APPENDIX

"On the Correction of Marx's Fundamental Theoretical Construction in the Third Volume of *Capital*," by

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CRITICS of Marx have hitherto shown little inclination to examine more closely the procedure which is used in the third volume of *Capital* [1] for the transformation of values into prices of production and for the determination of the average rate of profit, in order to see whether this procedure is free of contradictions.

Tugan-Baranowsky provides an exception in this respect.[2] He has shown specifically that the way Marx calculates the average rate of profit is not valid. Moreover, Tugan-Baranowsky has pointed out how with given prices of production and a given average rate of profit it is possible to calculate correctly the corresponding values and the rate of surplus value. In this case there is posed a problem which is the opposite of that which Marx tried to solve.

It is nevertheless interesting to show that Marx erred, and in what way, without reversing his way of posing the problem. For this purpose, it will be convenient, in order not to complicate the presentation, to introduce the same limiting assumption which Tugan-Baranowsky made use of, namely, that the entire advanced capital (including the constant capital) turns

[1] Vol. III, pp. 182-203.

[2] *Theoretische Grundlagen des Marxismus* (Leipzig, 1905), pp. 170-188.

over once a year and reappears again in the value or the price of the annual product.[1] Insofar as it is a question of demonstrating Marx's errors it is quite unobjectionable to work with limiting assumptions of this kind, since what does not hold in the special case cannot claim general validity.

In still another respect the procedure followed here agrees with that of Tugan-Baranowsky. The different spheres of production from which Marx composes social production as a whole can be put together into three departments of production. In Department I means of production are produced, in Department II workers' consumption goods, and in Department III capitalists' consumption goods. At the same time we shall assume that in the production of all three groups of means of production, that is, those which are used respectively in Departments I, II, and III—the organic composition of capital is the same.

Finally, we shall assume "simple reproduction."

Let c_1, c_2, c_3 stand for the constant capital, v_1, v_2, v_3 for the variable capital, and s_1, s_2, s_3 for the surplus value in Departments I, II, and III respectively. The conditions of simple reproduction are expressed in the following system of equations:

$$c_1 + v_1 + s_1 = c_1 + c_2 + c_3 \qquad (1)$$

$$c_2 + v_2 + s_2 = v_1 + v_2 + v_3 \qquad (2)$$

$$c_3 + v_3 + s_3 = s_1 + s_2 + s_3 \qquad (3)$$

If we now designate the rate of surplus value by r, then we have

$$r = \frac{s_1}{v_1} = \frac{s_2}{v_2} = \frac{s_3}{v_3}$$

and equations (1), (2), and (3) can be rewritten as follows:

$$c_1 + (1 + r)v_1 = c_1 + c_2 + c_3 \qquad (4)$$

$$c_2 + (1 + r)v_2 = v_1 + v_2 + v_3 \qquad (5)$$

$$c_3 + (1 + r)v_3 = s_1 + s_2 + s_3 \qquad (6)$$

[1] This assumption is also found, for example, in Kautsky, *Karl Marx' Ökonomische Lehren* (Stuttgart, 1903), p. 98.

The problem now is to convert these value expressions into price expressions which conform to the law of the equal rate of profit.

Marx's solution consists, first, in forming the sums

$$c_1 + c_2 + c_3 = C \tag{7}$$

$$v_1 + v_2 + v_3 = V \tag{8}$$

$$s_1 + s_2 + s_3 = S \tag{9}$$

next, in determining the sought-for average rate of profit, which will be designated by ρ, from the formula

$$\rho = \frac{S}{C + V} \tag{10}$$

and, finally, expressing the production prices of the commodities produced in the three departments by

$$c_1 + v_1 + \rho\,(c_1 + v_1)$$
$$c_2 + v_2 + \rho\,(c_2 + v_2)$$
$$c_3 + v_3 + \rho\,(c_3 + v_3)$$

from which it emerges that the sum of these three price expressions, or the total price, is identical with the sum of the corresponding value expressions, or the total value $(C + V + S)$.

This solution of the problem cannot be accepted because it excludes the constant and variable capitals from the transformation process, whereas the principle of the equal profit rate, when it takes the place of the law of value in Marx's sense, must involve these elements.[1]

The correct transition from value quantities to price quantities can be worked out as follows:

Suppose that the relation between the price and the value of the products of Department I is (on the average) as x to 1, in the case of Department II as y to 1, and in the case of Department III as z to 1. Furthermore let ρ be the profit rate which

[1] For a closer examination of this point, see the second article of my work "Wertrechnung und Preisrechnung im Marxschen System," *Archiv für Sozialwissenschaft und Sozialpolitik,* Vol. XXV, No. 1 (July, 1907).

is common to all departments (though now formula (10) can no longer be regarded as the correct expression for ρ).

