The 21st century's solution to the old transformation problem

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> By far [Bortkiewicz's] most important achievement is his analysis of the theoretical framework of the Marxian system, much the best thing ever written on it and, incidentally, on its other critics.
>
> Schumpeter[1]

Abstract:

This paper reviews the controversy over the transformation problem stemming from von Bortkiewicz's critique (1907) of Marx's transformation process. It covers the contribution from Winternitz (1948) and later papers by Samuelson (1971) and others. The key assumption of this paper is that competition will reallocate capital in such a way that the rates of profit are equalized, and Marx's two aggregate equalities are satisfied. These two constraints must be the starting point which determines how to allocate the capital. Thus the general form of the transformation problem is bi-linear – price structure and allocation of capital are endogenous variables. With the allocation of capital regarded as variable, it becomes possible to make two of Marx's equalities hold, that between total value and total price and between total surplus value and total profit.

Introduction

Over the 125 years since K. Marx's *Das Capital* was first published, the so-called "*transformation problem"* has inspired a good number of economists. Marx's problem was to convert the values of commodities into prices in accordance with the law which establishes an identical rate of profit for all "departments"[2], determining the prices from values and providing an explanation of profit on the basis of surplus values. Can the price structure, under the conditions of industrial capitalism and competition, be logically derived from values determined by labor, as postulated by D. Ricardo and K. Marx?

The transformation problem controversy began 125 years ago with the charge against Marx that he committed some mathematical mistakes in his way to solve the transformation procedure. The best known of these early critics was L. von Bortkiewicz (1907) who used a simultaneous method of determination of values and prices and assumed "simple reproduction".

This method, however, seems unsatisfactory to Winternitz (1948), as the normal case in a capitalist system is expanded reproduction when there is accumulation of capital. The transformation process induces a change of the value of commodities against the price of production, and a change of price may require readjusting the capital allocation to restore the equilibrium.
But all these contributions do not attach enough importance to the flows of capital across

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1 A. Schumpeter, Ladislaus von Bortkiewicz: 1868-1931, in *Ten great economists from Marx to Keynes* (New York, 1960), 302-305.

2 The term "department" refers to the three types of units of production. In Department I, means of production (plant and machines) and raw materials are produced, in Department II, workers' consumption goods, and in Department III, capitalists' consumption goods (luxury goods).

departments during the transformation process. They also assume that the entire capital employed (including the constant fixed capital) turns over once a year and reappears again in the value or the price of the annual product[1]. This allows them to use a homogeneous system and compute a rate of profit as the root of the characteristic equation of the matrix, but seems to be an inadequate solution, as the profit rate in this theorical system is invariant of the capital allocation, while it is not the case in real world.

In this paper, we will examine the transformation problem, not by doing the history of the controversy once again[2,] but rather by focusing on the following topics:
1/ The necessity of using independent equations in a price determination system,
2/ The capitalist production is production for profit. Hence, capital flows from departments with lower profits towards departments with higher ones. These movements lead to an equalization of the rate of profit between different departments. Thus the transformation process consists not only of redistributing surplus values between departments of production – as Marx did – but also of allocating the capital across departments. This second aspect has been totally neglected by previous commentators[3].
3/ The main problem with prevailing mathematical solutions, from a Marxist point of view, is that they all produced results where either the total value was not equal to the total price of production, or the total surplus value was not equal to the total profit. We think that these two constraints must be the starting point which determines how to allocate the capital. Thus the general form of the transformation problem is bi-linear[4].
4/ Von Bortkiewicz's and Winternitz's methods are not contradictory but complementary. However, their numeric examples are more restrictive than the algebraic model. Mainstream economists fail when they say that once the inputs were included in the transformation process, either the total price would not be equal to the total value, or the total profit would not be equal to the total surplus value. Their methods are incomplete. If we consider a competitive economy constituted of independent departments, an allocation of capital exists – although in a homogeneous system – which validates Marx's fundamental constraints and his method for computing the profit from the surplus-value, and the price from the value.

This paper divides into VII sections.
Section I introduces the traditional solution to the transformation problem (von Bortkiewicz, Samuelson, Winternitz).
Section II deals with competition, allocation of capital and equalization of profit rate.
Section III: The paradox of the two numerical examples of von Bortkiewicz.
Section IV: Solving the transformation problem.
Section V: The determination of the allocation of capital.
Section VI: The reconciliation of the mainstream problematic and Marx's method.
Section VII: Concluding remarks.

In agreement with Bortkiewicz and against Marx, we argue that the value of inputs should be transformed. In our solution, the values of inputs and outputs are transformed into price of production. We assume that the cost-prices of a commodity are equal to the prices of production of the commodities consumed in its production.
We also assume that there is no technical change during the period.

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1 This point is not discussed in this text. But this remark is the starting point of the criticism of the mainstream theory.
2 Dostaler G. 1978, Jorland, G. 1995.
3 Except Heimann, E., *“Kapitalismus und socialismus”,* Potsdam: Alfred Protte, 1931; *“Methodo-logisches zu den Problemen des Wertes und des wirtschaftlichen Prinzips”,* Archiv für Sozialwissenschaft und Socialpolitik, XXXVII (1913) 758-807 ; see Jorland, 1995, p. 220 and following.
4 F (Y, X) = [Y] [A] [X] where [Y] – capital employed – is a row vector [Y_1, Y_2], [X] a column vector of transformation coefficients [x_i] and [A] the n x n technical coefficient matrix.

The notations are:

C_i = Total constant capital in department "i" ($C_i = Y_i * c_i$)

c_i = The proportion of constant capital in a unit of capital employed

V_i = Total variable capital in department "i" ($V_i = Y_i * v_i$)

v_i = The proportion of variable capital in a unit of capital employed

W_i = The value of the production of department "i"

x_i = The transformation coefficients

S_i = The surplus-value in department "i"

e_i = The rate of surplus-value ($e_i = S_i / V_i$)

r = The profit rate

g_i = The organic composition of department "i"

G = The social organic composition

Y_t = The total capital employed in all departments

Y_i = The capital employed in department "i"

y_i = The proportion of capital employed in department "i" ($y_i = Y_i/Y_t$)

$\mu_i = V_i / V_1 + V_2$

I The old presentation of the transformation problem: [Bortkiewicz 1907], [Meek 1956] , [Samuelson 1970], [Sweezy 1942][Winternitz 1948]

