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CHAPTER III

The Effect of Alternative Lag Distributions

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FISCAL AND MONETARY POLICY MAKERS share the desire to influence the flow of business fixed investment. Their overall purposes may be for the short run (for example, to counteract a business cycle) or the long run (to affect the rate of growth of potential output). In the first case, it is desirable to apply a policy tool with prompt, highly concentrated impact, for the cyclical situation may have drastically changed by the time a policy with a long lag takes hold. In the second, it is desirable to affect the flow of investment more gradually, or else the policy itself may create short-run instability. In either case, as Griliches and Wallace have emphasized, "whether or not a particular stabilization or growth policy will actually do more harm than good depends crucially on the form of the lag function." [1]

Through tax changes in 1954, 1962, and 1964, the federal government has sought to encourage investment spending, presumably with long-run goals in mind. On the other hand, monetary changes since the Treasury-Federal Reserve accord in 1951 have presumably aimed at prompt and reversible influence. Without doubt, the 1966 and 1967 offsetting changes in the tax treatment of investment were explicitly designed to have short-run effects on investment.

[1] Zvi Griliches and Neil Wallace, "The Determinants of Investment Revisited," *International Economic Review*, Vol. 6 (September 1965), p. 328.

This chapter seeks to evaluate the direct effects of fiscal and monetary instruments, as well as other determinants, on expenditures for producers' durable equipment, the largest component of business fixed investment.[2] In particular, it is concerned not only with *how much* a particular policy affects investment, but also *when* the effect occurs. Jorgenson, in a series of papers with several colleagues, has drawn the striking conclusion that "any measures which result in a once-over change in demand for capital will result in a relatively short and sharp boom in investment demand followed by a lengthy period of steadily worsening stagnation induced by a decline in total investment expenditures relative to their peak levels." [3] This conclusion, if correct, has important policy implications. However, a controversial feature of the model from which it is derived is the assumption that all of the determinants of investment act with the same distributed lag. I have attempted to relax this restrictive assumption.

The model discussed in this chapter is heavily influenced by Jorgenson's neoclassical investment model, in the extended version developed by Hall and Jorgenson to study the effects of tax policy on investment. All fiscal and monetary parameters in their model affect investment by means of changes in the imputed rent on the services of capital goods. By assumption, in their model the elasticity of investment demand with respect to this rent is unity, and thus the elasticity of investment demand

[2] By the direct effect of a change in a policy parameter, I mean its effect with all other determinants of investment unchanged. A tax credit stimulates investment indirectly through feedbacks on the other determinants of investment, but the magnitude of these feedbacks can be discussed only in the context of a complete model.

[3] Dale W. Jorgenson, "Anticipations and Investment Behavior," in James S. Duesenberry, Gary Fromm, Lawrence R. Klein, and Edwin Kuh (eds.), *The Brookings Quarterly Econometric Model of the United States* (Chicago: Rand McNally and Co., 1965; Amsterdam: North-Holland, 1965), pp. 85–86. See also Dale W. Jorgenson, "Capital Theory and Investment Behavior," in American Economic Association, *Papers and Proceedings of the Seventh-fifth Annual Meeting, 1962* (*American Economic Review*, Vol. 53, May 1963), pp. 247–59; Dale W. Jorgenson and James A. Stephenson, "The Time Structure of Investment Behavior in United States Manufacturing, 1947–1960," *Review of Economics and Statistics*, Vol. 49 (February 1967), pp. 16–27; Dale W. Jorgenson and James A. Stephenson, "Investment Behavior in U.S. Manufacturing, 1947–1960," *Econometrica*, Vol. 35 (April 1967), pp. 169–220; Robert E. Hall and Dale W. Jorgenson, "Tax Policy and Investment Behavior," *American Economic Review*, Vol. 57 (June 1967), pp. 391–414; Robert E. Hall and Dale W. Jorgenson, "The Role of Taxation in Stabilizing Private Investment," in Vincent P. Rock (ed.), *Policymakers and Model Builders: Cases and Concepts* (Washington: Gordon and Breach, 1969); and Chap. 2 in this volume.

with respect to each of the determinants of the rent is an assumed, rather than estimated, value. In removing this second restrictive assumption, I provide estimates of the direct effects of various policy changes that are less dependent on the particular way the model is put together.

The next section sets forth a rationale for the estimation of separate lag distributions for different determinants of investment. The rationale is stated in terms of a neoclassical model in which factor proportions are variable only up to the point when new machines are installed.[4] With the addition of assumptions about how machines wear out and how expectations about future prices are formed, this model implies that the short-run elasticity of investment demand with respect to changes in the rent will never exceed the long-run elasticity. This rationale, while plausible, is not, however, the only way in which a divergence in lag distributions could be justified.

The model is used to explain quarterly data for aggregate expenditures on producers' durable equipment. The long-run price elasticity of demand for equipment is estimated to be very close to unity, with the short-run elasticity considerably smaller. At the same time, the short-run elasticity of equipment demand with respect to output substantially exceeds the long-run elasticity. Tax parameters, the interest rate, and the yield on equities all appear to be important determinants of the level of expenditures, although the quantitative impact of the investment tax credit, adopted in 1962, appears to be greater than that of any of the other policy measures studied.

Specification of a Distributed Lag Model

To provide policy makers with knowledge about the speed with which their actions can affect investment, it is necessary to estimate the parameters of a model of investment demand in which explicit attention is given to monetary and fiscal parameters. This section discusses the specification of

[4] For theoretical developments of this model, mostly in the context of long-run growth, see Leif Johansen, "Substitution versus Fixed Production Coefficients in the Theory of Economic Growth: A Synthesis," *Econometrica*, Vol. 27 (April 1959), pp. 157–76; Robert M. Solow, "Substitution and Fixed Proportions in the Theory of Capital," *Review of Economic Studies*, Vol. 29 (June 1962), pp. 207–18; Benton F. Massell, "Investment, Innovation, and Growth," *Econometrica*, Vol. 30 (April 1962), pp. 239–52; Edmund S. Phelps, "Substitution, Fixed Proportions, Growth and Distribution," *International Economic Review*, Vol. 4 (September 1963), pp. 265–88; and Murray C. Kemp and Phạm Chí Thānh, "On a Class of Growth Models," *Econometrica*, Vol. 34 (April 1966), pp. 257–82.

such a model, in which a number of terms, reflecting relative prices, capital costs, and the effects of taxes, are combined into a single expression for the imputed rent on a piece of equipment. In the long run, it is assumed that the demand for equipment is a log-linear function of this rent, which is identified with the price of the services provided by a unit of equipment. In addition, the demand for equipment is assumed to be proportional to the sum of (a) desired changes in capacity and (b) replacement of capacity.

A crucial feature of this model is the assumption that changes in the determinants of the rent may affect investment expenditures with a lag distribution different from that which exists for changes in the determinants of desired capacity. Given the current state of knowledge about possibilities in the real world for changes in factor proportions, it cannot be assumed a priori that all of the determinants of investment affect expenditures with the same time pattern.

The stylized world on which the model of investment behavior is based is one in which a firm, at time t, must make investment decisions η periods ahead. Any equipment that is ordered in period t is delivered in period $t+\eta$ and can be put immediately into production. From the vantage point of period t, the firm knows that even if it orders no new equipment in period $t+\eta$ it will have a certain amount of equipment on hand. This equipment includes the machinery available in period t, plus the equipment previously ordered that will be delivered between t and $t+\eta$, less the equipment that will be retired during this same period. The factor proportions on all of this equipment are assumed to be fixed, although in general they will differ from machine to machine. Each machine, if operated for a period by the appropriate amounts of the cooperating factors, can produce a certain amount of output,[5] which is defined as the capacity of the machine.[6] The aggregate capacity of all the machines that will be in existence at the beginning of period $t+\eta$, with no new investment, is Q_{K_0}.

With the assumption that the firm's objective in fixing factor proportions is the minimization of cost, it must decide (a) how much new capacity to order for delivery η periods later, and (b) what blueprint should be used to specify the factor proportions on the new capacity. I assume that the two

[5] For the present it is assumed there is only a single homogeneous output.

[6] In fact, unless fixed capital is used on a twenty-four-hour basis, there is still no unique measure of capacity. It does not seem unreasonable, however, to assume that institutional patterns set a "normal" degree of utilization that can be altered only by incurring higher costs.

decisions are separable, and, given the amount of new capacity to be ordered, consider first the question of the blueprint.

Choice of Factor Proportions

In a situation in which relative prices will change over the lifetime of the capacity, it will be impossible, in general, to choose proportions that use the optimal amount of all factors at all times. In order to simplify the problem of choice, it is assumed that starting from an initial rent $c(0)$ earned by a new machine, the flow of rent is expected to decline exponentially, so that $c(t) = c(0)e^{-\delta t}$. This decline may result from physical deterioration of the capacity so that the flow of output it can produce becomes less over time, from rises in the wages or rents on nonfixed factors, from changes in the price of the output produced, or from other causes. This assumption is a very crude approximation, and may compromise the results.[7] If it is granted, however, the present value of the stream of rents earned (after deduction of income taxes) by a unit of new capacity over its lifetime may be written (ignoring subscripts)

$$\int_0^\infty e^{-rt}(1-u)c(0)e^{-\delta t}dt + kq + q(1-k')u\int_0^T e^{-rs}D(s)ds, \qquad (3.1)$$

in which

q = the price of the ith capital good (when new)
k = the rate of tax credit on investment in the ith good
k' = the rate of tax credit that must be deducted from the depreciation base
r = the appropriate discount rate (including adjustment for risk)
u = the rate of direct taxation of business income
T = the lifetime of the ith good prescribed for tax purposes
$D(s)$ = the proportion of the depreciation base for an asset of age s that may be deducted from taxable income
δ = the exponential rate of decline in the value of services provided by a unit of the ith good.

The first term in (3.1) is the discounted stream of quasi-rents, after deduction

[7] This assumption, in particular, implies that the quasi-rent will approach zero only asymptotically. However, if the reason for the expected decline in quasi-rent is, for example, an expected rise in all wages at a constant rate $\dot{w}$, then $c(t) = c(0)(1-w_0e^{\dot{w}t})$ (where w_0 is the initial wage share), and this function declines at an increasing rate and reaches zero in finite time.

of direct taxes. The second term allows for tax credits of a certain proportion of the cost of the machine. The third term is the discounted value of the stream of taxes saved as a result of a deduction of depreciation expenses from taxable income.

If this present value is equated to q, the price of a new machine, it is possible to solve for $c(0)^*$, the quasi-rent that must be earned by a new machine to justify its purchase. If z denotes $\int_0^T e^{-rs}D(s)ds$, the present value of the depreciation deduction, the equality is

$$q(1-k-uz+uzk') = \int_0^\infty e^{-(\delta+r)t}c(0)^*(1-u)dt; \tag{3.2}$$

integrating the right-hand side leads to [8]

$$c(0)^* = \frac{q(r+\delta)(1-k-uz+uzk')}{1-u}. \tag{3.3}$$

The blueprints tell how much of the malleable output must be molded into each of the models of machinery that are available. In order to proceed further, it is assumed that at any particular point of time, the available blueprints correspond to a neoclassical production function of the special constant elasticity of substitution (CES) class [9] (in the first degree homogeneous version as originally developed by Arrow, Chenery, Minhas, and Solow).[10] With m factors, this function, with suitable normalization,[11] may be written

$$Q = \left[\sum_{i=1}^{m} \alpha_i X_i^{(\sigma-1)/\sigma}\right]^{\sigma/(\sigma-1)}, \tag{3.4}$$

[8] This derivation closely follows that of Hall and Jorgenson in "Tax Policy and Investment Behavior," and the formula arrived at differs from theirs only because I have distinguished between k and k'.

[9] This m-factor constant-elasticity-of-substitution function, though itself a special case, is still relatively general compared with the two most popular alternatives, the Cobb-Douglas and Leontief production functions, both of which are special cases of the class of CES functions.

[10] Kenneth J. Arrow, Hollis B. Chenery, B. S. Minhas, and Robert M. Solow, "Capital-Labor Substitution and Economic Efficiency," *Review of Economics and Statistics*, Vol. 43 (August 1961), pp. 225–50.

[11] As originally written by Arrow, Chenery, Minhas, and Solow, the right-hand side was multiplied by a scaling (or so-called efficiency) parameter v, with the sum of the α's set equal (normalized) to unity. With no loss of generality, the efficiency parameter may be set equal to unity. If the alternative normalization is adopted, the sum of all the distribution parameters (the α's) is then equal to $v^{(\sigma-1)/\sigma}$.

in which

Q = output in physical terms
X_i = the input of the ith factor [12]
σ = the elasticity of substitution
α_i = the distribution parameter for the ith factor (as noted below, these parameters may change over time).

Suppose, for the moment, that no factor price is expected to vary over the lifetime of the capacity to be installed. In this case, the assumed decline in rent could come about if, for example, a certain portion of the capacity disappeared each period, thus reducing both the output and the costs by exactly the same proportion. For this special case, a single set of factor proportions will minimize costs at all times. Given knowledge of prices for a unit of the services of each of the factors (c_i, $i = 1, \ldots m$), conditions for cost minimization subject to the production constraint (3.4) may be derived by forming the Lagrangian expression

$$COST = \sum_{i=1}^{m} c_i X_i + \Lambda \left[Q - \left(\sum_{i=1}^{m} \alpha_i X_i^{(\sigma-1)/\sigma} \right)^{\sigma/(\sigma-1)} \right], \tag{3.5}$$

and, as necessary conditions, setting the m first derivatives with respect to the X_i equal to zero. This leads to the equations

$$c_i - \Lambda \alpha_i (Q/X_i)^{1/\sigma} = 0, \qquad i = 1, \ldots m. \tag{3.6}$$

From (3.6) it would be possible to derive $m-1$ equations expressing the cost-minimizing ratios of each of the other $m-1$ factors to X_1 (numbering the factors so that equipment, the factor being studied, is the first);

$$X_i/X_1 = \alpha_i^\sigma c_1^\sigma / \alpha_1^\sigma c_i^\sigma, \quad i = 2, \ldots m. \tag{3.7}$$

Substituting these ratios into the production function leads to the expression

$$X_1/Q = \left[\alpha_1 + \sum_{i=2}^{m} \frac{\alpha_i^\sigma c_1^{\sigma-1}}{\alpha_1^{\sigma-1} c_i^{\sigma-1}} \right]^{\sigma/(\sigma-1)} \tag{3.8}$$

[12] This input may be thought of as a flow of services per unit of time, or, without loss of generality, as a stock of fixed factor that provides a flow of services proportional to the stock, with the proportionality factor normalized to unity by appropriate choice of units. Changes in units will, of course, change the distribution parameter for the factor in question, but as long as σ does not change over time, this will not cause any difficulty. The input of the factor "machinery" in what follows is the number of units of the homogeneous output that are frozen into the machine of the particular blueprint chosen.

for the amount of the first factor to be embodied in (or needed to cooperate with) each unit of new capacity.

Not only is (3.8) very awkward, especially if the number of factors is greater than two, but also it is necessary to make assumptions about the time paths of each of the α_i in order to use it to derive the demand for a factor. Unless it is assumed that each of the distribution parameters is constant over time, or that each changes according to some exponential time pattern, (3.8) leads to hopeless difficulties. In addition, the choice ot units in which to measure each of the other factors presents problems. In computing a wage rate, for example, should labor be measured in man-hours, or man-hours adjusted for education, or some other unit? Although it will certainly prove necessary to make arbitrary assumptions about smoothness with regard to technical changes in the distribution parameter for equipment, it seems desirable to make as few as possible.

Fortunately, the number of such assumptions can be decreased by using the economic interpretation of the Lagrangian multiplier Λ, and adding one plausible behavioral assumption. Note first that Λ represents the minimum average total cost of output produced with the newest technology. If the firm is investing in new equipment, this average cost must represent its marginal cost as well.[13] The interpretation of Λ as a minimum average cost may be verified by rewriting equation (3.6) in the form

$$c_i = \Lambda \alpha_i X_i^{-(1/\sigma)} \left[\sum_{i=1}^{m} \alpha_i X_i^{(\sigma-1)/\sigma} \right]^{1/(\sigma-1)}, \quad i = 1, \dots m, \tag{3.9}$$

multiplying both sides of each equation by X_i, and summing all m equations, to get

$$\begin{aligned} \sum_{i=1}^{m} c_i X_i &= \Lambda \left[\sum_{i=1}^{m} \alpha_i X_i^{(\sigma-1)/\sigma} \right] \left[\sum_{i=1}^{m} \alpha_i X_i^{(\sigma-1)/\sigma} \right]^{1/(\sigma-1)} \\ &= \Lambda \left[\sum_{i=1}^{m} \alpha_i X_i^{(\sigma-1)/\sigma} \right]^{\sigma/(\sigma-1)} \\ &= \Lambda Q. \end{aligned} \tag{3.10}$$

[13] When the firm is investing in new machinery, the average *variable* costs on the oldest machinery in use cannot exceed the appropriate calculated average *total* cost using new machinery. The trick lies in calculating average total cost, which must include allowance for expected decreases in value of new machinery, without knowledge about expectations of future prices. The simplified rules of thumb I have assumed make the calculation easy, but they may not be at all appropriate.

Thus

$$\Lambda = \frac{\sum_{i=1}^{m} c_i X_i}{Q}, \tag{3.11}$$

the average cost with the latest technology, assuming that static cost-minimizing factor proportions are used.

With the addition of the behavioral assumption that price p is set as a constant markup factor M on marginal or minimum average cost, so that

$$p = M\Lambda, \qquad M \geq 1, \tag{3.12}$$

it is possible to use observed output price as a proxy for minimum average cost. If (3.12) is substituted into (3.6), only this one equation is required to write the demand for the first factor per unit of new capacity as a function of its rental, its distribution parameter, the price of output, and M. Thus

$$X_1/Q = [\alpha_1 \, p/c_1 M]^{\sigma}. \tag{3.13}$$

The derivation of (3.13) has proceeded under the assumption of a static world, with no changes in prices or technology. In a time series analysis, it is unrealistic to assume that the underlying production function will not change over time. As long as technical change is factor-augmenting only with respect to factors other than machinery, the marginal product of machinery will not, however, be changed.

Factor augmentation, in the sense used here, may be defined as follows. A change in one of the distribution parameters α_i cannot be distinguished from a change in the units in which the factor X_i is measured. Thus the production function (3.4), governing additions to capacity at time t, may be written with unchanged distribution parameters, but with scale factors that multiply the quantities of each factor when these quantities are measured in time-invariant units. These scale factors are functions of time, and thus the production function may be written

$$Q_t = \left[\alpha_1 \{h_{1_t} X_{1_t}\}^{(\sigma-1)/\sigma} + \sum_{i=2}^{m} \alpha_i \{h_{i_t} X_{i_t}\}^{(\sigma-1)/\sigma}\right]^{\sigma/(\sigma-1)}, \tag{3.4.1}$$

where the first factor is equipment. In (3.4.1) technical change that increases h_{i_t} is factor-augmenting with respect to the ith factor; a single unit of that factor goes further. Each of the derivatives of (3.4.1) with respect to any factor involves only the scaling of that particular factor, so that

$$\partial Q_t/\partial X_{1_t} = \alpha_1 h_{1_t}(Q_t/h_{1_t}X_{1_t})^{1/\sigma}. \tag{3.14}$$

For equipment, therefore, a change in h_{i_t} $(i = 2, m)$ is Harrod-neutral.[14]

If, however, there is a change in h_{i_t} (including a Hicks-neutral technical change, in which all the h_{i_t} rise by the same proportion), the marginal product of a unit of equipment will change over time.[15] Without loss of generality, all equipment-augmenting technical progress may be treated as embodied in new equipment, for Hall has shown that "given rates of embodied and disembodied technical change and a given deterioration function cannot be distinguished from a lower rate of embodied technical change, a higher rate of disembodied change, and a higher rate of deterioration."[16] This means that the rate of disembodied equipment-augmenting change can be arbitrarily normalized to zero; ways to make old equipment more productive will simply show up as slower deterioration of the capacity embodied in that equipment.