The counterpart of equations (4), (5), and (6) is now the following system:

$$(11) \qquad (1 + \rho)(c_1x + v_1y) = (c_1 + c_2 + c_3)x$$

$$(12) \qquad (1 + \rho)(c_2x + v_2y) = (v_1 + v_2 + v_3)y$$

$$(13) \qquad (1 + \rho)(c_3x + v_3y) = (s_1 + s_2 + s_3)z$$

In this manner we obtain three equations with four unknowns (x, y, z, and ρ). In order to supply the missing fourth equation we must determine the relation between the price unit and the value unit.

If we were to choose the price unit in such a way that total price and total value are equal, we would have to set

$$(14) \qquad Cx + Vy + Sz = C + V + S$$

where

$$(15) \qquad C = c_1 + c_2 + c_3$$

$$(16) \qquad V = v_1 + v_2 + v_3$$

$$(17) \qquad S = s_1 + s_2 + s_3$$

If, on the other hand, the price unit and the value unit are to be regarded as identical, then we have to consider in which of the three departments the good which serves as the value and price unit is produced. If gold is the good in question, then Department III is involved and in place of (14) we get

$$(18) \qquad z = 1$$

Let us follow this last procedure. In this fashion the number of unknowns is reduced to three (x, y, and ρ).

To arrive at the simplest possible formulas, let us form the following expressions:

$$\frac{v_1}{c_1} = f_1 \quad , \quad \frac{v_1 + c_1 + s_1}{c_1} = g_1$$

$$\frac{v_2}{c_2} = f_2 \quad , \quad \frac{v_2 + c_2 + s_2}{c_2} = g_2$$

$$\frac{v_3}{c_3} = f_3 \quad , \quad \frac{v_3 + c_3 + s_3}{c_3} = g_3$$

and

$$1 + \rho = \sigma$$

Equations (11), (12), and (13) can be rewritten, taking account of (1), (2), and (3), as follows:

$$\sigma\,(x + f_1 y) = g_1 x \tag{19}$$

$$\sigma\,(x + f_2 y) = g_2 y \tag{20}$$

$$\sigma\,(x + f_3 y) = g_3 \tag{21}$$

From equation (19) we get:

$$x = \frac{f_1 y\,\sigma}{g_1 - \sigma} \tag{22}$$

If we substitute this value for x in equation (20) the result is

$$(f_1 - f_2)\,\sigma^2 + (f_2 g_1 + g_2)\,\sigma - g_1 g_2 = 0 \tag{23}$$

from which it follows that

$$\sigma = \frac{-(f_2 g_1 + g_2) + \sqrt{(f_2 g_1 + g_2)^2 + 4\,(f_1 - f_2)\,g_1 g_2}}{2\,(f_1 - f_2)} \tag{24}$$

or, otherwise written,

$$\sigma = \frac{f_2 g_1 + g_2 - \sqrt{(g_2 - f_2 g_1)^2 + 4\,f_1 g_1 g_2}}{2\,(f_2 - f_1)} \tag{25}$$

It is easy to show that in this case the quadratic equation (23) yields only one solution which is relevant to the terms of the problem. If $f_1 - f_2 > 0$, we get $\sigma < 0$ by putting a minus sign in front of the square root in formula (24). If on the other hand $f_1 - f_2 < 0$, the result of putting a plus sign in front of the square root in formula (25) is

$$\sigma > \frac{g_2}{f_2 - f_1}$$

and *a fortiori*

$$\sigma > \frac{g_2}{f_2}$$

This contradicts equation (20) which yields

$$\sigma < \frac{g_2}{f_2}$$

From equations (20) and (21) we find:

$$y = \frac{g_3}{g_2 + (f_3 - f_2)\,\sigma} \tag{26}$$

and when we have solved for σ and y, x can be calculated according to formula (22).

