. Then:

(10)

$$w_y = 1$$

In order to find the other missing equation, one must start with the real wage, which Marx assumes to be given. The real wage is formed by certain quantities of *some* of the  $n$  goods. One can, however, equally well say that the real wage is constituted by certain quantities  $(\mu_1, \mu_2, \mu_3, \dots, \mu_n)$  of all the  $n$  goods, where some of these quantities equal zero. The value of this complex of goods known as the real wage, is clearly identical with the money wage. We therefore get:

$$(11) \quad \mu_1 w_1 + \mu_2 w_2 + \dots + \mu_n w_n = l$$

We thus reach a system of  $n + 2$  equations with just as many unknowns. The simplest solution of these equations is as follows: On the basis of (9), we deduce from (11):

$$(12) \quad (1 + r)l(\mu_1 A_1 + \mu_2 A_2 + \dots + \mu_n A_n) = l$$

We then introduce the description:

$$(13) \quad \mu_1 A_1 + \mu_2 A_2 + \dots + \mu_n A_n = U$$

$U$  clearly means the amount of labour embodied in the complex of goods forming the real wage.  $U$  is what Marx calls “the necessary labour”<sup>20</sup>. Formulas (12) and (13) then yield the simple relationship

$$(14) \quad (1 + r)U = 1$$

or

$$(15) \quad r = \frac{1 - U}{U}$$

It is this particular expression of the rate of surplus value which plays an important part in *Das Kapital*<sup>32</sup>. The rate of surplus value appears here as the proportion between “surplus labour” and “necessary labour”, or as the ratio between the two parts of the working day in which the surplus value and the worker’s necessities, or the equivalent of these necessities, are produced. Marx uses the term “necessary labour time” for this second part of the working day.

If for instance, the length of the working day is 12 hours, whilst the necessary labour time amounts to 8 hours, then, in formula (15),  $U$  would be equal to  $2/3$  and one would find:  $r = 1/2$ , *i.e.* a rate of surplus value of 50%.

To find  $l$ , one need only write equation (10) as follows:

$$(1 + r)lA_y = 1$$

which, considering (15), yields the relationship:

$$(16) \quad l = \frac{U}{A_y}$$

Marx assumes in his numerical examples<sup>23</sup>, that “a mass of gold of 12 shil-

<sup>20</sup> *Das Kapital*, I, p. 198.

<sup>32</sup> *Das Kapital*, I, pp. 542-546.

<sup>23</sup> *Das Kapital*, I, p. 166 *et seq.*

lings” is “the product of 24 hours of labour or two working days”. If, then, both the value of goods and the wage rate are expressed in shillings,  $A_y$  must be taken to be equal to  $1/6$ , and if, as earlier on,  $U = 2/3$  one gets  $l = 4$ . Should  $U = 1/2$  (an assumption commonly made by Marx), then  $l = 3$ . This means that the wage rate is 3 shillings<sup>23</sup>.

Having ascertained the two unknowns  $r$  and  $l$ , we need only introduce them into the equations of set (9) in order to find the commodity values ( $w_1, w_2, etc.$ ) we are seeking, since the amounts of labour  $A_1, A_2, etc.$  are assumed to be given. As  $w_y = (1+r)lA_y = 1$ , these commodity values can also be ascertained in a direct fashion, *i.e.* without the detour over  $r$  and  $l$ , namely on the basis of:

$$(17) \quad w_1 = \frac{A_1}{A_y}, \quad w_2 = \frac{A_2}{A_y}, \quad \dots \dots w_n = \frac{A_n}{A_y}$$

These equations show that the values of commodities depend exclusively on the amounts of labour required for their production, and that commodity value is consequently unaffected by the level of the wage rate and of the rate of surplus value. Marx attaches the utmost importance to this point and contrasts it with the view which takes the outlays on wages and the capitalist’s profit as the starting point for the determination of commodity values. In this connection, Marx speaks of the “illusion that value arises out of its own components”<sup>24</sup>, and of the “fine erroneous circular reasoning” which he believes to be constituted by the fact that the value of a commodity is gained by adding up “revenues”, the very size of these revenues being made to depend on the commodity values<sup>25</sup>.

The above derivation shows, however, that Marx’s alleged contrast does not, in fact, exist. Formula (7), which has been our starting point, is based on this very idea that the value of a commodity is created by the sum of wages and profit. To express first commodity values as functions of wages (in set (9)), and then to express wages as a function of commodity values (in equation (11)), is by no means circular reasoning. For anyone to regard this procedure as an offence against logic, would only reveal his utter ignorance of algebra.

We shall now consider prices. According to Marx’s model, reproduced at the beginning of this article, prices would be equal to values, were it not for constant capital. This would, however, be valid only on the assumption that the turnover period of variable capital is the same in all lines of production. At this point, when we are trying to reach a wider generalization of our theoretical study, we must emancipate ourselves from this assumption.

We shall first look for the price of the unit of any commodity which is produced with the help of variable capital only. Let  $p$  be this price,  $A$  the amount of labour required (just as in the value-calculation),  $\lambda$  the wage rate,  $\rho$  (as formerly) the rate of profit, and  $t$  the turnover period. As we have in view

<sup>23</sup> *ibid.*, p. 170.

<sup>24</sup> *Das Kapital*. III, 2, p. 382; cf. II, pp. 383-385.

<sup>25</sup> *ibid.*, pp. 378, 382, 398. I am here disregarding rent, which Marx also considers in these passages. Cf. *Theorien über den Mehrwert*, II, 1, p. 80, where he speaks, not of a “circular reasoning” but of a “miserable seesaw”.

the final price, *i.e.* the price at which the commodity will be sold to the consumer, we must regard the turnover period as starting with the moment when wages are paid, and ending at the moment of the sale of the commodity to its final buyer. Whether the industrial and the commercial function are united in the person of one and the same capitalist, or whether a division of these functions takes place, is, theoretically, immaterial to the magnitude of the price. We shall later pay regard to the complication arising from the fact that the outlay on wages caused by the production of a certain commodity, may be spread over different points of time. We provisionally assume that total wages ( $A\lambda$ ) are paid at one moment of time.

This sum of wages forms one component of price. The other is the capitalist's gain or the profit. A turnover period of 1 year would make profit equal  $\rho A\lambda$  (since  $\rho$  is the yearly rate of profit). Were the turnover period 2, 3 *etc.* years, profit would be not  $2\rho A\lambda$ ,  $3\rho A\lambda$ , *etc.*, but (because of compound interest)  $\{(1+r)^2 - 1\}A\lambda$ ,  $\{(1+r)^3 - 1\}A\lambda$ , *etc.* In the theory of prices, just as in other regions of theory, there is nothing to prevent one from applying the principle of compound interest also to the case where the period after which interest is due, or the turnover period, is no longer expressed by an integral, but by a fractional number of years. Thus, for each value of  $t$ , profit would be expressed by  $\{(1+r)^t - 1\}A\lambda$

Within the system of price-calculation, the equations:

$$(18) \quad p = A\lambda + \{(1 + \rho)^t + 1\}A\lambda$$

correspond to equations (7) and (8).

Thus, whilst the values of two commodities which embody the same amount of labour equal each other, this is not generally the case with the prices of such commodities, except on the condition that the turnover period should be identical for both commodities. When this is not the case, the commodity with the longer turnover period will command a higher price. We thus receive a confirmation for an earlier assertion that prices do not equal values even when constant capital is completely absent<sup>26</sup>.

Let us now consider the case where the total wages  $A\lambda$  are not paid out at one moment, but at  $m$  different moments which precede the moment of the completion or of the sale of the product by  $t_1, t_2, t_3 \dots \dots t_n$  units of time (*i.e.* years or fractions of years). Let  $a_1\lambda, a_2\lambda, a_3\lambda, a_4\lambda \dots \dots a_n\lambda$  be the amounts paid out in wages at each of these moments. Formula (19) will then obviously be replaced by:

$$(20) \quad p = (1 + \rho)^{t_1} \lambda a_1 + (1 + \rho)^{t_2} \lambda a_2 + \dots + (1 + \rho)^{t_m} \lambda a_m$$

and naturally

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<sup>26</sup> The variety in turnover periods, or, more precisely, the different durations of the turnover periods, imply in the system of value—calculation that the *annual rate of surplus value* varies with the lines of production. See *Das Kapital*, II, pp. 279-295. It must always be borne in mind that in the equations (17)  $r$  represents not the annual rate of surplus value, but, in Marx's words (*loc. cit.*, p. 291) "the real rate of surplus value".

$$(21) \quad a_1 + a_2 + \dots + a_m = A$$

We shall now show that *formula* (20) does not cease to be valid when expenditures for raw materials and for the depreciation of equipment are added to the capitalist's outlay on wages. Expenditure on raw materials is represented by circulating constant capital, expenditure on depreciation by a part of fixed constant capital.

Here too, as in value-calculation, and for the same reasons, we need only consider the case where constant capital—circulating as well as fixed—is in its turn exclusively the result of direct labour.

So far as circulating constant capital is concerned, there is no need for mathematical proof to show that the productive participation of this kind of capital does not alter the structure of formula (20). All that happens here is that the production of a commodity goes through several independent stages, in which different capitalists are active consecutively. All of these, with the exception of the first, add mark-ups not only to their own outlays on wages, but to the wage-outlays of their predecessors. The period of time for which the mark-up is calculated, equals in each case the period of production in the corresponding stage. These periods of time are added up, so that in applying formula (20) to this case, one must so-to-speak date back appropriately some of the wage payments, namely those which the "last" producer has not effected himself.

The case is not quite as straightforward with fixed constant capital. Let us assume that the piece of capital equipment ( $K$ ) concerned, e.g. a machine or a building, embodies an amount of labour  $E$ . The outlay on wages occasioned by the production of  $K$  consequently equals  $\lambda E$ . To begin with, let this wage-outlay be conceived of as a once for all outlay. Let  $\tau$  be the lapse of time between the moment at which the wage payment is made, and the moment at which  $K$  is put into the service of production. The price  $C_0$  of  $K$  at this moment is, on the basis of (19):

$$(22) \quad C_0 = (1 + \rho)^\tau \lambda E$$

Let furthermore  $C_1, C_2, C_3, \dots$  be the price of  $K$  after the expiration of 1 year, of 2, 3 etc. years. After the expiration of a certain period of time,  $K$  is completely used up, and, if this period lasts  $\omega$  years,  $C_\omega$  must equal zero. Let also  $b_1, b_2, b_3, \dots, b_\omega$  be the amounts which—in proportion to  $K$ 's share in production—enter into the price of the output produced with the aid of  $K$  in the 1st, 2nd, 3rd etc. years; then, according to the "capitalistic calculation" the following relationships ensue<sup>27</sup>.

$$(23) \quad \left\{ \begin{array}{l} b_1 = \rho C_0 + C_0 - C_1 \\ b_2 = \rho C_1 + C_1 - C_2 \\ \dots\dots\dots \\ b_\omega = \rho C_{\omega-1} + C_{\omega-1} - C_\omega \end{array} \right.$$

<sup>27</sup> For the sake of simplicity, we assume here yearly periods of production

It can be proved that if the price components  $b_1, b_2, \text{ etc.}$  are reduced to formula (19), *i.e.* if one makes

$$(24) \quad \left\{ \begin{array}{l} b_1 = (1 + \rho)^{\tau+1} l e_1 \\ b_2 = (1 + \rho)^{\tau+2} l e_2 \\ \dots\dots\dots \\ \dots\dots\dots \\ b_w = (1 + \rho)^{\tau+\omega} l e_\omega \end{array} \right.$$

one will get

$$e_1 + e_2 + e_3 + \dots + e_\omega = E$$

This would mean that K's contribution to production is calculated to have exactly the same significance as if the amount of labour  $E$ , which is embodied in  $K$ , had been expended *directly* on the production of this output.

(23) does in fact yield:

$$\begin{aligned} \frac{b_1}{1 + \rho} &= C_0 - \frac{C_1}{1 + \rho} \\ \frac{b_2}{(1 + \rho)^2} &= \frac{C_1}{1 + \rho} - \frac{C_2}{(1 + \rho)^2} \\ \frac{b_3}{(1 + \rho)^3} &= \frac{C_2}{(1 + \rho)^2} - \frac{C_3}{(1 + \rho)^3} \\ &\dots\dots\dots \\ &\dots\dots\dots \\ \frac{b_{\omega-1}}{(1 + \rho)^{\omega-1}} &= \frac{C_{\omega-2}}{(1 + \rho)^{\omega-2}} - \frac{C_{\omega-1}}{(1 + \rho)^{\omega-1}} \\ \frac{b_\omega}{(1 + \rho)^\omega} &= \frac{C_{\omega-1}}{(1 + \rho)^{\omega-1}} - \frac{C_\omega}{(1 + \rho)^\omega} \end{aligned}$$

The sum of these  $\omega$  equations gives (since  $C_\omega = 0$ ):

$$(26) \quad C_0 = \frac{b_1}{1 + \rho} + \frac{b_2}{(1 + \rho)^2} + \dots + \frac{b_\omega}{(1 + \rho)^\omega}$$

and if one inserts in this formula for  $b_1, b_2, \text{ etc.}$  the values indicated in (24), then:

$$C_0 = (1 + \rho)^\tau \lambda (e_1 + e_2 + \dots + e_\omega)$$

from which follows, on the basis of (22), the formula (25) which we set out to prove.

That part of the price of a product which reflects the contribution of fixed capital to its production, can therefore be expressed according to formula (19), provided we suitably split up the amount of labour embodied in the fixed capital.

This is, however, true only on the qualifying assumption—which we have

so far stipulated—that the outlay on wages occasioned by the production of this particular piece of capital equipment, occurs at one single moment.

On the other hand, in the normal case, where the wage outlay is spread over several moments, the price of the piece of capital equipment, as well as that part of the price of the product which is caused by the contribution of this particular capital to production, are both represented by a formula of the shape of (20).

The shape of formula (20) is not altered by the insertion, on the right-hand side, of fresh items which accord—with regard to their shape—with the old items or with their sums. Formula (20) thus proves to be the general expression for the price of a commodity. This is so independently of the circumstance whether, and to what degree, the production of this commodity has required not only variable capital, but also the use of constant capital, either circulating or fixed.

Insofar as this theorem refers specifically to fixed capital, it agrees essentially with Ricardo's theorem that all differences between commodities with regard to the greater or smaller contribution made by fixed capital in their production, be traced back to differences in the length of their periods of production<sup>28</sup>. Marx appreciates the considerable progress which Ricardo has thereby brought about us the analysis of the formation of prices, and gives him credit for it as a "great merit"<sup>29</sup>. It is therefore all the more remarkable that Marx does not himself take this step too, but consistently clings to the distinction between two or three forms of capital. This distinction drags on through all the three volumes of *Das Kapital* and has obstructed rather than fostered Marx's purposes. He clung to his strict distinction between variable and constant capital<sup>29</sup>, because one of his major considerations was to disprove the erroneous view that the capitalist's profit was due to the "productivity of capital". Our subsequent discussion will prove, however, that to eliminate all distinctions between the various forms of capital, as was done in setting up our basic formula (20), does not, by any means, entail support of the "productivity theory".

The system of value-calculation yielded, for the  $n$  commodities in the market, an equivalent number of value-equations (see set (9)). Similarly,  $n$  price-equations of form (20) can be set forth in the system of price-calculation. The number of items on the right-hand side of each of these  $n$  equations may vary. Furthermore, the quantities  $a_1, a_2, etc.$  and  $t_1, t_2, etc.$  are obviously different in each equation. On the other hand,  $\rho$  and  $\lambda$  (like  $r$  and  $l$  formerly) do not vary from one equation to another. These two quantities are unknowns which join the  $n$  unknowns represented by the  $n$  prices ( $p_1, p_2, . . . p_n$ ) of the units of the commodities concerned. The missing two additional equations are found in the same way in which, earlier on, we found equations (10) and (11). We thus get:

<sup>28</sup> "D. Ricardo. Principles of Political Economy and Taxation, edited by E. C. K. Gonner. London, 1903, Ch. I. Section IV, p. 31.

<sup>29</sup> Theorien über den Mehrwert, II<sub>1</sub>, p. 18.

<sup>29</sup> It is sometimes overlooked (e.g. by W. Liebknecht. Zur Geschichte der Werttheorie in England, 1902. p. 31) that the Marxian division of capital into constant and variable capital is by no means the same as Ricardo's division into fixed and circulating capital.

$$(27) \quad p_y = 1$$

and

$$(28) \quad \mu_1 p_1 + \mu_2 p_2 + \dots + \mu_n p_n = \lambda$$

Here too, we get a system of  $n + 2$  equations with  $n + 2$  unknowns. These equations are solved by first inserting in equation (28) for  $p_1, p_2, \text{ etc.}$  the terms standing on the right-hand aide of the corresponding price-equations. Equation (28) is thus transformed into an equation of form (20), which shall be written as follows:

$$(29) \quad (1 + \rho)^{\tau_1} \lambda u_1 + (1 + \rho)^{\tau_2} \lambda u_2 + \dots + (1 + \rho)^{\tau_s} \lambda u_s = \lambda$$

$\tau_1, \tau_2, \text{ etc.}$  are here the turnover periods, and  $u_1, u_2, \text{ etc.}$  the amounts of labour required to produce the commodity-complex which forms the real wage-rate. By eliminating  $\lambda$  on both sides of the last equation, we find:

$$(30) \quad (1 + \rho)^{\tau_1} u_1 + (1 + \rho)^{\tau_2} u_2 + \dots + (1 + \rho)^{\tau_s} u_s = 1$$

This equation corresponds to equation (14). The following relationship obviously holds good:

$$(31) \quad u_1 + u_2 + u_3 + \dots + u_s = U$$

If the turnover period were constant and equal to 1 year, then (30) would merge into (14), and  $\rho$  would equal  $r$ . In this special case, there would be no difference at all between value-calculation and price-calculation.