In Bortkiewicz's work, the determination of prices is simultaneous. The different spheres of production from which Marx composes social production as a whole are put together into three departments of production. At the same time he assumes that in Departments I, II, and III the surplus rate ($e_i = S_i / V_i = 2 / 3$) is uniform, but that the composition of capital is different ($g_1 \neq g_2 \neq g_3$). In term of Marxian categories of constant capital (C_i), variable capital (V_i) and surplus value (S_i) the problem can be written:

$C_1 + V_1 + S_1 = W_1$

$C_2 + V_2 + S_2 = W_2$

$C_3 + V_3 + S_3 = W_3$

In the canonical model of "simple reproduction":

department 1: $C_1 + V_1 + S_1 = C_1 + C_2 + C_3$

department 2: $C_2 + V_2 + S_2 = V_1 + V_2 + V_3$

department 3: $C_3 + V_3 + S_3 = S_1 + S_2 + S_3$

L. von Bortkiewicz supposes that the correlation between the price and the value of the products of Department I is (on the average) as x_1 to 1, in the case of Department II as x_2 to 1, and in the case of Department III as x_3 to 1 (transformation coefficients).

In his 1907 article, L von Bortkiewicz[1] argued that:

[1] In order for these three homogeneous linear equations to have a non-zero solution, it is necessary that the first two of them have such a solution.

$x_1 C_2 + x_1 C_3 = x_2 V_1 + r (x_1C_1 + x_2V_1)$

$x_2 V_1 + x_2 V_3 = x_1 C_2 + r (x_1C_2 + x_2V_2)$

then : $-(C_2 + C_3) \quad x_1 + \quad V_1 \quad x_2 \quad + r (x_1C_1 + x_2V_1) = 0$

$C_2 \quad x_1 \quad -(V_1 + V_3) \quad x_2 \quad + r (x_1C_2 + x_2V_2) = 0$

(1) using Marx’s method of transformation, the equilibrium conditions for simple reproduction would break down,

(2) both inputs and outputs are transformed simultaneously whereas in Marx’s solution, capital utilized in production is expressed in terms of value while outputs are expressed in prices, and

(3) once the inputs were included in the transformation procedure, either the total price would not be equal to the total value, or the total profit would not be equal to the total surplus value.

This previous method of transformation, however, seems unsatisfactory to Winternitz. Von Bortkiewicz bases his analysis of the transformation problem on Marx's scheme of simple reproduction, i.e. a continuation of production on the same scale. With Marx's method of transformation, the equilibrium of simple reproduction is obtained by an exchange of equal values. It would not possibly be maintained, however, if prices of production are used. Winternitz obviously finds this result unsatisfactory. A change in the price structure – he says – can only disturb an existing equilibrium. And the transformation process induces a change of the value of commodities against the price of production, and a change of price may require readjusting the capital allocation to restore the equilibrium.

Von Bortkiewicz assumes that gold, the money commodity, is one of the luxury goods so that prices in the third department are not affected by change over from values to prices of production. This assumption is arbitrary and unjustified – Winternitz says – and makes the total price deviate from the total value.

Bortkiewicz's conclusion is irrelevant because it doesn't take into account the necessary independence of equations in a price system. L. von Bortkiewicz bases his calculations on the equations of simple reproduction. In fact they are not relevant to the transformation problem. And a transformation method which was valid only under this assumption would be insufficient, as the normal case in a capitalist system is expanded reproduction when there is accumulation of capital. [Winternitz, 1948]

A fundamental price determination system must have only independent equations. Assuming simple reproduction leads to an interdependence of the values created in the departments, emphasizes K. May[1] in 1948. In linear algebra, a family of equations is **linearly independent** if none of them can be written as a linear combination of finitely many other vectors in the collection. In Bortkiewicz simple reproduction system the third equation can be derived algebraically from the others[2]. Equations are not independent. The third equation can not be maintained, so simple reproduction assumption can not be preserved in a price determination system.

In this text we shall consider the two first independent equations of von Bortkiewicz's system. We have:

If we use the L. von Bortkiewicz's notations [$f_i = V_i / C_i$, $g_i = (C_i + V_i + M_i) / C_i$ et $t = r + 1$] we have the system :

$$\begin{bmatrix} t-g_1 & tf_1 \\ t & tf_2 - g_2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = 0$$

<=> [A] [X] = O => det A = 0

This *homogeneous* system of linear equations has a unique non-trivial solution if and only if its determinant is zero, which requires the following determinantal quadratic to vanish:

$(t-g_1)(tf_2 - g_2) - t^2 f_1 = 0$ and then: $(f_1 - f_2)\, t^2 + (f_2 g_1\ g_2)t - g_1 g_2 = 0$. Which is a quadratic equation, where:

$$t = \frac{f_2 g_1 + g_2 - \sqrt{(g_2 + f_2)^2 + 4(f_1 - f_2) g_1 g_2}}{2(f_2 - f_1)} \quad ; \quad x_1 = \frac{f_1}{g_1 - t} * x_2 \quad \text{and} \quad x_2 = \frac{g_3}{g_2 + (f_3 - f_2)t} x_3$$

If gold is the good which serves as the value and price unit, then we get $x_3 = 1$ and total surplus value in all departments equals total profit.

1 Kenneth May, Value and Price of Production: A note on Winternitz's Solution”, Economic Journal, LVIII (1948) p. 596-599.

2 $C_3 = W_1 - C_1 - C_2$, $V_3 = W_2 - V_1 - V_2$ and $S_3 = e * V_3 = e * [W_2 - V_1 - V_2]$

$(C_1 x_1 + V_1 x_2) * (1+r) = W_1 * x_1$

$(C_2 x_1 + V_2 x_2) * (1+r) = W_2 * x_2$

As the rate of profit must be equal in department I and department II, we have:

$$(1+r)=\frac{W_1 x_1}{C_1 x_1+V_1 x_2}=\frac{W_2 x_2}{C_2 x_1+V_2 x_2}$$

From this an equation of the second degree can easily be derived for $x = x_1/x_2$

$$(W_1 C_2)x^2+(W_1 V_2-W_2 C_1)x-W_2 V_1=0$$

which resolves as:

$$x = \frac{-W_1 V_2+W_2 C_1 \text{-/+} \sqrt{(W_1 V_2-W_2 C_1)^2+4 W_1 W_2 V_1 C_2}}{2 W_1 C_2}$$

Ignoring negative solution, x is now given and the average rate of profit[1] is already determined:

$$r = \frac{W_1 x}{C_1 x+V_1}-1$$

The invariance conditions: This solution is somewhat restrictive. As we have a homogeneous system, the transformation problem has been solved in relative prices; additional aggregate characteristics are required to determine absolute price.