Let us now see by several numerical examples how these formulas can be used to transform values into prices. Suppose for example that the given value expressions are the following:

TABLE 1: VALUE CALCULATION

Dept. of Production	*Constant Capital*	*Variable Capital*	*Surplus Value*	*Value of Product*
I	225	90	60	375
II	100	120	80	300
III	50	90	60	200
Total	375	300	200	875

From this we derive the following numerical values:

$c_1 = 225$, $c_2 = 100$, $c_3 = 50$, $v_1 = 90$, $v_2 = 120$, $v_3 = 90$, $s_1 = 60$, $s_2 = 80$, $s_3 = 60$, and further: $f_1 = \frac{2}{5}$, $f_2 = \frac{5}{6}$, $f_3 = \frac{9}{5}$, $g_1 = \frac{5}{3}$, $g_2 = 3$, $g_3 = 4$.

Formulas (25), (26), and (22) yield:

$$\sigma = \frac{5}{4}, \text{ therefore } \rho = \frac{1}{4}, y = \frac{16}{15}, x = \frac{32}{25}, \text{ and we get:}$$

TABLE 2: PRICE CALCULATION

Dept. of Production	*Constant Capital*	*Variable Capital*	*Profit*	*Price of Product*
I	288	96	96	480
II	128	128	64	320
III	64	96	40	200
Total	480	320	200	1,000

In Department I the price expression for constant capital (288) comes from multiplying the corresponding value expression (225) by $\frac{32}{25}$, and the price expression for variable capital

(96) from multiplying the corresponding value expression (90) by $\frac{16}{15}$. The profit in this department consists of the sum of the two price expressions $(288 + 96)$ multiplied by the profit rate $(\frac{1}{4})$. The figures for the other departments are calculated in exactly the same way.[1]

That the total price exceeds the total value arises from the fact that Department III, from which the good serving as value and price measure is taken, has a relatively low organic composition of capital. But the fact that total profit is numerically identical with total surplus value is a consequence of the fact that the good used as value and price measure belongs to Department III.

It is not without interest to compare the price and profit relations of Table 2 with the price and profit relations which Marx would have obtained in this case. According to formula (10) Marx would have written $\rho = \frac{200}{675} = \frac{8}{27}$, since (according to Table 1) $S = 200$, $C = 375$, $V = 300$.

We get:

TABLE 3: PRICE CALCULATION ACCORDING TO MARX

Dept. of Production	*Constant Capital*	*Variable Capital*	*Profit*	*Price of Product*
I	225	90	$93\frac{9}{27}$	$408\frac{9}{27}$
II	100	120	$65\frac{5}{27}$	$285\frac{5}{27}$
III	50	90	$41\frac{13}{27}$	$181\frac{13}{27}$
Total	375	300	200	875

[1] Table 1 is taken from the above-mentioned work of Tugan-Baranowsky, and all figures in Table 2 are related to the corresponding figures of Tugan-Baranowsky (*ibid.*, p. 171) as 8 to 5. Tugan-Baranowsky sets up his value schema in terms of labor units instead of money units. This is legitimate enough, but it turns attention away from the real difference between value calculation and price calculation.

There thus emerges a discrepancy between the prices of the quantities produced in the various departments ($408\frac{9}{27}$, $285\frac{5}{27}$, $181\frac{13}{27}$) and the numerical expressions for constant capital, variable capital, and profit. As already indicated, Marx would have had to determine the average rate of profit in this case to be $\frac{8}{27}$, or 29.6 percent, while according to the correct procedure it amounts to $\frac{1}{4}$, or 25 percent.[1]

But Marx not only failed to indicate a valid way of determining the rate of profit on the basis of given value and surplus value relations; more, he was misled by his wrong construction of prices into an incorrect understanding of the factors on which the height of the rate of profit in general depends.[2] He took the position that with a given rate of surplus value the rate of profit is greater or smaller according as the total social capital, including all spheres of production, has a lower or higher organic composition. This view follows from the fact that Marx expressed the rate of profit by formula (10). If we designate, as before, the rate of surplus value by r and the relation of the value of constant capital to total capital by q_0, according to which

$$r = \frac{S}{V} \text{ and } q_0 = \frac{C}{C + V}$$

we should then have:

$$\rho = (1 - q_0)\, r \tag{27}$$

According to this, with a given rate of surplus value the only circumstance which affects the height of the rate of profit is whether the share of constant capital in total capital, the quotient q_0 is larger or smaller; and it would make no difference at all what differences existed between the organic composition of the capitals in the different spheres of production.