As soon as the possibility of technical change is admitted, the assumption that relative prices and factor costs do not change becomes untenable. In addition, if the model is to be applied empirically, there must be some recognition that prices are, in fact, constantly changing. If the production function and the costs of time t were to persist forever, the factor proportions that would minimize costs would be

$$X_{1_t}/Q_t = (1/h_{1_t})(h_{1_t}\alpha_1\, p_t/c_{1_t}M)^\sigma \equiv V_t, \tag{3.13.1}$$

but these will generally be the wrong proportions in a world of changing prices.

For purposes of estimation, it is assumed that entrepreneurs choose their factor proportions by a simple suboptimal rule of thumb. This rule involves factor proportions V_t^* which are calculated as a distributed lag

[14] Technical change is said to be Harrod-neutral if, at a constant rate of interest, technical change leaves the capital-output ratio unchanged. This condition will be satisfied if the marginal product of capital, as a function of the capital-output ratio, is unaffected by the technical change.

[15] Technical change is said to be Hicks-neutral if, for a given set of factor proportions, the technical change does not affect the ratio of the marginal products of any pair of factors. Since the marginal product of the jth factor includes h_{j_t} raised to the power $(\sigma-1)/\sigma$, equal proportional changes in h_{i_t} and h_{j_t} will not affect the ratio of the marginal product of the ith factor to the marginal product of the jth factor.

[16] Robert E. Hall, "Technical Change and Capital from the Point of View of the Dual," *Review of Economic Studies*, Vol. 35 (January 1968), p. 38.

function of past values of V, the *static* optimum amount of equipment per unit of capacity defined in output terms. Thus,

$$V_t^* = \sum_{k=0}^{\infty} \chi_k V_{t-k}. \tag{3.15}$$

There is no particular reason why the weights χ should add up to one.

This rule is undoubtedly a vast oversimplification. It would be desirable to consider explicitly the ways in which expectations about interest rates, equipment prices, and prices of other factors are formed, and the choice of optimum proportions in the face of such expectations. However, such an explicit model would require many more specific assumptions than I have been willing to make.

Gross Additions to Capacity

The decision as to how much capacity to order at time t for delivery η periods later depends on how much is needed for replacement, and how much is desired for net additions. Replacement temporarily aside, desired net additions to capacity should equal the difference between the capacity the firm desired to have, and placed orders to achieve, in period $t+\eta-1$ and the capacity desired in period $t+\eta$. Desired capacity Q_K^* is assumed to be a roughly constant [17] multiple ($\zeta^* \geq 1$) of output planned for period $t+\eta$, $Q_{t+\eta}^*$, that is,

$$Q_{K_{t+\eta}}^* = \zeta^* Q_{t+\eta}^*. \tag{3.16}$$

Furthermore, Q^* may, it is assumed, be approximated by a function of past outputs

$$Q_{t+\eta}^* = \sum_{i=0}^{\infty} \xi_i Q_{t-i}. \tag{3.17}$$

[17] With fixed proportions prevailing ex post, there will be an optimal degree of excess capacity which depends on the certainty with which demand expectations are held, the costs, due to lost sales or overtime work, of not having enough capacity, and the cost of holding excess capacity. Thus, in general the desired capacity-output ratio ζ will be a function of relative prices, among other things, but for simplicity I assume it is approximately constant. For approaches to the problem of the optimal degree of excess capacity for relatively special cases, see Alan S. Manne, "Capacity Expansion and Probabilistic Growth," *Econometrica*, Vol. 29 (October 1961), pp. 632–49, and Kenneth R. Smith, "The Determinants of Excess Capacity," paper presented to the North American Regional Conference of the Econometric Society, Washington, 1967.

Then net additions to desired capacity are

$$Q^*_{K_{t+\eta}} - Q^*_{K_{t+\eta-1}} = \zeta^*\left[\sum_{i=0}^{\infty} \xi_i (Q_{t-i} - Q_{t-i-1})\right]. \tag{3.18}$$

For different firms, and for different types of equipment within a firm, the lead time η will vary. Therefore, in deriving an aggregate model, it is desirable to specify that actual net additions to capacity are a distributed lag function of past desired net additions, with the weights adding up to one and with each particular weight ψ_j indicating the proportion of the capacity ordered in period t that is installed in period $t+j$. In other words, ψ_j represents the proportion of the equipment aggregate for which the waiting period η is equal to j. Thus an aggregate equation for actual net additions to capacity would be

$$Q_{K_t} - Q_{K_{t-1}} = \zeta^*\left[\sum_{j=0}^{\infty}\sum_{i=0}^{\infty} \psi_j \xi_i (Q_{t-i-j} - Q_{t-i-j-1})\right]. \tag{3.19}$$

This equation can be written in general form as

$$Q_{K_t} - Q_{K_{t-1}} = \zeta^*\left[\sum_{k=0}^{\infty} \varphi_k (Q_{t-k} - Q_{t-k-1})\right].$$

The above equation, however, is nothing but a first-order difference equation with a very simple solution,

$$Q_{K_t} = \zeta^* \sum_{k=0}^{\infty} \varphi_k Q_{t-k} + Constant, \tag{3.20}$$

where the constant is arbitrary and must be zero if actual capacity is to equal desired capacity in a steady state.

If net additions to capacity actually follow such a pattern (either without error, or with very small, serially independent errors), current capacity can be adequately approximated as a function of past output levels. This approximation is not likely to be very good in the case of an individual firm or industry, or even in the aggregate in the face of a substantial downswing, for the lag between desired and actual net decreases in capacity cannot be expected to resemble the lag for net increases.[18] Nevertheless,

[18] This is the well-known problem of asymmetry in the operation of the acceleration principle. Aggregation, of course, does not solve the problem, except to the extent that some firms can sell their excess capacity to others. Nevertheless, the declines in aggregate net new orders for machinery in the post-Korean period, even in the worst recessions, have been moderate enough to encourage the feeling that the bias from this source will not be serious.

in the absence of any direct measures of capacity or capacity utilization for the economy as a whole, an approximation of this sort is perhaps the best available measure.

The conceptual stock of capacity specified in equation (3.20) is the analogue in this model to the stock of capital that ordinarily appears in investment functions. The crucial difference is that in this model the amount of investment necessary to replace a unit of capacity that wears out or becomes obsolete will depend on recent relative prices rather than on the amount of investment that originally took place.

As to replacement demand, by far the simplest assumption is that the flow of services from a unit of capacity declines exponentially from the time the capacity is installed. Under static conditions, such a pattern would imply exponential decline in the prices of used machinery. Since existing evidence suggests that exponential decline may be an adequate approximation to the true pattern,[19] this assumption can be rationalized, and leads to the conclusion that the proportion of existing capacity to be replaced in a given time period may be adequately approximated by a constant (δ).[20] Then gross additions to capacity, or gross investment in capacity units I_{QK}, may be written as

$$I_{QK_t} = \zeta^* \sum_{k=0}^{\infty} \varphi_k(Q_{t-k} - Q_{t-k-1}) + \delta\zeta^* \sum_{k=0}^{\infty} \varphi_k Q_{t-k-1}, \tag{3.21}$$

using (3.20) to express the net stock of capacity at the end of period $t-1$ (the beginning of period t).

[19] See, for example, George Terborgh, *Realistic Depreciation Policy* (Machinery and Allied Products Institute, 1954), Chaps. 4 and 5; Zvi Griliches, "Capital Stock in Investment Functions: Some Problems of Concept and Measurement," in Carl F. Christ and others, *Measurement in Economics: Studies in Mathematical Economics and Econometrics in Memory of Yehuda Grunfeld* (Stanford University Press, 1963), pp. 121–23; and Gregory C. Chow, *Demand for Automobiles in the United States: A Study in Consumer Durables* (Amsterdam: North-Holland, 1957).

[20] A simplification of this sort may not be tenable, however, in a model with ex post fixed proportions, to the extent that rising variable costs eliminate the quasi-rents on old equipment even before it is physically worn out. In such circumstances, the proportion of capacity retired in any given period would be a function of relative prices, among other things. On the other hand, if there is some possibility of choosing the durability of machinery, and more durable machines cost more, one would expect that market forces would lead to the manufacture of capital goods in which the physical life is normally *less* than the economic life.

Equipment Expenditures

To derive an investment function, gross additions to capacity must be scaled by a factor that represents the incremental ratio of equipment to capacity considered optimal at the time the plans are made final. The rule of thumb for choosing this marginal ratio is given in (3.15). As given in (3.15), V_t^* must be multiplied by net additions to desired capacity in period $t+\eta$, as given by (3.17), and also by the amount of replacement to take place in period $t+\eta$, which will be $\delta\zeta^*$ times the output planned for $t+\eta$. Thus, desired gross additions to capacity I_{QK}^* are given by

$$\begin{aligned} I_{QK_{t+\eta}}^* &= \zeta^* \sum_{i=0}^{\infty} \xi_i (Q_{t-i} - Q_{t-i-1}) + \delta\zeta^* \sum_{i=0}^{\infty} \xi_i Q_{t-i-1} \\ &= \zeta^* \sum_{i=0}^{\infty} \xi_i [Q_{t-i} - (1-\delta) Q_{t-i-1}], \end{aligned} \tag{3.22}$$

and planned expenditure $I_{t+\eta}^*$ is

$$I_{t+\eta}^* = \zeta^* \sum_{k=0}^{\infty} \chi_k V_{t-k} \sum_{i=0}^{\infty} \xi_i [Q_{t-i} - (1-\delta) Q_{t-i-1}]. \tag{3.23}$$

If a lag is introduced between plans and expenditures, as before, *aggregate* equipment expenditures in period t are expressed in terms of past output levels and relative price ratios as

$$I_t = \zeta^* \sum_{j=0}^{\infty} \sum_{k=0}^{\infty} \sum_{i=0}^{\infty} \psi_j \chi_k \xi_i V_{t-k-j} [Q_{t-i-j} - (1-\delta) Q_{t-i-j-1}]. \tag{3.24}$$

This expression may be thought of as a special case of the general form

$$I_t = \zeta^* \sum_{j=0}^{\infty} \sum_{i=0}^{\infty} \beta_{ij} V_{t-i} Q_{t-j}. \tag{3.25}$$

The relationship between the coefficients β_{ij} in equation (3.25) and the coefficients of the equation that immediately precedes it is as follows:

$$\beta_{ij} = \sum_{k=0}^{\min[i,j]} \psi_k \chi_{i-k} [\xi_{j-k} - (1-\delta) \xi_{j-k-1}]. \tag{3.26}$$

Equation (3.25) involves a large—indeed, in principle, infinite—matrix of coefficients β_{ij} of relative price and output terms. Unrestricted estimation of the matrix, however, is neither possible nor desirable.

The common-sense interpretation of (3.25) is that equipment spending is a complex weighted sum of the effects of lagged relative prices and output levels, interacting multiplicatively. But if quarterly changes in V and Q are small relative to their levels, (3.25) is approximately proportional to

$$I_t \approx \zeta^* \sum_{i=0}^{n} \lambda_i V_{t-i} \tilde{Q}_t + \zeta^* \sum_{j=0}^{n} \mu_j Q_{t-j} \tilde{V}_t, \tag{3.27}$$

in which $\tilde{Q}_t$ and $\tilde{V}_t$ are approximations to the levels of Q and V over the period $t-n$, t. On this interpretation

$$\lambda_i \approx \sum_j \beta_{ij} = \sum_{k=0}^{i} \delta \psi_k \chi_{i-k} \tag{3.28}$$

and

$$\mu_j \approx \sum_i \beta_{ij} = \sum_{k=0}^{j} \psi_k [\xi_{j-k} - (1-\delta)\xi_{j-k-1}]; \tag{3.29}$$

that is, the λ's represent row sums of the β_{ij} coefficient matrix and the μ's represent column sums. Since all of the ψ's and χ's are assumed positive, the λ's should all be positive.[21] With respect to the μ's, only the first nonzero μ_j weight can be specified a priori to be positive; nevertheless it seems likely that the first few μ_j would be positive, while the weights applied to more distant output effects would be negative. In other words, both recent and distant relative prices are expected to enter the equation with the same sign, but recent output levels are expected to have a positive effect on investment while past levels should have a negative effect. This simply reflects the familiar acceleration principle; for a given level of current output a higher level of past output means a smaller output rise and hence less investment. The distinction between the expected pattern of λ's and the μ's arises because without ex post substitution, investment reacts not to a change in relative prices but only to the level of the ratio.[22] In this model there is no acceleration effect with respect to relative prices.

Using (3.26), Table 3-1 sets out a numerical example of what the full coefficient matrix β would be, with δ arbitrarily set at 0.10, and given the

[21] This statement requires amplification. The χ weights, which partly represent expectations about future relative prices, might conceivably contain negative terms. The assumption that they should all be positive implies that V_t^* is a positively weighted sum of past values of V_t, which may not be true.

[22] To the extent that more capacity is replaced when V is rising, because old capacity (put in place when V was low) becomes uneconomical, I might react to changes in V even when ex post substitution does not exist. As noted earlier, the assumption of exponential retirement patterns excludes this possibility, but the assumption may not be valid.

arbitrarily chosen sets of weights [23]

$$
\begin{aligned}
[\xi_0\ \xi_1\ \xi_2] &= [0.5\ \ 0.4\ \ 0.15] \\
[\chi_0\ \chi_1\ \chi_2\ \chi_3] &= [0.4\ \ 0.3\ \ 0.2\ \ 0.1] \\
[\psi_0\ \psi_1\ \psi_2] &= [0.25\ \ 0.5\ \ 0.25]. \qquad (3.30)
\end{aligned}
$$

In the example, the row sums are all positive, while the column sums are first positive and then negative. The same pattern is evident in each row across; the coefficients are first positive and then negative.

The Standard Neoclassical Model

The functional forms (3.25) and (3.27) are particularly appropriate for testing the hypothesis that a more general functional form is needed than is provided by the standard neoclassical model, because the standard model can also be fitted into these forms. I identify the "standard" model with the path-breaking work of Jorgenson, mentioned in the introduction to this study; [24] the most important particular in which it differs from the model I have specified is that in it factor proportions are assumed to be freely variable at all times. In this case, (3.13.1) would give the appropriate

TABLE 3-1. Example of Beta Matrix with Ex Post Fixed Proportions

Number of periods factor proportions are lagged	*Number of periods output is lagged* 0	1	2	3	4	5	*Total* [a] λ_j
0	0.050	—0.005	—0.021	—0.014	0.000	0.000	0.010
1	0.038	0.096	—0.026	—0.052	—0.027	0.000	0.029
2	0.025	0.072	0.032	—0.043	—0.041	—0.014	0.032
3	0.012	0.049	0.027	—0.028	—0.029	—0.010	0.021
4	0.000	0.025	0.022	—0.013	—0.017	—0.007	0.010
5	0.000	0.000	0.012	—0.001	—0.006	—0.003	0.003
Total μ_i [a]	0.125	0.237	0.048	—0.151	—0.120	—0.034	0.105

Source: Equation (3.26).
[a] Details may not add to totals due to rounding.

[23] These weights imply (a) that output planned at $t+\eta$ is one-half of output at t, plus 40 percent of output at t—1, plus 15 percent of output at t—2; (b) that $V_t^* = 0.4V_t+0.3V_{t-1}+0.2V_{t-2}+0.1V_{t-3}$; and (c) that 25 percent of orders placed in period t are filled in that period, 50 percent in period $t+1$, and 25 percent in period $t+2$.

[24] See note 3, p. 62.

conditional operating rule for the cost-minimizing ratio of capital stock to output.[25]

Jorgenson recognizes, however, that there exists a lag—or series of lags—between the making of plans and the delivery of equipment. In the most rigid form of this model, plans are made without regard to this lag; one interpretation of this assumption would be that expectations are static,[26] that is,

$$Q^*_{t+\eta} = Q_t \tag{3.31}$$

and

$$V^*_t = V_t. \tag{3.32}$$

A desired stock of equipment K^* is computed as

$$K^*_t = V^*_t Q^*_{t+\eta} = V_t Q_t. \tag{3.33}$$

If equipment is ordered to cover any change in desired capital stock, and a proportion ψ_j of the aggregate total of equipment ordered in period t is delivered in period $t+j$, then

$$K_t - K_{t-1} = \sum_{j=0}^{\infty} \psi_j (V_{t-j} Q_{t-j} - V_{t-j-1} Q_{t-j-1}) \tag{3.34}$$

and

$$K_t = \sum_{j=0}^{\infty} \psi_j V_{t-j} Q_{t-j}, \tag{3.34.1}$$

where K_t is the aggregate stock at the end of period t.

Then, since (3.34) provides an expression for net investment in period t, and replacement in that same period will be δK_{t-1} if exponential retirement is assumed, this means

$$I_t = \sum_{j=0}^{\infty} \psi_i (V_{t-j} Q_{t-j} - V_{t-j-1} Q_{t-j-1}) + \delta K_{t-1}. \tag{3.35}$$

[25] I am also assuming that the standard model includes enough assumptions to assure the existence of an aggregate capital stock. See Franklin M. Fisher, "Embodied Technical Change and the Existence of an Aggregate Capital Stock," *Review of Economic Studies*, Vol. 32 (October 1965), pp. 263–88. The operating rule does not, however, specify how the optimum output is chosen.

[26] Nothing would be changed if expectations embodied some sort of constant trend so that, for example, expected output Q^* was a constant multiple of current output. The constant trend factor could not, in the estimation, be untangled from the other coefficients.

If (3.35) is substituted for K_{t-1}, then

$$\begin{aligned} I_t &= \sum_{j=0}^{\infty} \psi_j(V_{t-j}Q_{t-j} - V_{t-j-1}Q_{t-j-1}) + \delta \sum_{j=0}^{\infty} \psi_j V_{t-j-1}Q_{t-j-1} \\ &= \sum_{j=0}^{\infty} \psi_j(V_{t-j}Q_{t-j} - [1-\delta]V_{t-j-1}Q_{t-j-1}). \end{aligned} \tag{3.36}$$

This corresponds to (3.25) with the matrix β simply

$$\begin{bmatrix} \psi_0 & 0 & 0 & \cdots \\ 0 & \psi_1-(1-\delta)\psi_0 & 0 & \\ 0 & 0 & \psi_2-(1-\delta)\psi_1 & \\ \cdot & & & \cdot \\ \cdot & & & \cdot \\ \cdot & & & \cdot \end{bmatrix}.$$

The row and column sums are obviously identical. The expected sign pattern of the weights on the main diagonal would be $\beta_{11} > 0$, $\beta_{22} \gtrless 0$, depending on whether or not $\psi_1 > (1-\delta)\psi_0$. All the weights could be positive if $\psi_{i+1} \geq (1-\delta)\psi_i$ for all i. But this would appear unlikely, for this restriction implies that the mean lag between orders and deliveries of equipment would have to be at least $1/\delta$ periods (that is, six to twelve years for plausible values of δ).