L. von Bortkiewicz suggests to choose Marx's proposition that total surplus value equals total profit. But the fact that total profit is numerically identical to total surplus value is a consequence of the fact that the goods used as value and price measure belong to Department III (x_3=1). That the total price exceeds the total value arises from the fact that Department III, from which the goods serving as value and price measure are taken, has a relatively low organic composition of capital.

However Winternitz notes that there is a second invariance condition that has equal theoretical merit, namely that the “sum of prices is equal to the sum of values”. But now the total surplus in value terms is not equal to profits in price terms.

In the general case – the mainstream authors say – we could maintain one equality or the other, but cannot maintain both at the same time. They believe that the system of value and that of prices are separately determined and are two substitutable but unharmonious systems.

Therefore, Marx was wrong, they say. Relative prices and profit rate are determined independently of value magnitudes, making value production “redundant” [Steedman 1977]. Also, in luxury industries, the general profit rate is determined independently of production conditions. In the prevailing system, very little of the quantitative dimension of Marx's value theory is left intact.

L. von Bortkiewicz's and Winternitz's solutions don't take into account the flows of capitals through departments. It is important to realize that the divergence between total value and total price in a traditional system is simply a question of capital allocation, and is not dependent on the method used to calculate the price, as we shall see now.

II Competition transfers capital through departments and levels different rates of profit.

“*Every change in the price structure normally disturbs an existing equilibrium. A change of prices may necessitate a changed distribution of social labor to restore the equilibrium”* Winternitz.

Marx's transformation problem asks how can equal magnitudes of capital with variable quantities of labor, give equal rates of profit if the law of value is operative. The process of transforming the value of commodities into the price of production is the process of formation of a general profit rate under competition. Profits are only a secondary, derivative and transformed form of surplus-value.

[1] Von Bortkiewicz gives an other definition of r but arithmetically the result is the same.

Total profit, which is surplus-value computed differently, can neither increase nor decrease through this transformation of values into prices of production. What is modified is not its magnitude, but only its distribution among capitals.[1] So that, «*the sum of all profits in all spheres of production must be equal to the sum of the surplus values, and the sum of the prices of production of the total social product be equal to the sum of its value.*"[2]

In volume III of *Capital,* K. Marx used an equal magnitude of capital employed in each department in his numerical examples (Y_i =100 or y_1=y_2=y_3=y_4=y_5=20 %, five departments).

In Bortkiewicz's first numeric example (example A) – that is present in section II - the total magnitude of capital employed is $ 675 billion and the allocation of capital is y_1=46.6 %, y_2=29.6 % and y_3=20.7 %.

In Bortkiewicz's second example (example B), the total magnitude of capital employed is $ 740 billion and the allocation of capital is y_1= 56.76%, y_2=23.78 % and y_3=19.46 %. He doesn't use equal magnitudes of capital employed (Y_i) in each department as Marx did . Why?

Are these allocations of capital correct? Nobody seems to wonder but us.

L. von Bortkiewicz obtains three equations with four unknowns (x, y, z, and r) in his system of three equations. In order to supply the missing fourth equation he must determine the correlation between the price unit and the value unit (x_3=1).

In fact, Bortkiewicz's three equations are not independent. In the canonical model of "simple reproduction", only two of them are independent, which are:

$C_1 + V_1 + S_1 = W_1$

$C_2 + C_2 + S_2 = W_2$

The third equation is a linear combination of the two first equations; it is redundant and can not be kept in a fundamental system of independent equations[3].

There is no general solution for the equations which preserves the equality of total value and total price, and total surplus value and total profit, if there is no competition and no capital movement. Every change in the price structure can only disturb an existing equilibrium. A change of prices may necessitate a new distribution of social labor to restore the equilibrium, as Winternitz says.

The main paradox of the L. von Bortkiewicz's solution is his incapacity to construct a numeric example in conformity with the generality of his algebraic model (section III).

III The paradox of the two numerical examples of von Bortkiewicz.

L. von Bortkiewicz gives several numerical examples of simple reproduction with three departments, but all of them are more restrictive than his algebraic system, and none is coherent with Marx's constraints.

The first famous example (example A) of L. von Bortkiewicz is the following:

[1] *Grundrisse*, Penguin Books, 1973, p 595 and 760.

[2] *Capital*, Volume III, Laurence and Wishart/Moscow, 1962, p.170, p. 157 and p. 165.

[3] We shall see further in this text that if we assume that the allocation of capital is unknown, the new fundamental systems will have only two independent equations and four unknowns (x_1, x_2, Y_1 and Y_2). We need two more equations to solve it.

Value calculation

Example A	C_i	V_i	S_i	W_i
Department I	225	90	60	375
Department II	100	120	80	300
Department III	50	90	60	200
Total	**375**	**300**	**200**	**875**

In matrix terms:

$$[Y][A]=[W]=[315 \quad 220 \quad 140]\begin{bmatrix} 0.7143 & 0.2857 & 0.1905 \\ 0.4545 & 0.5455 & 0.3636 \\ 0.3371 & 0.6429 & 0.4286 \end{bmatrix}=\begin{bmatrix} 375 \\ 300 \\ 200 \end{bmatrix}$$

For the above numerical calculation, t = 5/4 and r* = 1/4, and x_1= 32/25 = 1.28, x_2 = 16/15 = 1.067, x_3 = 1 so that the price calculation must be proportional to:

Example A	C_i	V_i	S_i	x_i W_i
Department I	228	96	96	480
Department II	128	128	64	320
Department III	64	96	40	200
Total	**480**	**320**	**200**	**1000**

Thus, authors of the prevailing theory believe that the system of value and that of prices are separately determined, and are two substitutable but unharmonious systems.

That the total price exceeds the total value arises from the fact that Department III, from which the good serving as value and price measure is taken, has a relatively low organic composition of capital. The fact that total profit is numerically identical with total surplus value, however, is a consequence of the fact that the goods used as value and price measure belong to Department III (x_3=1).

The second Bortkiewicz's example (example B) is the following.