[1] See the first article of my work "Wertrechnung und Preisrechnung," in *Archiv für Sozialwissenschaft und Sozialpolitik*, Vol. XXIII, No. 1, p. 46.

[2] By rate of profit we understand here and in what follows, unless the contrary is expressly stated, the average rate of profit.

It is true that in *Capital* we read that the general rate of profit is determined by two factors: (1) the organic composition of the capitals in the different spheres of production, hence the different profit rates of the individual spheres, and (2) the distribution of the total social capital among these different spheres.[1] But the way Marx works these two factors into his calculation schema is such as to allow us to reduce them to one single factor, namely the organic composition of the total social capital.

Let q_1 represent the relation of constant capital in our Department I to the total capital of that department, γ_1 the share of the latter in the total social capital. Similarly let q_2, γ_2 and q_3, γ_3 represent the analogous quantities in Departments II and III. These designations can be expressed in the following formulas:

$$\frac{c_1}{c_1 + v_1} = q_1, \frac{c_2}{c_2 + v_2} = q_2, \frac{c_3}{c_3 + v_3} = q_3;$$

$$\frac{c_1 + v_1}{C + V} = \gamma_1, \frac{c_2 + v_2}{C + V} = \gamma_2, \frac{c_3 + v_3}{C + V} = \gamma_3$$

From these formulas it appears that:

$$\frac{c_1 + c_2 + c_3}{C + V} = \gamma_1 q_1 + \gamma_2 q_2 + \gamma_3 q_3$$

or also, since $c_1 + c_2 + c_3 = C$ and $\frac{C}{C + V} = q_0$,

$$q_0 = \gamma_1 q_1 + \gamma_2 q_2 + \gamma_3 q_3 \tag{28}$$

If one now substitutes this formula for q_0 in (27) and takes account of the fact that $\gamma^1 + \gamma^2 + \gamma_3 = 1$, one gets:

$$\rho = \frac{\gamma_1 (1 - q_1) r + \gamma_2 (1 - q_2) r + \gamma_3 (1 - q_3) r}{\gamma_1 + \gamma_2 + \gamma_3} \tag{29}$$

This formula expresses the Marxian standpoint very clearly: the general rate of profit (ρ) appears as the arithmetic average

[1] Vol. III, pp. 191-192.

of the particular rates of profit $(1 - q_1)r$, $(1 - q_2)r$, and $(1 - q_3)r$, which contribute to the formation of the average with the respective "weights" $\gamma_1, \gamma_2, \gamma_3$. And of the two factors which in Marx's view determine the general rate of profit, one, according to formula (29), is represented by q_1, q_2, q_3 and the other by $\gamma_1, \gamma_2, \gamma_3$. It is, however, obvious from formula (28) that these two factors can be reduced to one single factor, that is to say, to the organic composition of the total social capital which is represented by q_0.

In opposition to this view we shall now show by means of a suitably constructed numerical example that, because formulas (27) and (29) are false, cases are possible in which, with a given rate of surplus value, one and the same rate of profit is compatible with different organic compositions of the total social capital. Take the following value schema as a starting point:

TABLE 4: VALUE CALCULATION

Dept. of Production	*Constant Capital*	*Variable Capital*	*Surplus Value*	*Value of Product*
I	300	120	80	500
II	80	96	64	240
III	120	24	16	160
Total	500	240	160	900

If we compare this table with Table 1 we find that the rate of surplus value is the same (66⅔ percent), while the organic composition of capital is higher. According to Table 1, $q_0 = \frac{375}{675} = .556$; while according to Table 4, $q_0 = \frac{500}{740} = .676$. Marx would say that the rate of profit must fall from 29.6 percent to 21.6 percent.

If we now apply to this table the correct method of transformation, as we did in going from Table 1 to Table 2, we find $x = \frac{32}{35}$, $y = \frac{16}{21}$, $\rho = \frac{1}{4}$, and as a complete result:

TABLE 5: PRICE CALCULATION

Dept. of Production	*Constant Capital*	*Variable Capital*	*Profit*	*Price of Product*
I	274 2/7	91 3/7	91 3/7	457 1/7
II	73 1/7	73 1/7	36 4/7	182 6/7
III	109 5/7	18 2/7	32	160
Total	457 1/7	182 6/7	160	800