In the general case, however,  $\rho$  may be either smaller or larger than  $r$ , according to whether all values  $\tau_1, \tau_2, \text{ etc.}$  are greater or less than 1.

It is clear furthermore that, speaking generally equation (30), does not admit of any solution in the sense of lower algebra, since the quantities  $\tau_1, \tau_2, \text{ etc.}$  may be expressed by any kind of integral or fractional number. In order really to be in a position to determine  $\rho$  from a numerical equation of form (30), one would have to have recourse to the methods of higher algebra, by the help of which  $\rho$  could be calculated to the desired degree of approximation.

The unknown  $\lambda$  is then determined with the help of the price-equation which contains  $\rho$ , on the left-hand side, when  $p_y$  can (according to equation (27)) be replaced by 1, and  $\rho$  by its value, which, as we have said, can be proximately determined from (30). One thus gets an equation of the first degree with *one* unknown ( $\lambda$ ).

The remaining unknowns ( $p_1, p_2, \text{ etc.}$ ) finally, can be determined without further ado from the corresponding price-equations. Incidentally, instead of calculating  $\lambda$ , one could form the quotients  $\frac{p_1}{p_y}, \frac{p_2}{p_y} \text{ etc.}$ , whereby  $\lambda$  would be

eliminated. Since  $p_y = 1$ , one would get for  $p_1, p_2, \text{ etc.}$  fractions, of which the numerators would contain—apart from  $\rho$ —the amounts of labour and the turnover periods appertaining to the commodity concerned, whilst the denominators would contain—apart from  $\rho$ —the amounts of labour and the

turnover periods appertaining to the commodity which serves as measure of price.

The above algebraic solution of the price-problem has been taken, in its essentials, from a work by W. K. Dmitrieff<sup>31</sup>. I have merely simplified his exposition somewhat: and have furthermore—by inserting the discussion of how the price of fixed capital gradually enters into the price of the product—freed it from the qualification that the fixed capital is completely used up during the course of production<sup>32</sup>.

Although Dmitrieff himself makes no attempt whatever to connect his system of equations with that of Marx's model—he therefore ignores value-calculation as a contrast to price-calculation—but tries rather to link up with Ricardo, one is nevertheless entitled to say that he has presented a theoretical model which keeps entirely to the Marxian way of posing the problem. Just like Marx, Dmitrieff's model shows as ultimate and exclusive determinants of price the technical conditions of production of commodities, including the technical conditions of production of the commodity labour, the latter finding their expression in a given real wage.

When, however, we compare the methods of solving the price-problem which has been set out in these terms, we find the following fundamental differences between Marx and Dmitrieff:

1. The separation of two different forms of capital, which is characteristic of Marx, is not found in Dmitrieff. By tracing all the capitalist's outlays back to wage-outlays, he makes the qualitative differences between the two forms of capital disappear in the quantitative difference between longer and shorter turnover periods. We have already discussed this point. Since, however, Marx and his disciples consider the strict distinction between variable and constant capital to be a scientific achievement of the first order, and regard as a lost person anyone who tries to disregard this distinction, we shall add the following supplementary remarks.

Marx believes the essential difference between variable and constant capital to lie in the fact that the former does, while the latter does not, yield a profit to its owner. This is, however, admittedly not true of the system of price-calculation, for the size of the capital profit is here determined by the total capital<sup>33</sup>. In the system of value-calculation, the individual capitalist does not "earn" anything on the constant capital. But is this not a matter of an "internal concern of the capitalist class"? And does not Marx, on the other hand, proclaim that as long as he is moving on the ground of value-calculation,

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<sup>31</sup> The title of this remarkable work, which appeared (in Russian) in Moscow in 1904, is *Economic Studies, 1st series: Attempt at an organic synthesis of the Labour Theory of Value with the Theory of Marginal Utility*. Separate discussion is devoted to: (1) Ricardo's Theory of Value, (2) A. Cournot's Theory of Competition, and (3) The Theory of Marginal Utility. Since the author employs algebraic and geometrical means of exposition and of demonstration, it is hardly surprising that his publication (apparently a first work!) has received very little notice (I mean of course from Russians), although it bears evidence of an exceptional theoretical talent and presents something really new. My attention was drawn to it by a (very favourable) review by A. A. Tschuprow in *Mitteilungen des St. Petersburg Polytechnischen Instituts*, 1905.

<sup>32</sup> Dmitrieff, *loc. cit.*, p. 11.

<sup>33</sup> This consideration suffices to cover Dmitrieff—who anyway is concerned with price-calculation alone—against the objection that he confused constant and variable capital. Our further remarks in the text are intended to forestall this objection, if it should ever be raised against my set equations (9).

he is not at all concerned with the basic principles governing the “distribution of the spoils” among the capitalists? From such a standpoint, should it not be immaterial whether the capital profit is in the past tense, or lies in the future? The former case is that of constant capital, which has already absorbed all the surplus value accruing to it, whilst the latter applies to variable capital which is destined to serve in the future, too, as the means of creating and appropriating surplus value.

If as has already been remarked, the strict separation of variable and constant capital is intended to prevent the emergence of the idea that the material factor of production—and not the personal one—is the source of profit, then one might think that both forms of capital are equally situated with regard to their (alleged) productivity. After all, the gain of capital, whether surplus value or profit, derives, according to Marx, from labour and not from capital<sup>34</sup>.

2. Dmitrieff works from the start on the assumption of an equal rate of profit, whilst Marx introduces this assumption only at a later stage of the discussion (in the third volume of *Das Kapital*. One must expect Marxists to object that Dmitrieff has thus betrayed the “objective point of view”, since the law of the equal rate of profit is, after all, connected with “motivation”, *i.e.* with the capitalists’ pursuit of maximum profit, and with competition. As against this, it may be pointed out that the Marxian law of value is in fact rooted in motivation as much as is the law of equal profit and could not impose itself otherwise than through competition (under certain conditions). A psychological principle thus heads even Marx’s whole theory<sup>35</sup>.

However, even if one were to admit the notorious absurdity that the law of value has a basis exterior to motivation and competition, “objectivism” would nevertheless not be saved even in the Marxian system, because the law of the equal rate of profit, here too, plays its part, though at a later stage of the discussion.

So far as the safe-guarding of the “objective” standpoint is concerned, Dmitrieff’s procedure, which reaches the rate of profit, or the equal rate of profit, not by detours, but postulates it *ab initio*, does not lag behind Marx’s method. Both procedures can be termed objective only in the sense that they avoid entering more closely into the play of motives.

3. Dmitrieff presents his exposition in an algebraic form, by treating unknown quantities as known and by reducing the quantitative relationships in question to a system of equations. Marx, on the contrary, always proceeds arithmetically: he assumes certain quantities to be known and deduces from them, by a series of successive operations, the unknowns which interest him.

This difference is by no means of a merely formal nature. Rather does the Marxian method rest on an unfounded view of the character of economic relations. Alfred Marshall<sup>36</sup> said once of Ricardo: “He does not state clearly,

<sup>34</sup> It is therefore just from the Marxian point of view that it is incorrect to say, as Marx occasionally does, that surplus value arises from the variable part of capital. *Das Kapital*, I, p. 414.

<sup>35</sup> It is not without interest to note that, with regard to the equality of the rate of surplus value (an essential point for the system of value-calculation!), Marx explicitly admits that this equality can only be brought about by competition (amongst the workers). *Das Kapital*, III, pp. 153—154.

<sup>36</sup> *Principles of Economics*, I. London, 1898, p. 565. Cf. p. 597.

and in some cases lie perhaps did not fully and clearly perceive how, in the problem of normal value, the various elements govern one another mutually, and not *successively* in a long chain of causation.” This description applies even more to Marx.

Admittedly Marx was enough of a realist not to close his mind against the fact that the various economic factors or elements condition each other mutually. We need only refer to his discussions, in the first volume of *Das Kapital*, of the manner in which the organic composition of capital depends on the level of the rate of surplus value<sup>37</sup>. Marx ought to have replaced the rate of surplus value by the general rate of profit also in the third volume; he would then have found that the general rate of profit, which, according to him, is significantly influenced by the average organic composition of the social capital, in its own turn affects the organic composition of individual capital and consequently also the average organic composition of the social capital<sup>38</sup>. But although Marx did not fail to notice even such complicated instances of a mutual dependence of economic elements or of the quantities in which these elements are expressed, yet when it came to the real model for the formation of prices and incomes, he nevertheless held firmly to the view that the elements concerned must be regarded as a kind of causal chain, in which each link is determined, in its composition and its magnitude, only by the preceding links. Following the words quoted from Marshall, one might denote this trait of the Marxian system as *successivism*”.

Modern economics is beginning to free itself gradually from the successivist prejudice, the chief merit being due to the mathematical school led by Léon Walras<sup>39</sup>. The mathematical, in particular the algebraic method of exposition clearly appears to be the satisfactory expression for this superior standpoint, which does justice to the special character of economic relations.

There is thus a decided advantage in the fact that Dmitrieff has recourse to algebraic procedure.

So much on the basic divergences of Dmitrieff’s model from Marx’s own.

One may well ask whether the greater generality and rigour of the former model has not been too dearly bought. The question is whether Dmitrieff’s set of equations sheds any light on those particular points which, as Marx maintained, could be placed in their proper perspective with the very help of the Marxian model, and *only* with its help. Were Dmitrieff’s model to indicate nothing further than that the question of price-formation (including wage-formation), regarded as a mathematical problem, is soluble, given the technical conditions of production of commodities (including the commodity labour power), then there would be some justification in saying of this model: elegant, but sterile. Fortunately this is not the case, and we shall now indicate the con-

<sup>37</sup> Pp. 398-400.

<sup>38</sup> This particular influence of the rate of profit on the organic composition of capital finds, incidentally, no expression in Dmitrieff’s model either, since the amounts of labour and the turnover periods concerned are regarded as given quantities, as is implied in the very nature of the way in which the problem is posed.

<sup>39</sup> The dispute between the followers of the theory of costs of production and those of the theory of marginal utility is mainly a result of the successivist prejudice. On this, cf. G. Cassel. “Grundriss einer elementaren Preislehre”, *Tübinger Zeitschrift*, Vol. 55 (1899), pp. 448-449, and “Die Produktionskostentheorie Ricardos etc.”, *ibid.*, vol. 57 (1901), p. 81.

sequences resulting from those algebraic formulae, firstly for the theory of prices, and then for the theory of profit.

We have seen (above p. 21) that the price of a commodity can be represented by a mathematical expression from which  $\lambda$  (the wage-rate) has been eliminated.

We are thus entitled to say with Ricardo that wages are not a direct determinant of the exchange-relationships of commodities, but that they only influence these relationships indirectly, to the extent to which the level of wages bears a certain relation to the level of the rate of profit ( $\rho$ ).

Yet more can be learned from a study of the numerator and of the denominator in that price-formula. Amongst the  $n$  commodities, let  $i$  be the ordinal number of the commodity, the price of which is under discussion. If the duration of the turnover period of  $i$  equals that of the commodity serving as a measure of price, then the price of  $i$  will equal its value. The equality of the turnover periods would find expression in the fact that the magnitudes  $t_1, t_2, etc.$  would be equal for both commodities, and that the amount of labour  $A_i$  which is embodied in one unit of commodity  $i$  is distributed over the turnover periods concerned in the same manner as is the amount of labour  $A_y$  which is embodied in one unit of price. Let this distribution be expressed by the formulae

$$A_i = a_{i,1} + a_{i,2} + \dots + a_{i,m}$$

and

$$A_y = a_{y,1} + a_{y,2} + \dots + a_{y,m}$$

where the following ratios are valid:

$$\frac{a_{i,1}}{A_i} = \frac{a_{y,1}}{A_y}, \quad \frac{a_{i,2}}{A_i} = \frac{a_{y,2}}{A_y} \text{ etc.}$$

On the basis of (20), we then get:

$$p_i = \lambda \left\{ (1 + \rho)^{t_1} a_{i,1} + (1 + \rho)^{t_2} a_{i,2} + \dots + (1 + \rho)^{t_m} a_{i,m} \right\}$$

and

$$p_y = \lambda \left\{ (1 + \rho)^{t_1} a_{y,1} + (1 + \rho)^{t_2} a_{y,2} + \dots + (1 + \rho)^{t_m} a_{y,m} \right\}$$

from which, owing to the above ratios, there follows:

$$p_i = \frac{A_i}{A_y} p_y$$

or

$$p_i = \frac{A_i}{A_y}$$

and finally, having regard to (17)

$$p_i = w_i$$

If, on the contrary, the turnover periods of commodity  $i$  do not equal those of commodity  $y$  (which serves as measure of price), then the price  $p_i$  will be greater or smaller than the value  $w_i$ , according to whether the turnover periods of  $i$  are generally longer or shorter than those of  $y$ . It is not possible to formulate this relationship more precisely. It would, for instance, not be correct to assert that what is decisive for the relationship between  $p_i$  and  $w_i$  is the cir-

cumstance whether the average duration of the turnover periods is greater for one or the other of the two commodities.

Such an assertion does, however, approximate fairly closely to the truth, at least in those cases where  $\rho$  is so small that one is justified in neglecting its second and higher powers. Availing oneself of this justification, formula (20) assumes the following form:

$$(32) \quad p = \lambda \{ (1 + t_1 \rho) a_1 + (1 + t_2 \rho) a_2 + \dots + (1 + t_m \rho) a_m \}$$

If we follow formula (21) and if we introduce the new designation

$$\frac{a_1 t_1 + a_2 t_2 + \dots + a_m t_m}{a_1 + a_2 + \dots + a_m} = d$$

then (32) becomes

$$(33) \quad p = \lambda A (1 + \rho d)$$

The magnitude  $d$  represents nothing other than just this average duration of the turnover periods for the production and distribution of the commodity concerned. Since  $d$  differs for each commodity, it is necessary to show by an index, the commodity to which a particular  $d$  refers. We thus get the equations:

$$(34) \quad p_i = \lambda A_i (1 + \rho d_i)$$

and

$$(35) \quad p_y = \lambda A_y (1 + \rho d_y)$$

Since  $p_y = 1$  and  $\frac{A_i}{A_y} = w_i$ , we get:

$$(36) \quad p_i = \frac{1 + \rho d_i}{1 + \rho d_y} w_i$$

Thus  $p_i$  would be greater or smaller than  $w_i$ , according to whether  $d_i$  is greater or smaller than  $d_y$ . But, as we have already said, the validity of this simple relationship is not absolutely strict.

The transition from value-calculation to price-calculation thus alters the exchange-relationships in favour of those commodities the production (and distribution) of which are marked by relatively long turnover periods, and to the disadvantage of those commodities where the contrary prevails.

Marx saw this state of affairs correctly insofar as he related the size of the excess of the price of a commodity over its value to the organic composition of the capital engaged in its production. And, conversely, according to Marx, when the price does fall below the value of a commodity, it falls all the more, the lower is the organic composition of the corresponding capital.

However, Marx goes quite wrong in his determination of the starting point of an increase of price over value, or a decrease of price below value, in order that the price of a commodity should equal its value, the organic composition of the capital engaged in its production should—according to Marx—equal the average composition of the total social capital<sup>40</sup>. What matters in reality,

<sup>40</sup> *Das Kapital*, III, pp. 185-187; cf. above, formula (6).

however, is not the comparison with this average composition, but the comparison with the composition of the capital engaged in the production of the commodity serving as measure of value and of price. This appears clearly from formula (36). Were, for instance,  $d_y$  the smallest of all  $n$  magnitudes  $d_1, d_2, \text{etc.}$  then all prices would be higher than the corresponding values, and vice versa: were  $d_y$  the largest of these magnitudes, then all prices would be lower than the corresponding values<sup>41</sup>.

Marx's error is a consequence of the fallacious method which he used for the transformation of values into prices. Ricardo, contrary to Marx, saw the relationship in question quite plainly<sup>42</sup>.

Furthermore, one cannot approve of Marx's formulation, according to which the quantitative relationship between value and price is made to depend, not on the duration of the turnover periods or of the processes of production, but on the organic composition of each capital. The following consideration, among others, argues against this formulation. Let us assume that in the production of a commodity, the various stages of production, which had hitherto not been separated, now acquire an independent character. In such a case there will be an increase in the share of constant capital in the total capital engaged directly or indirectly in the production of this commodity, because the constant capital will now also comprise certain intermediary products (semi-manufactured goods), which do not figure at all as capital in an undivided production process. Such a change in the conditions of production, however, cannot, in theory, affect the price of the final product. Looking at the situation from the standpoint of Marx's model, one would then have to say that although in such a case the organic composition of the capital concerned has indeed changed, the effect of the change on price has been compensated by an acceleration of the turnover. This example should demonstrate how inappropriate it is to regard the organic composition of capital as a separate factor apart from the turnover period.

Formula (36) and the precise formulae on which it is based, show also that a rise in the rate of profit ( $\rho$ ) will raise the prices of those goods, the production of which is marked by turnover periods of relatively long duration<sup>43</sup>, and depress the prices of the others. A fall in the rate of profit has exactly the opposite effect on prices. These findings, too, are contained in Ricardo.

In discussing the theory of value, Ricardo directed his attention primarily to the movement of prices conditioned by a changing rate of profit. The divergence of prices from values interested him far less. In fact, the establishment of price-calculation in place of value-calculation appears in Ricardo only, as it were, as the special case of an increase in the rate of profit, where the latter rises from 0 to some positive amount.