The lag pattern derived above depends not only on ex post variability of factor proportions but also on the existence of static expectations. There is no reason why the two assumptions should be connected, and relaxing the assumption of static expectations leads to a much different lag pattern. If, for example, price and output expectations are generated by processes of the form

$$Q^*_{t+\eta} = \sum_{i=0}^{\infty} \xi_i Q_{t-i}, \tag{3.37}$$

and

$$V^*_{t+\eta} = \sum_{k=0}^{\infty} \chi_k V_{t-k}, \tag{3.38}$$

then an argument precisely analogous to the one just given leads to the expression

$$I_t = \sum_{j=0}^{\infty} \sum_{k=0}^{\infty} \sum_{i=0}^{\infty} \psi_j \chi_k \xi_i [V_{t-k-j}Q_{t-i-j} - (1-\delta)V_{t-k-j-1}Q_{t-i-j-1}]. \tag{3.39}$$

If equation (3.39) is also considered a special case of

$$I_t = \sum_{j=0}^{\infty} \sum_{i=0}^{\infty} \beta_{ij} V_{t-i} Q_{t-j}, \tag{3.39.1}$$

the correspondence between the coefficients β_{ij} and the coefficients of (3.39) is

$$\beta_{ij} = \sum_{k=0}^{\min[i,\, j]} \psi_k [\chi_{i-k}\, \xi_{j-k} - (1-\delta)\chi_{i-k-1}\, \xi_{j-k-1}]. \tag{3.40}$$

In this case the row and column sums of the β matrix will not be identical unless the coefficients χ_k are identical with the coefficients ξ_i. A condition that is sufficient, though not necessary, to guarantee that all the row sums in the β matrix corresponding to (3.39) will be positive is that $\chi_{i+1} \geq (1-\delta)\chi_i$ for all i.

Table 3-2 gives a numerical example of a β matrix derived from equation (3.40) using the same sets of ψ, χ, and ξ weights that were used in deriving Table 3-1. In this example, both the row sums and the column sums are first positive and then negative, which would be the normal case.

TABLE 3-2. Example of Beta Matrix for Standard Neoclassical Model with Nonstatic Expectations

Number of periods factor proportions are lagged	*Number of periods output is lagged*						*Total* [a]
	0	1	2	3	4	5	λ_j
0	0.050	0.040	0.015	0.000	0.000	0.000	0.105
1	0.038	0.085	0.055	0.016	0.000	0.000	0.194
2	0.025	0.061	0.000	—0.020	—0.012	0.000	0.055
3	0.012	0.038	—0.004	—0.061	—0.045	—0.014	—0.074
4	0.000	0.014	—0.009	—0.045	—0.033	—0.010	—0.083
5	0.000	0.000	—0.010	—0.030	—0.021	—0.007	—0.068
6	0.000	0.000	0.000	—0.011	—0.009	—0.003	—0.024
Total [a] μ_i	0.125	0.237	0.048	—0.151	—0.120	—0.034	0.105

Source: Equation (3.40).
[a] Details may not add to totals due to rounding.

To summarize, I have shown that in the most extreme form of the standard model (with the implicit assumption of static expectations) the jth row sum of the coefficient matrix of equation (3.25) would be identical to the jth column sum. In the less restrictive case in which expectations about both V and Q are formed as weighted averages of past observations on V and Q, with all the weights having positive signs, the presumption is that both the row sums and the column sums would be positive at first and then negative. This contrasts to the hypothesized sign pattern of row sums derived from a theoretical model in which ex post substitution is not possible. In that case, all the row sums would be positive.[27]

As noted in the introduction, policy makers should be concerned with whether a measure that affects the implicit rent on equipment sets off a short-lived investment boom. Thus, for the purposes of short-run policy, the most important thing is to be able to estimate the parameters of (3.25) or (3.27), and to derive a qualitative (and quantitative) description of the λ weights. It would also, however, be desirable to be able to interpret the estimates and to draw conclusions about the sort of capital-theoretic model that generated the data. I have noted that an unambiguous interpretation will not generally be possible. All that can be said is that, if all the λ weights turn out to have the same sign, if only a few of them are negative, or if they differ very significantly from the distribution of the μ weights, then these results are not inconsistent with a model in which ex post substitution is not possible. In addition, this pattern would seem improbable in a model in which factor proportions could be easily altered even after fixed equipment was in place.

Estimation of the Parameters of the Model

This section describes an attempt to use the model specified in equations (3.3), (3.13.1), and (3.25) to explain the U.S. Department of Commerce's quarterly series of expenditures for producers' durable equipment in the United States (see Appendix Table 3-A-1, pages 128–30). This aggregate is the most comprehensive series available on equipment expenditures by American firms; it is also the only quarterly series available that separates expenditures for equipment from expenditures for new construction. In a sense, the estimation of the lag distributions corresponding to the model provides a test of the putty-clay hypothesis that choice of technique is possible only up to the point at which new machinery is put into place.[28]

[27] This is true, at least given the assumptions that V_t^* is a positively weighted average of past values of V, and that the amount of capacity desired is not a function of V.

[28] Models of this sort have become known as "putty-clay" models because, in one stylized version, machinery is assumed to be made of "putty," which can be shaped into any given form until it is put into place, after which it becomes hard-baked "clay."

Choice of Variables and Parameters

In the development of the model, a single homogeneous output was assumed. For empirical applications, however, it may be more realistic to think of the production function (3.4) as an aggregate approximation, entailing a summation over many products for each of which there exists at any moment several blueprints defining the feasible methods of production.[29] The approximation will be useful if the demand for investment goods, as a function of relative prices and gross additions to capacity, behaves as predicted by (3.13.1). In particular, the separate estimation of σ, which represents the long-run elasticity of equipment demand with respect to the inverse of product rent, will provide an important test of the degree to which the demand for equipment is sensitive to the price of a unit of equipment services, and thus to fiscal and monetary parameters that affect the rent.

The central hypothesis of this analysis is contained in the equation

$$I_t = \zeta^* \sum_{j=0}^{\infty} \sum_{i=0}^{\infty} \beta_{ij} V_{t-i} Q_{t-j} + \varepsilon_t . \qquad (3.41)$$

For the estimation, the variables in (3.41) are defined as follows:

I_t = deflated expenditures for producers' durable equipment (in 1958 dollars, seasonally adjusted quarterly totals at annual rates)[30]

Q_t = business gross product (gross national product less the gross product of government, households and institutions, and rest of world, in 1958 dollars, seasonally adjusted at annual rates)

V_t = a variable that is proportional to the equilibrium ratio of equipment to output, given the prices and technology of period t

ε_t = an independently distributed random error.

As specified in equations (3.13) and (3.13.1),

$$V_t = h_{1_t}^{\sigma-1} (\alpha_1 p_t)^{\sigma} (c_{1_t} M)^{-\sigma} . \qquad (3.13.2)$$

Two of the parameters, α_1 and M, are constants independent of time; α_1 is the distribution parameter for equipment in the production function and M is the markup proportion over minimum average cost. In the estimation these parameters are absorbed into the coefficients β, and it is not possible to identify them separately. Thus for purposes of estimation, a new

[29] Of course, all of the usual problems associated with index numbers arise.

[30] Unless otherwise stated, all seasonally adjusted variables are used as provided by the U.S. Department of Commerce. The basic data are given in App. Table 3-A-1, pp. 128–30.

variable V' is defined as follows[31]

$$V_t' = h_{1_t}^{\sigma-1} p_t^{\sigma} c_{1_t}^{-\sigma}. \tag{3.42}$$

In this expression

σ = the price elasticity of demand for equipment
h_{1_t} = the technical change parameter for equipment
p_t = the implicit price deflator for business gross product (seasonally adjusted)
c_{1_t} = the imputed rent per unit of new equipment.

The technical change parameter h_{1_t} is assumed to follow a smooth trend.[32] A parameter h' is defined such that

$$h_{1_t}^{\sigma-1} = e^{h'\text{TIME}}, \tag{3.43}$$

with TIME equal to zero at the midpoint of 1958 and incremented by one each quarter. The rate (per year) of equipment-augmenting technical change is then

$$4h'/(\sigma-1), \tag{3.44}$$

since

$$h_{1_t} = e^{h'\text{TIME}/(\sigma-1)}. \tag{3.45}$$

The expression for the imputed rent on new equipment is derived from (3.3),

$$c_{1_t} = \frac{q_t(r_t+\delta)(1-k_t-u_t z_t+u_t z_t k_t')}{1-u_t}. \tag{3.46}$$

In this expression

δ = annual rate of decline of value of the services provided by a unit of fixed equipment
q_t = implicit price deflator for producers' durable equipment (seasonally adjusted)
r_t = discount rate (per year)
u_t = general rate of income taxation for corporations

[31] After V' has been substituted for V in equation (3.41), the sum of all the distributed lag weights $(\Sigma_j \Sigma_i \beta_{ij})$ will correspond to $\delta(\alpha_1/M)^{\sigma}(\Sigma_j \psi_j)(\Sigma_k \chi_k)(\Sigma_i \xi_i)$, whereas in the examples shown in Tables 3-1 and 3-2, $\Sigma_j \Sigma_i \beta_{ij} = \delta(\Sigma_j \psi_j)(\Sigma_k \chi_k)(\Sigma_i \xi_i)$. The parameter ζ^* is also assumed constant and cannot be estimated separately; it is subsumed in the estimated values of the β_{ij} weights.

[32] Unless some such assumption about the smoothness of technical change is made, there is no possible way to distinguish between the effects of technical change and those of relative prices.

z_t = present value of the depreciation deduction
k_t = effective rate of tax credit against equipment purchases
k_t' = rate of tax credit deducted from depreciation base.

Although it was initially my intention to estimate δ, this did not prove feasible, and it was specified to be 0.16 (see below). The choice of measures for r_t, z_t, and k_t requires more extensive comment.

The appropriate empirical measure of the discount rate is subject to a great deal of disagreement, but from the point of view of policy the choice is crucial. The discount rate implicit in investment decisions made in an uncertain world might not be highly correlated with any observed market yields. It might be beyond even slight influence by the monetary authorities.

In a riskless world, the market rate of interest would provide an adequate approximation to the unobservable discount rate appropriate to investment decisions.[33] Yields on equities might differ from the interest rate, but only to the extent that growth in earnings (either in real terms or because of inflation) was anticipated. The risk factor, however, cannot be ignored, and inasmuch as the discount rate fluctuates cyclically due to changing market interpretations of the risk involved, the yield on equity may be more closely correlated with the discount rate. As a very rough approximation, the discount rate might be represented by a weighted sum of Moody's industrial bond yield RM_{MBCIND}, Moody's industrial dividend-price ratio $DIV/PRICE$,[34] the corporate income tax rate u, and a time trend $TIME$ (equal to zero at mid-1958—see Appendix Table 3-A-1, pages 128–30). Thus,

$$r_t = (r_0 + r_1\, RM_{\mathrm{MBCIND}} + r_2\, DIV/PRICE + r_3\, TIME)(1 - r_4 u_t). \quad (3.47)$$

The general formula for z_t, the present value of the depreciation deduction, is

$$z_t = \int_0^T e^{-rs} D(s)\,ds. \quad (3.48)$$

[33] In their most recent study of the electric utility industry, Miller and Modigliani found that, for the three years studied (1954, 1956, 1957), the long-term interest rate seemed to be the variable most closely related to their measure of the cost of capital. See Merton H. Miller and Franco Modigliani, "Some Estimates of the Cost of Capital to the Electric Utility Industry, 1954–57," *American Economic Review*, Vol. 56 (June 1966), pp. 333–91.

[34] If dividend payouts are an approximately constant proportion of expected earnings (when earnings have been adjusted for overstatement of depreciation) but adjust only slowly to earnings fluctuations, the dividend-price ratio will be a more accurate representation of actual market discounting of expected earnings than the more volatile ratio of actual earnings to price.

In this formula,

r = the discount rate
$D(s)$ = the proportion of the cost basis for an asset of age s that may be deducted from income for tax purposes
T = the lifetime of the asset for tax purposes.

Until the end of 1953, the dominant method of depreciation in the United States was the straight-line method, for which

$$D(s) = \begin{cases} 1/T & \text{for } 0 \leq s \leq T, \\ 0 & \text{otherwise.} \end{cases} \tag{3.49}$$

According to Hall and Jorgenson, the present value of the deduction for straight-line depreciation is

$$z_{\mathrm{SL}_t} = (1-e^{-r_t T_t})/(r_t T_t) \tag{3.50}$$

if the current discount rate is expected to persist.[35]

Starting in 1954, two accelerated depreciation methods were permitted by law, the sum-of-the-years-digits method, and the double-declining-balance method. The sum-of-the-years-digits method, as Hall and Jorgenson show, "dominates the double-declining-balance and straight-line formulas in the range of discount rates and lifetimes with which we are concerned."[36] The advantage over the double-declining-balance method, however, is small, especially for assets with relatively short lives, and this factor, along with the computational ease of the double-declining-balance method, may account for the fact that the latter has been preferred. Because the present values for the two accelerated methods are close together for the relevant range of lifetimes and discount rates, with the sum-of-the-years-digits method dominant, I have chosen to use the sum-of-the-years-digits formula to represent all depreciation taken by accelerated methods. A continuous approximation to the formula for the deduction is

$$D(s) = \begin{cases} 2(T-s)/T^2 & \text{for } 0 \leq s \leq T, \\ 0 & \text{otherwise,} \end{cases} \tag{3.51}$$

and the present value of the deduction according to the sum-of-the-years-digits method is

$$z_{\mathrm{SYD}_t} = [2/(r_t T_t)][1-(1-e^{-r_t T_t})/(r_t T_t)]. \tag{3.52}$$

[35] Hall and Jorgenson, "Tax Policy and Investment Behavior," p. 394.
[36] *Ibid.*, p. 395.

The present values of the depreciation deduction for the two methods for selected lifetimes and discount rates, as calculated by Hall and Jorgenson, are given in Table 3-3.[37]

TABLE 3-3. Present Values of Depreciation Deduction for Selected Depreciation Methods, Lifetimes, and Discount Rates

		Present values	
Lifetime (years)	*Discount rate*	*Straight-line*	*Sum-of-the-years-digits*
5	0.06	0.864	0.907
5	0.12	0.752	0.827
10	0.06	0.752	0.827
10	0.12	0.582	0.696
25	0.06	0.518	0.643
25	0.12	0.317	0.456

Information about the extent to which accelerated methods have actually been adopted is given by Ture.[38] He found that, according to the U.S. Treasury Department's special compilation, "Life of Depreciable Assets," for taxable year 1959, 29.3 percent of the production machinery, transportation vehicles and equipment, and furniture and office machinery and related equipment that had been acquired since 1953 was being depreciated by the double-declining-balance method, and 23.1 percent was being depreciated by the sum-of-the-years-digits method. On the basis of this information, I have chosen to represent the present value of the depreciation deduction (per dollar of equipment purchased) as

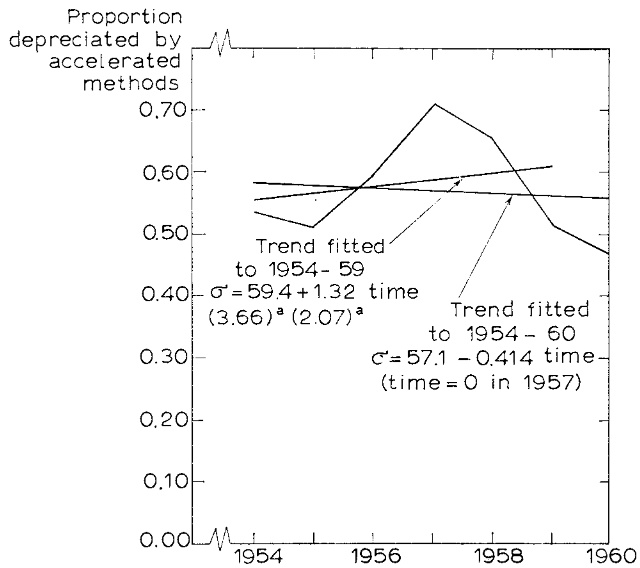
$$z_t = \varsigma z_{\mathrm{SYD}_t} + (1-\varsigma) z_{\mathrm{SL}_t}, \tag{3.53}$$

with ς_t equal to 0.524 starting in the first quarter of 1954. As Figure 3-1 shows, there is apparently no trend (or learning curve) associated with the adoption of accelerated depreciation by corporations. From this evidence, the assumption that ς was zero before 1954 and then rose immediately to some constant value does not seem unreasonable. It should be noted, however,

[37] *Ibid.*

[38] Norman B. Ture, *Accelerated Depreciation in the United States, 1954–60* (Columbia University Press for the National Bureau of Economic Research, 1967), Table A-12, pp. 147–55.

FIGURE 3-1. Proportion of Depreciable Assets of Corporations Depreciated by Accelerated Methods, by Year of Purchase, 1954–60



Source: Norman B. Ture, *Accelerated Depreciation in the United States, 1954-60* (Columbia University Press for the National Bureau of Economic Research, 1967), Table A-8, p. 133.
[a] Standard error.

that the sample for this part of the Treasury survey was rather small, and also that this conclusion is quite the opposite of the one reached by Wales, using different data and methods.[39] Further work is needed to clear up the inconsistency.

Equipment lifetimes, represented by T, have been adjusted to take account both of changes in guideline lives and of what is known about actual practice. These adjustments accord with those of Hall and Jorgenson;[40] it is assumed that the new depreciation guidelines shortened average tax lifetimes from 15.1 to 13.1 years, starting in the third quarter of 1962.

The Long amendment to the initial 1962 tax credit legislation provided that any credit claimed had to be deducted from the depreciation base, but it was repealed by the Revenue Act of 1964; starting in the first quarter of 1964, k' is assumed to be zero.

[39] Terence J. Wales, "Estimation of an Accelerated Depreciation Learning Function," *Journal of the American Statistical Association*, Vol. 61 (December 1966), pp. 995–1009.

[40] Hall and Jorgenson, "Tax Policy and Investment Behavior," p. 400.

Although business confidence that a credit would be passed must have built up after passage by the House on March 29, 1962, substantial assurance about the effective date and terms of the credit could not have existed before the bill was reported by the Senate Finance Committee in August.[41] The parameter k, representing the effective rate of tax credit, is arbitrarily assumed to be zero until the third quarter of 1962, and to be equal to 5 percent for later periods.

The estimated effective rate is lower than the maximum statutory rate, 7 percent, for several reasons: (a) restrictions on the applicability of the credit to short-lived equipment; (b) confinement of the maximum credit for public utility companies to 3 percent; and (c) restrictions on the amount of credit taken in any one year. The limitations on the credit taken in any one year have now been amended, but there remains applicable the most important restriction in the original law, which provided that the amount of the credit taken in any one year could not exceed the first $25,000 of tax liability plus one-fourth of any remaining tax liability.

My estimate of the actual effective rate is based on information for 1963, the only year for which the tax credit was fully in effect and for which relatively complete data have been published. Expenditures for producers' durable equipment in 1963 amounted to $34.8 billion (current dollars). When all the figures for "cost of eligible property" that can be found in the 1963 Treasury data reported in *Statistics of Income*[42] are added together and allowance is made for some double counting of partnerships and small corporations, the total is about $32.98 billion. Considering that the Treasury data are not quite complete, and more important, that they are based on the bookkeeping years of the taxpaying units, the two numbers are remarkably close. For practical purposes it seems unlikely that identifying producers' durable equipment and cost of eligible property as one and the same thing will lead to serious error.