The reason why Table 4 gives the same rate of profit as Table 1 (25 percent) is that according to formula (25) the rate of profit ($\rho = \sigma - 1$), given a certain rate of surplus value, depends exclusively on the organic composition of the capitals in Departments I and II (in this connection it is necessary to keep in mind the meaning of the quantities f_1, f_2, g_1, and g_2), and that in this respect Tables 1 and 4 are identical. But the circumstance that the ratio of constant capital to total capital in Department III has grown from about 36 percent to about 83 percent has no bearing on the height of the rate of profit. For the rest, however, this result is hardly surprising from the point of view of the theory of profit which sees the origin of profit in "surplus labor." Ricardo had already taught that a change in the relations of production which touches only such goods as do not enter into the consumption of the working class cannot affect the height of the rate of profit.[1]

Let us now consider a case where the rate of profit changes in spite of the fact that the organic composition of the total social capital remains the same. This happens if one contrasts with Tables 1 and 2 the following tables:

[1] For a closer examination of this point, see the third article of my work "Wertrechnung und Preisrechnung."

TABLE 6: VALUE CALCULATION

Dept. of Production	*Constant Capital*	*Variable Capital*	*Surplus Value*	*Value of Product*
I	205	102	68	375
II	20	168	112	300
III	150	30	20	200
Total	375	300	200	875

Following formulas (25), (26), and (22) we get

$$\sigma = \frac{415 - 5\sqrt{409}}{216} = 1.453, y = .432, x = .831$$

and as a complete result:

TABLE 7: PRICE CALCULATION

Dept. of Production	*Constant Capital*	*Variable Capital*	*Profit*	*Price of Product*
I	170.3	44.1	97.1	311.5
II	16.6	72.6	40.5	129.7
III	124.6	13.0	62.4	200
Total	311.5	129.7	200	641.2

Marx's method of transformation would have produced the same rate of profit again, 29.6 percent (instead of 45.3 percent), and the distribution of the total profit among the three departments would have been as follows: Department I, $90\frac{26}{27}$ (instead of 97.1), Department II, $55\frac{19}{27}$ (instead of 40.5), and Department III, $53\frac{9}{27}$ (instead of 62.4).

The erroneous character of Marx's transformation method comes out even more clearly in the special case where there is no constant capital in Department II. We have this case in the following table:

TABLE 8: VALUE CALCULATION

Dept. of Production	Constant Capital	Variable Capital	Surplus Value	Value of Product
I	180	90	60	330
II	0	180	120	300
III	150	30	20	200
Total	330	300	200	830

In this case we can no longer use formula (25) for the purpose of calculating ρ or σ, because $f_2 = \infty$ and $g_2 = \infty$. We have instead to go back to equations (11), (12), and (13). We find from (12), since $c_2 = 0$, that

$$1 + \rho = \frac{v_1 + v_2 + v_3}{v_2}$$

By reason of formula (2) we can also write (again because $c_2 = 0$):

$$1 + \rho = \frac{v_2 + s_2}{v_2}$$

and finally

$$\rho = \frac{s_2}{v_2}$$

or

$$\rho = r$$

The rate of profit is equal to the rate of surplus value, thus according to Table 8 equal to ⅔ or 66⅔ percent. If we put this value of ρ into formulas (11) and (13) we get two equations of the first degree with two unknowns (x and y), since here too $z = 1$, and we find: $x = 10/13$, $y = 2/13$. The conversion of values into prices and of surplus value into profit gives:

TABLE 9: PRICE CALCULATION

Dept. of Production	Constant Capital	Variable Capital	Profit	Price of Product
I	138 6/13	13 11/13	101 7/13	253 11/13
II	0	27 9/13	18 6/13	46 2/13
III	115 5/13	4 8/13	80	200
Total	253 11/13	46 2/13	200	500

According to Marx, however, the relevant quantity relations would be as follows:

TABLE 10: PRICE CALCULATION ACCORDING TO MARX

Dept. of Production	*Constant Capital*	*Variable Capital*	*Profit*	*Price of Product*
I	180	90	85 5/7	355 5/7
II	0	180	57 1/7	237 1/7
III	150	30	57 1/7	237 1/7
Total	330	300	200	830

The rate of profit would be $\frac{200}{630}$ or 31.8 percent (instead of 66⅔ percent!).

In this case, characterized by the absence of constant capital in Department II, the incorrectness of Marx's derivation of prices and profit is particularly obvious. For it is clear that here in Department II, where the outlay of capitalists consists solely of variable capital and indeed of the very commodities which are produced in that department, the gain of the capitalists must always remain in the same relation to their outlay whether the prices of the relevant commodities are higher or lower. There is no way, either through exchange of commodities or through "price regulation," by which this relation could be reduced from 66⅔ percent to 31.8 percent.