Such a point of view is justified by the fact that Ricardo does not recognize

<sup>41</sup> Cf. above, pp. 10-II.

<sup>42</sup> *Principles*, Ch. I, Section V, last paragraph, p. 36. When considering the price movements of any commodity. Ricardo uses—as standard of comparison for the durability of the capital engaged in the production of that commodity—that capital which is engaged in the production of the commodity which functions as the measure of prices.

<sup>43</sup> *i.e.* again in comparison with the commodity serving as measure of prices.

value-calculation in the Marxian sense. Like Marx, Ricardo does indeed start with a situation where the law of value, as expressed in (27), is valid. Whilst Marx, however, assumes unequal rates of profit for this initial situation, Ricardo considers that there is no profit at all. This difference between the two theoretical models is not entirely without significance for the theory of profit, as will be seen in the further course of these discussions. This difference is, on the other hand, without any importance for the question of exchange-relationships, because in the system of value-calculation, even as conceived by Marx, the exchange-relationships do not in any way depend on the level, nor even on the existence, of profits. One need only take  $r$  to be equal to 0, and values remain unchanged.

We cannot therefore agree with Marx when he objects against Ricardo—as he does on innumerable occasions—that the latter had confused prices with values<sup>44</sup>. Admittedly, Ricardo does not use the terms “value” and “price” in the Marxian sense of a contrast between value-calculation and price-calculation. However, in discussing the exchange-relationships of goods, or prices, Ricardo displays complete knowledge of the conditions on which this contrast rests<sup>45</sup>. Not only was he well aware that the intervention of the general rate of profit in these exchange-relationships brings about a modification of the (original) law of value, and thus, in Marxist terms, divergences of prices from values, but he correctly judged the direction and the extent of these divergences.<sup>46</sup>

Marx denies this: he repeatedly asserts that Ricardo had only examined the “very secondary question” of the manner in which changes in the rate of profit affect prices, whilst he had altogether overlooked the much more important point that the very existence of the rate of profit suspends the law of value. According to Marx, Ricardo would thus have assumed that prior to a change in the rate of profit, prices were proportional to the amounts of labour embodied in the goods concerned.<sup>47</sup>

What basis can there be for such an assertion, however, in view of the following words of Ricardo, with which he draws the conclusion from one of his imaginary numerical examples: “Here then are capitalists employing precisely the same quantity of labour annually on the production of their commodities, and yet the goods they produce differ in value on account of the different quantities of fixed capital, or accumulated labour, employed by each respectively”.<sup>48</sup>

Marx’s comments on the discussions from which Ricardo draws this conclusion, are as follows<sup>49</sup>: “This exceedingly ponderous illustration of an ex-

<sup>44</sup> *Theorien über den Mehrwert*, II<sub>1</sub>, pp. 16, 17, 32-33, 38, 43, 60, 97, 111, 140, 143, 151; *Das Kapital*, III<sub>1</sub>, p. 183, footnote.

<sup>45</sup> The question of the origin of profit is not considered at this stage, but will be discussed further on.

<sup>46</sup> The same cannot be said of Marx—without an essential qualification. Cf. above, pp. 26—27.

<sup>47</sup> *Das Kapital*, III<sub>1</sub>, pp. 183-184; *Theorien über den Mehrwert*, II, pp. 40-44. The same baseless objection against Ricardo is to be found in A. C. Whitaker, *History and Criticism of the Labour Theory of Value in English Political Economy*, New York, 1904, pp. 55-56.

<sup>48</sup> *Principles*, Ch. I. Section IV, p. 28. Cf. p. 26.

<sup>49</sup> *Theorien über den Mehrwert*, II<sub>1</sub>, p.25[p. 221]. A footnote adds: “Not for this reason, but because these fellows have the fixed ideas that each of them ought to get the same spoils out of the ‘support they give to labour’, in other words, that their commodities, whatever their values may be must be sold at production prices which always yield the same rate of profit.” As if the law of value were being thrown over

ceedingly simple affair is built up in such a complicated way in order to avoid saying simply: Since capitals of equal size, whatever the relation between their organic constituent parts, or their periods of circulation, yield profits of equal size—which would be impossible if commodities were sold at their value, etc.—there exist prices of production which are different from the values of commodities. And this is in fact involved in the concept of a general rate of profit.”

These words contain a criticism, not of the substance of Ricardo’s conclusion, but merely of his mode of expression and of his demonstration. One might almost be led to believe that Marx admits that Ricardo had correctly recognized the nature of the conditions which Marx calls a divergence of prices from values.

But no! Marx does not go thus far to meet Ricardo. He does indeed comment once on the passage concerned that it shows some signs of the “correct surmise of the differences between production prices and values”.<sup>50</sup> Marx thinks, however, that Ricardo “forgets” this distinction already at the end of Section IV of the first chapter (from which the passage quoted has been taken)<sup>51</sup>. And on what does Marx base this assertion? Simply on the fact that in the last paragraph of this Section (and later in Section V), Ricardo no longer discusses the divergences of prices from values, but the price changes brought about by an alteration in the rate of profit<sup>52</sup>. This is indeed a strange method of criticism!

When Marx speaks of “forgetting”, he is using his mildest language. In another passage<sup>53</sup>, he resolutely asserts that Ricardo failed ever to attain the conclusion that prices diverge from values. Elsewhere he says: “this most important point of view simply does not exist for Ricardo”<sup>54</sup>. Ricardo is alleged “not to have had the slightest suspicion of the general change which takes place in the price of goods owing to the emergence of a general rate of profit”.<sup>55</sup>

Considering how close Marx came to admitting that Ricardo had known, in essence, the distinction between value and price, we may wonder, when faced with these last quotations, whether the accusation of all-too-short a

because the capitalists are its enemies! Nor do their efforts aim at all—as Marx assumes here—at earning a profit *proportional* to the total capital employed. What the capitalists want, is the *maximum* profit and an equal rate of profit comes about because this aim is common to them all, but meets with a resistance on the part of the buyers of their produce. Price-calculation does not spring solely from the will of the capitalists, and still less does it arise from their distorted mentality (“idées fixes”)! Quite apart from this, ‘Marx’s comment falls to attain its polemical purpose for the following reason: Ricardo assumes in his discussions *ab initio* that the rate of profit is the same in all lines of production. Having come to the conclusion that his example contained an incongruity with the (original) law of value, why need Ricardo then particularly point out that this conclusion rests on that assumption?

<sup>50</sup> *Theorien über den Mehrwert*, II<sub>1</sub>, p. 44. Cf. *Das Kapital*, III<sub>1</sub>, p. 158 [p. 211], where he talks of Ricardo “who doubtless realized that his prices of production differed from the value of commodities”.

<sup>51</sup> *Theorien über den Mehrwert*, II<sub>1</sub>, pp. 42-43.

<sup>52</sup> In the paragraph concerned, Ricardo names as cause of the price changes, not an alteration in the rate of profit, but an alteration in the value of labour. A rise in the value of labour, however, always implies for him a fall in the rate of profit, and a fall in the value of labour always implies a rise in the rate of profit. For details see later on.

<sup>53</sup> *Theorien über den Mehrwert*, II<sub>1</sub>, p. 47.

<sup>54</sup> *ibid.*, p. 111.

<sup>55</sup> *ibid.*, p. 161.

memory—raised by Marx against Ricardo—does not recoil on Marx himself<sup>56</sup>.

As regards Marx's polemics on this matter, it might at most be conceded that Ricardo should have kept more sharply separate the two questions of, firstly, the incongruity between value and price, and secondly, the influence on prices of changes in the rate of profit. But even this objection would, strictly speaking, be beside the point, since the first question presents itself, as it were, as a special case of the second one<sup>57</sup>. Apart from this point of criticism, there only remains the form of the exposition, in particular its arrangement.

That Ricardo often falls in this respect, and that he is not always *fortunate* with his numerical examples, is, of course, notorious<sup>58</sup>. When, however, with regard to this very incongruity of value and price, Marx talks of a "lack of ability to abstract", of "confusion" and "inner lack of clarity" in Ricardo<sup>59</sup>, he invites the most energetic contradiction<sup>60</sup>.

Coming now to the problem of the influence exercised on prices by changes in the rate of profit, Marx accuses Ricardo of having not only pushed this

<sup>56</sup> "Marx discussions of Ricardo in *Theorien über den Merhwert* rebound in unjust, petty, wordsplitting remarks (the footnote on p. 46 is characteristic in this regard). And yet Ricardo is practically the only author who, in certain sense, finds any mercy in Marx's eyes at all. Cf. *Das Kapital*, I, p. 49, footnote.

<sup>57</sup> Cf. above, p. 27.

<sup>58</sup> "Whether Marx leaves less to wish for in this respect, la yet another question.

<sup>59</sup> *Theorien über den Mehrwert*, II<sub>1</sub>, pp. 37, 42, 35.

<sup>60</sup> "On the other hand, it can hardly be said of Ricardo's interpreters that they had a clear insight into the matters discussed in the fourth and fifth Sections of the first Chapter of the *Principles*. Thus in a newly published essay by Dimitri Kalinoff, "David Ricardo and the theory of marginal value" (*Zeitschrift für die gesamte Staatwissenschaft*, edited by L. Bucher, supplementary volume XXII, Tübingen, 1907), Ricardo's theorem that a rise in wages alters exchange-relationships in the disadvantage of goods, in the production of which fixed capital has a relatively large share, (Cf. above, p. 27 and footnote 52) is related to the fact that "with a rise in the level of civilization, the costs of subsistence and reproduction the personal human labour force rise, whilst there is a fall in the cons of the production of the existence (*sic!*) and of the formation of the existence of the machine" (*loc. cit.*, p. 50). Whitaker (*loc. cit.*, p. 52) introduces his discussion of Sections IV and V of Chapter I of Ricardo's *Principles* with these words: "We may now turn our attention to what is perhaps as difficult a passage as was ever incorporated into a treatise on economics" It obviously never occurred to Kalinoff that this part of Ricardo's work needs to be studied with somewhat greater attention, and he gave a completely untenable interpretation to a most important component of Ricardo's theory of value. Kalinoff's essay is altogether rich in arbitrary associations of ideas which occasionally touch the borderline of conceptual confusion. He tries, for instance, to trace the distinction between direct (living) and indirect (accumulated) labour to the distinction between quality and quantity. He says, "Ricardo's concept of labour thus comprises on the one hand the whole past, crystalized, accumulated labour, the principally *quantitative productive power* of society; and on the other hand, the living, direct labour, society's *qualitative productive power*, developed on the basis of an existing technico-economic civilisation" (p. 30). According to Marx (*Das Kapital*, I, p. 167), these two forms of labour are related to each other as the pluperfect is to the perfect, and in my opinion, too, the category time would have been more appropriate here than the categories quantity and quality. Here, as in many other instances, Kalinoff is struggling to establish contact with Kant. But quite modern "philosophic" concepts also get dragged in, e.g. Simmel's "Superadditum of wealth." Kalinoff (p. 117) calls this (thrice !): "Superadditum". This may not be blamed on the printer. The author cannot, however, escape the blame that he, as an economist, should have allowed himself to be governed, somewhat excessively, by philosophers. He should not, furthermore, talk, in connection with Ricardo, of the "cognitive value of his views" (p. 32)! This philosophic whitewash does nothing to render the work any more respectable. The reader is also disturbed by the *epitheta ornantia* with which the writer smothers the authors he discusses (Cf. p. 105 on Rodbertus). Altogether, there is no lack of fine words and phrases in Kalinoff's essay. But as regards its basic ideas, namely that value is determined simultaneously by marginal utility and marginal cost, it contains nothing new. When Kalinoff (p. 139) claims to have "followed not the well-trodden roads of current theoretical opinion, but the cliffpaths of a higher synthesis of the apparently contradictory", he might risk sounding a little presumptuous had he not added that he was striving "to follow to the best of his ability, Adolph Wagner's research method". Has not Kalinoff, however, deceived himself as regards his solidarity with Wagner? He has taken over from the latter certain points of view on economic and financial policy, and we are not, here, objecting to this. These points of view appear to me, however, to have little in common with the question of "methods of research". We shall, incidentally, not deprive the reader of the information that Wagner, under whose eyes Kalinoff's work was accomplished (*loc. cit.*, p. 139), does not repudiate his disciple. Cf. A. Wagner, *Theoretische Sozialökonomik*, I, Leipzig, 1907, p. 217.

question too much into the background—this Las been fully discussed above—but of having treated it in a manner both “one-sided and erroneous”<sup>61</sup>. The following main points must here be considered<sup>62</sup>.

(1) Ricardo is alleged to have failed to recognize the fundamental distinction between variable and constant capital, and to have often left the latter completely out of account<sup>63</sup>. This objection was answered earlier on<sup>64</sup>.

(2) Ricardo is accused of having wrongly assumed that a rise in the rate of profit always went hand in hand with a fall in wages, and a fall in the rate of profit with a rise in wages. We shall discuss this point more fully later on..

(3) Ricardo is supposed to have assumed a general rate of profit as given. Marx writes: “He assumes a *general rate of profit*, or an *average profit of equal magnitude* for different investments of capital of equal size, or for different spheres of production in which capitals of equal size are employed—or, what is the same thing, a profit proportionate to the amounts of capital employed in the various spheres of production. Instead of assuming this general rate of profit in advance, Ricardo should rather have investigated how far its existence is in any way consistent with the determination of value by labour time; and he would then have found that instead of being consistent with it, *prima facie* it contradicts it, and its existence has therefore to be explained through a number of intermediary stages—an explanation which is something very different from merely including it under the law of value.”<sup>65</sup> In discussing a numerical example of Ricardo’s, Marx accuses him of having assumed an equal yearly rate of profit of 10% as “necessity and law”<sup>66</sup>. In another passage we find: “All of Ricardo’s illustrations only serve him to smuggle in the assumption of a general rate of profit”.<sup>67</sup>

Now it is true that if one believes, with Ricardo, that prices depend on the rate of profit, then the problem of price-formation cannot be considered to have been solved as long as one has not brought to light the factors determining the level of the rate of profit. Ricardo’s discussions—which Marx attacks here—do not, however, pretend in the least to give a complete solution of the price problem; their sole purpose, on the contrary, is to show how

<sup>61</sup> *Das Kapital*, III<sub>1</sub>, p. 184.

<sup>62</sup> We are not considering in our text any of the obviously void objections. Thus Marx finds it “most peculiar” that it did not occur to Ricardo to consider a fall as well as a rise in wages, or, a rise as well as a fall in the rate of profit, (Cf. above, footnote 52) as factors affecting prices. Marx continues: “and the *servum pecus imitatorum* did not even progress so far as to make this most self-evident, in fact tautologous, application”. (*Das Kapital*, II<sub>1</sub>, p. 183, footnote). The truth of the matter is that Ricardo reveals, quite unambiguously, in Section V of Chapter I of the *Principles*, that a fall in wages affects prices in a manner directly opposite to that corresponding to a rise in wages. He gives, it is true, a numerical example only for the case of a rise in wages. And the “miserable Peter MacCulloch (*Theorien über den Mehrwert*, II<sub>1</sub>, p. 38), whom Marx presumably chiefly meant by the *servum pecus imitatorum*, says quite explicitly: “If wages, instead of rising, has fallen, the opposite effects would have been produced”. (*The Principles of Political Economy*, Reprint of 1825 Edition, London, Murray, 1870, p. 162.) The following should here not be forgotten: Ricardo, J. R. McCulloch, and J. S. Mill, when discussing the effects on prices of changes in wage rates, have a practical purpose in view: they are fighting against the views of interested parties who were claiming that a rise in wages would injure (domestic) industry. It is therefore no great wonder that these authors pay more attention to a rise in wages than to the opposite case.

<sup>63</sup> *Theorien über den Mehrwert*, II<sub>1</sub>, pp. 14,27-28,29,33, 35, 46-47. Cf. *Das Kapital*, II, pp.383-384.

<sup>64</sup> Above, p. 20 *et seq.*

<sup>65</sup> *Theorien über den Mehrwert*, II<sub>1</sub>, pp. 14[p. 212].

<sup>66</sup> *ibid.*, p. 25, footnote 1.

<sup>67</sup> *ibid.*, p. 37, Cf. also p. 60.

changes in the rate of profit affect prices. For this reason, Ricardo is fully entitled, in his numerical examples, not only to assume that profits have been equalized, but also to adopt an arbitrary numerical value for the rate of profit. In an algebraic treatment, this arbitrary choice would be shown by leaving the rate of profit ( $\rho$ ) indeterminate. Dmitrieff's set of equations does, indeed contain an equation which allows  $\rho$  to be determined. This equation, however, remains quite beyond the scope of the discussion of the special case which preoccupies Ricardo in Sections IV and V of the first Chapter of his *Principles*.<sup>68</sup>

Ricardo's "fraudulent substitution" or smuggling" of the rate of profit is attacked by Marx from yet another, more general, standpoint. He alleges that Ricardo thus favoured the cost of production theory<sup>69</sup>, which Marx considers to be untenable for two reasons: firstly, because it ends in a vicious circle, and, secondly, because it gives rise to the view that the use of capital is an independent element of value, separate from labour.

The first objection is indeed applicable to sundry supporters of the cost of production theory, but not to the theory as such<sup>70</sup>.

The second objection hinges on the nature of profit. Equation (30) indicates that the rate of profit depends only on those amounts of labour and those turnover periods which concern the production and distribution of the goods forming the real wage rate. This theoretical result agrees entirely with Ricardo's thesis that the rate of profit cannot possibly be affected by the conditions of production of those goods which do not enter into real wages. A change in the conditions of production or of acquisition of wine, velvet, silk, or any other goods consumed only by the wealthy, does not bring about a change in the level of the rate of profit<sup>71</sup>.