Second example: Value calculation

Example B	C_i	V_i	S_i	W_i
Department I	300	120	80	500
Department II	80	96	64	240
Department III	120	24	16	160
Total	**500**	**240**	**160**	**900**

If we compare this table with example A we find that:

	Example A	Example B
Organic composition c_1/v_1	225/90 = 2.5	300/120 = 2.5
Organic composition c_2/v_2	100/120 =0.833	80/96 = 0.833
Rate of surplus value e	200/300 =2/3	160/240 =2/3

The organic compositions of departments I and II and the rate of surplus value are the same in both examples, while the social organic composition of capital is higher in example B (500/240 = 2.08 > 375/ 300 =1.25) and the organic composition of department III is different (c_{3B}/v_{3B} = 120/24 = 5 instead of $c_{3A}/v_{3A} = 50/\ 90 = 0.55$).
In matrix terms:

$$[Y][A]=[W]=[420 \quad 176 \quad 144]\begin{bmatrix} 0.7143 & 0.2857 & 0.1905 \\ 0.4545 & 0.5455 & 0.3636 \\ 0.8333 & 0.1666 & 0.1111 \end{bmatrix}=\begin{bmatrix} 500 \\ 240 \\ 160 \end{bmatrix}$$

And the price calculation must be proportional to:

Example B Price calculation	**Constant capital**	**Variable capital**	**Profit**	**Price of production**
Department I	274 2/7	91 3/7	91 3/7	457 1/7
Department II	73 1/7	73 1/7	36 4/7	182 6/7
Department III	109 5/7	18 2/7	32	160
Total	**447 1/7**	**186 6/7**	**160**	**800**

In example B, capital employed is $ 740 billion. It was $ 750 billion in first example. Von Bortkiewicz doesn't explain this change of scale.

So this example differs from the first because the scale is different and because a part of capital employed has been transfered through departments.

For an easier comparison, it's useful to consider a same capital employed magnitude in the two numerical examples ($ 750 billion), so that :

Example C	C_i	V_i	S_i	W_i
Department I	304.05	121.62	81.08	506.75
Department II	81.08	97.30	64.86	243.24
Department III	121.62	24.32	16.22	162.16
Total	**506.75**	**243.24**	**162.16**	**912.16**

Total capital employed is 506.75 + 243.25 = 735.
In matrix terms:

$$[Y][A]=[W]=[425.67 \quad 178.38 \quad 145.94]\begin{bmatrix} 0.7143 & 0.2857 & 0.1905 \\ 0.4545 & 0.5455 & 0.3636 \\ 0.8333 & 0.1666 & 0.1111 \end{bmatrix}=\begin{bmatrix} 506.75 \\ 243.24 \\ 162.16 \end{bmatrix}$$

As previously the price calculation must be proportional to:

Example C Price calculation	**Constant capital**	**Variable capital**	**Profit**	**Price of production**
Department I	277.99	92.66	92.66	463.32
Department II	74.13	74.13	37.07	185.33
Department III	111.20	18.53	32.43	162.16
Total	**453.18**	**189.38**	**162.16**	**810.81**

If we compare example C with example A we find that the first two lines of matrix A are the same. It is no surprise then that r = 0.25, as in the first example, because it depends exclusively on the organic composition of capitals in department I and II, even though allocation of capital is different.

What is different? In the first and second examples, the total magnitude of capital employed is Y_t = 735 and in Department I, in first example Y_{1A}= 215 + 90 = 315 and in second example Y_{1c} = 304.05 + 121.62 = 425.67.

In department II, in the first example Y_{2A}= 220 and in the second example Y_{2C} = 178.38. In department III, $Y3_A$= 140 and in second example Y_{3C} = 145.94.

We have a transfer of capital of 41.62 from Department II to Department I and Department III.

But why such a transfer? Von Bortkiewicz doesn't explain. Why not $ 50.4 billion, $ 36.1 billion or $ 20.3 billion?

For von Bortkiewicz these two examples are not reasonable because in each of them total value is not equal to total price.

Two departments	**Total Value**	**Total Price**	Distance
Example A	875	1000	875 < 1000
Example C	912.16	840.81	912.16 > 840.81

In the first example, with the allocation of capital Y_A =[315; 220; 140], total value is **lower** than total price.

In the second example, with the allocation of capital Y_C = [425.67; 178.38; 145.94], total value is **higher** than total price.

If we assume a progressive transfer of capital from department II to department I, **there should exist** an allocation of capital where total value equals total price, an equilibrium where the two fundamental equalities are checked.

Von Bortkiewicz failed to built a correct numerical example, coherent with Marx's constraints, while his algebraic model allows it. And for this reason, he is not based to reject the methods of Marx of calculating price and profit .

Does this equilibrium exist? Von Bortkiewicz gives in his numerical example two arbitrary distributions of capital Y_A =[315; 220; 140] and Y_C = [425.67; 178.38; 145.94]. The efficient allocation may be somewhere in-between these two allocations. What is the equilibrium allocation of capital between the departments? We shall respond to this question in next sections.

This does not mean that we consider the transformation as an empirical process of successive adjustments up to a point of balance, which would correspond to the consciousness which the capitalists have of the rate of profit to be realized.

Von Bortkiewicz – his numerous examples in the support – asserted that there was no solution which satisfies the constraints of Marx. To invalidate this proposition we just have to show that there is at least a solution which satisfies simultaneously the equations of Von Bortkiewicz and the constraints of Marx. It is what we are now going to establish.

IV Determining the allocation of capital

A fundamental price determination system must have only independent equations. When the equations are independent, each equation contains new information about the variables, and removing any of the equations increases the size of the solution set. In linear algebra, a family of equations is **linearly independent** if none of them can be written as a linear combination of finitely many other vectors in the collection. In the “simple reproduction” system, the third equation can be derived algebraically from the others. The three equations are therefore not independent. The third equation of von Bortkiewicz's system can not be preserved.

We remain with the following for the fundamental price production system:

Department I $Y_1 (x_1c_1 + x_2v_1) (1+r) = Y_1 x_1 w_1$ (1)

Department II $Y_2 (x_1c_2 + x_2v_2) (1+r) = Y_2 x_2 w_2$ (2)

In the first famous example of L. von Bortkiewicz Y_1 = $ 315 billion, Y_2 = $ 220 billion and total capital employed is $Y_1 + Y_2$ = $ 535 billion.

We shall now assume that total capital employed in all departments is always $ 535 billion.