Following Table 9 we can represent commodity exchanges as follows:[1]

	The capitalists of Department I	II	III
(1) hold commodities priced at:	138 6/13	27 9/13	80

[1] For the sake of simplicity it is assumed that the capitalists advance consumption goods to their workers *in natura* so that the workers take no direct part in commodity exchanges.

(2) buy commodities priced at:

from	I	—	—	$115\frac{5}{13}$
	II	$13\frac{11}{13}$	—	$4\frac{8}{13}$
	III	$101\frac{7}{13}$	$18\frac{6}{13}$	—

(3) sell commodities priced at:

to	I	—	$13\frac{11}{13}$	$101\frac{7}{13}$
	II	—	—	$18\frac{6}{13}$
	III	$115\frac{5}{13}$	$4\frac{8}{13}$	—

As can be seen, in the case of each group of capitalists the sum of the prices at which commodities are bought is the same as the sum of the prices at which commodities are sold. Table 10 would show a different picture:

The capitalists of Department

		I	II	III
(1) hold commodities priced at:				
		180	180	$57\frac{1}{7}$
(2) buy commodities priced at:				
from	I	—	—	150
	II	90	—	30
	III	$85\frac{5}{7}$	$57\frac{1}{7}$	—
(3) sell commodities priced at:				
to	I	—	90	$85\frac{5}{7}$
	II	—	—	$57\frac{1}{7}$
	III	150	30	—

Here the capitalists of Departments I and III would take in less than they pay out, while contrariwise the capitalists of Department II would take in more than twice what they pay out.

The case where $c_2 = 0$ is, however, useful not only for showing up very clearly to what paradoxes Marx's method of converting values into prices leads, it is also very well suited to serve as a starting point for an essential supplement to our previous exposition.

One would be inclined to conclude from the fact that in this particular special case the rate of profit is simply equal to the rate of surplus value, and also from the fact that it is entirely independent of the organic composition of capital in Departments I and III, that the organic composition in these two departments could be of any height without there ensuing a decline in the rate of profit. If this were true, and regardless of its being a special case, one could hardly suppress a strong doubt about the correctness of explaining profit by the principle of "surplus labor."

The truth of the matter, however, is that the share of constant capital in the total investment of Departments I and III cannot exceed a certain limit if the rate of profit in these two departments is also to equal r. If we substitute r for ρ in equation (11) and take account of equation (4), we get:

$$(1 + r)(c_1x + v_1y) = [c_1 + (1 + r)v_1]x$$

from which follow

$$c_1xr < (1 + r)v_1x$$

and also

$$c_1 < \frac{1 + r}{r} v_1$$

On the other hand, by reason of equation (1), with $c_2 = 0$, we have

$$c_3 = (1 + r)v_1$$

Let us introduce the new expressions

$$\frac{(1 + r)^2}{r} = \beta \qquad \text{and} \qquad \frac{c_1 + c_3}{c_1 + v_1 + c_3 + v_3} = q'$$

We now have the inequality

$$c_1 + c_3 < \beta v_1 \tag{30}$$

Therefore

$$1 + \frac{v_1 + v_3}{c_1 + c_3} > 1 + \frac{v_1 + v_3}{\beta v_1}$$

or

$$\frac{1}{q'} > \frac{(1 + \beta)\, v_1 + v_3}{\beta v_1}$$

and as a consequence

(31) $$q' < \frac{\beta v_1}{(1 + \beta)\, v_1 + v_3}$$

We then have *a fortiori:*

$$q' < \frac{\beta}{1 + \beta}$$

or

(32) $$q' < \frac{1 + 2r + r^2}{1 + 3r + r^2}$$

The quantity q' is, however, the expression for the organic composition of the combined capitals of Departments I and III. The independence of the rate of profit from the organic composition of the capitals in I and III, in the case where there is no constant capital in II, therefore, does not at all mean that the organic composition of capital in the other two departments can be indefinitely high. The truth of the matter is rather that if the share of constant capital in these departments, the quantity q', exceeds a certain limit, the equalization of the rate of profit becomes impossible.