Adding up all the tentative credits for corporations, sole proprietorships, and partnerships (again adjusting for double counting) results in a total of $1.917 billion.[43] This is 5.5 percent of $34.8 billion, and can be called the

[41] This summary of events is based on U.S. Treasury Department, Office of Tax Analysis, "Investment Credit: Log of Actions and Events" (mimeograph; February 1, 1967), supplied by Melvin I. White.

[42] U.S. Treasury Department, Internal Revenue Service, *Statistics of Income—1963, U.S. Business Tax Returns* (1967).

[43] The tentative credit is the amount that could be claimed if there were no restrictions on amounts claimed.

"effective rate of tentative credit." It is lower than 7 percent because (a) about 20 percent of the equipment was purchased by public utilities, which could claim only 3 percent credit, thus lowering the effective rate by 12 percent; and (b) for corporations (the only unit for which the two figures are available) qualified investment is only about 90 percent of cost of eligible property, since much of the eligible property has a tax life of less than eight years.[44] These two adjustments account for virtually all of the difference between 7 percent and 5.5 percent.

Restrictions on the maximum credit that could be claimed mean that actual credits taken amounted to only about \$1.39 billion, or 4.0 percent of spending for producers' durable equipment. The difference, however, was not lost. Because of carry-back and carry-forward provisions, it seems likely that most of this credit could eventually be claimed, although the delay would make the present value of a dollar's credit amount to less than a dollar. Furthermore, it must be assumed that corporations, when planning their investments, *expect* to make profits in the relatively near future. It is very hard to derive an expression to introduce correctly the portion of the tentative credit not immediately claimed. But it will not do to ignore it completely. On a priori grounds the true effective rate of tax credit should be at least as close to the rate of tentative credit as it is to the rate of actual credit. I have chosen 5 percent as a compromise between 4 percent and 5.5 percent.

Restrictions on Lag Distributions

At the close of the first section, it was argued that the interesting parameters in equation (3.41) are the row and column sums of the β matrix —the λ and μ weights. One feasible method of approximation involves estimating two coefficients out of each row and column of the coefficient matrix β to act as proxies for all the rest of the coefficients in the row or column. Experimentation with various patterns reveals that the results are insensitive to the choice of coefficients to be estimated, as long as at least two coefficients in each row are estimated.[45] The practice of estimating two diagonal sets of coefficients $\beta_{i,i}$ and $\beta_{i,i-1}$ for $i = 2, \ldots, n$ has been adopted for all regressions that are reported here. The maximum lag n has been chosen equal to 12, implying that the estimated value of equipment

[44] Qualified investment differs from cost of eligible property because assets with short lives receive weights of less than one in determining qualified investment.

[45] Estimating more than two coefficients in each row and column produced negligible improvements in the unadjusted coefficient of determination and reduced the adjusted coefficient of determination in all cases in which it was tried.

spending is based on values of V' and Q for the preceding three years.[46] Since $\mu_j = \beta_{jj}+\beta_{j+1,j}$ and $\lambda_k = \beta_{kk}+\beta_{k,k-1}$, the coefficients to be estimated determine μ_j for $j = 1, 12$, and λ_k for $k = 2, 12$.[47] Thus, the equation to be estimated, referred to as the expenditures equation, is

$$I_t = \sum_{i=2}^{12} \beta_{i,i-1} V'_{t-i}Q_{t-i+1} + \sum_{i=2}^{12} \beta_{i,i} V'_{t-i}Q_{t-i}+\varepsilon_t. \tag{3.54}$$

Multicollinearity would not permit the estimation of all twenty-two of the $\beta_{i,i-1}$ and $\beta_{i,i}$ coefficients, and to overcome this difficulty, I have used the technique developed by Shirley Almon.[48] The $\beta_{i,i-1}$ coefficients are constrained to be values of a third-degree polynomial in i, with the additional constraint that the value of the polynomial for $i = 13$ should be zero. This last constraint has the effect of forcing the $\beta_{i,i-1}$ coefficients to approach zero as the index approaches 13. In mathematical form the constraints applied amount to

$$\beta_{i,i-1} = A_3(i^3-2197)+A_2(i^2-169)+A_1(i-13) \text{ for } 2 \leq i \leq 13.$$

A similar constraint has been applied to the estimated $\beta_{i,i}$ coefficients:

$$\beta_{i,i} = B_3(i^3-2197)+B_2(i^2-169)+B_1(i-13) \text{ for } 2 \leq i \leq 13.$$

With the insertion of these constraints into (3.54) all but six of the linear coefficients may be eliminated. The coefficients A_1, A_2, A_3, and B_1, B_2, B_3 could be estimated, or, as I have done, the constraints could be applied

[46] In the theoretical development, all lag weights were specified to extend over the infinite past. For the purposes of estimation, an infinite lag specification could have been adopted. In this case, finite lags are more convenient, especially because there are several different lag distributions to be estimated. A priori, it is at least as plausible to assume that the effects of a change in a particular variable will become negligible after a finite period as it is to assume that the effects will continue forever. In addition, even though only a finite number of lagged values of Q enter equation (3.54), it is shown in the section beginning on p. 109 that this does not generally imply that the *complete* adjustment of capacity to a change in output takes place within a finite period (see Table 3-12).

[47] This estimation procedure requires the first row sum λ_1 to be zero. Alternative regressions in which this requirement was removed produced no evidence that λ_1, the row sum representing the effects of V' lagged only one quarter, was significantly different from zero. The regressions reported below show estimated values of λ_2 as well which are essentially zero; no significant effects of variations in relative prices are found until the third quarter after the change.

[48] Shirley Almon, "The Distributed Lag between Capital Appropriations and Expenditures," *Econometrica*, Vol. 33 (January 1965), pp. 178–96.

by making use of Lagrangian interpolation weights.[49] The six coefficients actually estimated, A'_1, A'_2, A'_3, and B'_1, B'_2, B'_3, are linear functions of, respectively, A_1, A_2, A_3, and B_1, B_2, B_3.

Preliminary Exploration of the Parameter Space

Even with these simplifications, estimation of the parameters of (3.54) presents a complicated nonlinear problem. The variable V' is a nonlinear function of σ, δ, r_0, r_1, r_2, r_3, r_4, and h'. If it is assumed that the error term ε in equation (3.54) is normally distributed with zero mean, constant variance, no serial correlation, and independent of all the right-hand variables, then maximum likelihood estimates of the parameters of (3.54) may be obtained by minimizing the sum of squared residuals with respect to all eight nonlinear parameters and the six linear A' and B' parameters. The next section describes the simultaneous nonlinear estimation of all these parameters (with the exception of r_3, r_4, and δ).

It is a considerably simpler task to obtain maximum likelihood estimates of the lag parameters alone, conditional on some assumed values of the nonlinear parameters. This estimation involves only linear methods. Because there is no guarantee that the nonlinear maximization technique will produce a global maximum of the likelihood function, it is generally a good idea to conduct some preliminary exploration of the parameter space. With so many nonlinear parameters, a systematic search of the parameter space is not feasible. The discussion of the nonsystematic preliminary search which follows is useful because it provides an impression of the sensitivity of the estimated error variance to the various parameters. It is possible that more can be learned in this way about the role and interactions of each of the variables than from reference only to a final set of estimates (complete with asymptotic standard errors whose interpretation is not at all clear).

Twelve preliminary trials have been carried out. For each trial, particular values of all the nonlinear parameters except σ have been guessed, and then an exhaustive search has been made in the interval (0, 2) for the value of σ that maximized the unadjusted coefficient of determination. The parameter values assumed in the various preliminary trials are indicated in Table 3-4. The assumption that h' is equal to zero implies that all technical change is other-factor augmenting. It should be noted that there is no particular

[49] For a more extensive discussion of the use of these weights, see Almon, "The Distributed Lag between Capital Appropriations and Expenditures." Except for rounding error in computation, the alternative methods for eliminating all but six coefficients will produce identical results.

reason why the weights r_1 and r_2 should sum to one, or any other specific value. Trials 1 and 2 represent constant after-tax discount rates. Trial 3 corresponds to the assumption Hall and Jorgenson made originally that the before-tax rate of return was constant at 0.14 throughout the period.[50] In the other trials, the assumed value for r_4 of 0.2 is an approximation to the desired proportion of debt in capital structure, included in the cost of capital in the manner suggested by the Modigliani-Miller theory.[51] Trials 7 and 12 use a discount rate based only on Moody's industrial bond yield, but the constants r_0 (the intercept) and r_3 (the time trend in the discount rate) are adjusted so that the discount rate actually used in the calculations has no trend, and has a mean value of 6 percent.[52] Similar adjustments have been made in the constants for trial 8, in which the discount rate is based only on the dividend-price ratio.

In all of the preliminary trials, the accelerated depreciation proportion parameter ς is assumed to be equal to 0.4, while the effective rate of tax credit k is assumed to be 7 percent. As several experiments discussed on pages 123–24 below make clear, the model is not at all sensitive to the assumed value of the proportion of accelerated depreciation. Only the estimated value of the elasticity of substitution σ is sensitive to the assumption about k; the experiments suggest that if, in the preliminary trials, k had been assumed equal to 5 percent instead of 7 percent, the estimated values of σ would have been about 20 percent higher.

For all of the trials, the sample period includes quarterly data from the third quarter of 1951 through the fourth quarter of 1965 (a total of fifty-seven observations; the third quarter of 1952 was eliminated because of abnormalities caused by the steel strike and seizure by the federal government).[53] For the same sample period, two of the most popular alternative models of investment behavior have also been used to explain equipment spending. The first of these models is an extension of the acceleration principle, with investment specified to be a function of net capital stock and changes in output. The model

[50] Hall and Jorgenson, "Tax Policy and Investment Behavior," p. 400.

[51] Miller and Modigliani, "Some Estimates of the Cost of Capital."

[52] This formulation might be interpreted as implying that the trend in RM_{MBCIND} does not affect the discount rate, but is instead offset by an opposite trend in the risk adjustment, and thus that only deviations about the trend are of significance.

[53] Observations on all the variables were available from 1947, but the equation could be fitted only for periods for which n lagged values were available. The time period starting in 1951 was chosen because some preliminary experiments were carried out assuming values for n as high as 18, but the right-hand variables with very long lags turned out to have insignificant coefficients.

TABLE 3-4. Trial Values of Parameters for Preliminary Estimates, Assuming a Depreciation Rate of 0.16 and No Technical Change

Trial number	r_0	r_1	r_2	r_3	r_4
1	0.07	0	0	0	0
2	0.04	0	0	0	0
3	0.14	0	0	0	1.0
4	0	1.0	0.5	0	0.2
5	0	0.75	0.75	0	0.2
6	0	0.5	1.0	0	0.2
7	0.01907	1.0	0	—0.0003178	0
8	0.03202	0	1.0	0.0004180	0
9	0	0.667	0.333	0	0.2
10	0.03	0.667	0.333	0	0.2
11	—0.03	1.5	0.5	0	0.2
12	0.01907	1.0	0	—0.0003178	0.2

Note: See p. 83 for definition and use of symbols.

estimated is

$$I_t = \alpha' \sum_{i=1}^{17} \beta_i \Delta Q_{t-i} + \delta K_{t-1} + \varepsilon_t . \tag{3.55}$$

The second alternative model corresponds to the standard neoclassical model with static expectations (see pages 76–80). In this model desired capital stock is assumed to be the product of V_t' and Q_t, and investment expenditures are assumed to be the sum of (a) a distributed lag function of changes in desired capital stock, and (b) replacement. The relationship is specified to be

$$I_t = \alpha' \sum_{i=1}^{17} \beta_i (V'_{t-i} Q_{t-i} - V'_{t-i-1} Q_{t-i-1}) + \delta K_{t-1} + \varepsilon_t . \tag{3.56}$$

In calculating V' for use in equation (3.56), nonlinear parameter values have been chosen to make it correspond as closely as possible to the model of Hall and Jorgenson. These parameters are the same as those used in trial 3, except that the proportion of depreciation by accelerated methods ς in equation (3.53) is set equal to 1 after 1954, in accordance with the assumption made by Hall and Jorgenson that the best proxy for the present value of depreciation deductions taken after 1954 is the present value for the sum-of-the-years-digits pattern. The assumed value of δ, 0.16, is between the value Hall and Jorgenson used for manufacturing equipment (0.1471) and the value they used for nonfarm nonmanufacturing equipment (0.1923).[54]

Both (3.55) and (3.56) are simply special cases of the more general model I have presented (except for the addition of the net capital stock as a right-hand variable).[55] In (3.55), σ is zero, and all of the terms involving relative prices drop out of the equation. In (3.56), σ is 1.0, and all of the elements of the β matrix not on the main diagonal are assumed to be zero. The more general form of the standard neoclassical model (with nonstatic expectations) can best be represented by estimating the lag distributions in equation (3.54). It is possible for the sign pattern of the estimated row sums to conform either to the predictions of the putty-clay model (with all of the row sums positive, as in Table 3-1) or to the general form of the standard neoclassical model (with the row sums positive for short lags and negative for long lags, as in Table 3-2).

In estimating both (3.55) and (3.56), the Almon polynomial technique has been used to restrict the lag weights; the restriction involves a fourth degree polynomial, according to the equation

$$\beta_i = C_4(i^4 - 104976) + C_3(i^3 - 5832) + C_2(i^2 - 324) + C_1(i - 18)$$

for $1 \leq i \leq 18$.

Table 3-5 reveals that it is possible to explain equipment spending quite well under a number of different discount rate assumptions, including the assumption that nothing can affect the discount rate used in investment decisions. However, the best fitting trials (4, 7, 9, 10, and 12) are those that emphasize the bond yield as the most important indicator of the prevailing discount rate. The best of these (trial 9) explains 35 percent of the variance left unexplained by the best constant rate equation (trial 3). It is also the

[54] Despite these similarities, the results are not comparable to the original results of Hall and Jorgenson because they used the consumer durables deflator and estimated the lag by a considerably different method, and because they carried out the estimation on net rather than gross investment. In a more recent paper, they present more comparable results, in terms of both the deflator used and the way in which the lag distributions are estimated. See Chap. 2 in this volume.

[55] The argument in the section beginning on p. 76 suggests that if either (3.55) or (3.56) were the correct specification, and if the errors were not autocorrelated, the net capital stock term would not add anything to the equation. In fact, it proved highly significant in both (3.55) and (3.56) and the errors were still autocorrelated. Several different net capital stocks, developed from historical data going back to 1909, were used; those calculated on the basis of an assumed δ equal to 0.16 provided the best fits. This same capital stock, when added to the better fitting versions of (3.54), added virtually nothing to the explanatory power of the equation.

best trial from the point of view of serial correlation, which is undesirably high for most of the cases. Given the crudeness of the linear formulation and the fact that no systematic method of choosing the weights for the discount rate estimate has been used, the improvement is surprisingly large.

Inclusion of the dividend-price ratio in the cost of capital seems to produce a slight improvement over formulations based solely on the trend-adjusted bond yield (compare trials 4 and 9 with trials 7 and 12); but the results are hardly conclusive. When the cost of capital is based heavily on

TABLE 3-5. Elasticities of Substitution and Goodness of Fit Statistics Derived from Preliminary Trials

Model and trial	*Elasticity of substitution*	*Coefficient of multiple determination*[a]	*Adjusted standard error of estimate*[b] *(in millions of 1958 dollars)*	*Coefficient of variation*[c]	*Durbin-Watson statistic*
Expenditures equation (3.54)					
1	0.87	0.97936	$ 804.9	0.027	0.97
2	0.94	0.98286	732.7	0.024	1.17
3	0.98	0.98370	714.4	0.024	1.24
4	0.85	0.98859	597.7	0.020	1.74
5	0.65[d]	0.97356	910.0	0.030	0.79
6	0.35[d]	0.95406	1,199.4	0.040	0.50
7	0.74	0.98737	628.8	0.021	1.58
8	0.65[d]	0.96359	1,067.7	0.036	0.56
9	0.90	0.98936	577.1	0.019	1.88
10	0.89	0.98739	628.4	0.021	1.57
11	0.60[d]	0.98039	783.5	0.026	1.10
12	0.77	0.98793	614.8	0.020	1.66
Accelerator model (equation 3.55)					
...	(0.00)[e]	0.95233	1,218.0	0.041	0.46
Standard neoclassical model with static expectations (equation 3.56)					
...	(1.00)[e]	0.92269	1,541.0	0.051	0.43

Source: Statistics for trials 1–12 are computed using the model given in equation (3.54), with trial values of various parameters as specified in Table 3-4. The specification for the accelerator model is given in equation (3.55) and the specification for the standard neoclassical model is given in equation (3.56).

[a] Unadjusted.

[b] Square root of sum of squared residuals divided by the number of observations minus the number of estimated parameters, equal to 7 for trials 1–12 and 5 for the accelerator model and the standard neoclassical model.

[c] Adjusted standard error of estimate divided by the mean of the dependent variable.

[d] Price elasticity of demand is estimated only to the nearest 0.05; other estimates are to the nearest 0.01.

[e] By assumption.

stock market yields, however, not only does the statistical fit worsen markedly, but also the estimates of the price elasticity of demand are much lower. This implies that many of the fluctuations in the stock market (which are reflected in the denominator of the dividend-price ratio) produce no corresponding change in investment; the only way the model permits less weight to be given to these fluctuations is through a lower estimate of the elasticity of substitution σ.

The possibility that the role of the stock market has been fundamentally misspecified in this equation cannot, however, be ruled out. It could be that movements in the dividend-price ratio are primarily reflections of expectations, or confidence, in which case it is a mistake to include them as part of the user cost of capital. Such an expectational variable might well have great effect on investment in the short run but little in the long run; but if it is included with relative price variables, the impact of which is the reverse, the various lag distributions will be muddled.

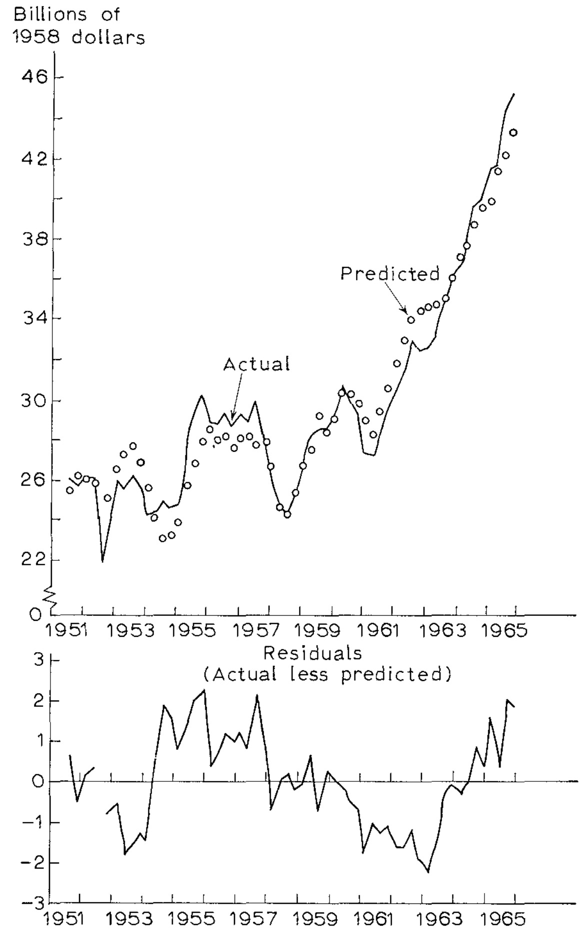
Some support for such a hypothesis may be found in the residuals for trial 9. There was, for example, a deceleration in equipment spending in late 1962 and early 1963. Because the dividend-price ratio in the equation is constrained to act with the same lag as all the other user cost variables, the sharp (20 percent) rise in dividend yields as a result of the market break in prices in 1962 does not help much in explaining this investment dip. At this time, allowing for more than two lag distributions does not seem fruitful, but it might be explored at some later date.