Until then – as von Bortkiewicz – we have considered that constant capital and variable capital where given items, in example A: C_1=$ 225 billion, V_1 = $ 90 billion, C_2=$ 100 billion, V_2 = $ 120 billion. We have noted them with capital letters (C_1, V_1, C_2, V_2).

Marx problematics:

As a matter of fact this notation, inherited from Tugan-Baranovsky, is not Marx's method, as he assumes that the capital employed in each department is equal to $ 100 billion (80C+20V, 75C+25V and so on.). Marx assumes the organic composition of each department, but he doesn't assume how the capital is allocated across departments.

As a matter of fact, in Von Bortkiewicz's example A, as the total capital employed in department I is 315, when we write C_1 = 225 and V_1 = 80, it is the product of the multiplication of a magnitude of capital employed by the percentage of constant capital or variable capital. For example:

Department I: C_1= 225 = 315 * 71.43% and V_1 = 90 = 315 * 28.57% and

Department II: C_2=100 = 220 * 45.45 % et V_2 = 120 = 220 * 54.54 %.

As the total capital employed is $Y_t = Y_1 + Y_2$ = $ 535 billion, the allocation of capital through the two departments is:

y_1 = 315 / 535 = 58.88 %

y_2 = 220 / 535 = 41.12 %

Finally:

Department I: C_1= 225 = 535 * 58.88 % * 71.43 % and V_1 = 80 = 535 *58.88 * 28.57% and

Department II: C_2=100 = 535 * 41.12% * 45.45 % et V_2 = 120 = 535 * 41.12 % * 54.54 %.

Or $C_i = Y_t y_i c_i$ and $V_i = Y_t y_i v_i$.

Now c_i, v_i and e_i are definite for 100 billion of capital employed (c_1 =225/315 = 71.43 %, v_1 = 90/315 = 28.57 % , e_1 = 60/315 = 2/3 ; c_2 =100/220 = 45.45 % , v_2 = 120/220 = 54.54 % and e_2 = 80/220 = 2/3). We would write :

Fundamental system	Y_i	c_i	v_i	s_i	w_i
Department I	**100**	71.43	28.57	19.05	119.05
Department II	**100**	45.45	54.54	36.36	136.36

In Marx's model Y_t = 200 and y_1 = 50% and y_2 = 50%.

$$[Y][A]=[W]=200\begin{bmatrix}0.5 & 0.5\end{bmatrix}\begin{bmatrix}0.7143 & 0.2857 & 0.1905\\ 0.4545 & 0.5455 & 0.3636\end{bmatrix}=\begin{bmatrix}119.05\\ 136.36\end{bmatrix}$$

Von Bortkiewicz's problematics:

In L. von Bortkiewicz's first famous example y_{1A} = 315/535 = 58.88 % and y_{2A} =220/315= 41.12 % .

$$[Y][A]=[W]=535[0.5888 \quad 0.4112]\begin{bmatrix}0.7143 & 0.2857 & 0.1905\\ 0.4545 & 0.5455 & 0.3636\end{bmatrix}=\begin{bmatrix}375\\ 300\end{bmatrix}$$

In his second example: y_{1B} = 70,5 % and y_{2B} = 29,5 % .

$$[Y][A]=[W]=535[0.7050 \quad 0.2950]\begin{bmatrix}0.7143 & 0.2857 & 0.1905\\ 0.4545 & 0.5455 & 0.3636\end{bmatrix}=\begin{bmatrix}506.75\\ 243.24\end{bmatrix}$$

All these allocations of capital are inconsistent with Marx's constraints. These allocations of capital are assumptions smuggled by von Bortkiewicz in his numerical examples – with no theoretical comment. But such assumptions don't exist in his algebraical construction, and we can say that his algebraic model is more general than his numerical examples. He cannot afford to say that Marx's equalities are never verified. Under this simple notation is hidden a fundamental assumption that must be demonstrated: how is the allocation of capital determined? Nobody knows and nobody seems to wonder but us.

New problematics:

The allocation of capital (y_1, y_2) is unknown, so that:

$$[Y][A]=[W]=535[y_1 \quad y_2]\begin{bmatrix}0.7143 & 0.2857 & 0.1905\\ 0.4545 & 0.5455 & 0.3636\end{bmatrix}=535[y_1 \quad y_2]\begin{bmatrix}w_1\\ w_2\end{bmatrix}$$

The social organic composition is therefore unknown. Its magnitude also depends on the allocation of capital (y_1 and y_2). Does a well-balanced allocation of capital between departments exist?

In the next section we develop a method to determinate the well-balanced allocation of capital.

V Solving the transformation problem

In this section we give the solution of the bi-linear transformation problem.

As we have seen above, the mainstream solution, in order not to complicate the presentation, introduces a limiting assumption concerning the constant capital, namely, that the entire advanced constant capital turns over once a year and reappears again in the value or the price of the annual product. The consequence of this hypothesis is the use of an **homogeneous system of equations.** In homogeneous systems, the profit rate can be determinate as solution of the quadratic equation of the matrix A (det A = 0). Bortkiewicz's profit rate is not dependent of the allocation of capital, as opposed to Marx's two constraints.

The main problem of mainstream mathematical solutions, from a Marxist point of view, is that they all produce results where either the total value was not equal to the total price, or where the total surplus value was not equal to the total profit.

The respect for the constraints of Marx depends on how the capital is allocated. The determination of allocation of capital (y_i) must be now considered. Marx's, von Bortkiewicz's and Winternitz's solutions are not inexact but only incomplete.
The general form of the transformation problem is bi-linear.

F (Y, X) = [Y] [A] [X]

where Y is a row vector of capital employed (Y_1, Y_2), X a column vector of transformation coefficients (x_1, x_2) and A the technical coefficient matrix.

In von Bortkiewicz's fundamental example of two departments, for an capital employed equal to 100:

Value calculation

Example A	C_i	V_i	S_i	W_i
Department I	71.43	28.57	19.05	119.05
Department II	45.45	54.55	36.36	136.36

Where $Y_1 = Y_2 = 100$

But in the new problematics: $Y_T = 535$, y_1 and y_2 are unknown.

$$[Y][A]=[W]= 535\ [y_1 \quad y_2]\begin{bmatrix}0.7143 & 0.2857\\ 0.4545 & 0.5455\end{bmatrix}\ t\ \begin{bmatrix}x_1\\ x_2\end{bmatrix}= 535\ [y_1 \quad y_2]\begin{bmatrix}1.1905\,x_1\\ 1.3636\,x_2\end{bmatrix}$$

x_1, x_2, y_1, y_2 and t are the five unknowns.