Marx holds this thesis to be false, and believes it to be based on a confusion of the rates of profit and of surplus value. For—according to Marx—Ricardo's assertion would indeed be true of the latter. "The general rate of surplus value is, therefore," says Marx, "ultimately affected by the whole process, only when the increase in the productiveness of labour has seized upon those branches of production that are connected with, and has cheapened those commodities that form part of, the necessary means of subsistence, and are therefore elements of the value of labour-power."<sup>72</sup> This is not so, however, continues Marx, for the general rate of profit, which is an arithmetical average of the rates of profit in individual branches of production. These particular rates of profit depend on the (common) rate of surplus value and the (varying) organic composition of the capital concerned. All capital, forming the total social capital, plays its part in the formation of this average, and therefore also that capital which is

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<sup>68</sup> Cf. above, p. 27 *et seq.*

<sup>69</sup> *Theorien über den Mehrwert*, II<sub>1</sub>, p. 71.

<sup>70</sup> Cf. above, p. 16.

<sup>71</sup> *Principles*, Ch. VI, p. 96 and Ch. VII, pp. 112-113. This point of view is developed by Ricardo also in Ch. XV, dealing with the taxation of profits, p. 186.

<sup>72</sup> *Das Kapital*, I, pp. 316-317. [Translation from the .3rd German edition by Moore and Aveling, ed. by F. Engels, London, Allen and Unwin, 1938, p. 308.] Cf. *Theorien*, II<sub>1</sub>, pp. 104, 147. Cf. also, above, formula (15).

invested in the production of luxury goods. Should, therefore, any change take place in the organic composition of the latter capital, then this would

necessarily have a certain effect on the general rate of profit<sup>1</sup>.

This argument would be beyond reproach, were Marx's view correct that the general rate of profit is an average of the individual rates of profit, where each individual rate of profit could be expressed as a ratio of surplus value and the value of the total capital concerned  $\left(\frac{m}{c+v}\right)$ . We know, however, that critics have shown this view to be untenable<sup>74</sup>, and we now see how far-reaching are the "arithmetical errors" committed by Marx when transforming values into prices.

For it is just these arithmetical errors, namely a confusion of values and prices, which led Marx to his erroneous construction of the general rate of profit, and thence to his unjustified attack on Ricardo's thesis, the essential significance of which cannot be rated sufficiently highly. If it is indeed true that the level of the rate of profit in no way depends on the conditions of production of those goods which do not enter into real wages, then the origin of profit must clearly be sought in the wage-relationships and not in the ability of capital to increase production. For if this ability were relevant here, then it would be inexplicable why certain spheres of production should become irrelevant for the question of the level of profit.

In other words, much better than Marx's contrary view, does Ricardo's thesis fit into that theory of profit which regards profit as a withholding of some of the produce of labour, i.e. into the "Withholding Theory" (as I should like to call it instead of "Exploitation Theory").

Compared with Ricardo's standpoint, Marx's view represents a decided retrogression. Nevertheless, as regards Marx's attempt, which is connected with his own view, to reduce the rate of profit to a certain mathematical expression, we must not a priori reject the posing of the problem on which this attempt was based. A closer examination of the relevant quantitative relationships shows, however, that it is quite impossible to represent the rate of profit ( $\rho$ ) as an explicit function of those magnitudes on which it depends<sup>75</sup>. This proves to be feasible only if one either makes certain qualifications<sup>76</sup>, or if one contents oneself with a method of approximation based on the neglect of the second and higher powers of  $\rho$ .

In the latter case,—just as formula (33) was derived from formula (20)—formula (30) yields:

$$(37) \quad 1 = 1(1 + \delta\rho)U$$

where

$$(38) \quad \delta = \frac{u_1\tau_1 + u_2\tau_2 + \dots + u_s\tau_s}{u_1 + u_2 + \dots + u_s}$$

<sup>1</sup> *Theorien über den Mehrwert*, II<sub>1</sub>, pp. 109-110, 147, 157, 164, 166. Cf. *Das Kapital*, III<sub>1</sub>, pp. 58, 81.

<sup>74</sup> On this point, Cf. my article "Zur Berichtigung der grundlegenden theoretischen Konstruktion Von Marx in dritten Bande des 'Kapitals'", which is due to appear shortly in *Conrads Jahrbücher*. [*Translator's note*: this article has appeared in English under the title "On the correction of Marx's Fundamental Theoretical Construction in the Third Volume of Capital", in a volume edited by Paul Sweezy, New York, 1949).

<sup>75</sup> Cf. above, p. 21.

<sup>76</sup> Cf. my essay mentioned in footnote 74.

$\delta$  is the average duration of the turnover periods of the production and distribution of the complex of goods which constitute the real wage-rate.

(37) gives:

$$(39) \quad \delta = \frac{1-U}{\delta U}$$

This latter formula could also have been deduced as follows. On the basis of formula (36), one finds:

$$(40) \quad \lambda = \frac{1 + \rho\delta}{1 + \rho\delta_y} l$$

On the other hand, (35) gives:

$$l(1 + \rho\delta_y) = \frac{1}{A_y}$$

In view of this relationship as well as of formula (16), formula (40) becomes

$$\frac{1}{A_y} = (1 + \rho\delta) \frac{U}{A_y}$$

or formula (37), from which, finally, follows formula (39).

Formula (40) merits a certain attention, not only as the starting-point for this second derivation, but also for its own sake. It affords a relationship between money wages as they appear under price-calculation ( $\lambda$ ) and money wages as they appear under value-calculation ( $l$ ). Formula (40) reveals the error of believing that the mere transition from value-calculation to price-calculation implies a fall in money wages (for the reason, say, that in value-calculation the capitalist's wage outlay only is burdened with a mark-up, whilst in price-calculation his total outlay is thus affected). Such a transition can, on the contrary, just as well bring about a rise in money-wages, namely if  $\delta$  is greater than  $d_y$ .

Starting with the formula of approximation (39), we shall now pursue the question—of greatest importance to the theory of profit—of the connection between the level of the rate of profit, and the amount of labour required to produce the real wage-rate (the absolute value of labour power). We shall examine, besides Marx's positive views on this question, also his discussion of Ricardo. As, however, Ricardo's relevant remarks have frequently been misunderstood—for which his own loose way of expression is partly to blame—it is essential first to ascertain their true sense.

To begin with, it must not be overlooked that in Ricardo's oft-repeated phrase that every rise in wages must necessarily be accompanied by a fall in profits, and *vice-versa*<sup>77</sup>, one must understand by wages neither money wages, nor real wages, but that amount of labour which is needed to produce the

<sup>77</sup> e.g. in *Principles*, Ch VI, p. 96: "Whatever increases wages, necessarily reduces profits." Cf. *D. Ricardo's Kleinere Kldnere Schriften*, herausgegeben von E. Leser, Jena 1905, p. 83: "There is no other way of keeping profits up but by keeping wages down" (Translator's Note: *On Protection to Agriculture*, Section VI [P. Sraffa's Edition, Vol. IV, p. 237]); Cf. also *Letters of D. Ricardo to J. R. McCulloch*, edited by Hollander, New York, 1895, pp. 71—72.

complex of goods forming real wages<sup>78</sup>. In Marxist phraseology, this is the (absolute) value of labour power.<sup>79</sup>

Furthermore, it is noteworthy that, although Ricardo does not speak of *rate* of profit, but simply of profit, and although he often measures the level of profit by the capitalist's share in the value or the price of the product, yet he nevertheless intends his assertion of the antagonism between wages and profit to be extended specifically to the rate of profit<sup>80</sup>.

We cannot, finally, stress sufficiently strongly that there is no mention in Ricardo of this antagonism prevailing only insofar as there is no change in the value or the price of the product. Such a qualification, which he proposed as a correction by various people<sup>81</sup>, not only contradicts Ricardo's own words<sup>82</sup>; it deprives his thesis—that high wages *always* go with low profits, and high profits with low wages—of its point and reduces it to triviality<sup>83</sup>.

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78 This can be seen particularly clearly in the following passage: "Profits, it cannot be often repeated, depend on wages; not on nominal, but real wages; not on the number of pounds that may be annually paid to the labourer, but on the number of days' work, necessary to obtain those pounds" (Ch. VII, p. 124). At the end of this passage, Ricardo might equally well have said: "necessary to obtain the commodities on which those pounds are expended by the labourer". Cf. Ch. VI, p.105: "Profits depend on the quantity of labour requisite to provide necessaries for the labourer, on that land or with that capital which yields no rent." Cf. also Ch. I. Section VII, p. 42. In the text, I always assume that we are dealing with the least favourable conditions of production, whereby rent is eliminated. When Diehl speaks of the antagonism between wages and profit, he does not pay due regard to the real sense in which Ricardo uses the words "real wages" or "wages". Diehl knows only the categories money wages and real wages, or, in his own words, "nominal" and "real" wages, and this accounts for his wholly inapplicable remarks on Ricardo's theory of this antagonistic relation (*Sozialwissenschaftliche Erläuterungen zu Ricardo*, II, pp. 176-177). It is, in any case, surprising that Diehl has only a few words to say on this important theory. Generally speaking, Diehl treats, in his *Social science comments on Ricardo*, both of more and of less than is promised by the title. Ricardo's theories often recede into the background, whilst more recent publications are treated all the more fully. Would Diehl not have done better to call his book "Ricardo as an Educator"? A correct interpretation of Ricardo's expression "real wages" is to be found in J. S. Mill *Principles*, Book II, Ch. XV, §7.

<sup>79</sup> Ricardo, too, often says "value of labour" instead of "wages" (e.g. Ch. I. Section IV, p. 28). The position of those who allege that Ricardo constructed an antagonism between rate of profit and real wages, is untenable if only for the reason that in Ricardo's numerical examples (Ch. V. p. 78, Ch. VI, pp. 94, 96) a fall in the rate of profit goes hand in hand, not with a rise, but with a fall in real wages.

<sup>80</sup> *Principles*, Ch. VI, pp. 94-95, 101.

<sup>81</sup> Thus, Adolph Wagner (*Theoretische Sozialökonomik*, I, 1907, pp. 345-346) says that profits must fall under the influence of a rise in wages, unless the burden of this rise can be shifted on to the consumer by raising the price of the product of labour. This is, of course, obvious to any businessman, but it has nothing to do with Ricardo's thesis that rising wages always go with falling profits, since Wagner's addition ("unless the burden etc.") degrades this thesis to a mere private business maxim. Malthus too, (*Principes d'économie politique*, traduit par Constancio, Paris, 1820, I, p.481 *et seq.*). (*Translator's note*: T. R. Malthus, *Principles of Political Economy*, London, 1820, p. 327 *et seq.*), connects this thesis of Ricardo's with the assumption of a constant price for the product (or for corn, as the chief consumption good of the working-class) and bases on this his polemics against Ricardo. These polemics, to which Diehl (II, p. 179) accords the mark "good", cannot however touch Ricardo, if only because Malthus disregarded the fact that "real wages" (or "wages") have a special meaning in Ricardo. Alfred Marshall (*Principles of Economics*, I, p. 632, footnote 2) is quite right in finding it "regrettable" that Ricardo did not invent a new term for his concept of wages. Marshall adds: His artificial use of a familiar term has seldom been understood by others, and was in some cases even forgotten by himself."

<sup>82</sup> *Principles*, Ch. VI, pp. 98-94.

<sup>83</sup> Ricardo's thesis becomes a self-evident proposition also in the hands of those who interpret it in the sense that wages and profits are nothing but the worker's or the capitalist's shares in the proceeds (disregarding rent). This mistaken interpretation is to be found, e.g., in McCulloch (*loc. cit.*, pp. 193-194), who has generally done much to water down Ricardo's theory of profit, in Böhm-Bawerk (*Kapital und Kapitalzins*, I, pp. 106-107), who, however, also made a very pertinent remark on Ricardo's thesis (Cf. below, footnote 106), and in Whitaker (*loc. cit.*, p.54). A. Wagner (*Theoretische Sozialökonomik*, I, pp. 285-286), too, appears to view Ricardo's theory of the antagonism between wages and profit in the same light as the three above-mentioned authors. Edwin Cannan (*A History of the theories of production and distribution In English political economy from 1776 to 1848*, second edition, London, 1903, pp. 276-310), gives a detailed account of the controversies connected with Ricardo's views on the antagonism between wages and profit. Cannan's outlook, however, is biased as he supports the theory of marginal productivity even when applied

This concludes our contribution to the clarification of Ricardo's theory of the antagonism between wages and profits.

Marx had a correct understanding of this theory, *i.e.* he interpreted it in Ricardo's sense, but he refused to accept its validity.

His objection is similar to the one he raised against another of Ricardo's theories which we discussed earlier; here again, Marx says that there would be nothing to object to in Ricardo's theory if one substituted the rate of surplus value for the rate of profit. In effect, the former could neither rise nor fall without wages—in Ricardo's sense—falling or rising simultaneously. This follows from formula (15), as the magnitude  $U$ , which appears in this formula, represents nothing other than wages in Ricardo's sense, or  $m$  Marxian terminology, the value of labour power.

The relationship between the rate of profit and the value of labour power is, however, according to Marx, a somewhat complicated one. Formula (5), which is supposed to express the rate of profit, can be transformed, on the basis of (15), into

$$(41) \quad \rho = \frac{(1 - q_0)(1 - U)}{U}$$

It is from this last formula that Marx derives his main argument against Ricardo's theory.

According to Marx, the rate of profit could alter without any change taking place in the value of labour power. The organic composition of capital—*i.e.* that of the total social capital—need only alter, and the rate of profit would move up or down, the value of labour power remaining constant. In fact, it would even be possible for the value of labour power to rise and for the rate of profit to increase at the same time, if the “average organic composition of the total capital belonging to a certain society” fell correspondingly, *i.e.* if  $q_0$  became smaller. And conversely: even with a fall in the value of labour power (*e.g.* through a rise in the productivity of labour whilst real wages remained constant or did not rise in the same proportion), it would be possible for the rate of profit to be lowered by a rise in the organic composition of social capital to a correspondingly higher degree, *i.e.* by  $q_0$  becoming greater.

In Marx's opinion, it is just this latter case which prevails in reality. With progressive techniques of production,  $q_0$  increases, and this brings about a tendency for the rate of profit to fall, without any need for  $U$  to rise. Only a correspondingly sharp fall in  $U$  could arrest, let alone outweigh, the tendency of the rate of profit to fall.

Marx alleges that Ricardo was as little able as any other “bourgeois” economist to discover this “simple” connection. Various authors had offered varying

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to the problem of the origin of interest (*loc. cit.*, pp. 308-309); he therefore does justice neither to Ricardo, nor, in particular, to J. S. Mill. Specially noteworthy is Mill's paper “On profits and interest” (in the *Essays on some unsettled questions of political economy*, written in 1829-30, first published in 1844, second edition 1879), which is a very valuable contribution to this theory of Ricardo's. That Marx expressed his disapproval of the *Essays* (*Das Kapital*, I, p. 97, footnote), is all the more characteristic, since here Mill comes even closer to the Marxian construction of profit than did Ricardo. Cannan (p. 301, footnote) believes the concept of the rate of surplus value to be latent in Mill.

explanations for the fact that the rate of profit was falling, but not one of them had found the right one.

This failure, according to Marx, was due to the fact that “political economy up to the present has been tinkering with the distinction between constant and variable capital without ever defining it accurately; that it never separated surplus value from profit, and never even considered profit in its purely theoretical form, that is, separated from its different sub-divisions, such as industrial profit, commercial profit, interest, ground rent; that it never thoroughly analyzed the differences in the organic composition of capital, and for this reason never thought of analyzing the formation of an average rate of profit”.<sup>84</sup>

Ricardo is specifically accused of having disregarded the dependence of the rate of profit on the organic composition of capital. Sometimes Marx asserts that Ricardo had completely ignored constant capital, at other times Marx accuses him of having assumed the organic composition of capital to be constant (namely in studying the changes which occur in the rate of profit over a period of time). In other words, Ricardo is alleged to have taken  $q_0$  either to be equal to nought or to be a constant. The first leads to an identification of the rate of profit with the rate of surplus value, the second to the assumption of a constant ratio between these two magnitudes.<sup>85</sup>

We have already shown that Marx based himself on formula (41) both when formulating his own views of the factors determining the level of the rate of profit, and in his polemics on this point. If we consider, however, that this formula is the issue of formula (5), which we have seen to be false, then we might be inclined to reject offhand both those positive views and those attacks of Marx.

Nevertheless, we must not overlook the fact that the correct formula (39) leads, if not to a result identical with that yielded by (41), yet to a similar one.

In (39), the factor  $\frac{1}{\delta}$  replaces the factor  $(1 - q_0)$  and as a relatively higher or

lower organic composition of capital coincides *praeter propter* with a relatively long or short duration of the processes of production (and circulation) concerned, one might think that the error involved in formula (41) is, in fact, immaterial. Nevertheless, the case is not quite so favourable for Marx.

We must here consider, above all, that while  $q_0$  refers to the total social capital or, in other words, to the totality of all lines of production,  $\delta$  depends only on the conditions prevailing in some lines of production, namely in those which are directly or indirectly related to the production of real wages. Marx's error, which we discussed earlier on, makes itself felt here.

Apart from this, by no means inessential, point, we must consider the following.

<sup>84</sup> *Das Kapital*, III, pp. 193-194 [p. 250].

<sup>85</sup> *Theorien über den Mehrwert*, II, pp. 135, 150-152, 157, 161, 166. Cf. I, p. 177 and *Das Kapital*, III, pp. 39.