In order to determine the upper limit for q_0, in other words for the share of constant capital in the total social capital, it is most convenient to start from the inequality (30) which can also be written as follows (with $c_2 = 0$):

$$C < \beta v_1$$

We have

$$q_0 = \frac{C}{C + V}$$

and therefore:

$$q_0 < \frac{\beta v_1}{\beta v_1 + V} \tag{33}$$

From the relation

$$\frac{V}{v_2} = 1 + r \tag{34}$$

we get, however,

$$V = v_2 + rv_2$$

and since on the other hand

$$V = v_1 + v_2 + v_3$$

it emerges that:

$$v_1 + v_3 = rv_2$$

and as a consequence

$$v_1 < rv_2$$

If we now substitute rv_2 for v_1 in (33), we get *a fortiori*

$$q_0 < \frac{\beta rv_2}{\beta rv_2 + V}$$

or also, taking account of (34),

$$q_0 < \frac{1 + r}{2 + r} \tag{35}$$

Hence if the rate of surplus value is 66⅔ percent, as we have assumed in the foregoing examples, then the constant capital invested in Departments I and III can in no case exceed ⅝ of the total social capital.

So much for the case in which $c_2 = 0$, that is to say in which constant capital is absent from Department II.

Likewise if $c_1 = 0$ it is impossible to determine the rate of profit by means of formulas (24) or (25), because here $f_1 = \infty$ and $g_1 = \infty$. If we take equations (11) and (12) as a basis for the determination of ρ or σ, we easily find:

$$\frac{1}{1 + r}\sigma^2 + f_2\sigma - g_2 = 0 \tag{36}$$

where r, as formerly, signifies the rate of surplus value $\left(\frac{s_1}{v_1}\right)$. This last equation can also be derived from equation (23) if one divides its coefficients by g_1. With $c_1 = 0$,

$$\frac{f_1}{g_1} = \frac{v_1}{v_1 + s_1} = \frac{1}{1 + r}$$

It would be entirely wrong to assume from the fact that r appears in (36) and not in (23) that in the case where c_1 is not zero the rate of profit is independent of the rate of surplus value. This is because the quantities g_1 and g_2 depend on r. We have:

$$g_1 = 1 + (1 + r)f_1$$

and

$$g_2 = 1 + (1 + r)f_2$$

If we eliminate the quantities f_1, f_2, g_1, g_2 from equations (23) and (36) by introducing the quantities q_1, q_2, and r, then the following relations emerge:

$$f_1 = \frac{1 - q_1}{q_1}, f_2 = \frac{1 - q_2}{q_2}$$

$$g_1 = \frac{1 + r\,(1 - q_1)}{q_1}, g_2 = \frac{1 + r\,(1 - q_2)}{q_2}$$

From this it is at once apparent that the rate of profit depends only on the rate of surplus value (r) and the organic composition of the capitals invested in Departments I and II.

The rate of profit is always smaller than the rate of surplus value, if we abstract from the special case where $c_2 = 0$. This can be proved as follows:

From equation (11) we find

$$c_1x + v_1y < (c_1 + c_2 + c_3)x$$

and, taking account of (4),

$$v_1y < (1 + r)v_1x,$$

from which it follows that

$$x > \frac{y}{1 + r}$$

From equation (12) there thus emerges the inequality:

$$(1 + \rho)\left(\frac{c_2 y}{1 + r} + v_2 y\right) < (v_1 + v_2 + v_3)y$$

or, taking account of (9),

$$(1 + \rho)\left(\frac{c_2}{1 + r} + v_2\right) < c_2 + (1 + r)v_2$$

and finally

$$1 + \rho < 1 + r$$

and

(37) $$\rho < r$$

Another upper limit for ρ can be derived from (11) in the following way. We have:

$$(1 + \rho)c_1 x < (c_1 + c_2 + c_3)x$$

and hence

(38) $$\rho < \frac{c_2 + c_3}{c_1}$$

This inequality allows us to conclude that with a given rate of surplus value (r) and a given quantity of variable capital (V), an unlimited growth of constant capital cannot take place without bringing about a decline in the rate of profit.

It follows from (4) that:

$$c_2 + c_3 = (1 + r)v_1$$

and this means that the growth of constant capital in Departments II and III finds a limit in the height of the rate of surplus value and in the size of the total disposable variable capital. It is to be remembered, too, that v_1 forms a part of V.