This system is composed of two sub-systems:

1. the transformation coefficients sub-system,
2. the allocation of capital sub-system.

A. The determination of the rate of profit and of the transformation coefficients

The transformation coefficients sub-system is AX =0.

In one hand we have the transformation coefficients homogeneous **sub-system I** :

Department I $(x_1c_1 + x_2v_1)(1+r) = x_1 w_1$ (1)

Department II $(x_1c_2 + x_2v_2)(1+r) = x_2 w_2$ (2)

And as t = 1+ r:

Sub-system I [1]:

$$\begin{bmatrix}c_1 t - w_1 & v_1 t\\ c_2 t & (v_2 - w_2)\end{bmatrix}\begin{bmatrix}x_1\\ x_2\end{bmatrix}=\begin{bmatrix}0\\ 0\end{bmatrix}$$ Sub-system (1)

This sub-system is: A X = 0 form. It is a homogeneous system and it has a non trivial[2] solution only is Det [A] = 0, or:

$$Det\,A=(c_2 v_1)t^2+(w_2-v_2)c_1 t+w_1(v_2+w_2)=0$$

It determines the rate of profit "t" as solution of this quadratic equation. So:

$$r+1\ =\ t\ =\ \frac{(v_2-w_2)c_1-\sqrt{[(w_2-v_2)c_1]^2+4[c_2 v_1 w_1(v_2+w_2)]}}{2c_2 v_1}$$

The second element is the determination of the ratio $x = x_1/x_2 = x^*$ [3]. As we have seen upper the Winternitz's method (in each department the profit rate is equal) gives x the ratio of the

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[1] $(x_1c_1 + x_2v_1)(1+r) = x_1 w_1$
$(x_1c_2 + x_2v_2)(1+r) = x_2 w_2$; as t = r+1

$(c_1 t - w_1)\ x_1 + v_1 t\ x_2 = 0$

$c_2 t\ x_1 + (v_2 - w_2)\ x_2 = 0$; and:

$$\begin{bmatrix}c_1 t - w_1 & v_1 t\\ c_2 t & (v_2 - w_2)\end{bmatrix}\begin{bmatrix}x_1\\ x_2\end{bmatrix}=\begin{bmatrix}0\\ 0\end{bmatrix}$$

[2] This method is an algebraic form of Borkiewicz's method.

[3] See Winternitz demonstration page 3

transformations coefficients[1]:

$$x = \frac{-w_1 v_2 + w_2 c_1 \text{-/+} \sqrt{(w_1 v_2 - w_2 c_1)^2 + 4 w_1 w_2 v_1 c_2}}{2 w_1 c_2}$$

The determination of the transformation coefficients x_1 and x_2 is not total. And $x_1 = a\ x_2$ and $x_2 = \lambda$.

The rate of profit ($r = r^*$) and x are two “structural” parameters .

Their magnitudes don't depend of the magnitude of total capital employed (here 535). And their magnitudes are independent of the allocation of capital.

It can be pointed out that the conclusion that the rate of profit is independent of the allocation of capital across industries, is the conclusion reached by both Bortkiewicz and mainstream economists. This conclusion is clearly contrary to what Marx himself said. Marx emphasized the opposite: that the rate of profit depends in part on the “distribution of the total social capital between these different spheres” [between spheres with a high composition of capital and spheres with a low composition of capital][2].

This method was introduced in price theory by L. Walras. If a homogeneous system is used the rate of profit is always independent of the allocation of capital across industries. Neo-classic economists, neo-marxists economists and some of the temporalists do so. Our main criticism is to show that this prevailing conclusion is not general. It depends of the kind of algebraic system of equations used to represent a price system of equations (homogeneous system or non-homogeneous system)[3].

Let us say that the unknowns “x” and “r” are ***structural items.***

Numerical application:

As in Bortkiewicz's example $r = r^* = 0.25$ and $x_1/x_2 = x^* = 1.2 \Leftrightarrow x_1 = 1.2\ x_2$

We have two equations and five unknowns (r, x_1, x_2, y_1 and y_2) now some of them (x and r) are determined by first sub-system.

In such a system of two independent equations we do not have three unknowns (x_1, x_2 and r), as Bortkiewicz believes, but five, which are the two deviation prices from values, x_i, the profit rate (r) and the allocation of capital[4] (y_1 and y_2). We have, however, only two independent equations. To find a solution we need more equations. Let us assume that the total surplus value equals the total profit, and the total value equals the total prices. These two constraints of Marx are the two equations which constitute the second sub-system.

B. Determining the allocation of capital

We have a second homogeneous sub-system (BY = 0) in which x and r are pre-determined ($x = x^*$ and $r = r^*$).

First constraint: The total value equals the total price:

$W_1 + W_2 = x_1 W_1 + x_2 W_2 \quad \Leftrightarrow Y_1 w_1 + Y_2 w_2 = x_1 Y_1 w_1 + x_2 Y_2 w_2$

Elimination of the Parameter Y_T:

$Y_T (w_1 + w_2) = x_1 Y_T w_1 + x_2 Y_T w_2 \quad \Leftrightarrow Y_T (y_1 w_1 + y_2 w_2) = Y_T(x_1 y_1 w_1 + x_2 y_2 w_2)$

$y_1 w_1 + y_2 w_2 = x_1 y_1 w_1 + x_2 y_2 w_2$

Second constraint: The total surplus value equals the total profit:

$Y_1 s_1 + Y_2 s_2 = [x_1(Y_1 c_1 + Y_2 c_2) + x_2 (Y_1 v_1 + Y_2 v_2)]r^*$ and:

$y_1 s_1 + y_2 s_2 = [x_1(y_1 c_1 + y_2 c_2) + x_2 (y_1 v_1 + y_2 v_2)]r^*$

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[1] See page 2

[2] Vol. 3, Ch. 9, p. 263, Vintage edition

[3] Very few economists used non-homogeneous system of linear equations in price systems. In such systems, the rate of profit is never independent of the allocation of capital across industries but depends in part on the rate of surplus-value and in part on the social composition of capital. And this latter depends on the allocation of capital. Laure van Bambeke V., *“Des valeurs aux prix absolus. Essai de théorie économique rationnelle”,* Innovations, 2006-2 , p. 171 à 198.

[4] We assume the magnitude of total capital employed, $Y_T = 535$.