Marx connects a rise in the organic composition of capital, *i.e.* a rise in  $q_0$ , with an increase in the productivity of labour. Only thus does the law of the falling rate of profit, “discovered” by Marx, attain the importance which he

himself and his disciples attach to it. An increasing productivity of labour (by raising  $q_0$ ) is supposed to bring about a fall in  $\rho$ , and in this they see a necessary internal contradiction of capitalist production. For whilst capitalist production includes—independently of the social conditions in which it takes place—a tendency or progressive development of the productive forces, yet profit, which after all is the driving force in capitalist production, is depressed by this very same progressive development<sup>86</sup>. In Marx's opinion<sup>87</sup>. It is here demonstrated in a purely economic way, that is, from a bourgeois point of view, within the confines of capitalist understanding, from the standpoint of capitalist production itself, that it has a barrier, that it is relative, that it is not an absolute, but only a historical mode of production corresponding to a definite and limited epoch in the development of the material conditions of production”.

The law of the falling rate of profit, or, in more general terms, Marx's theory of the factors determining the level of the rate of profit, is thus clearly a matter of eminent importance. To judge this theory, one must—among other things—arrive at a clear view concerning the criterion of an increased productivity of labour. Productivity of labour is measured by the ratio of the quantity which is produced of any commodity and the amount of labour employed in its production. If production is divided into several stages, then productivity can be measured for each stage separately by disregarding the amount of labour employed in the preceding stages. An increased productivity of labour will then be equivalent to “an operation of a larger mass of means of production by fewer labourers”.<sup>88</sup>

For reasons which it would take us too far to discuss here, it is advisable, in measuring the productivity of labour, to base oneself on the quantity of the *end-product* concerned not only for the last (highest) stage of production, but for all stages. From this standpoint, one can speak of an increase in the productivity of labour, *e.g.* at the stage of machine-production, only if a reduction takes place in the (absolute) amount of labour which the labour embodied in the machine transmits to an end-product of a given quantity and quality. We have, here, to consider not only the conditions of the production of the machine, but also of its use.

On the basis of this definition, the question whether the productivity of labour is higher or lower, can obviously be answered without regard to any relations of value or price.<sup>89</sup>

Marx believes that, in reality, the increase in the productivity of labour shows itself not only in the fact “that there is a decrease in the total amount of labour embodied in the commodity” (were this not so, there could be no question of an increase in the productivity of labour !), but also in the fact that the share of living labour in this total decreases whilst that of past labour increases.<sup>90</sup> In our exposition, this assertion would correspond to the assump-

<sup>86</sup> *Das Kapital*, III, pp. 231, 241.

<sup>87</sup> *ibid.*, p. 242 [pp. 304-305]. Cf. pp. 223 and 232.

<sup>88</sup> *Das Kapital*, III, p. 32 [p. 72], Cf. p. 52.

<sup>89</sup> In order to express this, one might say “technical” or “physical” productivity. Cf. Irving Fisher, *The Nature of Capital and Income*, New York, 1906, p. 186.

<sup>90</sup> *Das Kapital*, III, p. 243, Cf. pp. 52, 228, etc.

tion that  $\delta$  increases. I shall presuppose this to be true, and, on this basis, enquire into the effects on the rate of profit of an increase in  $\delta$ .

In doing this, it will be convenient to distinguish the two following ways in which an increase in  $\delta$  can take place. The increase in the magnitude  $\delta$ , *i.e.* in the average duration of the turnover periods involved in the production of real wages, is connected with the fact that in any one, or in several, of the relevant branches of production, there occurs either the addition of a new, preliminary, stage of production, or else a change in the conditions of productivity in the various stages of production.

The first case occurs when a new means of production is introduced (*e.g.* a machine), where formerly one made do with mere “manual” labour. The manufacture of this new means of production can then be regarded as a new, preliminary, stage in the production of the end-product. Such a change in the conditions of production may involve an increase in  $\delta$  (although this need not necessarily be so). Assuming an increase in  $\delta$ ,  $U$ , *i.e.* the amount of labour embodied in the given real wage, must clearly diminish, since the introduction of the new means of production will only take place if it increases the productivity of the labour serving to produce real wages. Now this increase in productivity will find expression in just this diminution in  $U$ . Under capitalistic production, however, this constitutes merely a necessary, but not a sufficient, condition for the introduction of the new means of production. The capitalist, who takes the decision regarding this process of production, must anticipate an additional profit from the new means of production before he feels the urge to introduce it. On no account must the new conditions of production yield a rate of profit lower than the previous one. Not productivity, but profitability, is here decisive. If therefore, the lengthening of the processes of production (the increase in  $\delta$  depends on the addition of a new preliminary stage of production, then it can not possibly bring about a fall in the rate of profit. Capitalism itself guards against this.

Marx knows very well that the capitalist has in view, not productivity, but profitability<sup>91</sup>, but he believes that this circumstance cannot prevent a fall in the rate of profit. Marx bases this opinion on the following line of argument, which, by the way, concerns not only the case we are considering.

We read in *Das Kapital*<sup>91</sup> : “No capitalist voluntarily introduces a new method of production, no matter how much more productive it may be, and how much it may increase the rate of surplus value, so long as it reduces the rate of profit. But every new method of production of this sort cheapens the commodities. Hence the capitalist sells them originally above their prices of production, or, perhaps, above their value. He pockets the difference, which exists between these prices of production and the market prices of the other commodities produced at higher prices of production. He can do this, because the average labour time required socially for the production of these other commodities is higher than the labour time required under the new methods of production. His method of production is above the social average. But

<sup>91</sup> *Das Kapital*, I, pp. 398-400.

<sup>91</sup> III, p. 247 [pp. 310—311].

competition generalizes it and subjects it to the general law. Then follows a fall in the rate of profit.—perhaps first in this sphere of production, which gradually brings the others to its level—which is, therefore, wholly independent of the will of the capitalist.”

These remarks could be expressed in our method of exposition as follows. We start with an initial situation in which the rate of profit is  $\rho$  the average turnover period  $\delta$ , the value of labour power  $U$ , and the wages  $\lambda$ , and go over to an ultimate situation in which the corresponding magnitudes are  $\rho'$ ,  $U'$  and  $\lambda'$ . Here,  $U'$  is smaller than  $U$  (because of the increased productivity of labour) and  $\delta'$  greater than  $\delta$  (because of the lengthening of the processes of production). We have to prove that  $\rho'$  must be smaller than  $\rho$  On the basis of formula. (14), we get:

$$(42) \quad \lambda = (1 + \delta\rho)U\lambda$$

and

$$(43) \quad \lambda' = (1 + \delta\rho')U'\lambda'$$

We also construct a transition stage during which the new means of production is introduced. This stage is marked by the fact that the prices (thus also  $\lambda$  as the price of the complex of goods forming real wages) are still the old ones, whilst some of the capitalists have already realized the labour-saving in question. These capitalists are supposed to obtain an additional profit, for the very reason that they employ the amount of labour  $U'$ , whilst the price of the products reflects the greater amount of labour  $U$ .

In their calculations, these capitalists will, however, clearly not leave out of account the fact that the lower amount of labour  $U'$  is linked to the longer period of production  $\delta'$ . They will therefore see to it that the inequality

$$(44) \quad \lambda > (1 + \delta\rho)U'\lambda$$

is fulfilled. For otherwise, they would have to expect to lose from the application of the new means of production.

Thus, on the one hand, we have, on the basis of formula (43)

$$(45) \quad (1 + \delta\rho')U' = 1$$

and on the other hand, on the basis of (44)

$$(46) \quad (1 + \delta\rho)U' < 1$$

$\rho'$  is therefore not smaller, but greater than  $\rho$ . This refutes Marx's line of argument quoted above.

A double error is involved in Marx's argument. Firstly, it is wrong to connect a change in the rate of profit with a change in prices, since, as can be seen from our formulae, the potential price movements affect the capitalist's product to the same degree as they do his outlay. This first error is due to Marx's choosing an arbitrary commodity, instead of the complex of goods which forms real wages. The second error lies in the fact that in the passage cited above, from the third volume of *Das Kapital*, Marx makes his capitalist cal-

culate according to the principles of the first volume. Were the capitalist to consider only the amount of labour used or the total wage bill, 'without paying regard to the period of turnover, then he would indeed go over to the method of production even if inequality (44) were not fulfilled. He would be ruled rather by the inequality  $U > U'$ . But however "fatuous" capitalist calculation may be, it nevertheless prevails in the competitive world. The principles governing calculation in the first 'volume of *Das Kapital* are held in sovereign contempt by "the capitalist mind". On this occasion, too, Marx is guilty of a gross confusion of value-calculation and price-calculation.

It might perhaps be objected against my arguments that they are based on an idea which is unjustified, since it is contradicted by the facts. This idea, namely, is that each single capitalist produces the whole complex of goods which forms real wages. One might possibly reach a different conclusion, were to pay regard to the independence of the various branches of production. The answer to this would be as follows.

To the extent to which different components of real wages correspond to different branches of production, it would be essential to consider that particular product, the production of which is altered by the introduction of the new means of production. Let  $\mu_i$  be the amount of this product contained in real wages,  $p_i$  the price of one unit of this product,  $A_i$  the amount of labour embodied in this unit, and  $d_i$  the average turnover period in the production of this product. Owing to the introduction of a new means of production, the amount of labour falls from  $A_i$  to  $A_i'$ , whilst the turnover period rises from  $d_i$  to  $d_i'$ . Inequality (44) will here be replaced by

$$(47) \quad p_i > (1 + d_i' \rho) A_i \lambda_i$$

which, on the basis of formula (33) becomes

$$(48) \quad (1 + d_i \rho) A_i > (1 + d_i' \rho) A_i'$$

For the first and final stages, the following formulae are then true<sup>93</sup>:

$$(49) \quad (1 + d_1 \rho) A_1 \mu_1 + (1 + d_2 \rho) A_2 \mu_2 + \dots + (1 + d_i \rho) A_i \mu_i + \dots + (1 + d_n \rho) A_n \mu_n = 1$$

and

$$(50) \quad (1 + d_1 \rho') A_1 \mu_1 + (1 + d_2 \rho') A_2 \mu_2 + \dots + (1 + d_i' \rho') A_i' \mu_i + \dots + (1 + d_n \rho') A_n \mu_n = 1$$

Formula (50) differs from formula (49) in that all the terms on the left-hand side contain  $\rho'$  instead of  $\rho$  whilst in the  $i$ th term  $A_i'$  replaces  $A_i$ , and  $d_i'$ , replaces  $d_i$ .

With the help of these two formulae, it is now easy to show that  $\rho'$  cannot possibly be smaller than  $\rho$ . For if this were the case, then the condition

$$(1 + d_i' \rho') A_i' > (1 + d_i \rho) A_i$$

and, *a fortiori*, the condition

<sup>93</sup> Cf. formula (27).

$$(51) \quad (1 + d_i' \rho) A_i' > (1 + d_i \rho) A_i$$

would necessarily be fulfilled. Inequality (51), however, contradicts inequality (48). On the other hand, having regard to (48),  $\rho'$  cannot equal  $\rho$  since otherwise the sum of the terms on the left side of equation (50) would add up to less than 1. Consequently,  $\rho'$  must be greater than  $\rho$ .

The independence of the various spheres of production manifests itself, however, not only in the manner we have considered so far, but also in the division of the process of production into different successive stages, in which different capitalists are active. One might ask whether the introduction of a new means of production in any one stage does not affect the production of the capitalist in the next higher stage. This doubt can very easily be removed, without having to modify Marx's construction in any way.

If namely, in studying the alteration occurring in the conditions of production at the lower stage, one assumes that there is no change in the productivity of labour at the higher stages, then that alteration can be seen to reduce—and not to raise—the organic composition of the capital employed at the higher stage. Either the machine or the raw materials will become cheaper, and—according to Marx—this is a factor counteracting the tendency of the rate of profit to fall.

This confirms that a lengthening of the processes of production (an increase in  $\delta$ , due to the introduction of a new means of production, cannot in any circumstance endanger the rate of profit.

We shall now proceed to the second manner in which an increase in  $\delta$  can take place. This consists in a shift in the conditions of productivity, *i.e.* in a change in the productivity of labour, which does not affect all stages of production equally.

If the productivity of labour changes in the same proportion at all stages, then this obviously does not affect the average duration of the turnover periods. Furthermore, insofar as such a change occurs in the spheres relevant to the production of real wages, it raises or lowers the rate of profit according to whether the productivity of labour increases or diminishes, since  $U$  becomes smaller in the first case and larger in the second<sup>94</sup>.

If, on the contrary, the productivity of labour does not increase or diminish in the same proportion at all stages of production, then a change in  $\delta$  may result. An increase in  $\delta$  will, in particular, occur where the productivity of labour either grows at an increasing rate or diminishes at a decreasing rate, or if it first diminishes at a decreasing rate and then—starting at some stage—grows at an increasing rate. We shall now examine more closely this last case, which may be termed a “shift of the conditions of productivity in favour of the higher stages of production”. Our main aim will be to show that this shift is linked to an increase in  $\delta$ .

In order to define the circumstances mathematically without introducing many fresh symbols, we shall assume that the turnover periods  $\tau_1, \tau_2, \text{etc.}$  in formula (30) (or several of these magnitudes forming a continuous partial

<sup>94</sup> Cf. formula (39).

series) correspond to different successive stages of production in such a way that  $\tau$  becomes into the lowest stage and  $\tau_1$  into the highest one. We thus have

$$(52) \quad \tau_1 > \tau_j + 1$$

Let the turnover periods ' $\tau_1, \tau_2$  etc. involve the amounts of labour

$$u_1, u_2, \dots, u_s$$

Let an altered productivity of labour be expressed by replacing this series by

$$ku_1, ku_2, \dots, ku_s$$

where, in our case

$$(53) \quad k_j > k_j + 1$$

for all values of  $j$  from 1 to  $s - 1$ .

As several values for  $\tau$  or  $u$  may fall into one and the same stage, one should —to IX quite precise—say that  $k_1$  is greater than  $k_{j+1}$  or equal to  $k_{j+1}$

Under the former conditions of productivity, the average turnover-period was<sup>95</sup>:

$$(54) \quad \delta = \frac{u_1\tau_1 + u_2\tau_2 + \dots + u_s\tau_s}{u_1 + u_2 + \dots + u_s}$$

Under the new conditions of productivity, this becomes:

$$(55) \quad \delta' = \frac{k_1u_1\tau_1 + k_2u_2\tau_2 + \dots + k_su_s\tau_s}{u_1 + u_2 + \dots + u_s}$$

We have to prove that  $\delta'$  is greater than  $\delta$ .

Formerly<sup>96</sup> we had

$$(56) \quad u_1 + u_2 + \dots + u_s = U$$

and we shall now introduce:

$$(57) \quad k_1u_1 + k_2u_2 + \dots + k_su_s = U'$$

Let further  $k_0$  signify the proportion in which productivity at all stages has changed on the average, so that we get

$$(58) \quad k_0 = \frac{k_1u_1 + k_2u_2 + \dots + k_su_s}{u_1 + u_2 + \dots + u_s}$$

and also

$$(59) \quad k_0 = \frac{U'}{U}$$

If now, on the right-hand side of formula (55), we divide numerator and

denominator by  $k_0$  and if we put  $\pi_j$  instead of  $\frac{k_j}{k_0}$  then we find:

<sup>95</sup> Cf. formula (38).

<sup>96</sup> Cf. formula (31).

$$(60) \quad \delta'U = \pi_1 u_1 \tau_1 + \pi_2 u_2 \tau_2 + \dots + \pi_s u_s \tau_s$$

We also have the inequality

$$(61) \quad \pi_j > \pi_{j+1}$$

which corresponds to inequality (53).

The magnitude  $\delta$  is an average of  $\tau_1, \tau_2, \text{ etc.}$  Consequently, some of these values of  $\tau$  must be greater and others smaller than  $\delta$ . Let the greater ones be  $\tau_1, \tau_2, \text{ etc.}$  up to  $\tau_m$ , the smaller ones  $\tau_{m+1}, \tau_{m+2}, \text{ etc.}$  up to  $\tau_s$ . Then construct the positive expression

$$D = u_1(\tau_1 - \delta) + u_2(\tau_2 - \delta) + \dots + u_m(\tau_m - \delta)$$

On the basis of formula (54) we are obviously also entitled to express D as follows:

$$D = u_{m+1}(\delta - \tau_{m+1}) + u_{m+2}(\delta - \tau_{m+2}) + \dots + u_s(\delta - \tau_s)$$

Furthermore, because of inequality (61), we have:

$$\pi_1 u_1(\tau_1 - \delta) + \pi_2 u_2(\tau_2 - \delta) + \dots + \pi_m u_m(\tau_m - \delta) > \pi_m D$$

and

$$\begin{aligned} \pi_{m+1} u_{m+1}(\delta - \tau_{m+1}) + \pi_{m+2} u_{m+2}(\delta - \tau_{m+2}) + \dots \\ + \pi_s u_s(\delta - \tau_s) < \pi_{m+1} D \end{aligned}$$

Therefore, *a fortiori*:

$$\pi_1 u_1(\tau_1 - \delta) + \pi_2 u_2(\tau_2 - \delta) + \dots + \pi_s u_s(\tau_s - \delta) > (\pi_m - \pi_{m+1})D$$

This latter inequality can, on the basis of formulas (60) and (58), be transformed into

$$\delta'U - \delta U > (\pi_m - \pi_{m+1})D$$

from which it finally follows that

$$\delta' > \delta.$$

It cannot therefore be denied that a shift in the conditions of productivity in favour of the higher stages of production brings with it a lengthening of the average turnover period, the result of which—the rate of surplus value remaining constant—must, owing to formula (39), be a fall in the rate of profit.