The improvement of trial 9 over the two alternative models is due to many factors. However, the difference between trial 3 and the standard neoclassical model can be accounted for almost completely by the additional flexibility in the lag structure allowed by the estimation of more elements in the β matrix, for the nonlinear parameters in both trials are virtually identical.[56] Nearly 80 percent of the variance left unexplained by the standard neoclassical model is explained when the more general lag structure is allowed.

Allowing for substitution significantly improves the explanation, as is demonstrated by a comparison of trial 3—in which tax parameters and the price of equipment relative to that of output are the only relative price varia-

[56] The only difference is in the accelerated depreciation proportion parameter ς which affects the fit hardly at all. The addition of the net capital stock to trial 3 does not improve its explanatory power. The superior fit in trial 3 cannot be attributed to the free estimation of σ; the estimated value (0.98) is very close to 1, with an asymptotic standard error of about 0.07.

FIGURE 3-2. Equipment Expenditures, Third Quarter 1951 through Fourth Quarter 1965, Actual, and Predicted Using Accelerator Model



Sources: App. Table 3-A-1, pp. 128–30, for actual data, and equation (3.55) for predicted values. A predicted value for the third quarter of 1952 is omitted because of the effects of the steel strike.

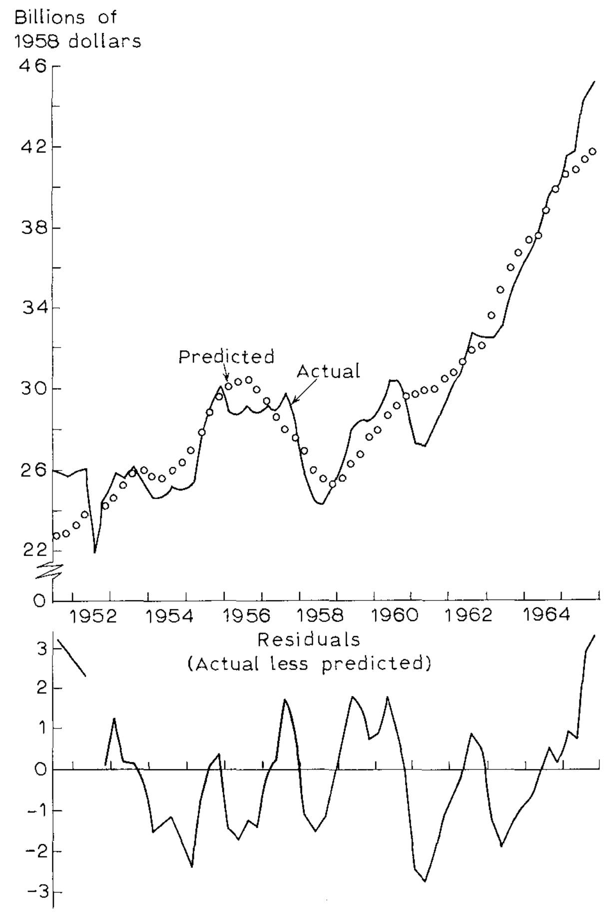
bles—with the accelerator model—in which the equipment-output ratio is constant. This is true even without any consideration of monetary variables. But the improvement occurs only if the relative price variable V' is allowed a lag structure distinct from that for output Q. The standard neoclassical model as stated in equation (3.56), which does not allow this freedom, explains less of the variance of investment than does the accelerator model.[57]

Fitted and actual values of expenditures, as predicted by the accelerator model and the standard neoclassical model, are given in Figures 3-2 and 3-3. As the Durbin-Watson ratios confirm, the errors are highly serially correlated. Although the accelerator model based on output alone explains the major qualitative movements of the investment series, Figure 3-2 shows that, without relative price effects, the investment booms of the mid-fifties and mid-sixties are underpredicted and the investment slowdown of the early sixties is not sufficiently reflected. Figure 3-3 demonstrates that the introduction of relative price effects without a separate lag smooths out the peaks and troughs in the series of estimated values even more. An explanation for this pattern can be derived from a comparison of the estimated lag structures for equations (3.55) and (3.56). When relative price effects are added without a separate lag structure, the apparent misspecification seems to bias the estimated accelerator coefficients downward in the first few periods. The mean lag is increased as the lag structure lengthens, and the sensitivity of predicted investment to changes in output is considerably reduced.

The lag distributions estimated for the various trials with the general lag specification are of particular interest. In every case they conform to the qualitative pattern represented in Table 3-1 and suggested by the putty-clay model, as opposed to the pattern represented in Table 3-2 and suggested by the standard neoclassical model with nonstatic expectations. For all trials, the row sums (λ weights) are all either positive or insignificantly different from zero, while the column sums (μ weights) are first positive and then negative. For trials 1, 2, 3, 5, and 7, the first λ weight has the wrong sign; otherwise the estimated row sums are all positive. The lag distributions are discussed at greater length below; clearly, however, the results of the preliminary estimation strongly support the hypothesis that, no matter how the discount rate is specified, the lag structure between changes in relative prices

[57] This striking result does not hold, however, when the two alternative models are reestimated using the consumer durables deflator as the deflator for investment expenditures and as the price index for equipment in calculating the rent. The coefficient of variation for the accelerator model was virtually unchanged when this deflator was used, but the fit for the standard neoclassical model improved markedly, so that it provided a slightly better explanation than the accelerator model. The sum of squared residuals, however, was still nearly four times the residual sum of squares for a version of trial 9 using the same deflator. These suggestive results were not explored further, but one interpretation is that the rigid form of the standard neoclassical model is much more sensitive to precise specification than the other equations.

FIGURE 3-3. Equipment Expenditures, Third Quarter 1951 through Fourth Quarter 1965, Actual, and Predicted Using Standard Neoclassical Model



Sources: App. Table 3-A-1, pp. 128–30, for actual data, and equation (3.56) for predicted values. A predicted value for the third quarter of 1952 is omitted because of the effects of the steel strike.

V' and equipment spending is significantly different from the lag structure leading from changes in output Q.

The reason for the insensitivity of the estimated lag structures to the way in which the discount rate is specified seems to be that changes in V' are dominated by changes in tax policy (see Figure 3-5 below) that affect all of the trials in the same way. The lag distributions estimated for trial 9 may be taken as typical. Table 3-6 gives the $\beta_{i,\,i-1}$ and $\beta_{i,\,i}$ coefficients and standard errors estimated for trial 9; in Table 3-7 the estimated weights are arranged in the form of a β matrix (but it should be recalled that the coefficients that are estimated are simply proxies for the complete set of coefficients in each row and column).

TABLE 3-6. Estimates of Lag Coefficients and Standard Errors for Trial 9

Coefficient	*Estimate*	*Standard error*
$\beta_{2,1}$	0.0363	(0.0032)
$\beta_{3,2}$	0.0334	(0.0024)
$\beta_{4,3}$	0.0300	(0.0023)
$\beta_{5,4}$	0.0262	(0.0024)
$\beta_{6,5}$	0.0223	(0.0025)
$\beta_{7,6}$	0.0182	(0.0025)
$\beta_{8,7}$	0.0142	(0.0025)
$\beta_{9,8}$	0.0104	(0.0025)
$\beta_{10,9}$	0.0069	(0.0024)
$\beta_{11,10}$	0.0040	(0.0020)
$\beta_{12,11}$	0.0016	(0.0013)
$\beta_{2,2}$	—0.0356	(0.0039)
$\beta_{3,3}$	—0.0315	(0.0027)
$\beta_{4,4}$	—0.0274	(0.0025)
$\beta_{5,5}$	—0.0234	(0.0027)
$\beta_{6,6}$	—0.0195	(0.0028)
$\beta_{7,7}$	—0.0158	(0.0028)
$\beta_{8,8}$	—0.0123	(0.0026)
$\beta_{9,9}$	—0.0091	(0.0024)
$\beta_{10,10}$	—0.0062	(0.0021)
$\beta_{11,11}$	—0.0037	(0.0017)
$\beta_{12,12}$	—0.0016	(0.0010)

Source: Data are derived from parameter estimates for equation (3.54) using trial values of certain parameters listed for trial 9, Table 3-4.

Nonlinear Estimates of the Model

An iterative technique has been used to obtain estimates of the parameters of the nonlinear version of the expenditures equation that maximize the likelihood function, at least locally. Because there may be several maxima, and because the iterative technique used cannot guarantee a global

TABLE 3-7. Estimates of Coefficients of $V'_{t-i}Q_{t-j}$ for Trial 9

i/j	1	2	3	4	5	6	7	8	9	10	11	12	$\lambda_i = \Sigma_j\beta_{ij}$
1													0.0000
2	0.0363	—0.0356											0.0007
3		0.0334	—0.0315										0.0019
4			0.0300	—0.0274									0.0026
5				0.0262	—0.0234								0.0028
6					0.0223	—0.0195							0.0028
7						0.0182	—0.0158						0.0024
8							0.0142	—0.0123					0.0019
9								0.0104	—0.0091				0.0013
10									0.0069	—0.0062			0.0007
11										0.0040	—0.0037		0.0003
12											0.0016	—0.0016	0.0000
Total[a]	0.0363	—0.0022	—0.0015	—0.0012	—0.0011	—0.0013	—0.0016	—0.0019	—0.0022	—0.0022	—0.0021	—0.0016	0.0174[b]

Source: Lag distributions are derived from parameter estimates for equation (3.54) using trial values of certain parameters listed in trial 9 of Table 3-4.

[a] $\mu_j = \Sigma\beta_{ij}$.

[b] $\Sigma\lambda_i = \Sigma\mu_j = 0.0174$.

maximum, the process has been started from a number of initial sets of estimates, including all of the sets of parameters used in the preliminary trials. All have led to the same local maximum.

The technique used is called the "maximum neighborhood" method by its originator, Donald Marquardt.[58] It combines the favorable features of two better-known techniques for solving nonlinear equations, Gauss's method and the method of steepest ascent. Gauss's method involves linearization of the model by expanding it in a Taylor series about the initial guesses of the parameters (and truncating after the first order terms). The normal equations for the linearized model can be solved, and they provide a new set of parameter estimates which usually give a larger value for the likelihood function when they are inserted into the original model. The method can break down, however, if the linear approximation is not sufficiently good in the neighborhood of the "corrected" parameter estimates: the new estimates may actually give a smaller value of the likelihood function.[59]

With the method of steepest ascent, however, it is always possible to improve the parameter estimates in such a way as to increase the likelihood function (except at a local maximum or a saddle point). The difficulty is that convergence may be extremely slow. Marquardt's method "in effect, performs an optimum interpolation between the Taylor series method and the gradient method, the interpolation being based upon the maximum neighborhood in which the truncated Taylor series gives an adequate representation of the nonlinear model."[60]

Convergence to a local maximum has been considered complete only when every one of the corrections estimated from the linearized model has passed the test

$$\frac{|b_i^{q+} - b_i^q|}{0.001 + |b_i^q|} < 0.00005,$$

where

b_i^q = the value of the ith parameter on the qth iteration

[58] Donald W. Marquardt, "An Algorithm for Least-Squares Estimation of Nonlinear Parameters," *Journal of the Society for Industrial and Applied Mathematics*, Vol. 11 (June 1963), pp. 431–41.

[59] In this case, Hartley recommends correcting the parameter estimates by only a fraction of the vector of corrections provided by the linearized model. See H. O. Hartley, "The Modified Gauss-Newton Method for the Fitting of Non-Linear Regression Functions by Least Squares," *Technometrics*, Vol. 3 (May 1961), pp. 269–80.

[60] Marquardt, "An Algorithm for Least-Squares Estimation of Nonlinear Parameters," p. 431. The program I used embodies the Marquardt algorithm and is a slight revision of IBM Share Program No. SDA-3094-01.

b_i^{q+} = the corrected value of the parameter.

Depending on the initial guesses, convergence for the model has involved as few as three iterations or as many as sixty.

Parameter Constraints and Estimates

In practice, it appears that convergence cannot be achieved if both the coefficient of the rate of technical change h' and the coefficient on the time trend in the discount rate equation r_3 are allowed to vary. Since both represent trend terms, the linearized model is too nearly singular to get meaningful results. Thus r_3 has been arbitrarily set at zero. Similarly, it has seemed necessary arbitrarily to normalize either the depreciation rate δ, the coefficient

TABLE 3-8. Nonlinear Estimates of Parameters, Asymptotic Standard Errors, and Summary Statistics for Expenditures Equation

Parameters and summary statistics	*Value*	*Asymptotic standard error*
Parameters		
r_0	—0.008	(0.002)
r_1 (bond yield)	0.535	(0.096)
r_2 (stock yield)	0.098	(0.028)
σ	1.022	(0.069)
h'	0.00182	(0.00030)
A_1'	0.0254	(0.0053)
A_2'	0.0184	(0.0032)
A_3'	0.0069	(0.0022)
B_1'	—0.0263	(0.0061)
B_2'	—0.0161	(0.0030)
B_3'	—0.0058	(0.0020)
Summary statistics		
Coefficient of multiple determination[a]		0.98954
Adjusted standard error of estimate[b]		0.597×10^9
Coefficient of variation[c]		2.00%
Durbin-Watson statistic		1.95
Sum of squared residuals		16.37×10^{18}
Mean of dependent variable		29.89×10^9
Number of observations		57

Source: Expenditures equation (3.54).
[a] Unadjusted.
[b] Square root of sum of squared residuals divided by number of observations minus 11.
[c] Adjusted standard error of estimate divided by mean of dependent variable.

on the industrial bond yield r_1, or the coefficient on the dividend-price ratio r_2, and to estimate, in effect, only the ratios r_1/δ, r_2/δ, and r_1/r_2. The parameter δ has therefore been set at 0.16 in all cases (some rough tests indicate that the results are not at all sensitive to the absolute value of δ, within the range $0.10-0.20$). No attempt has been made to estimate the coefficient of the tax rate r_4 in the discount rate equation, which plays a very small role in the model in any case. Instead, this parameter has been set at 0.2 on the basis of rather casual examination of movements in debt-equity ratios at market value and book value.

Nonlinear solution of the model thus has involved obtaining least squares estimates of the parameters r_0, r_1, r_2, σ, and h', and of the A' and B' parameters. Asymptotic standard errors have been computed, but they are in fact only the standard errors of the parameter estimates as computed from the linearized model. The Taylor series model can be written

$$\hat{I} = f(Q, V'; \hat{\beta}) + \partial(\beta - \hat{\beta}), \tag{3.57}$$

in which $f(Q, V'; \hat{\beta})$ is the $m_2 \times 1$ vector of predicted values of I as a nonlinear function of the matrix of right-hand variables Q, V', and the $m_3 \times 1$ vector of final parameter estimates $\hat{\beta}$; $(\beta - \hat{\beta})$ is an $m_3 \times 1$ vector; and ∂ is the $m_2 \times m_3$ matrix of partial derivatives of the estimated values of I with

TABLE 3-9. Estimates of Long-run Elasticities of Equipment Expenditures for Selected Determinants, at Selected Price Levels, 1953–65

Determinant	*1953 : 1 prices*	*1958 : 1 prices*	*1963 : 1 prices*	*1965 : 4 prices*
Output[a]	1.00	1.00	1.00	1.00
Price of output	1.02	1.02	1.02	1.02
Price of equipment	—1.02	—1.02	—1.02	—1.02
Bond yield	—0.20	—0.21	—0.22	—0.23
Dividend-price ratio	—0.07	—0.05	—0.04	—0.02
Corporate tax rate	—0.20	—0.18	—0.17	—0.06
Proportion of depreciation by accelerated methods	—	0.02	0.02	0.02
Service lifetime for tax purposes	—0.10	—0.09	—0.09	—0.09
Rate of tax credit	—	—	0.05	0.10
Time[b]	0.01	0.01	0.01	0.01

Source: Data are derived from nonlinear parameter estimates for expenditures equation (3.54).

[a] By assumption.

[b] Since the origin of the variable time is completely arbitrary, these elasticities have been calculated as $(\partial I/\partial t) \cdot (1/I)$ instead of $(\partial I/\partial t) \cdot (t/I)$. For the purposes of these calculations, time is measured in years, although for the other calculations it is measured in quarters.

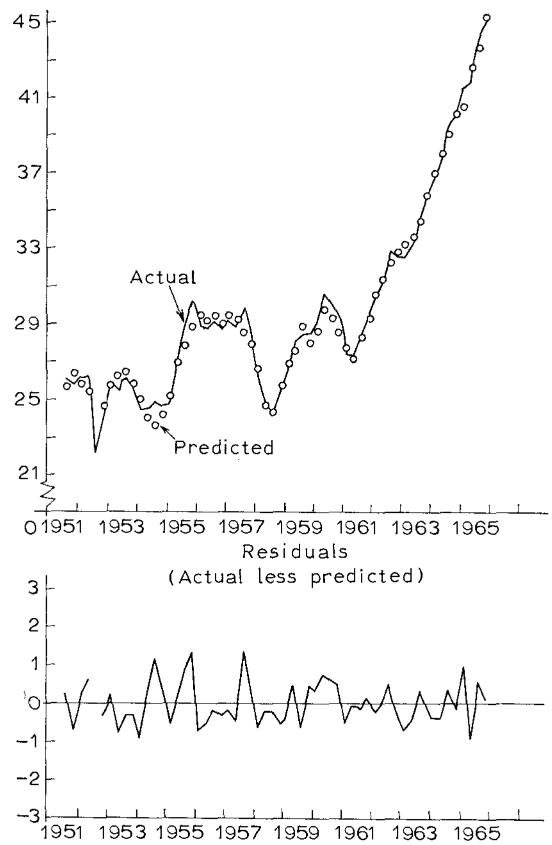
respect to each of the parameters, evaluated at $\hat{\beta}$, that is,

$$\partial_{ij} = \frac{\partial f_i}{\partial \beta_{j_{\hat{\beta}}}}. \tag{3.58}$$

In this case,

$$\frac{\sum_{t=1}^{m} (I_t - \hat{I}_t)^2}{m_2 - m_3} (\partial' \partial)^{-1}$$

FIGURE 3-4. Equipment Expenditures, Third Quarter 1951 through Fourth Quarter 1965, Actual, and Predicted Using Expenditures Equation



Sources: For actual data, App. Table 3-A-1, pp. 128–30; for predicted values, expenditures equation (3.54), using nonlinear estimates of the parameters. A value for the third quarter of 1952 is omitted because of the effects of the steel strike.

is the asymptotic variance-covariance matrix of the parameter estimates from which the asymptotic standard errors have been derived.