So we have the new homogeneous **sub-system II** :

$(w_1 - x_1 w_1)$ y_1 + $(w_2 - x_2 w_2)$ $y_2 = 0$

$[x_1c_1 r^*+x_2v_1 r^*- s_1]$ y_1 + $[x_1c_2 r^*+x_2v_2 r^*- s_2]$ $y_2 = 0$

This writing is possible because r* is a constant which doesn't depend of the allocation of capital. And it is the consequence of the fact that the first system is a homogeneous system.

$$\begin{bmatrix} w_1(1-x_1) & w_2(1-x_2) \\ x_1c_1r*+x_2v_1r*-s_1 & x_1c_2r*+x_2v_2r*-s_2 \end{bmatrix}\begin{bmatrix} y_1 \\ y_2 \end{bmatrix}=\begin{bmatrix} 0 \\ 0 \end{bmatrix} \quad \text{Sub-system II}$$

Which is a B Y = 0 homogeneous system.

And as r = r* and x = x_1/x_2 = x* are the solution of sub-system I and structural parameters, we can write: $x_1 = a * x_2$ and $x_2 = \lambda$, where "*a*" is a parameter and "λ" an unknown.

$$\begin{bmatrix} w_1(1-a\lambda) & w_2(1-\lambda) \\ a\lambda c_1r*+\lambda v_1r*-s_1 & a\lambda c_2r*+\lambda v_2r*-s_2 \end{bmatrix}\begin{bmatrix} y_1 \\ y_2 \end{bmatrix}=\begin{bmatrix} 0 \\ 0 \end{bmatrix} \quad \text{Sub-system III}$$

In matrix B, all the parameters are known except *λ*. This *homogeneous* system III of two linear equations has a unique non-trivial solution if and only if its determinant is zero.

Det B = 0 => λ is the solution of the quadratic equation of the B matrix, i.e.:

$$[B] = \begin{bmatrix} w_1(1-a\lambda) & w_2(1-\lambda) \\ a\lambda c_1r*+\lambda v_1r*-s_1 & a\lambda c_2r*+\lambda v_2r*-s_2 \end{bmatrix}$$

The general form of quadratic equation of the B matrix is:

$$A\lambda^2+B\lambda+C$$

where A, B and C are parameters, i.e.:

$$A = w_2r(ac_1+v_1)-w_1ar(ac_2+v_2)$$

$$B = w_1[r(ac_2+v_2)+as_2]-w_2[r(ac_1+v_1)-s_1]$$

$$C = w_1s_2-w_2s_1$$

As "A" equals zero, the solution of this quadratic equation is λ = - C /B:

$$\lambda=-\frac{w_1s_2-w_2s_1}{w_1[r(ac_2+v_2)-as_2]-w_2[r(s_1w_2+v_1)-s_1]}$$

And finally:

$$\lambda=\frac{w_2s_1-w_1s_2}{w_1[r(ac_2+v_2)-as_2]-w_2[r(s_1w_2+v_1)-s_1]}$$

We now know the value of λ, so system III can be solved with an usual method (substitution for example).

C. The numerical example:

These systems are all very abstract, so let us develop a numerical example with the five unknowns as letters (x_1, x_2 , y_1, y_2 and t) and the parameters as numbers.

The bi-linear system is:

$$[Y][A]=[W]= 535\begin{bmatrix} y_1 & y_2 \end{bmatrix}\begin{bmatrix} 0.7143 & 0.2857 \\ 0.4545 & 0.5455 \end{bmatrix} t \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}= 535\begin{bmatrix} y_1 & y_2 \end{bmatrix}\begin{bmatrix} 1.1905x_1 \\ 1.3636x_2 \end{bmatrix}$$

C1. The determination of the rate of profit and of the transformation coefficients

a) The determination of the rate of profit:

As the rate of profit (r or t = r+1) and the ratio of transformation coefficients ($x = x_1/x_2$) are independent of the allocation of capital, we have the sub-system I:

$$\begin{bmatrix} 0.7143t-1.1905 & 0.2857t \\ 0.4545t & 0.5455\text{t}-1.3636 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

This system is a homogeneous system and its solution is non trivial[1] only if Det [A] = 0, or:

$$Det\ A = 0.2597t^2 - 1.6234t + 1.6234 = 0$$

This equation resolves as t = 1.25 and r = 0.25

b) The determination of the transformation coefficients[2].

Let's examine system I on an other way. As t = r+1:

$(0.7143\ x_1 + 0.2857\ x_2)\ t = 1.1905\ x_1$
$(0.4545\ x_1 + 0.5455\ x_2)\ t = 1.3636\ x_2$

As the rate of profit must be the same in department I and department II, we have:

$$t = \frac{1.1905x_1}{0.7143x_1+0.2857x_2} = \frac{1.3636x_2}{0.4545x_1+0.5455x_2}$$

From this, an equation of the second degree can easily be derived for $x = x_1/x_2$

$$0.5411x^2 - 0.3246x - 0.3996 = 0$$

which has the positive result:

$x = 1.2$ and $x_2 = \lambda$

C2. The determination of λ and of the allocation of capital.

a) The determination of λ:

$$\begin{bmatrix} 1{,}1905(1-1.2\lambda) & 1.3636(1-\lambda) \\ 0.2857\lambda-0.1905 & 0.2727\lambda-0.3636 \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

This system is a homogeneous system and it has a non trivial solution only if Det [A] = 0, or:

$$Det\ A = 0\lambda^2 - 0{,}1948\lambda + 0{,}1732 = 0$$

As the first parameter is equal to zero, we find that λ= 0.1732/0.1948 = 0.8889.

Now the two transformation coefficients are:

$X_1 = 1.0667$
$x_2 = \lambda = 0.8889$

This solution is different from von Bortkiewicz's and Winternitz's results.
b) The determination of the allocation of capital (y_1 and y_2).
We can use these values in the upper sub-system II.

$$\begin{bmatrix} -0.07936 & 0.1515 \\ 0.06345 & -0.1212 \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

The solution of this sub-system is the ratio $y = y_1/y_2 = 1.9091$. And as $y_1+y_2 = 100\%$, the allocation of

1 This method is an algebraic form of Borkiewicz's method.
2 It is the Winternitz's method

capital is: y_1 = 65.63% and y_2 = 34.37%.
C) As we know the magnitude of the total capital employed ($ 535 billion) and the allocation of capital, we can compute Y_1 and Y_2:
Y_1 = 535*65.63% = 351.1
Y_2 = 535*34.37% =183.9

VI The reconciliation of the mainstream theoretical corpus and Marx's method.