Insofar as the capitalist is active at an independent stage of production, he will not offer any resistance against an increase in  $\delta$ . For if he has the possibility of increasing the productivity of labour in his own sphere, he has no reason to consider the possibility that a lengthening of the average turnover period may be closely linked to this<sup>97</sup>. Marx describes how the capitalist will, at first, gain an additional profit from the increased productivity of labour, *i.e.* for so as the old prices prevail. Later on, however when prices have adjusted themselves to the changed situation, this additional profit will vanish, and a

<sup>97</sup> Circumstances here are thus different from the first case, when the lengthening of the average turnover period was linked to the introduction of a new means of production.

fall in the rate of profit will, apparently, be inevitable if the conditions of productivity have shifted in favour of the higher stages of production.

One might therefore be inclined to accept Marx's explanation of the fall in the rate of profit as being at least partly sound. This would, indeed, be true only of one method of lengthening production, or of increasing the constant part of capital, but it is just this method which Marx regards as being highly characteristic of the real course of affairs. According to Marx, the typical case of an increase in the productivity of labour consists in producing—with the same amount of labour as formerly—a new machine which will enable one to process—with the same number of employees—greater quantities of raw materials and semi-manufactured products than would have been possible with the old machine. In such a case, there would be, for each worker, a greater amount of constant capital, both fixed and circulating<sup>98</sup>. In this process, circulating capital increases faster than fixed capital, so that each unit of the product contains a smaller amount—considered absolutely—of fixed capital<sup>99</sup>.

In dealing with greater quantities of raw materials and semi-manufactured products, the organic composition of the capital actively concerned changes only if no equivalent increase in the productivity of labour takes place in the lower stages of production, *i.e.* in the production of these raw materials and semi-manufactured products as well as of the machines involved. Marx sometimes omit to mention this, but must have realized it quite clearly. One may read in *Das Kapital* that the rate of profit can remain unchanged (for the very reason that the organic composition of capital would not change) “if the increase in the productivity of labour extended its effects uniformly and simultaneously to all the elements of the commodities, so that the total price of the commodities would fall in the same proportion in which the productivity of labour would increase, while on the other hand the mutual relations of the different elements of the price of commodities would remain the same”<sup>100</sup>. In this connection, we may also call to mind Marx's remarks on the cheapening of the elements of constant capital, which is alleged to slow down the fall in the rate of profit.<sup>101</sup>

We shall not here further pursue the question whether Marx's belief that the productivity of labour generally shifts in favour of the higher stages of production, represents a correct generalization of real events. I shall simply assume that this does really happen, or, in other words, that inequality (53) is fulfilled in the transition from an earlier stage, with the rate of profit  $\rho$ , to a later stage with the rate of profit  $\rho'$ .

<sup>98</sup> *Das Kapital*, III<sub>1</sub>, pp. 192, 203, 206, 228.

<sup>99</sup> This is clearly expressed in the words: “If five labourers produce ten times as many commodities as formerly, this does not increase the outlay for fixed capital tenfold; although the value of this part of the constant capital increases with the development of the productive forces, it does not increase by any means in the same proportion with them” (*Das Kapital*, III<sub>1</sub>, p.243 [p. 305]). In the case cited, *i.e.* where the workers produce ten times as many goods as formerly, the fixed capital which participates in the production would, for example, increase only sixfold. An unchanged amount of goods would thus lie produced by 1/10 of the amount of living labour formerly required, and by 6/10 of the amount of fixed capital formerly required. In *Das Kapital* Marx's words, quoted above, are followed by a passage by Engels (square brackets !) in which the contrary is asserted with regard to the fixed capital. and is then assumed in the numerical example on p. 244.

<sup>100</sup> *Das Kapital*, III<sub>1</sub>, p. 211 [p. 269].

<sup>101</sup> *ibid.*, p. 207.

We are dealing with the comparison between the two magnitudes

$$\rho = \frac{1-U}{\delta U}$$

and

$$(62) \quad \rho' = \frac{1-U'}{\delta' U'}$$

Marx's assertion that  $\rho'$  is smaller than  $\rho$  is based on the presupposition that the rate of surplus value remains constant, or that

$$(63) \quad \frac{1-U}{U} = \frac{1-U'}{U'}$$

Marx bases his law of the falling rate of profit explicitly on this presupposition and regards the rise in the rate of surplus value as a factor apart. "This factor", says Marx, "does not suspend the general law. But it causes that law to become more of a tendency, that is, a law whose absolute enforcement is checked, retarded, weakened, by counteracting influences."<sup>102</sup>

However, in discussing the question whether  $\rho'$  is greater or smaller than  $\rho$ , is it permissible to use equation (63) ?

From (63) we derive

$$U' = U$$

and, since the shift in the conditions of productivity in favour of the higher stages of production results in  $\delta' > \delta$ , we get

$$U' \delta' > U \delta$$

or, on the basis of (54) and (55),:

$$(64) \quad k_1 u_1 \tau_1 k_2 u_2 \tau_2 + \dots + k_s u_s \tau_s > u_1 \tau_1 + u_2 \tau_2 + \dots + u_s \tau_s$$

For the rate of profit to fall as a result of a shift in the conditions of production—the rate of surplus value remaining constant—it would therefore be necessary to comply with inequality (64). This, however, presupposes that at least one of the values of  $k$  is greater than 1. In other words, the productivity of labour would have to fall in at least one of the stages of production.

As we have seen, however, Marx assumes on the contrary that the productivity of labour increases at all stages of production, or, in other words, that all values of  $k$  are less than 1. We now see that this assumption cannot be reconciled with his presupposition that the rate of surplus value remains constant.

In fact, it is easy to demonstrate that this assumption leads necessarily, not only to a higher rate of surplus value, but also to a higher rate of profit. We have indeed on the one side, on the basis of formula (54):

$$(65) \quad \delta U = u_1 \tau_1 + u_2 \tau_2 + \dots + u_s \tau_s$$

and on the other side, on the basis on formula (55):

<sup>102</sup> *ibid.*, p. 215 [p.275].

$$(66) \quad \delta'U' = k_1u_1\tau_1 + k_2u_2\tau_2 + \dots + k_su_s\tau_s$$

As we have now assumed that all values of  $k$  are less than 1, we find

$$(67) \quad \delta U' < \delta U$$

and since, on the same assumption, we have

$$U' < U$$

It is clear that the formula (62) for  $\rho'$  the numerator is greater and the denominator smaller than in the corresponding formula for  $\rho$ . Therefore:

$$(69) \quad \rho' > \rho$$

This derivation also shows that if only any one of the values of  $k$  is less than 1 and no single one greater than 1, the inequalities (67) and (68) will have been fulfilled, and that this will lead to inequality (69). In other words: to bring about a rise in the rate of profit, it is sufficient for the productivity of labour to increase only at any one stage of production without a concurrent fall in the productivity of labour at other stages.

Marx is quite right in saying that such a rise in productivity, which is limited to one stage of production, will at first allow the capitalists engaged in that stage of production to benefit by a rise in the rate of profit above the general level, and that this supernormal rate of profit will later on be pressed down to the general level. This latter will, however, be a different one: not, as Marx thought, a lower, but a higher, level than the previous one.

Marx's own proof of his law of the falling rate of profit errs principally in disregarding the mathematical relationship between the productivity of labour and the rate of surplus value. He regards the latter as a factor apart. The absurdities to which one can be led by such a method of isolation, can be recognized from the following very simple example. Let  $a$  be a positive magnitude which is related to two other positive magnitudes  $b$  and  $c$  by the relationship  $a = \frac{b}{c}$ . The question is: in what direction will  $a$  change when

some fourth magnitude  $d$  alters, where each of them magnitudes  $b$  and  $c$  is a function of  $d$ . Let, e.g.,  $b = d^5$  and  $c = d^3$ . The correct solution of this problem is obviously as follows: one eliminates  $b$  and  $c$  from the expression for  $a$ , finds  $a = d^2$  and concludes therefrom that  $a$  will change in the same direction as  $d$ . If, however, one applies to this case Marx's method of isolation, one might

express  $a$ , e.g., by  $\frac{b}{d^3}$  and conclude from this formula that  $a$  diminishes with an increase in  $d$  and increases with a diminution of  $d$ . Were one to add that a change in  $b$  might indeed disturb this relationship, but that this would be a matter apart, then this would reveal all the more clearly the similarity of this method of procedure with Marx's method of isolation.

What is in fact true is thus the exact contrary of Marx's theory. An increase in the productivity of labour—whether it occurs at all stages of production, or only at some—leads to an increase in the rate of profit, with the sole pro-

vision that this increase in productivity should take place in those lines of production which are directly or indirectly relevant for the production of real wages.<sup>103</sup>

If, however, the increase in productivity were linked to a shift in the conditions of productivity in favour of the higher stages of production, the only effect would be that the rate of profit would increase in a lesser degree than the rate of surplus value. Let the old rate of surplus value be  $r$  and the new one  $r'$ . If productivity has increased on the average of all stages proportionately from  $k_0$  to 1, as would follow from (59), then, on the basis of (15) we get

$$(70) \quad r' = \frac{1 - k_0 U}{k_0 U}$$

and consequently

$$(71) \quad \frac{r}{r'} = \frac{\frac{1}{k_0} - U}{1 - U}$$

On the other hand, (39) and (62) lead to:

$$(72) \quad \frac{\rho}{\rho'} = \frac{\delta \left( \frac{1}{k_0} - U \right)}{\delta' (1 - U)}$$

But as the presupposed shift in the conditions of productivity expresses itself in  $\delta'$  being greater than  $\delta$ , we reach in fact the result:

$$(73) \quad \frac{\rho'}{\rho} < \frac{r'}{r}$$

This inequality represents the particle of truth which is contained in Marx's law of the falling rate of profit. A shift in the conditions of productivity in favour of the higher stages of production, if accompanied by an increase in productivity at all stages of production, will, in a certain sense, affect the rate of profit unfavourably, not indeed by lowering the rate of profit, but by only allowing it to increase less fast than the rate of surplus value.

It will be remembered that Marx accused Ricardo of having confused the rate of profit with the rate of surplus value in his theory of the antagonism between wages and profits. The above formulae shed a fresh light on this flatter too: if  $k_0$  is less than 1, *i.e.* if the productivity of labour increases on the average of all stages of production, then a rise will occur in the rate of surplus value. The rate of profit may nevertheless fall. To enable one safely to assert that the rate of profit will also rise, it will not be sufficient for the productivity of labour to increase in the average of all stages of production; it will, in addition, be necessary that productivity shall not fall in any stage of production. This correction of Ricardo's theory can be formulated as follows:

<sup>103</sup> As the formulae show, a fall in productivity of labour produces the contrary effect, *i.e.* a fall in the rate of profit.

the rate of profit rises when the wage in Ricardo's sense, *i.e.*  $U$ , falls, and the rate of profit falls when  $U$  rises, but only on condition that, in the first case, no fall in the productivity of labour occurs at any stage of production, and that, in the second case, no increase takes place at any stage of production. That Ricardo does not specifically mention this qualification is certainly an inaccuracy, but one of far less significance than the grave errors in Marx's theory of the movements of the profit rate.

With regard to Ricardo, we must not, however, overlook that one cannot always give a causal interpretation to the antagonism between  $\rho$  and  $U$ , which really does prevail with the above-mentioned qualification. For Ricardo very often talks as if a change in  $\rho$  could not be caused otherwise than by a change in  $U$ .<sup>104</sup> The truth is rather that although a change in  $U$  is always accompanied by a change in  $\rho$ <sup>105</sup>, this does not imply that all changes must start with the first of these two magnitudes. Apart from Alfred Marshall, Böhm-Bawerk, too, drew attention<sup>106</sup> to Ricardo's tendency to interpret in an exclusively causal sense the quantitative relationships between various economic factors such as the one we have just been discussing.

Here it may be worth pointing out once more that, in making wages and profits move in opposite directions, Ricardo means by wages not real wages themselves but the magnitude  $U$ , *i.e.* the amount of labour embodied in real wages.

In discussing Marx's views on the connection between the productivity of labour and the rate of profit, I have regarded the real wage as given. This would be expressed mathematically by assuming the magnitudes  $\mu_1, \mu_2, \text{etc.}$  in formula (28) to be constant. For it is only on this assumption that one can assert that an increase in the productivity of labour will diminish the magnitudes  $u_1, u_2, \text{etc.}$  appearing in formula (29). But if one drops the assumption of constant real wages, then one will certainly have to admit also that the rate of profit can fall whilst the productivity of labour increases. This does not, however, constitute the slightest concession to Marx, for the following reasons:

(1) To the extent to which one allows for the possibility that real wages may rise, even that theorem ceases to be valid which asserts that the rate of surplus value rises with an increasing productivity of labour (if, namely, the relevant lines of production are affected). That, however, is a theorem that Marx never questions.

(2) If, with an increasing productivity of labour and rising real wages, the rate of profit should happen to fall, this would occur not because, but in spite of, the rise in productivity.

(3) If the fall in the rate of profit goes hand in hand with a rise in real wages, then the fact that the productivity of labour is simultaneously increasing, loses all semblance of being a paradox. There can then be no more question of a contradiction between the historical mission of capitalism—which the “develop-

<sup>104</sup> Cf. also Marx, *Theorien über den Mehrwert*, II<sub>1</sub>, p. 139.

<sup>105</sup> I disregard here the condition discussed above which qualifies the validity of this statement.

<sup>106</sup> See footnote 83 and Böhm-Bawerk, *Capital und Kapitalzins*, I, pp. 106-107. On “causalism” in economics generally, cf. my remarks in *Schmoller's Jahrbücher*, 22nd year (1898), p. 1191.

ment of the material force of production” is claimed to be—and those social conditions of production which form the essence of capitalism<sup>107</sup>.

So far, I have disregarded one objection which Marx repeatedly raises against Ricardo. Marx criticizes Ricardo’s discussion of the factors determining the level of the rate of profit on the grounds that the working-day is, there, considered to be an extensively and intensively constant magnitude.<sup>108</sup>

It is true that Ricardo does not devote much space to the influence which a lengthening of the working day and an increase in the intensity of labour exercise on the rate of profit. However, his views on the antagonism between wages and profits make this influence appear self-evident. For the rate of profit is determined, according to Ricardo, by the number of working days required for the production of the worker’s means of subsistence. Is it not obvious that a reduction in the number of working days due to a lengthening of the working day or to an increase in the intensity of labour, must necessarily lead to a rise in the rate of profit? These two factors have just the same effect as an increase in the productivity of labour. Should the lengthening of the working day or the increase in the intensity of labour occur in a line of production neither directly nor indirectly relevant to the production of the worker’s means of subsistence, then this will not lead to a rise in profits. A fall in the price of the product concerned, or a rise of wages in that particular line of production, or the joint effect of both factors, would press the rate of profit down to that level which corresponds to the conditions under which the worker’s subsistence is produced.<sup>109</sup>

Thus, contrary to Marx’s allegations, Ricardo’s writings do indicate very clearly the effects produced by an alteration in the working day, whether in time or in intensity. Nevertheless, there is nothing to prevent one from expressing these effects by regarding the amount of labour (embodied in the product) as a three-dimensional magnitude, as is done by Marx.

Let  $Q$  be the amount of labour embodied in real wages. Let  $U$  be, as formerly, the corresponding number of working days, (so that, if real wages are calculated for the working day,  $U$  will prove to be a proper fraction). Let the new terms  $s$  and  $i$  be the length in hours of the working day and the intensity of labour respectively. We then get

$$Q = Usi$$

and formula (39) can be expressed as:

$$(74) \quad \rho = \frac{si - Q}{\delta Q}$$

<sup>107</sup> *Das Kapital*, III<sub>1</sub>, p. 232.

<sup>108</sup> *Das Kapital*, III<sub>1</sub>, pp. 222 and 39, cf. *Theorien über den Mehrwert*, II<sub>1</sub>, pp. 134 and 138.

<sup>109</sup> In the first volume of *Das Kapital* (p. 536), Marx asserts that a lengthening of the working day or an increase in the intensity of labour results in a “change in the magnitude of the surplus value”, irrespective of whether the products of the industries concerned do or do not form part of the worker’s customary consumption. Had Marx intended that the expression “magnitude of the surplus value” should include the rate of surplus value, then this assertion would conflict with my arguments in the text. At the same time, it would, however, also upset the assumption that the rate of surplus value is the same in all lines of production. According to Marx’s numerical examples on this point (p.536 and 418), only the surplus value produced by each single worker alters, (but not the rate of surplus value), because it is assumed that “the price of labour power” and surplus value increase equally. Marx, however, lets drop the imprudent remark that they can also increase “in unequal measure”.

Formula (74) indicates that if  $Q$  is given, the profit rate  $\rho$  will be greater or smaller according to whether one of the magnitudes  $s$  or  $i$  increases or diminishes.

The same result can be reached if one takes as the basis for discussion, not the formula of approximation (39), but the exact formula (30). Generally speaking, the laws of the rate of profit do not undergo any essential change if one formula is replaced by the other.

Marx did not succeed in formulating these laws correctly, let alone in completing or correcting Ricardo's formulations of them.<sup>110</sup>

If, however, there is any generally significant point on which Marx is to some extent superior to Ricardo, then it is the theory of the *origin of profit*.

Both take the view that profit, or speaking more generally, the gains of capital, originate in a withholding of some of the produce of labour. All efforts of the opponents of the "theory of withholding", to show that Ricardo does not base himself on this theory, rest on an untenable interpretation of occasional remarks of his<sup>111</sup> or on arbitrary additions to his ideas.<sup>112</sup> We must, however, admit that the theory of withholding is not expressed in Ricardo as clearly as one might wish<sup>113</sup>, whilst Marx succeeded in giving a pregnant and unequivocal expression to due view of the origin of profit which lies at the basis of this theory.