We could say with equal justification that the growth of constant capital in Departments II and III finds a limit in the quantity of labor which society has at its disposal in a given economic period. Let this quantity be H. Of this h_1 belongs to Department I, h_2 to II, and h_3 to III, so that $H = h_1 + h_2 + h_3$.

If we designate the quantity of labor contained in one unit of value as η then we have:

$$h_1 = (v_1 + s_1)\eta, h_2 = (v_2 + s_2)\eta, h_3 = (v_3 + c_3)\eta, \text{ and}$$
$$H = (V + S)\eta$$

We can now write

$$(c_2 + c_3)\eta = h_1$$

and since h_1 is a part of H, it appears that the constant capital invested in Departments II and III, measured in terms of the quantity of (stored-up) labor which it contains, is limited by the quantity of (living) labor which is available for use in production during the relevant economic period.

Nevertheless, so far as the constant capital invested in Department I (c_1) is concerned, one can imagine it as growing indefinitely without disturbing the conditions of economic equilibrium as they find expression in equations (4), (5), and (6). But, as formula (38) shows, sooner or later the consequence of the growth of constant capital in Department I must be a decline in the rate of profit. For the rest, the inequality (38) is valid even in the case where $c_2 = 0$.

It follows from what has been said that it would be entirely incorrect to state in opposition to Marx that the rate of profit does not depend in general on the organic composition of the total social capital. The simple relation between ρ and q_0 with which Marx operates—see equation (27)—does not exist, and cases can be constructed in which, with a given rate of surplus value (r), the rate of profit (ρ) remains unchanged although q_0 takes on different values, just as cases are possible in which ρ assumes different values although q_0 remains unchanged. But—and this should not be overlooked—such cases are based on the supposition that the organic composition of capital is different in the three departments. If, on the other hand, the condition $q_1 = q_2 = q_3$ is fulfilled, then values and prices are identical and formula (27) comes into force.

This last remark cannot serve to excuse Marx. For if the condition which would validate formula (27) is fulfilled, then the entire operation of converting values into prices is pointless, while Marx makes use of this formula precisely in connection with this operation.

The above remark is directed only against the criticism which holds that, regardless of whether the quantities q_1, q_2, and q_3 are equal or not, the Marxian thesis of the influence of the organic composition of the total social capital on the height of the rate of profit, as this thesis finds expression in formula (27), is false.

Tugan-Baranowsky in particular makes this mistake. The two numerical examples with which he tries to refute the Marxian thesis are precisely characterized by the assumption that the organic composition of capital is equal in all three departments, in other words that $q_1 = q_2 = q_3 = q_0$.

In one example,[1] r (the rate of surplus value) falls from 1 to $7/9$, while at the same time q_0 increases from $2/3$ to $20/29$, from which it emerges, entirely in keeping with formula (27), that ρ (the rate of profit) declines from $1/3$ to $7/29$.[2]

In the other example,[3] r rises from 1 to $81/44$, while at the same time q_0 increases from $2/3$ to $25/36$, from which, once again in keeping with formula (27), ρ increases from $1/3$ to $9/16$.

Tugan-Baranowsky concludes from the fact that in the one case a growth in the share of constant capital accompanies a fall and in the other case a rise in the rate of profit, that the general

[1] *Op. cit.*, p. 177.

[2] By q_0 I always understand the relation of the *value* of variable capital to the *value* of the total capital, while in Tugan-Baranowsky's examples it is a question of *price* expressions. In the place of q_0, which equals $\frac{C}{C+V}$, there thus appears $\frac{Cx}{Cx+Vy}$. But the latter expression is identical with q_0 if one assumes, as Tugan-Baranowsky does, that the organic composition of capital is identical in all three departments. For in this case we have $x = y$ or alternatively $x = y = 1$.

[3] *Ibid.*, pp. 180-181.

rate of profit is entirely independent of the organic composition of the social capital, and that therefore the Marxian theory of profit is false.[1]

As though such numerical examples could in any way touch the Marxian theory of the influence of the organic composition of the total social capital on the rate of profit! According to Marx, this influence makes itself felt in the indicated way only if the rate of surplus value remains unchanged.[2]

[1] See the first article in my work "Wertrechnung und Preisrechnung," pp. 48-49.

[2] *Capital,* Vol. III, for example p. 75 and p. 248. The extent to which this limiting condition figures in the Marxian law of the falling rate of profit I have discussed thoroughly in the third article of my work "Wertrechnung und Preisrechnung im Marxschen System."