The fruits of the nonlinear estimation are given in (a) Table 3-8, which presents the estimated parameters and various summary statistics; (b) Table 3-9, which shows the estimated long-run elasticities of equipment spending with respect to all of the important variables and policy parameters in the model; and (c) Figure 3-4, in which the fitted and actual values and the residuals are plotted. The unadjusted coefficient of variation is improved hardly at all over trial 9, and the adjusted standard error is actually slightly larger. The Durbin-Watson statistic, however, is slightly closer to 2.0.

CAPITAL COST. The estimated values of both r_1 and r_2 are large compared with their asymptotic standard errors, but this is misleading, for these coefficients were quite unstable when small changes were made in the model or sample. The estimate of r_0 is negative, and if equation (3.47) is taken as an estimate of the discount rate, the predicted result seems unreasonably low. But examination of the underlying model indicates that, of the two places in which the discount rate enters, its absolute level matters only when it is used to discount depreciation patterns in computing the present value of the depreciation deduction. In this role, a low discount rate may act primarily as an ad hoc adjustment to weaken the influence of changes in depreciation rules as a determinant of investment.[61] Thus, it decreases the elasticity of investment demand with respect to, for example, a change in guideline lives.

TECHNICAL CHANGE. The estimated trend term h' is positive, and indicates that with all other variables held constant, the ratio of equipment spending to output is estimated to rise at a rate of about 0.73 percent per year. Although h' was included in the equation to allow for technical change, it would not be proper to interpret this parameter as a reliable measure of the degree to which technical change is capital-augmenting or capital-using.

Instead, the estimate of h' seems to reflect primarily an offset to trends in other variables. For example, the higher the estimate of r_1, the greater the weight given to the bond yield, which has a very definite uptrend, in the determination of the discount rate. Other things equal, then, the rent will become higher over time and the equilibrium equipment-output ratio lower. But if this ratio is not to fall over time (and Figure 3-5 indicates that it was about as high in 1965 as it was in 1948) then the estimate of h' must be higher, as can be seen by substituting equation (3.43) into equation (3.42), other things

[61] At a discount rate of zero, all depreciation patterns have the same present value.

being equal, whenever r_1 is higher. If a high estimate of r_1 results from a high cyclical partial correlation between investment orders and the bond yield, the equilibrium effects are offset by an algebraically larger estimate of h'.

The relationship of all this to technical change seems rather remote. Despite the fact that it cannot be interpreted, it still seems useful to allow for a trend in order to minimize the danger of accepting one of the other variables as significant when it is really acting as a proxy for the trend.

PRICE ELASTICITY. The estimated price elasticity is close to 1, and also close to the preliminary estimates. The estimated effect of the tax credit is relatively large; with repeal of the Long amendment, the elasticities in Table 3-9 indicate that repeal of the investment tax credit would lead eventually to a permanent reduction of about 10 percent in equipment spending. Accelerated depreciation is estimated to have an effect that, while substantial, is considerably smaller than that of the tax credit. The impact of variations in guideline lives within the range that has been contemplated is also relatively small.[62]

The very large change over time in the elasticity with respect to the corporate tax rate requires some comment. This elasticity is proportional to the derivative of the rent with respect to u. From (3.3), the expression for the rent, the relevant derivative is

$$\frac{q(r+\delta)(1-k-z+zk')}{(1-u)^2}.$$

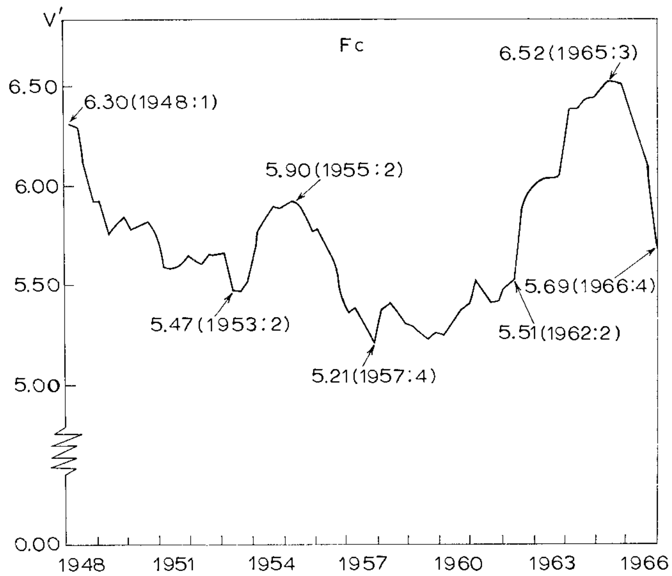
Thus the elasticity is proportional to $1-k-z+zk'$, and when z, the present value of the depreciation deduction, is close to 1, the elasticity is quite sensitive to the value of k, the rate of tax credit. For a piece of equipment on which the whole credit of 7 percent can be claimed, if z is greater than 0.93 and with k' equal to zero after repeal of the Long amendment, an increase in the tax rate should increase investment, for the discounted depreciation deductions exceed the cost of the machine (net of credit), and the higher the investment, the greater the savings on the excess deductions.

Relative Factor Proportions and Lag Distributions

The time path of V', which summarizes all of the relative price and trend effects, is plotted in Figure 3-5. Apart from the general downtrend of V' through most of the period, attributable to rising equipment prices (relative to the

[62] The simulations reported in pp. 116–21 give considerably more information about the estimated effects of tax policies.

FIGURE 3-5. Time Series of Conglomerate Relative Prices V' as Estimated from Expenditures Equation, 1948–66



Source: App. Table 3-A-1, pp. 128–30.

rest of output) and to rising interest rates, there are three major movements in the series, all primarily the results of changes in tax laws. In 1954–55, the adoption of accelerated depreciation provided a significant additional investment incentive. In the last half of 1962, adoption of the tax credit, along with liberalization of depreciation guidelines, provided another significant offset to rising costs; the repeal of the Long amendment in 1964, restoring the tax credit to the depreciation base, added to the value of this incentive. Finally, temporary repeal of the tax credit in late 1966 created a situation in which the indicated equilibrium ratio of equipment per unit of capacity was only slightly above its low levels of the late fifties and early sixties (with the improvement due to the relative stability of equipment prices since 1958).

Figure 3-5 is of particular interest in light of Bert Hickman's conclusion, based on investment functions fitted for the period 1949–60, that the capital-output ratio in the United States was declining.[63] If true, this proposition might

[63] Bert G. Hickman, *Investment Demand and U.S. Economic Growth* (Brookings nstitution, 1965).

mean that private investment demand would be insufficient to sustain full employment. Figure 3-5 indicates that the marginal capital-output ratio might well have declined during the period studied by Hickman, although not necessarily for technological reasons. But it also indicates that at least the desired ratio of equipment to output has been substantially affected by government policy since that time. Hickman did not allow for this effect (which was less important for his sample period), but it has played a significant part in the revival of investment demand since 1963.

LAG DISTRIBUTIONS. Table 3-10 gives estimates of the lag parameters derived from the nonlinear estimates of the $\beta_{i,\,i-1}$ and $\beta_{i,\,i}$ parameters. As suggested by the putty-clay model, and as illustrated in the example in Table 3-1, all of the row sums are either positive or insignificantly different from zero (the asymptotic standard error for the one row sum that is negative is 0.0012). Only the first column sum is positive while all the others are negative. One might have expected the first two or three column sums to be pos-

TABLE 3-10. Lag Distributions Obtained from Nonlinear Estimation of Expenditures Equation

Period i	Coefficients of $V'_{t-i}Q_{t-i+1}$ $\hat{\beta}_{i,\,i-1}$	Coefficients of $V'_{t-i}Q_{t-i}$ $\hat{\beta}_{i,\,i}$	Column sums of β matrix $\hat{\mu}_i$	Row sums of β matrix $\hat{\lambda}_j$
0	—	—	—	—
1	—	—	0.0254	—
2	0.0254	—0.0263	—0.0019	—0.0009
3	0.0245	—0.0238	—0.0011	0.0007
4	0.0227	—0.0210	—0.0007	0.0017
5	0.0203	—0.0181	—0.0007	0.0022
6	0.0174	—0.0151	—0.0009	0.0023
7	0.0143	—0.0121	—0.0011	0.0021
8	0.0110	—0.0093	—0.0014	0.0017
9	0.0079	—0.0067	—0.0016	0.0012
10	0.0050	—0.0043	—0.0017	0.0007
11	0.0026	—0.0024	—0.0015	0.0003
12	0.0009	—0.0009	—0.0009	0.0000
Total[a]	0.1520	—0.1400	0.0120	0.0120

Source: Data are derived from nonlinear parameter estimates for expenditures equation (3.54).
Note: $\Sigma\Sigma\hat{\beta}_{ij} = \Sigma\hat{\beta}_{i,\,i-1} + \Sigma\hat{\beta}_{i,\,i} = \Sigma\hat{\mu}_j = \Sigma\hat{\lambda}_i = 0.01197$.

[a] Details may not add to totals due to rounding.

itive, but the result may be occasioned by the particular approximating technique that was used.

Impact of Changes in Output and Factor Proportions

The sum of all the coefficients shows that a rise in output of $1 billion (1958 value) will eventually increase the flow of expenditures for producers' durable equipment by V' times $11.97 million, or, at the level of V' in the fourth quarter of 1965 (6.494), by roughly $78 million.[64] Due to the accelerator effect, this response will be exceeded as capacity is initially adjusted upward, and only after three years will the response die down to the steady-state effect, which is replacement on the equipment needed to produce $1 billion of output. Table 3-11 gives the increments in spending for producers' durable equipment that would stem from a sustained rise in output, as percentages of the steady-state increment.

The most disturbing lag coefficient is the relatively large response of investment in the first quarter following a change in output. Some investment functions have specified a priori that no stimulus lagged less than two quarters can have *any* effect, in view of the supposed accuracy of investment anticipations. But Eisner and Evans and Green have found that unexpected rises in sales, or simply changes in sales occurring *after* anticipations have been reported, can enhance explanations based on anticipations alone.[65] In principle, changes in output might well be reflected very rapidly in changes in orders (especially cancellations) and at least some of these orders could be promptly translated into expenditures. Although the elasticity of I_t with respect to Q_{t-1} seems too large to be reconciled with Eisner's estimates of the elasticity of I_t with respect to sales in period $t-1$, the constraint of this lag

[64] If the depreciation rate δ is 0.16, this would correspond to an equipment-output ratio of about 0.49; if δ is 0.10, the ratio would be 0.78. The Commerce Department data on net stocks of equipment in 1966 (in 1958 dollars, with straight-line depreciation) show $221 billion, compared with private business product of $579 billion in that year, a ratio of 0.38.

[65] Robert Eisner, "Realization of Investment Anticipations," in Duesenberry and others (eds.), *Brookings Quarterly Econometric Model*; and Michael K. Evans and Edward W. Green, "The Relative Efficacy of Investment Anticipations," *Journal of the American Statistical Association*, Vol. 61 (March 1966), pp. 104–16. Eisner's conclusions were tempered by the relatively unsuccessful results of tests based on extrapolation beyond the sample period. But extrapolations with the functions estimated by Evans and Green generally produced better predictions than the anticipations did.

TABLE 3-11. Effects of Sustained Rise in Output and Changes in Relative Prices on Equipment Expenditures during Next Twelve Quarters

(Percentage of steady-state response)

Quarter after change	*Effect of change in physical output*[a]	*Effect of change relative prices*[b]
0	—	—
1	212	—
2	197	—5
3	188	2
4	182	17
5	176	35
6	169	53
7	159	70
8	148	83
9	134	92
10	120	98
11	108	100
12	100	100

Source: Simulation of equation (3.54), using the nonlinear parameter estimates shown in Tables 3-9 and 3-10.

[a] Assuming no change in static optimum equipment per unit of capacity.

[b] Assuming constant 4 percent growth in physical output.

coefficient to zero, which is the principal feasible alternative, does not seem theoretically justifiable. In fact, even the requirement that changes in output cannot affect investment in the same quarter is hard to defend; this specification was adopted largely to minimize statistical problems resulting from simultaneous determination.

The distinction between output and sales should not, however, be overlooked. It is possible that changes in output (which, unlike sales, are under the control of the producer) are in fact correctly anticipated, and that their high correlation with nearly simultaneous investment is a result of this correct anticipation. Viewed in a slightly different way, changes in output may result from some external cause (say, changes in orders) which stimulates investment demand as well. If this is the case, orders themselves should be studied (though this can be done only on an industry level), but as a first approximation the linkage from aggregate demand (via orders) to output and then to investment may not be seriously misspecified if the intermediate orders stage is suppressed.

A less optimistic interpretation would point out that if serial correlation is present, lagging the endogenous output variable will not remove problems brought about by failure to consider all of the simultaneous equations in the underlying economic system. The presence of serial correlation in the empirical results must still be suspected because the Durbin-Watson test is not designed for use in equations that contain lagged endogenous variables. The difficulties may be compounded by the fact that the lagged output variable includes the dependent variable, equipment spending, as one of its components. Although estimation of a more complete system is not feasible in this study, a cautious interpretation of the lag distributions is certainly advisable.

Given relative prices, it is possible to compute the equilibrium increment to an imaginary net stock of equipment that will be brought about by a sustained unit change in output.[66] The lag weights, when combined with an a priori value of the depreciation rate, can be used to derive an expression for the proportion of the adjustment from one equilibrium stock to another that will be completed within n quarters after the change in output. For the estimated weights, the results, as a function of the assumed δ, are shown in Table 3-12. For reasonable values of δ, in the range from 0.08 to 0.16, the adjustment seems relatively slow; only 42 percent to 68 percent of the adjustment takes place within the first five years, and it is more than ten years before the adjustment even approaches 90 percent completion. Long lags in capital-stock adjustment models, of course, are nothing new, but it is these long lags that have led to criticism of many of the simpler versions of such models.

The speed of response of equipment spending to a change in V' varies with the rate of growth in output; the faster output grows, the faster substitution will take place. With investment running at a level of around \$50 billion and with a 4 percent growth in real output, a 1 percent change in the interest rate (from the fourth quarter 1965 level of 4.72 percent) would eventually change the flow of spending for producers' durable equipment by about 5 percent (or \$2.5 billion), but a year after the interest rate change, less than 20 percent of the eventual effect would have been felt. If the change were

[66] If both relative prices and technical change are held constant, it is possible to speak of a stock of equipment, since all machines are of the same model.

TABLE 3-12. Proportion of Adjustment of Stock of Equipment to Change in Physical Output Completed after Selected Number of Quarters, by Selected Depreciation Rates

(Percentage of adjustment)

Number of quarters	Depreciation rate				
	0.04	0.08	0.12	0.16	0.20
4	8	15	22	29	36
8	13	25	36	46	55
12	17	32	45	56	66
16	20	37	51	63	72
20	23	42	57	68	77
40	37	61	76	86	92

Source: Expenditures equation (3.54).

then reversed,[67] the effects of the original change would continue to build up and it would be more than a year after an equal deviation of interest rates in the opposite direction (from the normal level) before spending on equipment would return to the level that would have obtained had no changes taken place.[68] Table 3-13 gives the time pattern of effects for the sequence of

[67] In case the reversal merely restored interest rates to the original level (as in a sequence 4 percent—5 percent—4 percent, as opposed to 4 percent—5 percent—3 percent), it would take over a year for the second change simply to offset the effects of the first change sufficiently to restore spending to the level that existed when the second change took place. Such lag effects may not be reasonable, but they could result from the lag of shipments behind orders and also from a process in which steps once taken to adopt a new technique result in the old technique being "forgotten." Note must also be taken of the lag between the application of policy instruments and the effect on those factors directly determining investment. In the case of tax changes, the lag may be negligible, but in the case of interest rates, it may be substantial.

[68] This research contributes to a larger project on monetary policy, the FRB-MIT quarterly econometric model, described in more detail in Frank de Leeuw and Edward Gramlich, "The Federal Reserve-MIT Econometric Model," *Federal Reserve Bulletin*, Vol. 54 (January 1968), pp. 11–40, and in Robert H. Rasche and Harold T. Shapiro, "The F.R.B.-M.I.T. Econometric Model: Its Special Features," in American Economic Association, *Papers and Proceedings of the Eightieth Annual Meeting, 1967* (*American Economic Review*, Vol. 58, May 1968), pp. 123–49. I have also, consequently, been concerned with the possibility that the effects of rationing, or availability of credit, might lead to more rapid monetary influences on investment. A variety of variables which have been suggested as likely to reflect such effects have been experimentally introduced into the model with very short lags, but none has come close to making a significant contribution to the explanation.

TABLE 3-13. Effect on Equipment Spending of a Change in Relative Prices Followed after One Year by a Change in the Opposite Direction of Twice the Magnitude of the Original Change

(Percentage of steady-state effect of the original change)

Quarters after first change	*Change in equipment spending*[a]
1	—
2	—5
3	2
4	17
5	35
6	63
7	66
8	49
9	22
10	—8

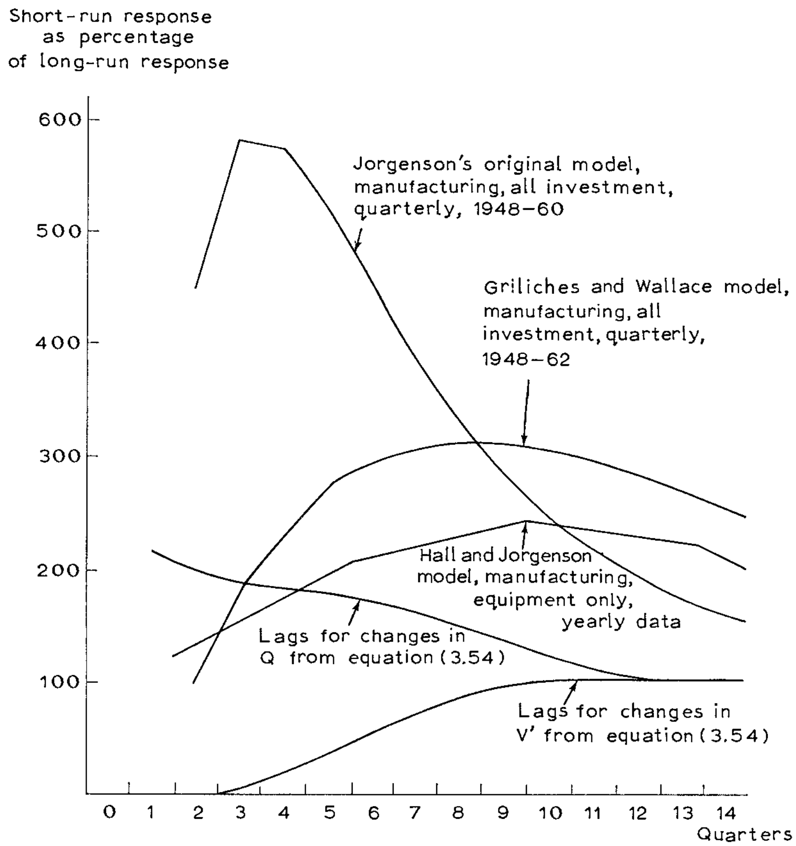
Source: Based on Table 3-11.
[a] Assuming a constant 4 percent growth in output.

interest rate changes assumed in this particular case; similar patterns can be computed from Table 3-11 for any desired sequence of changes.