<sup>110</sup> Cf. Marx remarks on the "false laws of the cause of the rise and fall of the rate of profit" in Ricardo, *Theorien über den Mehrwert*, II<sub>1</sub>, p. 97

<sup>111</sup> This is true, e.g., of Tugan-Baranowsky. In his *Theoretische Grundlagen des Maximums*, pp. 135-136, he refers to a letter of Ricardo's to McCulloch which says: "I sometimes think that if I were to write the chapter on value again which is in my work, I should acknowledge that the relative value of commodities was regulated by two causes instead of by one, namely by the relative quantity of labour necessary to produce the commodities in question, and by the rate of profit for the time that the capital remained dormant, and until the commodities were brought to market" (Letters of D. Ricardo to John Ramsay McCulloch 1816-1823, ed. by Hollander, New York, 1895, pp. 71-72). Tugan-Baranowsky, who incidentally translates "rate of profit" by "mass of profit" ("Masse des Profits"), infers from these words that "for Ricardo, time was another factor—completely independent of labour—of the value of those goods which could be increased at will". It would therefore be "a gross misunderstanding to see in Marx's theory of value a logical development of Ricardo's theories" (*loc. cit.*, p. 159). Since, however, he supposes Marx's explanation of profit to be based on his theory of value (*ibid.*, p. 167), Tugan-Baranowsky eliminates Ricardo from the ranks of the supporters of the "theory of withholding". In reality, however, the last thing that can be read into this letter of Ricardo's is a conflict between him and Marx on the question of the origin of profit. The doubts there expressed by Ricardo, concern merely the form of the exposition which he gave in the *Principles*. Ricardo says that it might have been better completely to disregard the (original) law of value instead of presenting the laws of the formation of prices as a modification of the former. But he immediately adds: "Perhaps I should find the difficulties nearly as great in this view of the subject as in that which I have adopted", and goes on—in direct sequence—to speak of his theory of the antagonism between wages and profits, from which he does not in the least retract in this passage. The rate of profit does indeed—according to him—assist in determining values, but, in its own turn, it is determined exclusively by the greater or lesser facility of producing the worker's subsistence. How could Ricardo have made such an assertion, if he regarded profit as the equivalent "of a factor of value quite independent of labour"? No, Ricardo nowhere even dreams of hypostatizing this or that expression for the use of capital in order thus to explain the origin of profit. His interpretation of profit is, on the contrary, entirely based on the idea of the yield of production being divided between workers and capitalists. The time (for which a capital is put into the service of production) is for him merely an element governing the amount of profit, the rate of profit being given. (Cf. P p. 27).

<sup>112</sup> Thus Marshall says that had Ricardo or not had a predilection for "short phrases", he would have said explicitly that time or waiting was just as an element in the cost of production as labour. In this connection, Marshall sharply criticizes those who trace Marx's theory of value and surplus back to Ricardo (*Principles of Economics*, I, pp. 565, 670-672).

<sup>113</sup> This explains that a supporter of the theory of withholding, such as J. Pierstorff (*Die Lehre vom nehmehrgewinn*, 1875, p.21) could adduce this theory against Ricardo, or that Böhm-Bawerk (*Kapital und Kapitalzins*, I, p. 111) finds it possible to speak of Ricardo's "undecided attitude on the question of the origin of profit" and even to assert that his writings "throw no light" on this subject (*loc. cit.*, p. 101).

For, in trying to make clear the origin of profit, Marx had the lucky inspiration to construct a model in which profit exists, without any norm other than the (original) law of value being decisive for the relationship in which products are exchanged for each other. Such a model made it obvious that profit could neither have its first cause in the mark-ups which were a phenomenon of an exchange-economy, nor needed to be regarded as a counterpart of the “productive services of capital”. In other words, by making value-calculation precede price-calculation, Marx succeeded—much more sharply and emphatically than Ricardo had done—in delimiting the theory of withholding against other theories of profit and in shaking off any common feature.

What is specific to Marx’s method can best be seen in a schematic arrangement, combining the contrast between value-calculation and price-calculation with the distinction between a state without profit and one with profit. We thus get the following four cases:

1. Value-calculation without profit,
2. Value-calculation with profit,
3. Price-calculation without profit,
4. Price-calculation with profit.

Price-calculation must here be conceived in a somewhat more general fashion than was done hitherto. Price-calculation would mean: To determine the price of a product according to the formula

$$(75) \quad p = (1 + \xi)^{t_1} \vartheta a_1 + (1 + \xi)^{t_2} \vartheta a_2 + \dots + (1 + \xi)^{t_m} \vartheta a_m$$

where  $p$ ,  $a_1$ ,  $a_2$ , etc. and  $t_1$ ,  $t_2$ , etc. have the same meaning as in formula (20), and where  $\xi$  and  $\vartheta$  are positive magnitudes, which need not be identical with  $\rho$  and  $\lambda$ .

We can describe Ricardo’s procedure as a direct transition from case 1 to case 4. Marx, on the other hand, before approaching case 4, devotes a detailed analysis to case 2, and thus does not permit even the germ of the idea that price-calculation might be the cause of profit<sup>114</sup>. Price-calculation appears in Marx, on the contrary, as the necessary consequence of the fact that profit exists and that it reveals the well-known tendency towards equalization.<sup>115</sup>

Hardly anyone will deny that when Marx set up his value-calculation, one of his aims—and by no means the least—was to throw the right light on profit.<sup>116</sup> But Marx was far from considering value-calculation only as a means to

<sup>114</sup> Ricardo’s exposition can, on the other hand, very well suggest this idea. Cf. Böhm-Bawerk, *Kapital und Kapitalizins*, I, p. 405.

<sup>115</sup> Case 3 also affords certain theoretical interest, but to discuss this case would lead us to far afield. We shall only mention that  $\vartheta < \lambda$  would here necessarily be the result.

<sup>116</sup> Marx was right in choosing as his starting point the fact that, in a capitalist exchange-economy, the origin of profit is obscured particularly by the nature of employment. Not surplus labour as such is—according to him—the *quid proprium* of the capitalist economy (“Capital has not invented surplus labour” says Marx, *Das Kapital*, I, p. 219), but the characteristic veil which in such an economy enshrouds surplus labour. To lift this veil in the very purpose of the device of “value-calculation”. (Cf., e.g. *Das Kapital*, I, pp. 142-143, footnote, or III, pp. 381-382). This device is indeed thoroughly useful, but when Marx presents matters as if the true nature of profit could not have been realized without it, does, the overestimates his personal service to the theory of withholding. Marx’s criticism of Ricardo, directed from this standpoint, is much too sharp (e.g. *Das Kapital*, II, pp. 198-200 or *Theorien*, II, p. 152). Still less does Marx do justice to J. S. Mill, whose views on the ultimate cause of profits he arbitrarily alters anti distorts. Cf. *Das Kapital*, I, pp. 528-530. Mill says (*Principles of Political Economy*, Book II, Chapter XV, §5): “The reason why capital

bring out more clearly the true nature of profit. He considered value-calculation to be, on the contrary, the indispensable basis for the theory of price and income formation in a capitalist economy, as well as the key to the explanation of a series of typical phenomena which his predecessors had wrongly interpreted, just because they lacked this basis.

Our arguments have shown to sufficiency, how slender is the foundation for such a view. It should, however, be well noted that if value-calculation does not form an essential stage in theory, yet it is, in itself completely harmless. No error can owe its existence to value-calculation, provided it is properly handled. Marx's erroneous doctrines, such as his law of the falling rate of profit, are thus due, not to his starting with value and surplus value in his construction of the general rate of profit, but rather to the circumstance that—as we have seen—he brought the general rate of profit into a mathematically wrong relationship to the given magnitudes of value and surplus value.<sup>117</sup>

This state of affairs has passed unnoticed by Marx's critics almost without exception. They are inclined to hold value-calculation, as such, responsible for various mistaken assertions to be found in Marx, without first examining whether he has correctly operated with value and surplus value.<sup>118</sup> On the other hand, Marx's apologists are just as little concerned with this important point. They say that value-calculation is as justified as every other scientific abstraction.<sup>119</sup> As if this settled the matter! Marx and his disciples are not content to use value-calculation merely to ascertain the nature of profit and to bring about a kind of contrasting effect by a comparison with price-calculation. They go beyond this and use value-calculation in an attempt to study

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yields a profit, is because food, clothing, materials and tools last longer than the time which was required to produce them; so that if a capitalist supplies a party of labourers with these things, on condition of receiving all they produce, they will, in addition to reproducing their own necessities and instruments, have a portion of their time remaining to work for the capitalist." Marx comments on this passage that "Mill here confuses the duration of the working-time with the duration of its products". In doing so, Marx suppresses the second half of the quotation, which removes any doubt of the fact that Mill deduces profit from surplus value, just as Marx does. Mill goes on to show that profit is conditioned, not by exchange, but by the productive power of labour. Were there no division of labour, says Mill, there would be neither selling nor buying, but profit would continue to exist. One would think that at least with this assertion, Mill would draw a word of approval from his severe critic. But no! Marx pretends to be shocked: "Here", says he, "exchange, sale and purchase, the general conditions of capitalist production, are thus sheer accessories, and yet there still subsist a profit without the sale or purchase of labour power!" It is, however, clear that Mill did not mean the buying and selling of labour power, but merely the buying and selling of products. One will not go wrong if one connects the ill will which Marx displays toward Mill, with the circumstance that Mill had, basically, anticipated Marx theory of surplus value. Cf. above, footnote 83.

<sup>117</sup> Marx is naturally quite wrong in using the basic principles of value-calculation in order to try to explain the behaviour of individual capitalists, which must necessarily be guided by price-calculation. See, e.g., *Das Kapital*, I, pp. 414-415, or III<sub>1</sub>, p. 51. Cf. G. Adler, *Die Grundlagen der Karl Marx'schen Kritik uder bestehenden Volkswirtschaft*, Tübingen, 1887, pp. 153-155, and *Handwörterbuch der Staatswissenschaften*, 2nd edition, VI, p. 827.

<sup>118</sup> Thus Peter von Struve (in the Russian periodical *Shism*, 1899, pp. 297-306) argues as follows: the rate of profit is nothing other than an expression of the productivity of social labour, independent of the social conditions of production, and must therefore not fall, but rise with the progressive development of the productivity of labour. Marx's law of the falling rate of profit asserts, however, the contrary. There could be no doubt, therefore, of the erroneous nature of "the mechanical labour theory of value", from which this law is derived. Thus, Struve tacitly assumes that this derivation is, in itself, immune to criticism.

<sup>119</sup> The simile of the laws of gravity, where abstraction is made of atmospheric resistance, is invariably cited here. See Kautsky, *Karl Marx Ökonomische Lehren*, 8th edition, 1903, pp. 100 and 110; W. Liebknecht, *Werttheorie in England*, pp. 110-111; Hugo Riekens, "Die Philosophische Wurzel des Marxismus", *Tübinger Zeitschrift*, 1906, pp. 417-418. This simile is, incidentally, to be found already in Thomas de Quincey (*Dialogues of three templars etc.*: Works, Vol. IV, 1862, p. 181), where it is applied to Ricardo's theory of value.

certain quantitative relationships which are peculiar to price-calculation and, in so doing, confer on value the character of an auxiliary quantity. One cannot, of course, deny a theoretician the right to form such an auxiliary quantity.

The question remains whether the analysis of these quantitative relationships is really facilitated—or perhaps even first made possible—by the introduction of this auxiliary quantity. Our discussions in the second half of this article lead us to answer: no. For not only can the reciprocal relationships of prices, wages and the rate of profit be reduced to their correct mathematical expression without the need to start with magnitudes of value and surplus value, but the latter magnitudes do not even appear in the calculation, if one employs the exact formulas. The *post factum* legitimation of value (in the Marxian sense) as an auxiliary quantity fails, because the quantitative relationships to be studied are too involved to be discerned by the somewhat rough and violent means of the Marxian law of value.

This means is particularly inappropriate for the refutation of the cost of production theory, as it is expounded by “popular” economists. Such a refutation was admittedly indispensable. For one can certainly not consider as above criticism a procedure which, in discussing exchange, treats wages, profits, and rent as given quantities from which prices can be calculated, and which then, in discussing distribution, enquires how wages, profits and rent are determined when prices are given.<sup>120</sup> Marx was not, however, the first to raise objection to this procedure<sup>121</sup>, and—what is far more important—he did not succeed in finding a substitute for this procedure by any creation of a self-consistent model.

This is achieved, on the contrary, by the mathematical method, which reduces the complicated quantitative relationships involved to a set of equations, where the number of equations is equal to the number of unknowns. The cost of production theory is thus freed from the faults which adhere to it in its “popular” version.<sup>122</sup>

The mathematical method, however, achieves still more: by its means, the cost of production theory can, ‘without any difficulty, be brought into harmony with the law of supply and demand or with the determination of prices by the subjective valuations of buyers (and, if need be, of sellers). Following the example of Walras, this is done by inserting the cost equations into a more comprehensive set of equations, in which regard is paid also to those subjective valuations.<sup>123</sup>

It is in this connection that the superiority of due mathematical method over the Marxian method appears particularly clearly. Marx was unable to grasp that the determination of prices by costs could perfectly well be reconciled

<sup>120</sup> See footnotes 24 and 25. Cf. Kautsky, *loc. cit.*, p. 106; Rodbertus, “Dritter sozialer Brief” in his *Schriften*, Berlin, 1899, VoL II, pp. 227-229.

<sup>121</sup> See, e.g., James Mill, *Elements of Political Economy*, London, 1821, p. 74.

<sup>122</sup> The term “labour theory of value” could, incidentally, be applied with same justification to the model given on pp. 16-22, although formula (20), which is basic to this model, corresponds entirely to the cost of production theory. Cf. below, footnote 142.

<sup>123</sup> It is nevertheless permissible—as we have done in this article—to take the cost equations out of this context and to treat them separately. Cassel seems to be of a different opinion (*Tübinger Zeitschrift*, 55th volume, p. 455).

with their determination by supply and demand. He therefore strove to explain away supply and demand as factors of value or price. The relevant passages of *Das Kapital* reveal clearly the scantiness of the author's familiarity with mathematical thought. He sees a contradiction in the procedure by which supply and demand are made to be equal and yet at the same time to influence prices, and enlightens us by telling us that their effect ceases<sup>124</sup> if they are equal. He furthermore believes he has detected a "confusion" in the method whereby, on the one side, prices are determined by supply and demand, whilst on the other side supply and demand are represented as functions of the level of prices.<sup>125</sup> Had Marx been in a position to follow the frequently given geometrical representation of the law of supply and demand<sup>126</sup> or the statement by J. S. Mill—whom he so grossly underestimates—that this law finds its mathematical expression, not in a ratio, but in an equation,<sup>127</sup> then Marx would have assigned to supply and demand as factors determining prices a position in his theory very different to the one he actually did.

When Engels says of Marx that he was a "thorough mathematician",<sup>128</sup> this sounds almost like a taunt. Kautsky calls him a "poor calculator",<sup>129</sup> meaning that Marx often makes arithmetical mistakes in his numerical examples. Marx was, however, poor calculator not only in this minor sense. He completely lacked understanding for somewhat complicated quantitative relationships<sup>130</sup> The best evidence for this is afforded by the derivation of prices from values—which has been discussed above in detail—and the conclusions he drew there—

<sup>124</sup> *Das Kapital*, I, pp. 133-134, and, very similarly, III<sub>1</sub>, p. 169. Böhm-Bawerk (*Zum Abschluss*.... p. 179) calls these arguments of Marx, a play on words.

<sup>125</sup> *Das Kapital*, III<sub>1</sub>, pp. 170-171; cf. p. 60.

<sup>126</sup> Such as is given, e.g. in Rau, *Grundsätze der Volkswirtschaftslehre*, 5th edition, 1847, pp. 578-580.

<sup>127</sup> *Principles*, Book III, Ch. II, 54. W. Liebknecht (*loc. cit.* p. 72) confused "equality" and "equation", and therefore gave quite a wrong interpretation of Mill's view of the law of supply and demand. The misunderstanding of this law appears to have been transmitted from Marx to his disciples. Cf. also Conrad Schmidt, "Die Durchschnittsprüfung und das Marx'sche Wertgesetz", *Neue Zeit*, 11th year, vol. 1, pp. 115 *et seq.*

<sup>128</sup> *Anti-Düring*, 4th edition, 1901, p. XIV.

<sup>129</sup> *Theorien über den Merkwert*, I, p. IX, cf., e.g., p. 204, footnote.