The mainstream theoretical corpus is based on a few hypothesis:

- both inputs and outputs are transformed simultaneously.
- the entire capital employed (including the constant fixed capital) turns over once a year and reappears again in the value or the price of the annual product.

The aim of Marx's conclusion is that the surplus value is based on the surplus labor even in a capitalist regime in which commodity prices are not proportionate to their respective labor values. The process of transformation of commodity-value into production price is the process of formation of a general profit rate under competition. Total profit, which is surplus-value computed differently, can neither increase nor decrease through this transformation of values into prices of production. What is modified is not its magnitude, but only its distribution of surplus-value among capitals, so that the sum of all profits in all spheres of production must equal the sum of the surplus values, and the sum of the prices of production of the total social product must equal the sum of its value.
We have modified these methods on two points:

1. the values of inputs and outputs are transformed into price of production as in Bortkiewicz's model, transformation is total.
2. the capital flows across departments and competition reallocates the capital in such a way that Marx's two aggregate equalities are satisfied.

Our numeric example is:

1. The value calculation:

The starting point is Y_t, the initial sum of money invested as capital to purchase means of production (constant capital) and labor-power (variable capital). The total capital is assumed to represent a definite quantity of abstract social labor (Y_t= $ 535 billion).
Let's take, for example, the following values:

Value calculation	Y_i	Y_ic_i	Y_iv_i	$Y_i\ s_i$	Y_iw_i
Department I	**351.10**	250.79	100.31	66.88	417.98
Department II	**183.90**	83.59	100.31	66.87	250.77
Total	**535.00**	**334.38**	**200.62**	**133.75**	**668.75**

This system differs from von Bortkiewicz's only because we are in a competitive economy and it is possible to transfer $ 36.1 billion from department II to department I.
The social organic composition of this system is: 334.38/200.62 = 1.667.
It is possible to provide a theory of profit as arising from surplus value. Marx's definition of profit rate is: r = M/ C+V =133.75/535 = 0.25).

2. The price calculation:

The predetermined general rate of profit is then multiplied by the capital invested in each industry in order to determine the profit component. This profit is then added to the cost price in order to determine the price of production of each commodity.

It is now possible to derive prices from values with the total transformation method. Inputs are also goods and should therefore be similarly transformed. We multiply inputs and outputs by the

price/value ratios (x_1 = 1.0667 and x_2 =0.8889). The price calculation has four steps :
a) The determination of capital employed in each department:
. capital employed in department I: x_1C_1 = 1.067*250.79 = 267.5 ; x_2V_1= 0.889 *100.31 = 89.17 and
. capital employed in department II : x_1C_2 =1.067*83.59 = 89.16 ; x_2V_2 = 0.889 *100.31 = 89.16.
b) The determination of the rate of profit[1]: Marx's definition is : total surplus-value / total engaged capital (r=133.75/535 = 0.25 = 25%).
c) The determination of profit in each department = capital employed in department "i" * profit rate:
department I = (267.5 + 89.17) * 0.25 = 89.17

department II = (89.16 + 89.16) * 0.25 = 44.58

d) The determination of production price = (x_1 C_i +x_2 V_i) (1+r).
department I = (267.5 + 89.17) * 1.25 = 445.84
department II = (89.16 + 89.16) * 1.25 = 224.91
So that the price calculation is:

Price calculation	**$x_1Y_ic_i$**	**$x_2Y_iv_i$**	**profit**	**x_i W_i**
Department I	267.5	89.17	89.17	445.84
Department II	89.16	89.16	44.58	224.91
Total	**356.67**	**178.33**	**133.75**	**668.75**

3.This numeric example of total transformation is relevant with von Bortkiewicz's analytic and results and Marx theories of value, price and profit because total value equals total price (668.75) and total profit equals total surplus-value (133.75).

Two departments	Values	Prices	Surplus-value	Profit
Third example	668.75	668.75	133.75	133.75

We have shown in this section – even though the inputs were included in the transformation procedure – that the consistency of the system depends of the allocation of capital across departments and not of the price calculation method as von Bortkiewicz says. And now it is possible to derive prices from values and to provide a theory of profits as arising from surplus value.
Academic authors disregard turnover time problems and assume all fixed capital to be depreciated within the production period. We shall see in an other text how different this system is when the entire advanced fixed capital doesn't turn over once a year.

VII Concluding remarks

We have developed in this text an internal critic on the main traditional theory of the old transformation problem, which uses homogeneous systems of linear equations.

Competition means capital transfers between departments.

The aim of this paper was to re-examine von Bortkiewicz's and Winternitz's answers to Marx's solution to the transformation problem between value and price.

Our transformation method doesn't exclude the constant and variable capitals (inputs) from the transformation process.

The allocation of capital across departments is an unknown. Its determination is an element of the transformation process as important as the process of distribution of surplus-value.

It has been demonstrated that, although Marx's solution to the transformation problem can be

[1] The profit rate can be calculated with two different methods: Marx's method and von Bortkiewick's method (see page 4). This two methods are here equivalent but the Marx's one is general and the von Bortkiewick's one is possible only because the price system is a **homogeneous** system.

modified, his basic conclusion remains valid. The interdependences between the surplus-value and the profit and between the values and the prices are maintained.

This internal critic of the mainstream theory must be completed by an external critic which used a non-homogeneous system of linear equation but is not developed in this text. Marx gives a more precise definition for production price: "*That price of a commodity which is equal to its cost price plus its share on the yearly average profit of the capital employed (not merely that consumed) in its production (regard being had the quickness or slowless of turnover) is its price of production (Marx, III,186).* In Marx's point of view, the capital advanced (i.e. the capital employed) is different from the consumed capital. The integration of the fixed capital can't be neglected any longer. The assumption that the entire capital employed (including the fixed capital) turns over a year and reappears again in the value or the price of the annual product can't be accepted any more. This assumption, inherited from Tugan-Baranowsky, is the only basis for using homogeneous systems of equations in price theory, and von Bortkewicz's erroneous and unrealistic method of calculation the profit rate damages the generality of the profit theory and must not be used anymore.

A break about this theoretical corpus is needed, so it is possible to transform the values of commodities in prices of production using the new transformation method and non-homogeneous system of linear equations. These points will be the subject of another text.

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