Figure 3-6 compares the estimated short- and long-run responses of equipment spending to changes in V' and Q, based on the nonlinear estimates, to some previous estimates of lags in the investment process. The results are not comparable, especially inasmuch as the Jorgenson and Griliches and Wallace models included structures as well as equipment.[69] The most recent estimates by Hall and Jorgenson, based on yearly data for equipment spending, conform only relatively well to my estimates of the lag for changes in output. But they exclude any possibility of interest rate effects; only changes in tax policy and the relative price of new equipment influence V' in their model. Their results are not inconsistent with the view that the rela-

[69] Jorgenson, "Capital Theory and Investment Behavior"; Griliches and Wallace, "The Determinants of Investment Revisited." In view, however, of the small proportion of construction in the total investment of manufacturers, the short-run response of construction to relative price changes would have to exceed the long-run response by a factor of ten or more to produce lag patterns for *total* investment like those implicit in the models cited. This seems unlikely.

FIGURE 3-6. Selected Lag Distributions for Capital Expenditures



Sources: Dale W. Jorgenson, "Capital Theory and Investment Behavior," in American Economic Association, *Papers and Proceedings of the Seventy-fifth Annual Meeting, 1962* (*American Economic Review*, Vol. 53, May 1963), Table 4, p. 259; Zvi Griliches and Neil Wallace, "The Determinants of Investment Revisited," *International Economic Review*, Vol. 6 (September 1965), Table 4, p. 321; and Robert E. Hall and Dale W. Jorgenson, this volume, Table 2-2, p. 36.

tively larger and more frequent fluctuations in output exerted a dominant influence in the estimation of the lag pattern. Granting this possibility, because no separate lag pattern was allowed for tax changes it would seem dangerous to base policy conclusions on the apparently large and rapid effects of tax policy found in their study.

Another way to judge the reasonableness of the estimates is to examine the equilibrium ratios of equipment stock (under idealized circumstances where such stocks make sense) to output, and also to compare the equilibrium shares of equipment spending in output. Both of these pairs of statistics are functions of the estimated values of V', and thus change over time.

The formula for the equilibrium ratio of equipment to capacity output, given the relative prices and technology of period t, is

$$V_t'(\sum_i \sum_j \beta_{ij})/\delta.$$

The estimate of $\Sigma\Sigma\beta_{ij}$ is 0.01197. Neither the absolute value of this sum nor the absolute value of the estimate of V' has any independent importance, but the product of V' and $\Sigma\Sigma\beta_{ij}$ represents the steady-state stream of replacements on the capacity needed to produce a unit of output. Thus for any assumed value of the rate of replacement δ, a certain ratio of the stock of equipment to output is implied.

TABLE 3-14. Equilibrium Equipment-Output Ratios, at Selected Price Levels

Period of price level	*Ratio*
1955 : 1	0.4406
1960 : 1	0.3961
1965 : 1	0.4837

Source: Data are derived from nonlinear parameter estimates for expenditures equation (3.54).

The implied ratios for $\delta = 0.16$ are given in Table 3-14.[70] In a stationary state with no growth in output, the proportion of equipment spending in gross business product, as estimated from the expenditures equation, would range between 5.7 percent and 7.0 percent for the range of values of V' observed during the sample period. With output growing at a steady 4 percent, the range would be about 7.5 percent to 9 percent. By comparison, the actual share of spending for equipment was 9.6 percent in the first quarter of 1948, then fell to 6.5 percent as a result of the steel crisis in 1952, and rose to 7.7 percent in the capital goods boom of 1955. At the low point in 1961, the ratio was only 6.3 percent, but it rose to 8.0 percent by the end of 1965.

[70] The actual ratio of the constant dollar net stock of equipment, consistent with exponential depreciation at 16 percent per year, was in the range of 35 percent of output throughout this period. But this ratio is not really comparable to the equilibrium ratios, which assume the existence of a steady state without growth in output. If output is growing, so that desired capital stock is greater than actual capital stock, equilibrium ratios will be higher than actual ratios, which is the result observed above.

Policy Simulations and Sensitivity Analysis of Tax Parameters

In order to evaluate more fully the effect of changes in tax policy, monetary policy, and relative prices, I have used the model, with the parameter estimates reported in Table 3-8, to simulate the time path of equipment expenditures under a number of alternative assumptions about tax parameters and other variables that affect V'. The simulated time paths of predicted equipment spending may be compared to the predicted values, given the actual policy parameters (or independent variables), in order to compute a measure of the direct effects of a policy such as accelerated depreciation or the investment tax credit. Without a complete model of national income determination, these computations are of limited meaning, for the path of output, as well as the paths of the other determinants of investment in equipment, must certainly be altered by induced changes in investment. Nevertheless, these computations might be conceived as the outcomes of controlled experiments in which the government sought to provide the same level of aggregate demand in two ways: (a) by direct government purchases of equipment, and (b) by indirect actions, such as tax incentives to encourage private equipment purchases, interest rate manipulations, and the like.

Policy Simulations

Seven sets of simulations have been carried out to represent the direct effects of the various combinations of tax policies that have been adopted since 1954. The results are tabulated in terms of constant dollar effects and proportions of actual expenditures in Table 3-15. The policies include the adoption of accelerated depreciation methods in 1954, the promulgation of new depreciation guidelines in 1962, adoption of an investment tax credit in 1962, the repeal of the Long amendment in 1964, and the reduction of general income tax rates for corporations in 1964 and 1965. There are interactions among the various policies—the depreciation guidelines would have induced slightly more investment if accelerated depreciation had not already been in effect, for example—and thus the effect of all of them is not the sum of the effects of each policy alone.

All of the policies taken together are estimated to have induced directly over \$17 billion (1958 value) of gross expenditures between 1954 and 1966, with the largest effects coming in 1964–66. The most important single policy is the tax credit. Even with the restriction imposed by the Long amendment, it would have directly induced over \$6 billion of gross investment by the end of 1966; with the Long amendment repealed, the effects add up to more

TABLE 3-15. Estimates of Direct Effects of Tax Policies on Equipment Expenditures, Dollar Amounts and Percentages of Actual Expenditures, 1954–66

Year	Accelerated depreciation	Depreciation guidelines	Investment tax credit with Long amendment	Repeal of Long amendment	Investment tax credit without Long amendment	Corporate income tax reductions of 1964–65	Total, all policies[a]
	Dollar amounts (in billions of 1958 dollars)						
1954	—0.01						—0.01
1955	0.22						0.22
1956	0.43						0.43
1957	0.44						0.44
1958	0.43						0.43
1959	0.51						0.51
1960	0.55						0.55
1961	0.57						0.57
1962	0.64						0.64
1963	0.67	0.05	0.22		0.22		0.95
1964	0.70	0.36	1.51	*	1.51	*	2.57
1965	0.77	0.54	2.14	0.90	3.04	0.05	4.41
1966	0.84	0.59	2.35	1.96	4.31	0.16	6.06
Total, 1954–66[a]	6.76	1.55	6.22	2.85	9.07	0.21	17.77
	Percentages of actual expenditures						
1954	*						*
1955	0.8						0.8
1956	1.5						1.5
1957	1.5						1.5
1958	1.7						1.7
1959	1.8						1.8
1960	1.8						1.8
1961	2.0						2.0
1962	2.0						2.0
1963	2.0	0.2	0.6		0.6		2.8
1964	1.8	1.0	3.9	*	3.9	*	6.7
1965	1.8	1.2	5.0	2.1	7.0	0.1	10.2
1966	1.7	1.2	4.8	4.0	8.8	0.3	12.4

Source: Estimated dollar effects of a policy are calculated as $(\hat{I}-\hat{I}_i^B)$ and estimated percentage effects are $(\hat{I}-\hat{I}_i^B)/I$, where $\hat{I}$ is the fitted value of I in equation (3.54), $\hat{I}_i^B$ is the value of $\hat{I}$ in a simulation in which the indicated policy is not carried out, and I is the actual value of equipment spending.

a. Details may not add to totals due to rounding and interactions.

* Less than $5 million, or less than 0.05 percent.

than $9 billion. The directly induced investment is smaller than the losses in tax revenue for 1962 and 1963, approximately equal to the reductions in 1964, and considerably in excess of the reductions for 1965 and 1966.[71] Accelerated depreciation policies are estimated to have had disappointingly small effects (amounting to roughly a 2 percent increase above what expenditures would otherwise have been for each year after 1955). The depreciation guidelines apparently increased expenditures by about 1 percent in each year after 1963.

The magnitude of the impact attributed to the tax credit from 1962:3 to 1965:4 is revealed in the seasonally adjusted quarterly breakdown of the estimated effects given in Table 3-16.

TABLE 3-16. Estimates of Direct Effects of Investment Tax Credit on Equipment Expenditures, by Quarters, 1962–65

(In billions of 1958 dollars, seasonally adjusted at annual rates)

Year and quarter	*Increment to equipment spending*
1962 : 3	0
4	0
1963 : 1	—0.09
2	0.02
3	0.29
4	0.66
1964 : 1	1.04
2	1.39
3	1.62
4	1.97
1965 : 1	2.33
2	2.86
3	3.27
4	3.69

Source: Expenditures equation (3.54).

The assumptions built into the model make it inappropriate for use in evaluating a change in tax law widely publicized as, and believed to be, temporary. One such assumption is equation (3.15), which states that entre-

[71] It should be recalled that the estimated effective rate of tax credit for 1963 is 4.0 percent; for later years it might be higher, but probably not much more than 5.0 percent.

preneurs average up past values of relative prices as embodied in V to form V^*, the capital intensity to be used in new capacity. If the tax change is temporary, it is unlikely that it will be given much weight in planning capital intensity. Second, and perhaps more important, there is likely to be a speculative acceleration of orders in response to a temporary tax benefit and a similar postponement effect in situations like the suspension of the investment tax credit in late 1966 and early 1967. This sort of behavior would violate the assumption that $Q_K^* - Q$, the desired degree of overcapacity, is not a function of relative prices. For these reasons there has been no attempt to simulate the effects of the tax credit suspension.

Two additional simulations have been made to predict what equipment expenditures might have been if (a) the bond yield had remained constant at 3.75 percent and the dividend-price ratio had remained constant at 4.39 percent throughout the postwar period, and (b) if V' had remained constant

TABLE 3-17. Estimates of Net Direct Effects of Variations in Interest Rates on Equipment Expenditures, 1950–66

(Percentage of actual expenditures)

Year	*First quarter*	*Second quarter*	*Third quarter*	*Fourth quarter*	*Annual average*
1950	2.6	2.2	2.0	2.3	2.3
1951	2.6	2.8	2.8	3.1	2.8
1952	2.9	2.8	3.0	2.6	2.8
1953	2.5	2.6	2.6	2.7	2.6
1954	2.8	2.3	1.7	1.4	2.1
1955	1.4	1.6	1.9	2.2	1.8
1956	2.7	3.0	3.0	3.1	3.0
1957	3.0	2.9	2.4	1.8	2.5
1958	1.4	0.8	—0.4	—1.2	0.2
1959	—1.6	—1.7	—1.8	—1.7	—1.7
1960	—1.7	—2.0	—2.4	—2.9	—2.3
1961	—3.6	—4.0	—4.2	—4.2	—4.0
1962	—4.1	—4.0	—3.8	—3.8	—3.9
1963	—3.8	—3.8	—3.6	—3.6	—3.7
1964	—3.4	—3.3	—3.1	—3.1	—3.3
1965	—3.0	—3.2	—3.2	—3.2	—3.2
1966	—3.3	—3.3	—3.2	—3.4	—3.3

Source: Estimated effects of interest rates are calculated as $(\hat{I}-\hat{I}^r)/I$ where $\hat{I}$ is the predicted value of I in expenditures equation (3.54), $\hat{I}^r$ is the value of I in a simulation in which the bond yield is held constant at 3.75 percent and the dividend-price ratio is held constant at 4.39 percent, and I is the actual value of equipment spending.

TABLE 3-18. Estimates of Net Direct Effects of Variation in Conglomerate Relative Prices V' on Equipment Expenditures, 1950–66

(Percentage of actual expenditures)

Year	First quarter	Second quarter	Third quarter	Fourth quarter	Annual average
1950	12.2	10.2	8.2	7.7	9.4
1951	7.4	6.8	6.4	6.1	6.6
1952	5.2	4.4	4.2	3.1	4.2
1953	2.4	2.1	1.8	2.1	2.1
1954	2.0	1.4	0.5	0.1	1.0
1955	0.3	1.0	1.7	2.5	1.4
1956	3.4	4.0	4.0	3.9	3.8
1957	3.3	2.6	1.4	—0.1	1.8
1958	—1.4	—2.5	—4.6	—6.2	—3.6
1959	—7.1	—7.3	—8.0	—7.8	—7.6
1960	—8.2	—8.6	—9.1	—9.7	—8.9
1961	—10.7	—10.9	—10.0	—10.3	—10.7
1962	—9.8	—9.2	—8.8	—8.8	—9.2
1963	—9.2	—8.6	—7.2	—5.6	—7.6
1964	—3.9	—2.4	—1.3	—0.1	—1.9
1965	1.0	2.4	3.3	4.2	2.8
1966	5.1	5.6	5.8	5.9	5.6

Source: Estimated effects of relative prices are calculated as $(\hat{I}-\hat{I}^{v'})/I$ where $\hat{I}$ is the predicted value of I in expenditures equation (3.54), $\hat{I}^{v'}$ is the value of I in a simulation in which V' is held constant at 5.79, its average value during the period 1947–66, and I is the actual value of equipment spending.

at its mean value throughout the sample period. These simulations are reported in Tables 3-17 and 3-18. In both cases, of course, the estimated effects are only partial: Total output is assumed to follow its actual time path.

The most striking characteristic of the interest rate simulation is the large negative stimulus arising between 1956 and 1961, apparently associated with the movement of the bond yield from a relatively low level before 1955 to a relatively high plateau after 1960.[72] If an attempt is made to match up turning points in the interest rate series with those in the estimated effect on equipment spending, the lags vary from two to eight quarters, with the aver-

[72] For the thirty-eight quarters from the first quarter of 1947 to the second quarter of 1956 inclusive, quarterly averages of Moody's industrial bond yield all fell in the range of 2.60 percent to 3.39 percent. The yield rose sharply between the second quarter of 1956 and the third quarter of 1959; for the twenty-six quarters starting with the third quarter of 1959, the quarterly averages remained in the range from 4.38 percent to 4.72 percent.

age about six quarters. The result of adding the lag between monetary changes and movements in long-term interest rates certainly does not encourage the use of monetary policy as a countercyclical influence on investment.

Sensitivity Analysis of Tax Parameters

Because tax policy—especially the tax credit—plays such an important role in the explanation of equipment spending, I have attempted to assess the sensitivity of the results to the assumed values of the parameters k and ς. The model has been completely reestimated with assumed values of k—the effective rate of the tax credit—varying all the way from 0 to 20 percent. The somewhat surprising result, recorded in Table 3-19, is that the estimated

TABLE 3-19. Summary Statistics for Selected Assumed Effective Rates of Investment Tax Credit

Assumed effective rate of tax credit (percent)	R^2	*Standard error of estimate*	*Durbin-Watson statistic*	*Estimate of price elasticity of demand*	*Estimated direct effect of tax credit (billions of 1958 dollars)*		
					1963	*1964*	*1965*
0	0.98682	669.71	1.55	1.112	0.00	0.00	0.00
1	0.98772	646.48	1.67	1.319	—$0.03	$0.29	$0.68
2	0.98848	626.13	1.78	1.305	—0.04	0.60	1.39
3	0.98898	612.37	1.85	1.224	0.035	0.94	2.05
4	0.98932	602.96	1.91	1.125	0.131	1.25	2.61
5	0.98954	599.63	1.95	1.022	0.219	1.50	3.04
6	0.98969	592.49	1.97	0.959	0.334	1.77	3.50
7	0.98978	589.89	1.98	0.879	0.409	1.95	3.80
8	0.98983	588.37	1.99	0.808	0.477	2.11	4.05
9	0.98986	587.57	2.00	0.740	0.519	2.21	4.20
10	0.98987	587.22	2.00	0.676	0.541	2.29	4.30
11	0.98987	587.16	2.00	0.619	0.554	2.34	4.36
12	0.98987	587.28	2.00	0.573	0.570	2.39	4.43
13	0.98986	587.52	2.00	0.533	0.582	2.44	4.47
14	0.98985	587.83	2.00	0.498	0.596	2.48	4.53
15	0.98984	588.18	2.00	0.470	0.614	2.53	4.59
16	0.98982	588.54	2.00	0.440	0.612	2.55	4.61
17	0.98981	588.93	2.00	0.415	0.618	2.57	4.62
18	0.98980	589.31	1.99	0.394	0.625	2.59	4.65
19	0.98978	589.70	1.99	0.374	0.633	2.62	4.68
20	0.98977	590.08	1.99	0.356	0.637	2.64	4.70

Source: See text.

price elasticity adjusts so as largely to offset even very extreme assumed values of k. The best explanation apparently is achieved with k assumed to be 11 percent, but the improvement in fit is quite small.

Because of the offsetting variations in the estimate of σ, the estimated direct stimulation of equipment spending due to the credit varies much less than the variations in k. Thus even if k is assumed to be 10 percent rather than 5 percent, a 100 percent increase, the estimated direct impact on equipment spending in 1965 increases by only 41 percent, from \$3.04 billion to \$4.30 billion.

It would be nice if the statistics in Table 3-19 could be used to construct a confidence interval for k (and for direct effects of the credit). But because of the nonlinearities in the model none of the standard tests for linear models is applicable. However, for large samples a likelihood ratio test can be used, and asymptotically this is equivalent to using a standard $\mathscr{F}$ test. To give some indication of the size of the reduction in error variance that comes as a result of relaxing the assumption $k = k^*$ (where k^* is an a priori value specified for k), I have tabulated values of the ratio

$$\mathscr{F}' = \frac{\Sigma(\hat{\varepsilon}_{k^*}^2 - \hat{\varepsilon}_k^2)}{\Sigma\hat{\varepsilon}_k^2/45}, \tag{3.59}$$

where $\hat{\varepsilon}_k$ are the residuals when k is estimated along with the other parameters, and $\hat{\varepsilon}_{k^*}$ are the residuals conditional on k equals k^*. I have labeled this ratio $\mathscr{F}'$ because it is computed in the same way as a statistic that would have an $\mathscr{F}$-distribution under the null hypothesis that k equals k^*, in the general linear model.[73] The ratio $\mathscr{F}'$ for various values of k^* is found in Table 3-20. Since little improvement in fit results from relaxing the prior assumption—that k is approximately 5 percent—I have adopted it. At the same time, Tables 3-19 and 3-20 show quite clearly that the time series examined provides little support for the hypothesis that k^* equals 0 (meaning that the tax credit has no effect at all). A substantial reduction in error variance can be achieved by adopting almost any hypothesis that implies that k is greater than 0.

The results suggest that the estimated price elasticity is not very sensitive to assumptions about k. But it should be noted that other regres-

[73] If the model were completely linear, this is exactly the ratio that would be computed, with fifty-seven observations, to test the hypothesis that one of twelve coefficients took a specified value, while the other eleven were estimated without restriction. See Franklin A. Graybill, *An Introduction to Linear Statistical Models*, Vol. 1 (McGraw-Hill, 1961), pp. 133–40.