<sup>130</sup> Cf. Ernst Lange: "Karl Marx als Volkswirtschaftlicher Theoretiker", *Conrads Jahrbücher*, 3rd sequence, vol. 14 (1897), pp. 551, 564, 578. Lange (p. 552) remarks ironically: "In order not to be unjust to Marx and Engels, we will, incidentally, mention explicitly that, in their quantitative judgments, they do nevertheless occasionally hit the nail on the head, as, for instance, when vol. III, part I, p. 203, says: 'If the rate of profit falls by 50%, then it falls by one half.'" Not on every occasion, however, on which Marx, after comparing two numerical values of some quantity, infers that this quantity has been halved, does his deduction correspond to the true facts. Thus, he at one time considers the case where formerly 20 labourers sufficed to accomplish a certain task whilst 30 labourers were now needed, and remarks that a "reduction in the productivity of labour by one half" had taken place here. In reality, however, the productivity of labour fell here in the ratio of 1/20 to 1/30, or relatively by 1/3 and not by 1/2. See *Das Kapital*, III<sub>1</sub>, p. 33. Marx is obviously not on the best of terms with arithmetic. Matters get still worse when he ventures into higher spheres. What can be said, for instance, when Marx tries to justify the omission of constant capital in his analysis of the process of the formation of surplus value, by referring to an alleged "Law of Mathematics", according to which, wherever mathematics "operates with variable and constant magnitudes, anti te constant magnitude is linked to the variable one merely by addition or subtraction", this constant magnitude is taken to be equal to zero? See *Das Kapital*, I, p. 195. On p. 191, he talks of a constant magnitude, which "is always transforming itself into a variable one". Marx's predilection for average values in characteristic of his purely surface relationship to mathematical concepts. He adduces average values even in cases where only certain maximum values are really, relevant. See *Das Kapital*, III<sub>1</sub>, pp. 161-163. Marx clearly regarded working with average values to be a sign highly scientific behaviour. Cf. his reference to Quételet in *Das Kapital*, III<sub>2</sub>, pp. 396—397 and I, p. 321, footnote. As regards mathematical endowment, Ricardo towers over Marx. Marshall (*Principles*, I, p. 722) says rightly: "Ricardo himself had no mathematical training. But his instincts were unique; and very few trained mathematicians could tread as safely as he over the most perilous courses of reasoning."

from. The relations of value-calculation to price-calculation have, after all, a completely mathematical character, and the inadequacy of Marx's treatment of this problem reflects the meagreness of his mathematical abilities

Our criticism of Marx's model of price and income formation in a capitalist economy may appear to be one-sided. In our discussion, we have certainly passed by a whole series of questions which are of a non-mathematical nature and which are generally apt to preoccupy the authors of works on Marxism. But for this very reason, this study—as its title was meant to indicate—does not claim to be an exhaustive criticism of Marx's model. On the other hand, it should not be overlooked that the originality of this model does mainly lie in the juxtaposition of value-calculation and price-calculation and in the derivation of prices from values, as well as of profit from surplus value, and that, compared to these, the other features peculiar to the system fade into the background.<sup>131</sup> In order to anticipate misunderstandings, we shall, however, briefly discuss some points which we have hitherto disregarded. These points concern: (1) the subordination of wages to the law of value; (2) the reduction of all labour to “simple average labour”; and (3) the distinction between productive and unproductive labour and the related concept of commercial profit.

(1): Wages appear in Marx system as the value (or the price) of the “commodity labour-power”, and Marxists praise it as a prominent scientific achievement that Marx replaced the “value of labour”, of which Ricardo and others had talked, by the “value of labour power”. This was supposed to have solved in one go a difficulty “which caused the downfall of Ricardo's School”.<sup>132</sup> This difficulty is alleged to consist in the impossibility of “reconciling the mutual exchange of capital and labour with Ricardo's law that value is determined by labour”. This is, however, a purely imaginary difficulty, or it is brought about artificially by simply equating—as Marx does—a certain quantity of a commodity to the amount of labour embodied in it<sup>133</sup> In this sense, one can write the formula

$$1 \text{ unit of commodity } A = a \text{ working days.}$$

But the sign of equality ( = ) which appears here, is employed elsewhere to express the fact that a certain quantity of some one commodity exchanges for a certain quantity of some other commodity. If labour were regarded as a commodity, then the above formula could also be interpreted to mean that a unit of A exchanges for a working days, which would be incompatible with that correct interpretation. With regard to Ricardo there cannot, however, even be any question of such a misunderstanding. For he distinguishes, from the very

<sup>131</sup> If one bears this in mind, one must be astonished that should have thought himself entitled to contrast his theory so markedly with Political Economy as such. All in all, we are dealing only with methodological device.

<sup>132</sup> Engels in his introduction to the second volume of *Das Kapital*, p. XX; cf. *Theorien über des Mehrvert*, II<sub>1</sub>, p.119, and W. Liebknecht, *loc. cit.*, pp. 90,93.

<sup>133</sup> *Das Kapital*, I, pp. 3-5.

start, very strictly between the amount of labour embodied in a commodity and the amount of labour which can be exchange for the same com-

modity in the market (through the intermediary of money).<sup>134</sup> It thus runs counter to historical justice to talk as if the confusion—prevailing amongst Marx’s predecessors—between the amount of labour embodied in the commodity and the amount of labour equivalent to the commodity, had only been removed by Marx through his substitution of the expression “labour power” for “labour”. It is no less wrong to ascribe to this Marxian neologism<sup>135</sup> the magical power of revealing the law governing the level of wages. A subordination of wages to the general law of value—as it is found in Marx—is not permissible for the reason that this law, insofar as its validity can be assumed is based on competition between producers, of which there can be no question for the “commodity labour power”.<sup>136</sup> If, however, one rejects the view that the wage-rate in its capacity as the value or price of a certain commodity—whether this be called labour or labour power—is subject to the general law of value or of price, then the wage-equations<sup>137</sup> acquire quite a different importance. There remains then simply the assumption that real wages are given—an assumption which was indicated in view of the special purposes of this study.

(2): The problem of reducing all labour to a “simple average labour” has been so well elucidated by anti-Marxists, notably G. Adler<sup>138</sup> and Böhm-Bawerk,<sup>139</sup> that it would be a work of supererogation to dwell once more on the inadequacy of Marx’s treatment of this question.<sup>140</sup> We need here only

<sup>134</sup> “Ricardo says (*Principles*, Chapter I, section I, p. 9): “If the reward of the labourer were always in proportion to what he produced, the quantity of labour bestowed on a commodity, and the quantity of a labour which that commodity would purchase, would be equal, and either might accurately measure the variations of other things: but they are not equal”.

<sup>135</sup> G. Simmel (*Philosophie des Geldes*, p. 432) tightly calls the introduction of “labour power” instead of “labour” a “matter of terminology”. It is not uninteresting to note that Marx himself speaks of labour as a commodity, e.g. in *Elend der Philosophie*, to which Engels (Introduction, p. XXV) specially refers. This language—contemned by Marx and Engels—is to be found occasionally also in *Das Kapital*, e.g. Vol. I, p. 361. Nor should the favourite term “unpaid labour” really be tolerated if the worker sells, not his labour, but his labour power. Marx seems indeed to have felt thus (see *Das Kapital*, I, p. 545), and if he did not renounce the use of this expression, which from his standpoint is illogical, this can only be ascribed to political purposes.

<sup>136</sup> See above, p. 23. Cf. von Wieser, *Der natürliche Wert*, pp. 179-184.

<sup>137</sup> See formulas (11) and (28).

<sup>138</sup> *Die Grundlagen der Karl Marx’schen Kritik der bestehenden Volkswirtschaft*, Tübingen, 1887, pp. 81-85.

<sup>139</sup> *Zum Abschluss etc.*, pp. 164-169.

<sup>140</sup> “Marx’s failure to solve this problem satisfactorily is admitted, incidentally, even by some Marxists, who, for this reason, are severely taken to task by Hilferding Hilferding himself (*Marx-Studien*, I, pp. 13-22) tries to save the situation by going back to the “cost of sustenance and training” required “to produce complicated labour power”. We need not here enquire whether he has remained loyal to his master with this attempted salvage, which, incidentally, is not original (cf. H. Dietzel: *Theoretische Sozialökonomik*, I, pp. 248—261, and Grabski, quoted by Böhm-Bawerk, *loc.cit.*, pp. 168-169). W. Liebknecht (*loc. cit.*, pp. 99-103) recommends taking the purely physiological concept of labour as a starting-point for the solution of the problem of reduction, although he is not blind to the difficulties of this course. Hugo Riekes (“Die philosophische Wurzel des Marxismus”, *Tübinger Zeitschrift*, 62nd year, 1906, p. 417) goes still further in this direction. Starting from some incidental observation by Marx, Riekes earnestly pronounces the law of value to be “a law of nature based on the principle of mechanical causality”, and infers from this principle “the undifferentiated qualitative equality of the value-substance”, i.e. of value-forming labour. Riekes further proclaims that the mechanical-causal point of view limits the field of economic laws to the factors corresponding to the concept of value. The truth of the Marxian theory of value could therefore not even be touched by the incongruity of value and price which is brought about by non-physical factors. There is no need to prove the arbitrary and sterile nature of such a “philosophy of nature” interpretation of Marx’s theory of value. This interpretation presents a singular contrast to the equally inappropriate “natural law” interpretation of the same theory, such as has recently been put forward, amongst others, by Rudolf Kaulla (*Die geschichtliche Entwicklung der modernen Wertheorien*, Tübingen, 1906, pp. 259-261 and 274).

take steps to prevent our positive observations on the relations between prices, wages and profits from being regarded as if they involved this Marxian “reduction-theory”. In these positive observations, the wage-rate has been considered to be a magnitude equal for all limes of production and for all professions. We have simply disregarded the existence of categories of workers receiving different wage-rates. The whole exposition thus acquires the character of a farreaching abstraction, without, therefore—it seems to me—losing all interest. For the essential problems of the relationship between prices, wages and profits remain; they even appear in their purest form just when one disregards differences in the levels of wages. Were one to allow for the latter, one would have to introduce in place of the one unknown  $\lambda$  some  $m$  unknowns  $\lambda_1, \lambda_2, \dots, \lambda_m$ , each of which would represent the money wage of a certain category of workers. The total number of unknowns in this set of equations<sup>141</sup> would equal  $n + m + 1$ , and in order to obtain the same number of equations, one would have to consider the real wage for each category of workers as given. Such a modification of the original model would not inconsiderably impair the transparency of the relevant mathematical relationships. The only way of avoiding this would be to proceed as follows: instead of fixing separately the real wage for each category of workers, one should follow Ricardo’s procedure and fix only the real wage of the lowest category of workers, as well as the ratio of the money wage of every other category of workers to the money wage of that lowest category. One could then gain, from the  $m + n + 1$  equations which would thus be formed, an equation of the form of (30), where the rate of profit ( $\rho$ ) will appear as the only unknown.<sup>142</sup>

<sup>141</sup> See above, pp. 20-21.

<sup>142</sup> Ricardo’s assumption that a constant ratio prevails between the various wage-rates, thus proves to be an element which greatly facilitates calculation. They go, however, decidedly too far, who attach so great an importance to the circumstance whether a thinker makes use of this assumption or whether he rejects it, that they trace dividing lines accordingly between the various schools and movements. This is done, e.g. by R. Zuckerkandl, who speaks of a ‘refutation of the labour theory of value by J. S. Mill’ (*Zur Theorie des Preises*, Leipzig, 1889, p. 270), and who calls Mill—in contrast to Ricardo—a supporter of the cost of production theory. Actually, Mill only objects to Ricardo’s assumption, because it appears to him to be too far removed from the facts. Incidentally, Zuckerkandl does not quite correctly reproduce Mill’s remarks on this point. At the very beginning of these remarks (*Principles*, Book III, Chapter IV, §2), Mill says: “...it would seem that the value of the product cannot be determined solely by the quantity of labour, but by the quantity together with the remuneration; and that values must partly depend on wages”. Zuckerkandl translates “it would seem” as if it meant “one must therefore say”. Zuckerkandl says further: “This is all the more important, as there is no foundation for the assumption that all changes in wages are universal” Zuckerkandl thus quite fails to express Mill’s point, namely that he would have no objection to Ricardo’s theorem—that prices are not affected by wages—if the only changes in wages were universal ones (cf. Mill, *loc. cit.*, §3, first sentence). Kaula (*loc. cit.*, p. 187) does just as little justice to Mill’s views. Mill’s attitude to Ricardo’s theory of value was much more accurately described by Whitaker (*loc. cit.*, p. 113), who says: “In the end, we may say that Mill placed more stress on qualifications of the labour theory than did Ricardo.” Were one to restrict the name “labour theory of value” to those theories which regard the amount of labour as the sole factor determining the exchange-relationships of (reproducible) commodities, then not only Mill, but Ricardo and Marx too (as author of the 3rd volume of *Das Kapital*) would have to be excluded from the ranks of the adherents of the labour theory of value. H Dietzel, who admits no qualification whatever to the sentence that “the value of the goods reproducible by labour is exactly proportionate to the amount of labour”, would then appear as the only representative of the labour of value, apart, perhaps, from Rodbertus. Dietzel believes that this sentence is not in fact affected by the unequal duration of the processes of production, nor by the unequal share of fixed capital in production, that Ricardo’s opinion to the contrary can be explained “by certain erroneous ideas of Ricardo’s concerning the effects of the spreading of machine technique, or more generally, concerning the effects on wages of an increase of fixed capital”. (*Theoretische Sozialökonomik*, I, p. 264.) Since these words were written, twelve years have gone by without Dietzel having found occasion—to the heat of my knowledge—to explain this strange utterance more fully.

(3): In the present study, we have dealt only with the value and price of physical goods. Of the various forms of labour which Marx pronounces to be unproductive, we are therefore directly concerned here only with the labour applied to commercial tasks,<sup>143</sup> because this is exercised on physical goods before these reach the hands of the consumer.<sup>144</sup> Marx does not deny that particular components in the price of a product correspond to the activities of the merchant and of the wage-earners he employs. Marx believes, however, that this does not constitute an increase in value, but a withholding of part of the value of the product which the industrial capitalist, who transfers his product to the merchant, must be content to suffer. And whilst he had formerly said that total price equals total value, this would now have to be defined more precisely in the sense that total price meant the sum, not of all “purchase prices”, but of all “sale prices”.<sup>145</sup> Since, however, value is determined according to the productive labour employed in the production of the goods, so that the commercial labour is not taken into account as well, the fact that the total sale price—which includes the merchant’s mark-ups—is equal to the total value, is considered to be a proof that these mark-ups are not due to a value or surplus-value created by the commercial worker, but that they represent a withholding of part of the surplus value created by the productive worker. The real facts are as follows: if commercial labour is not brought into account in value, then this constitutes a fresh cause for prices (*i.e.* sale prices) to diverge from values. Abstraction made of the other causes of such divergences, namely the differences in the organic composition and in the turnover periods of capital, the addition of commercial labour—which, in the system of price-calculation, will, just like any other labour, require its wages and give occasion to the formation of profits—will be seen to have raised price above value, or to have depressed price below value, according to whether relatively more or less commercial labour adheres to the product concerned than to that commodity which serves as the measure of value and price. Assume, as Marx generally does, that this commodity is gold, and assume that the same principles are valid for the ratio in which gold is exchanged for other goods, as for the ratios in which these goods are exchanged against each other, perhaps through the intermediary of gold; the total sale price would then certainly surpass total value, if the share of commercial labour in the total labour embodied in the product were smaller for gold than for all other goods. And the converse would be true, *i.e.* the total sale price would fall short of total value, if that share were highest in the case of gold.<sup>146</sup> In short, there is no need whatever for the total

<sup>143</sup> According to Marx, the transport of goods is productive. See *Das Kapital*, II<sub>1</sub>, pp. 272-273, *nd théorien über den Mehrwert*, I, pp. 427-428.

<sup>144</sup> As regards the rate of surplus value and the rate of profit, they are both depressed by the circumstance that, apart from wages, other sums are now added which are meant to cover certain expenses on material goods”, and this circumstance therefore exerts an indirect influence on the prices of material goods.

<sup>145</sup> Marx (*Das Kapital*, III<sub>1</sub>, pp. 269-270) calls “purchase price” the price at which the merchant buys the goods, and “sale price” that at which he sells them. Instead of “sale price”, he sometimes uses the term “real price of production” (p. 274), whilst the unqualified term “price of production” has the same meaning as purchase price”. But occasionally (*e.g.* bottom of p. 268) “price of production” is equivalent to “sale price”. When, on pp. 269-270, Marx presents the sale price as the sum of purchase price and commercial profit he disregards the wages of the commercial workers, which constitute part of the margin between purchase price and sale price. On this, see *ibid.*, pp. 276-286.

<sup>146</sup> Cf. above, pp. 10-11.

sale price to coincide with the total value. Contrary to Marx, there can therefore also be no question of some theoretical results having subsequently proved that it would have been correct to deny commercial labour recognition as value-forming (productive), or, in other words, to ignore commercial labour in the determination of value.<sup>147</sup> The special position to which Marx—in his model—relegates commercial labour and commercial profit, lacks all justification.<sup>148</sup> Our positive observations should therefore be interpreted in the sense that, apart from the labour employed in the production of goods, one must also always take account of that labour which enables the goods to reach the consumer.

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<sup>147</sup> In completion, we may mention that it is not always easy to define where industrial labour ceases and commercial labour begins. See Ernst Lange, *loc. cit.*, p. 554.

<sup>148</sup> Marx says (*Das Kapital*, III<sub>1</sub>, p. 289): “just as the worker’s unpaid labour directly procures surplus value to productive capital, so the unpaid labour of the commercial wage-earner procures to mercantile capital a share in that surplus value.” This fine distinction is arbitrary and superfluous. Marx goes quite wrong when he connects the behaviour of the individual capitalist with this distinction. See *Das Kapital*, III<sub>1</sub>, p. 284; cf. above, footnote 117. But although there is no justification for a different treatment of commerce in a purely abstract theory of value and price, yet the economic historian and the sociologist will inclined in reality manifold differences and contrasts between industrial and mercantile capital. Marx was inclined to believe that, in modern times, mercantile capital had been displaced from the leading position it had formerly occupied. It therefore makes a peculiar impression when Schmoller, in talking of the great importance of commerce in modern economic life, and of tise evils connected therewith, says of Marx: “What Marx calls capitalism and attacks as such, is really nothing but this dependence of the whole economy on the selfish desire for profit of the traders and the selfishly exploited power of their capital”. (*Grundriss der Allgemeinen Volkswirtschaftslehre*, II, Leipzig, 1904, p. 40.)