TABLE 3-20. $\mathscr{F}'$ Statistics for Alternative Assumed Effective Rates of Investment Tax Credit

Assumed effective rate of tax credit (percent)	$\mathscr{F}'$	Assumed effective rate of tax credit (percent)	$\mathscr{F}'$
0	13.54	10	0.01
1	9.55	11	0.00
2	6.17	12	0.02
3	3.95	13	0.05
4	2.45	14	0.10
5	1.46	15	0.16
5.5	1.03	16	0.21
6	0.82	17	0.27
7	0.42	18	0.33
8	0.19	19	0.39
9	0.06	20	0.45

Source: Equation (3.59).

sions (not reported here) fitted to samples that did not include post-1962 data all produced estimates of σ, the relative price elasticity of equipment demand, that were close to 1. Still, on the basis of this evidence the possibility that highly visible policies such as the tax credit may have larger effects than would be produced by an equivalent reduction in equipment prices cannot be ruled out. The key factor is the effect on expectations about eventual factor price changes, and this effect could occur in many ways.

Similar experiments with the parameter ς, the proportion of depreciation taken under accelerated methods, have revealed that the model is almost totally insensitive to this parameter within the entire admissible range (0 to 1). With k set at 5 percent and ς varied from 0 to 1, the maximum variation in the sum of squared residuals is only 1.3 percent. The best fit is for ς equals 0, while the worst is for ς equals 1. None of the estimates of the other parameters changes by more than a few percent as ς is varied.

The sensitivity of the likelihood function to ς is so slight that no firm conclusions about even the direct effects of accelerated depreciation can be derived from the data. Trying to estimate ς empirically would be futile; no significant results could be obtained. But this also means that, in this model, the data give no support to the hypothesis that accelerated depreciation has any effect at all on investment! Of course, the data do not deny the hypothesis either; they simply shed no light. Since my prior value for ς, derived from analysis of Ture's data,[74] is about 0.5, I cannot reject this hypothesis, and it remains a part of the model. Some other researcher, whose null hypothesis was ς equals 0, could not reject that either. This result on the statistically insignificant effects of accelerated depreciation in a neoclassical aggregative model is very disturbing. All of the reported effects of accelerated depreciation are conditional on the assumption that a given change in the imputed rent on equipment has the same effect whether it comes about via changes in the depreciation rules or a change in some other component of the rent. However, the results with respect to the tax credit are by no means so ambiguous. Given the evidence that *some* tax incentives matter, this only strengthens my view that the presumption should be that tax incentives make a difference, even where the evidence is ambiguous. But it should be made clear that, in the case of accelerated depreciation, the conclusion is drawn on the basis of presumptions, not empirical evidence.

Conclusions

The principal conclusions drawn from the empirical work presented in this study may be summarized as follows:

> The value and efficacy of the neoclassical approach—the inclusion of relative prices in the investment function—are substantially confirmed. Relative prices appear to have a statistically significant effect on equipment expenditures. At least one tax measure, the investment tax credit, is independently shown to have a statistically significant effect on equipment expenditures.[75]
>
> The general neoclassical model provides an explanation of aggregate equipment expenditures superior to that given by either of the two most popular alternatives—the standard neoclassical model and

[74] As discussed on p. 85, Ture, *Accelerated Depreciation in the United States*, indicates that 52.4 percent of the equipment purchased by American corporations between 1954 and 1959 was being depreciated, in 1959, by accelerated methods.

[75] These statements are based on likelihood ratio tests, which are only approximate and apply only for large samples, and in no way depend on the reported asymptotic standard errors. In research carried out after this chapter was already in proof, it has become apparent that the asymptotic standard errors reported in Table 3-8 are subject to a great deal of rounding error. Alternative, but not necessarily better, methods of

the flexible accelerator model. As suggested by the putty-clay hypothesis, relative prices (including tax credits, interest rates, depreciation rules, and so forth) apparently affect equipment spending with a much longer lag than do changes in output.

The long-run elasticity of equipment spending with respect to the rental price of equipment services is estimated to be very close to unity, but this elasticity cannot be estimated with any great precision.

The investment tax credit adopted in 1962 has probably directly stimulated more investment spending than the policy has cost the government in taxes.

Variations in measures of the cost of capital seem to have the negative partial effect on equipment spending suggested by standard theory, but stable estimates of the effects of these variables have not been obtained.

The marginal ratio of equipment spending to output is apparently sensitive to direct fiscal and monetary policy measures, and there is no evidence pointing to a secular decline in this ratio. Attempts to manipulate this ratio for short-run purposes may, however, prove difficult to implement, due to lags in the response to the policy measures.

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computing the asymptotic standard errors have led to estimates as much as six times the size of those reported. In any case, the standard errors cannot and should not be used for making inferences or testing hypotheses, for there is no accepted statistical theory of inference in nonlinear models based on asymptotic errors computed according to the formula on p. 104.

APPENDIX A TO CHAPTER III

Data Estimates

TABLE 3-A-1. Equipment Expenditures, Output, and Components of Conglomerate Relative Price Term, First Quarter 1947 through Fourth Quarter 1966

Year and quarter	Expenditures for producers' durable equipment[a] (billions of 1958 dollars, seasonally adjusted annual rate)	Business gross product[a] (billions of 1958 dollars, seasonally adjusted annual rate)	Conglomerate relative price term[b]	Implicit price deflator for business gross product[a] (1958=1.000)	Imputed rent per unit of new equipment[c] (1958=1.000)	Implicit price deflator for producers' durable equipment[a] (1958=1.000)	Moody's composite industrial bond yield[d, e] (percent)	Moody's industrial dividend-price ratio[d, e] (percent)	Effective rate of tax credit against equipment purchases[e, f] (percent)	Effective rate of tax credit to be deducted from depreciation base[e, g] (percent)	General corporate income tax rate[e, h] (percent)	Assumed lifetime of equipment for tax purposes[i] (years)	Proportion of equipment depreciated by accelerated methods[j] (percent)	Present value of depreciation deduction: Straight-line basis[k]	Present value of depreciation deduction: Sum-of-the-years-digits basis[l]	Present value of depreciation deduction: Basis used for tax purposes[m]
1947: 1	25.1	268.4	6.4950	0.7473	0.1106	0.619	2.62	4.57	0	0	38	17.5	0	0.9173	0.9441	0.9173
2	24.5	271.4	6.2984	0.7538	0.1151	0.642	2.60	5.10	0	0	38	17.5	0	0.9143	0.9420	0.9143
3	23.8	272.6	6.3156	0.7692	0.1174	0.654	2.64	4.97	0	0	38	17.5	0	0.9137	0.9416	0.9137
4	25.0	278.4	6.2633	0.7887	0.1216	0.668	2.84	5.35	0	0	38	17.5	0	0.9034	0.9345	0.9034
1948: 1	26.8	280.4	6.3026	0.8034	0.1233	0.673	2.90	5.74	0	0	38	17.5	0	0.8984	0.9311	0.8984
2	25.3	285.5	6.2916	0.8133	0.1252	0.689	2.82	5.34	0	0	38	17.5	0	0.9042	0.9351	0.9042
3	24.9	287.6	6.0861	0.8275	0.1319	0.721	2.87	5.73	0	0	38	17.5	0	0.8996	0.9319	0.8996
4	25.8	290.0	5.9347	0.8227	0.1346	0.730	2.88	6.56	0	0	38	17.5	0	0.8935	0.9277	0.8935
1949: 1	24.2	285.1	5.9195	0.8162	0.1341	0.728	2.79	6.95	0	0	38	17.5	0	0.8942	0.9282	0.8942
2	22.9	282.8	5.7539	0.8101	0.1371	0.743	2.78	7.18	0	0	38	17.5	0	0.8930	0.9273	0.8930
3	21.8	286.7	5.8172	0.8046	0.1350	0.738	2.71	6.59	0	0	38	17.5	0	0.8997	0.9319	0.8997
4	21.4	283.9	5.8528	0.8027	0.1341	0.735	2.67	6.54	0	0	38	17.5	0	0.9015	0.9332	0.9015
1950: 1	21.7	300.6	5.7810	0.7944	0.1346	0.733	2.63	6.46	0	0	42	17.5	0	0.9044	0.9352	0.9044
2	24.2	309.0	5.8024	0.8031	0.1358	0.741	2.65	6.17	0	0	42	17.5	0	0.9056	0.9360	0.9056
3	26.9	321.1	5.8216	0.8234	0.1390	0.755	2.68	6.49	0	0	42	17.5	0	0.9023	0.9337	0.9023
4	26.4	326.0	5.7659	0.8392	0.1433	0.774	2.70	6.92	0	0	42	17.5	0	0.8986	0.9312	0.8986
1951: 1	25.0	327.8	5.5861	0.8691	0.1533	0.809	2.73	6.59	0	0	50.75	17.5	0	0.9015	0.9332	0.9015
2	25.4	332.7	5.5774	0.8740	0.1547	0.807	2.92	6.56	0	0	50.75	17.5	0	0.8948	0.9286	0.8948
3	26.0	339.0	5.6081	0.8734	0.1540	0.807	2.93	6.13	0	0	50.75	17.5	0	0.8973	0.9303	0.8973
4	25.7	338.5	5.6351	0.8841	0.1555	0.815	2.97	5.86	0	0	50.75	17.5	0	0.8977	0.9306	0.8977
1952: 1	26.0	339.9	5.6345	0.8852	0.1560	0.815	2.99	5.64	0	0	52	17.5	0	0.8987	0.9313	0.8987
2	26.1	338.0	5.6086	0.8866	0.1572	0.822	2.97	5.69	0	0	52	17.5	0	0.8991	0.9315	0.8991
3	22.1	341.7	5.6493	0.8934	0.1576	0.824	3.00	5.52	0	0	52	17.5	0	0.8992	0.9316	0.8992
4	24.3	353.2	5.6627	0.8983	0.1583	0.827	3.05	5.36	0	0	52	17.5	0	0.8984	0.9311	0.8984

1953: 1	25.9	360.0	5.6603	0.8988	0.1588	0.827	3.11	5.27	0	0	52	17.5	0	0.8968	0.9300	0.8968
2	25.5	364.1	5.4694	0.8959	0.1639	0.837	3.38	5.55	0	0	52	17.5	0	0.8853	0.9220	0.8853
3	26.1	361.5	5.4699	0.8982	0.1646	0.839	3.39	5.66	0	0	52	17.5	0	0.8842	0.9212	0.8842
4	25.6	357.2	5.5292	0.8952	0.1627	0.835	3.29	5.56	0	0	52	17.5	0	0.8884	0.9242	0.8884
1954: 1	24.2	351.6	5.7668	0.9089	0.1588	0.842	3.13	5.21	0	0	52	17.2	52.4	0.8982	0.9309	0.9153
2	24.3	350.8	5.8327	0.9084	0.1572	0.841	3.07	4.80	0	0	52	17.0	52.4	0.9041	0.9350	0.9203
3	24.8	355.7	5.8946	0.9061	0.1555	0.835	3.08	4.50	0	0	52	16.7	52.4	0.9073	0.9372	0.9229
4	24.6	363.3	5.8918	0.9083	0.1562	0.843	3.06	4.27	0	0	52	16.4	52.4	0.9110	0.9397	0.9260
1955: 1	24.7	376.0	5.8959	0.9111	0.1569	0.846	3.11	4.14	0	0	52	16.2	52.4	0.9111	0.9398	0.9262
2	27.1	383.4	5.9030	0.9121	0.1571	0.848	3.16	3.93	0	0	52	15.9	52.4	0.9123	0.9406	0.9271
3	28.9	389.2	5.8463	0.9172	0.1598	0.862	3.23	3.72	0	0	52	15.6	52.4	0.9128	0.9410	0.9276
4	30.1	392.9	5.7673	0.9221	0.1631	0.878	3.24	3.95	0	0	52	15.4	52.4	0.9121	0.9405	0.9270
1956: 1	28.8	390.1	5.7768	0.9318	0.1648	0.890	3.22	3.90	0	0	52	15.1	52.4	0.9147	0.9423	0.9291
2	28.7	391.9	5.6692	0.9397	0.1696	0.908	3.39	3.83	0	0	52	15.1	52.4	0.9097	0.9388	0.9249
3	29.2	390.4	5.5872	0.9515	0.1745	0.926	3.55	3.85	0	0	52	15.1	52.4	0.9045	0.9352	0.9206
4	28.7	396.0	5.4187	0.9588	0.1815	0.947	3.84	3.99	0	0	52	15.1	52.4	0.8946	0.9284	0.9123
1957: 1	29.2	398.7	5.3675	0.9691	0.1855	0.961	3.94	4.17	0	0	52	15.1	52.4	0.8904	0.9255	0.9088
2	28.9	398.0	5.3773	0.9753	0.1867	0.967	4.00	3.86	0	0	52	15.1	52.4	0.8903	0.9255	0.9087
3	29.8	399.8	5.3046	0.9844	0.1913	0.976	4.26	4.01	0	0	52	15.1	52.4	0.8815	0.9193	0.9013
4	28.5	393.6	5.2139	0.9883	0.1957	0.994	4.26	4.46	0	0	52	15.1	52.4	0.8790	0.9176	0.8992
1958: 1	26.0	382.8	5.3838	0.9955	0.1914	0.990	3.92	4.41	0	0	52	15.1	52.4	0.8897	0.9250	0.9082
2	24.6	383.8	5.4144	0.9979	0.1911	0.998	3.80	4.10	0	0	52	15.1	52.4	0.8952	0.9288	0.9128
3	24.3	394.7	5.3762	1.0007	0.1933	1.003	4.00	3.68	0	0	52	15.1	52.4	0.8914	0.9262	0.9096
4	25.2	405.5	5.3191	1.0046	0.1964	1.010	4.24	3.32	0	0	52	15.1	52.4	0.8860	0.9225	0.9051
1959: 1	26.5	412.6	5.3023	1.0096	0.1984	1.018	4.29	3.25	0	0	52	15.1	52.4	0.8849	0.9217	0.9042
2	28.1	423.7	5.2641	1.0115	0.2005	1.022	4.45	3.09	0	0	52	15.1	52.4	0.8809	0.9189	0.9008
3	28.4	418.5	5.2342	1.0160	0.2029	1.022	4.69	3.02	0	0	52	15.1	52.4	0.8740	0.9141	0.8950
4	28.5	423.2	5.2606	1.0167	0.2024	1.018	4.70	3.12	0	0	52	15.1	52.4	0.8731	0.9135	0.8943
1960: 1	29.0	432.9	5.2512	1.0210	0.2040	1.023	4.70	3.42	0	0	52	15.1	52.4	0.8715	0.9123	0.8929
2	30.5	431.3	5.2998	1.0252	0.2033	1.023	4.63	3.47	0	0	52	15.1	52.4	0.8733	0.9136	0.8944
3	29.9	428.9	5.3505	1.0265	0.2021	1.022	4.52	3.51	0	0	52	15.1	52.4	0.8764	0.9158	0.8970
4	29.2	425.0	5.3870	1.0308	0.2019	1.021	4.52	3.54	0	0	52	15.1	52.4	0.8762	0.9157	0.8969
1961: 1	27.3	423.0	5.5178	1.0340	0.1982	1.009	4.46	3.16	0	0	52	15.1	52.4	0.8802	0.9184	0.9002
2	27.2	433.4	5.4723	1.0339	0.2002	1.018	4.49	3.09	0	0	52	15.1	52.4	0.8797	0.9181	0.8998
3	28.3	441.3	5.4149	1.0333	0.2025	1.025	4.60	2.98	0	0	52	15.1	52.4	0.8769	0.9162	0.8975
4	29.4	450.5	5.4280	1.0379	0.2033	1.030	4.59	2.93	0	0	52	15.1	52.4	0.8775	0.9166	0.8980
1962: 1	30.3	457.1	5.4949	1.0422	0.2020	1.025	4.55	3.02	0	0	52	15.1	52.4	0.8782	0.9171	0.8986
2	31.3	464.8	5.5105	1.0432	0.2020	1.026	4.44	3.51	0	0	52	15.1	52.4	0.8788	0.9175	0.8991
3	32.8	470.0	5.9054	1.0451	0.1895	1.022	4.49	3.61	5.0	5	52	13.1	52.4	0.8919	0.9265	0.9100
4	32.6	474.4	5.9785	1.0480	0.1881	1.020	4.40	3.49	5.0	5	52	13.1	52.4	0.8948	0.9286	0.9125

1963: 1	32.5	477.2	6.0191	1.0505	0.1876	1.020	4.38	3.33	5.0	5	52	13.1	52.4	0.8962	0.9296	0.9137
2	33.1	481.8	6.0373	1.0533	0.1879	1.022	4.40	3.17	5.0	5	52	13.1	52.4	0.8964	0.9297	0.9139
3	34.7	489.9	6.0380	1.0551	0.1885	1.023	4.45	3.16	5.0	5	52	13.1	52.4	0.8952	0.9288	0.9128
4	35.9	497.4	6.0518	1.0585	0.1890	1.025	4.47	3.14	5.0	5	52	13.1	52.4	0.8947	0.9285	0.9124
1964: 1	36.7	503.9	6.3684	1.0609	0.1806	1.030	4.49	3.07	5.0	0	50	13.1	52.4	0.8941	0.9281	0.9119
2	37.7	511.5	6.3926	1.0637	0.1807	1.029	4.54	2.98	5.0	0	50	13.1	52.4	0.8932	0.9275	0.9111
3	39.5	517.6	6.4327	1.0676	0.1806	1.030	4.52	2.91	5.0	0	50	13.1	52.4	0.8941	0.9281	0.9119
4	40.0	520.0	6.4354	1.0734	0.1818	1.036	4.53	2.96	5.0	0	50	13.1	52.4	0.8936	0.9277	0.9115
1965: 1	41.5	532.2	6.4719	1.0766	0.1817	1.038	4.52	2.96	5.0	0	48	13.1	52.4	0.8934	0.9276	0.9113
2	41.7	538.9	6.5084	1.0829	0.1820	1.038	4.56	2.98	5.0	0	48	13.1	52.4	0.8922	0.9268	0.9103
3	44.2	548.9	6.5185	1.0845	0.1824	1.036	4.63	3.01	5.0	0	48	13.1	52.4	0.8902	0.9253	0.9086
4	45.2	561.6	6.4944	1.0883	0.1840	1.041	4.72	2.98	5.0	0	48	13.1	52.4	0.8879	0.9238	0.9067
1966: 1	46.4	569.4	6.4578	1.0974	0.1869	1.045	4.94	3.12	5.0	0	48	13.1	52.4	0.8813	0.9192	0.9011
2	47.7	571.4	6.4013	1.1095	0.1909	1.055	5.15	3.33	5.0	0	48	13.1	52.4	0.8747	0.9046	0.8956
3	49.8	576.2	6.1191	1.1167	0.2012	1.056	5.51	3.71	3.3	0	48	13.1	52.4	0.8634	0.9067	0.8861
4	50.7	582.8	5.6861	1.1243	0.2180	1.073	5.62	3.67	0	0	48	13.1	52.4	0.8607	0.9048	0.8838

[a] Source: U.S. Department of Commerce.

[b] Source: Derived from equation (3.42).

[c] Source: See discussions, pp. 82–88.

[d] Source: *Survey of Current Business*, various issues.

[e] Used as ratio in estimating parameters.

[f] See discussion on pp. 87–88.

[g] See discussion on p. 86.

[h] Source: U.S. Treasury Department.

[i] See discussion on p. 86.

[j] See discussion on p. 85.

[k] Source: Derived from equation (3.50).

[l] Source: Derived from equation (3.52). Available for tax purposes beginning in 1954.

[m] Average of straight-line and sum-of-the-years-digits methods weighted according to the estimated proportion of equipment depreciated by accelerated methods (equation 3.53).